Dynamical study of the pentaquark antidecuplet

Fl. Stancu[a] *

[a]Université de Liège, Institut de Physique B.5,
Sart Tilman, B-4000 Liège 1, Belgium

Dynamical calculations are performed for all isomultiplets of the flavour antidecuplet to which the pentaquark Θ^+ belongs. The framework is a constituent quark model where the short-range interaction has a flavour-spin structure. In this model the lowest pentaquarks have positive parity. Each antidecuplet member is described by a variational solution with the Pauli principle properly taken into account. By fitting the mass of Θ^+ of minimal content $uudd\overline{s}$, the mass of Ξ^{--}, of minimal content $ddss\overline{u}$, is predicted at approximately 1960 MeV. The influence of the octet-antidecuplet mixing on the masses of the $Y = 1$ and 0 pentaquarks is considered within the same model and the role of the kinetic energy plus the hyperfine interaction in this mixing is pointed out.

1. Introduction

The existence of exotic baryons containing four quarks and an antiquark in their lowest Fock component has now a solid experimental support. The observation of a narrow peak at 1.54 ± 0.01 GeV/c^2, called Θ^+, as an $S = 1$ baryon resonance in the photo-production from neutron $\gamma n \rightarrow K^+K^-n$ [1], has been confirmed by several groups in various photo-nuclear reactions [2]. This has been followed by the observation in pp collisions [3] of other narrow resonances Ξ^{--} and Ξ^0 at about 1862 MeV, from which Ξ^{--} is interpreted as another pure exotic member of an SU(3) flavour antidecuplet. The work of Diakonov, Petrov and Polyakov [4] has played a particularly important role in these discoveries. In the context of a chiral soliton model they predicted a narrow pentaquark, with a width of less than 15 MeV, located at the about experimentally observed mass of Θ.

At the end of the '70's, following the observation of several signals, light pentaquarks were studied theoretically [5,6], but these signals were not confirmed. Charmed pentaquarks with strangeness, $uuds\bar{c}$ and $udds\bar{c}$ were also predicted [7,8], but experimental searches carried at Fermilab have remained inconclusive [9]. These pentaquarks were introduced in the context of the one-gluon exchange model (colour-spin interaction) and the heavy ones carried negative parity. On the other hand positive parity pentaquarks containing heavy flavours were proposed in the context of a pseudoscalar exchange model (flavour-spin interaction) [11] about ten years later [12]. In this model, the lowest ones,

*e-mail : fstancu@ulg.ac.be

$uudd\bar{c}$ and $uudd\bar{b}$, do not carry strangeness. Recently the H1 Collaboration at DESY [10] reported a narrow resonance of mass 3099 MeV, interpreted as a $uudd\bar{c}$ pentaquark.

The spins and parities of Θ^+ and Ξ^{--} are not yet known experimentally. In this new wave of pentaquark research, most theoretical papers take the spin equal to 1/2. The parity is more controversial. In chiral soliton or Skyrme models the parity is positive [4]. In constituent quark models it is usually positive. In the present approach, the parity of the pentaquark is given by $P = (-)^{\ell + 1}$, where ℓ is the angular momentum associated with the relative coordinates of the q^4 subsystem. We analyze the case where the subsystem of four light quarks is in a state of orbital symmetry $[31]_O$ and carries an angular momentum $\ell = 1$. Although the kinetic energy of such a state is higher than that of the totally symmetric $[4]_O$ state, the $[31]_O$ symmetry is the most favourable both for the flavour-spin interaction [12] and the colour-spin interaction [13]. In the first case the statement is confirmed by the comparison between the realistic calculations for positive parity [12] and negative parity [14], based on the same quark model [15]. In Ref. [12] the antiquark was heavy, c or b, and accordingly the interaction between light quarks and the heavy antiquark was neglected, consistent with the heavy quark limit. In Ref. [16] an attractive spin-spin interaction between $\overline{s}$ and the light quarks was incorporated and shown that a stable or narrow positive parity $uudd\overline{s}$ pentaquark can be accommodated within such a model. This interaction has a form that corresponds to η meson exchange [17] and its role is to lower the energy of the whole system.

The purpose of this letter is to perform dynamical calculations of all the members of the antidecuplet to which Θ^+ and Ξ^{--} are supposed to belong. To our knowledge this is the first attempt in this direction. The present study is a natural extension of Ref. [12] where the heavy antiquark c or b is now replaced by a light quark u, d or s. To describe the short range interaction we rely on the same model [15] as that used in [12]. That means that the quark-quark interaction has a flavour-spin structure [11] and that the parameters are fitted to the light non-strange and strange baryon spectra. Moreover we assume that the quark-antiquark interaction is proportional to a spin-dependent operator, but it is flavour independent, as in Ref. [16]. Its role is to introduce the same flavour independent shift for each member of the pentaquark antidecuplet of equal spin. We shall fix this shift by adjusting the mass of Θ^+ to the experimental value. There is no other free parameter in the Hamiltonian model used in this study. For the pure exotic Ξ^{--}, we predict a mass of 1960 MeV. For the antidecuplet members with $Y = 1$ and 0 we investigate the role of the octet-antidecuplet mixing. To some extent this study will be a comparative one.

We search for a variational solution of a five-body Hamiltonian, containing a kinetic energy term, a confinement term and a short range (hyperfine) interaction having a flavour-spin structure. The $SU_F(3)$ breaking is taken into account by the strange quark mass which appears in the mass term, in the kinetic part and in the hyperfine part. The latter also breaks $SU_F(3)$ through the masses of the pseudoscalar mesons exchanged among quarks.

2. The Hamiltonian

The Hamiltonian has the form [15]

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \frac{(\sum_i \vec{p}_i)^2}{2\sum_i m_i} + \sum_{i<j} V_c(r_{ij}) + \sum_{i<j} V_\chi(r_{ij}) \,, \tag{1}$$

with the linear confining interaction

$$V_c(r_{ij}) = -\frac{3}{8}\,\lambda_i^c \cdot \lambda_j^c \; C \;\, r_{ij} \,, \tag{2}$$

and the flavour–spin interaction

$$\begin{aligned} V_\chi(r_{ij}) \;\;=\;\; & \left\{ \sum_{F=1}^{3} V_\pi(r_{ij}) \;\lambda_i^F \lambda_j^F \right. \\ + \;\; & \left. \sum_{F=4}^{7} V_K(r_{ij}) \;\lambda_i^F \lambda_j^F + V_\eta(r_{ij}) \;\lambda_i^8 \lambda_j^8 + V_{\eta'}(r_{ij}) \;\lambda_i^0 \lambda_j^0 \right\} \;\vec{\sigma}_i \cdot \vec{\sigma}_j \;. \end{aligned} \tag{3}$$

The analytic form of $V_\gamma(r)$ ($\gamma = \pi, K, \eta$ or η') is

$$V_\gamma(r) = \frac{g_\gamma^2}{4\pi}\frac{1}{12 m_i m_j}\{\theta(r - r_0)\mu_\gamma^2 \frac{e^{-\mu_\gamma r}}{r} - \frac{4}{\sqrt{\pi}}\alpha^3 \exp(-\alpha^2 (r - r_0)^2)\} \;, \tag{4}$$

with the parameters:

$$\begin{gathered} \tfrac{g_{\pi q}^2}{4\pi} = \tfrac{g_{\eta q}^2}{4\pi} = \tfrac{g_{Kq}^2}{4\pi} = 0.67, \;\; \tfrac{g_{\eta' q}^2}{4\pi} = 1.206, \\ r_0 = 0.43 \;\text{ fm}, \;\; \alpha = 2.91 \;\text{ fm}^{-1}, C = 0.474\, fm^{-2}, \; m_{u,d} = 340 \;\text{ MeV}, \; m_s = 440 \;\text{ MeV}, \\ \mu_\pi = 139 \;\text{ MeV}, \;\; \mu_\eta = 547 \;\text{ MeV}. \; \mu_{\eta'} = 958\,\text{MeV}, \;\; \mu_K = 495 \;\text{ MeV}. \end{gathered} \tag{5}$$

which lead to a good description of low-energy non-strange and strange baryon spectra. Fixing the nucleon mass at m_N = 939 MeV, this parametrization gives for example m_Δ = 1232 MeV and $N(1440)$ = 1493 MeV. The lowest negative parity states appear at $N(1535) - N(1520)$ = 1539 MeV, i. e. above the Roper resonance, in agreement with the experiment.

3. The wave function Ansatz

We start with the q^4 subsystem and treat the quarks as identical particles in all cases. Then following Ref. [12] the orbital (O) part of the lowest totally antisymmetric state must carry the symmetry $[31]_O$. In the flavour-spin (FS) coupling scheme this state has the form

$$|1\rangle \;\; = \;\; |[31]_O [211]_C \, [1111]_{OC} \;\; ; [22]_F [22]_S [4]_{FS}\rangle \tag{6}$$

which means that the wave function is totally symmetric in the flavour-spin space and totally antisymmetric in the orbital-colour (OC) space and that the q^4 subsystem carries non-zero angular momentum and has zero spin. Then the $q^4\overline{q}$ state is obtained by coupling the antiquark to the state $|1\rangle$ of Eq.(6) which leads to either $\overline{10}_F$ or to 8_F and to a total spin 1/2. To derive the orbital part we denote the quarks by 1, 2, 3 and 4 and the antiquark by 5 and introduce the internal Jacobi coordinates

$$\begin{aligned} \vec{x} \; &= \; \vec{r}_1 \; - \; \vec{r}_2 \; , \quad \vec{y} \; = \; (\vec{r}_1 \; + \; \vec{r}_2 \; - \; 2\vec{r}_3) \, / \sqrt{3} \\ \vec{z} \; = \; (\vec{r}_1 \; + \; \vec{r}_2 \; + \; \vec{r}_3 \; - \; 3\vec{r}_4) \, / \sqrt{6} \; , &\quad \vec{t} \; = \; (\vec{r}_1 \; + \; \vec{r}_2 \; + \; \vec{r}_3 + \; \vec{r}_4 - \; 4\vec{r}_5) \, / \sqrt{10} \; . \end{aligned} \tag{7}$$

The key issue is to construct a wave function with correct permutation symmetry in terms of the above Jacobi coordinates. Assuming an s^3p structure for $[31]_O$, the three independent $[31]_O$ states denoted by ψ_i are [12]

$$\psi_1 = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} \end{array} = \langle \vec{x} \, |000\rangle \, \langle \vec{y} \, |000\rangle \, \langle \vec{z} \, |010\rangle \; , \tag{8}$$

$$\psi_2 = \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1} \end{array} = \langle \vec{x} \, |000\rangle \, \langle \vec{y} \, |010\rangle \, \langle \vec{z} \, |000\rangle \; , \tag{9}$$

$$\psi_3 = \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1} \end{array} = \langle \vec{x} \, |010\rangle \, \langle \vec{y} \, |000\rangle \, \langle \vec{z} \, |000\rangle \; , \tag{10}$$

where $|n \; \ell \; m\rangle$ are shell model wave functions and we took the quantum number $m = 0$ everywhere, for convenience. Thus each function carries an angular momentum $\ell = 1$ in one of the relative coordinate, which leads to a total parity $P = +\,1$ and a total angular momentum $J = 1/2$ or $3/2$. The degeneracy of these two states can be lifted by the introduction of a spin-orbit coupling.

The functions (8)-(10) are used to construct a totally antisymmetric orbital-colour state for the q^4 subsystem, in agreement with (6). The coefficients of the resulting linear combination are fixed by group theory, namely by the Clebsch-Gordan coefficients of the permutation group S_4. In this case, the absolute value of all three coefficients is equal to $1/\sqrt{3}$, which means that each of the states (8)-(10) contribute with equal probability.

The pentaquark orbital wave functions are obtained by multiplying each ψ_i by $\left\langle \vec{t} \, |000 \right\rangle$ which describes the motion of the q^4 subsystem relative to $\overline{q}$. The wave function associated to each relative coordinate is chosen to be a Gaussian. This gives

$$\psi_1 \; = \; \psi_0 \;\; z \, Y_{10} \;\; (\hat{z}) \tag{11}$$

$$\psi_2 \; = \; \psi_0 \;\; y \, Y_{10} \;\; (\hat{y}) \tag{12}$$

$$\psi_3 \; = \; \psi_0 \;\; x \, Y_{10} \;\; (\hat{x}) \tag{13}$$

where

$$\psi_0 = [\frac{1}{48\pi^5\alpha\beta^3}]^{1/2} \; \exp \left[- \, \frac{1}{4\alpha^2} \, \left(x^2 \; + \; y^2 \; + \; z^2 \right) \; - \; \frac{1}{4\beta^2} \; t^2 \right] \; . \tag{14}$$

The two variational parameters are α, the same for all internal coordinates of the q^4 subsystem, $\vec{x}$, $\vec{y}$ or $\vec{z}$, and β, for $\vec{t}$, the relative coordinate of q^4 to $\overline{q}$.

The algebraic structure of the state (6) is identical to that of Ref. [18]. The small overlap of the resulting $q^4\overline{q}$ state with the kinematically allowed final states could partly explain the narrowness of Θ^+.

Table 1
The hyperfine interaction V_χ, Eq. (3), integrated in the flavour-spin space, for four quark subsystems. The upper index indicates the flavour of every interacting qq pair.

q^4	$I,\ I_3$	V_χ
$uudd$	0, 0	$30\ V_\pi - 2\ V_\eta^{uu} - 4\ V_{\eta'}^{uu}$
$uuds$	1/2, 1/2	$15\ V_\pi - V_\eta^{uu} - 2\ V_{\eta'}^{uu} + 12\ V_K + 2\ V_\eta^{us} - 2\ V_{\eta'}^{us}$
$ddss$	1, -1	$V_\pi + \frac{1}{3}V_\eta^{uu} + \frac{2}{3}V_{\eta'}^{uu} + \frac{4}{3}V_\eta^{ss} + \frac{2}{3}V_{\eta'}^{ss} + 20V_K + \frac{16}{3}V_\eta^{us} - \frac{16}{3}V_{\eta'}^{us}$

4. Matrix elements

The expectation values of the hyperfine interaction V_χ, Eq. (3), in the flavour-spin space, are presented in Table 1 for the three q^4 subsystems necessary to construct the antidecuplet. They are expressed in terms of the two-body radial form (4) now denoted as $V_\gamma^{q_a q_b}$ where $q_a q_b$ specifies the flavour content of the interacting pair. The $\mathrm{SU}(3)_F$ is explicitly broken by the quark masses and by the meson masses. By taking $V_\eta^{uu} = V_\eta^{us} = V_\eta^{ss}$ and $V_{\eta'}^{uu} = V_{\eta'}^{us} = 0$, one recovers the simpler model described in Ref. [19] where one does not distinguish between the uu, us or ss pairs in the η-meson exchange. Moreover, in Ref. [19] one takes as parameters the already integrated two-body matrix elements of some radial part of the hyperfine interaction, as in Ref. [11]. Here we specify a radial form, which allows the explicit introduction of radial excitations at the quark level, whenever necessary. Then, from Table 1 one can easily reproduce Table 3 of [19] containing the coefficients x_1, x_2 and x_3, i. e. the multiplicities, or the fraction of the two body matrix elements associated to π, K and η exchange respectively, which appear in the expression for the mass. The first and last row of x_i, corresponding to Θ and Ξ^{--} are straightforward, inasmuch as their contents are $uudd\overline{s}$ and $ddss\overline{u}$ respectively. To get the x_i associated with N_5 and Σ_5, which we call here $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ respectively, one must construct the linear combinations

$$V_\chi(N_{\overline{10}}) = \frac{1}{3}V_\chi(uudd) + \frac{2}{3}V_\chi(uuds),$$
$$V_\chi(\Sigma_{\overline{10}}) = \frac{1}{3}V_\chi(uuss) + \frac{2}{3}V_\chi(uuds), \quad (15)$$

in agreement with the flavour wave functions given in the Appendix and the relation $V_\chi(uuss) = V_\chi(ddss)$. Moreover, in Ref. [19], for each exchanged meson, one assumed that the radial two-body matrix elements are equal irrespective of the angular momentum of the state, $\ell = 0$ or $\ell = 1$, which we won't do.

5. Results and discussion

In Table 2 we present the variational energy E of the model Hamiltonian (1) resulting from the trial wave function described by Eqs. (11)-(14) for various $q^4\overline{q}$ systems related to the antidecuplet or the octet. One can see that, except for the confinement contribution $\langle V_c \rangle$, all the other terms break $\mathrm{SU}(3)_F$, as expected: the mass term $\sum_{n=1}^{5} m_i$ increases, the kinetic energy $\langle T \rangle$ decreases and the short range attraction $\langle V_\chi \rangle$ decreases with the

Table 2
Expectation values (MeV) and total energy $E = \sum_{n=1}^{5} m_i + \langle T \rangle + \langle V_c \rangle + \langle V_\chi \rangle$ obtained from the Hamiltonian (1) for various $q^4\overline{q}$ systems. The mass M is obtained from E by subtraction of 510 MeV in order to fit the mass of Θ^+. The values of the variational parameters α and β are indicated in the last two columns.

$q^4\overline{q}$	$\sum_{n=1}^{5} m_i$	$\langle T \rangle$	$\langle V_c \rangle$	$\langle V_\chi \rangle$	E	M	$\alpha(fm)$	$\beta(fm)$
$uudd\overline{d}$	1700	1864	442	-2044	1962	1452	0.42	0.92
$uudd\overline{s}$	1800	1848	461	-2059	2050	1540	0.42	1.01
$uuds\overline{d}$	1800	1535	461	-1563	2233	1732	0.45	0.92
$uuds\overline{s}$	1900	1634	440	-1663	2310	1800	0.44	0.87
$ddss\overline{u}$	1900	1418	464	-1310	2472	1962	0.46	0.92
$uuss\overline{s}$	2000	1410	452	-1310	2552	2042	0.46	0.87

Table 3
The antidecuplet mass spectrum in MeV.

Pentaquark	$Y,\ I,\ I_3$	Present result	Carlson et al. [19]	Exp + GMO formula
Θ^+	2, 0, 0	1540	1540	1540
$N_{\overline{10}}$	1, 1/2, 1/2	1684	1665	1647
$\Sigma_{\overline{10}}$	0, 1, 1	1829	1786	1755
Ξ^{--}	-1, 3/2, -3/2	1962	1906	1862

quark masses. For reasons explained in the introduction, we subtract 510 MeV from the total energy E in order to reproduce the experimental Θ^+ mass.

For completeness, in the last two columns of Table 2 we also indicate the values of the variational parameters α and β appearing in the radial wave function (14) which minimize the energy of the systems displayed in the first column. The parameter α takes values around $\alpha_0 = 0.44$ fm. In the same quark model this is precisely the value which minimizes the ground state nucleon mass [14] when the trial wave function is $\phi \propto \exp[-(x^2+y^2)/4\alpha_0^2]$ where $\vec{x}$ and $\vec{y}$ are the first two of the Jacobi coordinates (7) defined above. The quantity α_0 gives a measure of the quark core size of the nucleon because it is its root-mean-square radius. The parameter β is related to the relative coordinate $\vec{t}$ between the center of mass of the q^4 subsystem and the antiquark. It takes values about twice larger than α, which is an indication that the four quarks cluster together, whereas $\bar{q}$ remains slightly separate in contrast to certain Ansaetze recently promulgated in the literature.

Table 3 reproduces the calculated antidecuplet mass spectrum obtained from the mass M of Table 2. The masses of Θ^+ and Ξ^{--} can be read off Table 2 directly. The other masses are obtained from the linear combinations

$$M(N_{\overline{10}}) = \frac{1}{3}M(uudd\bar{d}) + \frac{2}{3}M(uuds\bar{s}),$$
$$M(\Sigma_{\overline{10}}) = \frac{2}{3}M(uuds\bar{d}) + \frac{1}{3}M(uuss\bar{s}) \ . \qquad (16)$$

In comparison with Carlson et al. [19], where the mass of Θ^+ is also adjusted to the experimental value, we obtain somewhat higher masses for $N_{\overline{10}}$, $\Sigma_{\overline{10}}$ and Ξ^{--}, the latter being about 100 MeV above the experimentally found mass of 1862 MeV [3]. This is in contrast to the strongly correlated diquark model of Jaffe and Wilczek [20], where Ξ^{--} lies about 100 MeV below the experimental value. Note that the mass of Θ^+ is also fixed in that model. In the lowest order of $SU(3)_F$ breaking, one can parametrize the result by the Gell-Mann-Okubo (GMO) mass formula, $M = M_{\overline{10}} + cY$. In the present case one obtains $M \simeq 1829 - 145\ Y$. The fit to the measured masses of Θ^+ and Ξ^{--} gives $M \simeq 1755 - 107\ Y$. Accordingly, the masses assigned to $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ are 1647 MeV and 1755 MeV. They are indicated in the last column of Table 3. Starting from this fit, Diakonov and Petrov [21] analyzed the masses of the non-exotic members of the antidecuplet as a consequence of the octet-antidecuplet mixing due to $SU(3)_F$ breaking. [2] A new nucleon state at 1650-1690 MeV and a new Σ at 1760-1810 MeV have been proposed as mainly antidecuplet baryons with $Y = 1$ and $Y = 0$ respectively. Shortly after, Pakvasa and Suzuki [23] also considered the octet-antidecuplet mixing in a phenomenological way starting from the Gell-Mann-Okubo mass formulae. There, the resonance $N^*(1710)$ was taken as the $Y = 1$ partner of Θ^+, as in the original work of Ref. [4]. That analysis showed that the range of values for the mixing angle required by the mass spectrum of the $Y = 1$ baryons is not consistent with the range needed to fit strong decays. [3]

However the recent modified PWA analysis [24] reconsiders the antidecuplet nature of $N^*(1710)$ used to determine the mass of Θ^+ in Ref. [4]. As a result, instead of $N^*(1710)$, it proposes two narrow resonances 1680 MeV and/or 1730 MeV, as appropriate $Y = 1$ partners of Θ^+. This interpretation of the data clearly requires octet-antidecuplet mixing.

In the present model, which contains $SU(3)_F$ breaking, the mixing appears naturally and it can be derived dynamically starting from the Hamiltonian (1). Recall that Table 3, column 3 gives the pure antidecuplet masses. The pure octet masses are easily calculable using Table 2 and the octet wave functions (see Appendix). We obtain

$$\begin{aligned} M(N_8) &= \frac{2}{3}M(uudd\bar{d}) + \frac{1}{3}M(uuds\bar{s}) = 1568 \text{ MeV}, \\ M(\Sigma_8) &= \frac{1}{3}M(uuds\bar{d}) + \frac{2}{3}M(uuss\bar{s}) = 1936 \text{ MeV}. \end{aligned} \tag{17}$$

The octet-antidecuplet off-diagonal matrix element, denoted by V, has only two non-vanishing contributions, one coming from the mass (first) term of (1) and associated with the overlap of $\Phi(N_{\overline{10}})$ and $\Phi(N_8)$, or of $\Phi(\Sigma_{\overline{10}})$ and $\Phi(\Sigma_8)$, and the other coming from the hyperfine interaction. Using the Appendix one can obtain the analytic form of V as

$$V = \begin{cases} \frac{2\sqrt{2}}{3}(m_s - m_u) + \frac{\sqrt{2}}{3}\ [S(uuds\bar{s}) - S(uudd\bar{d})] = 166 \text{ MeV} & \text{for N} \\ \frac{2\sqrt{2}}{3}(m_s - m_u) + \frac{\sqrt{2}}{3}\ [S(uuss\bar{s}) - S(uuds\bar{d})] = 155 \text{ MeV} & \text{for } \Sigma \end{cases} \tag{18}$$

[2] A similar analysis, but restricted to the ideal mixing postulated by Jaffe and Wilczek [20], has been made in Ref. [22].

[3] A more extended representation mixing including the 8, 10, $\overline{10}$, 27, 35 and $\overline{35}$ were considered in Ref. [25] in the context of the chiral soliton model. The masses of $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ were predicted to be the same as those in the last column of Table 3. The estimated range for the pure exotic pentaquarks turn out to be 1430 MeV $< M(\Theta^+) <$ 1660 MeV and 1790 MeV $< M(\Sigma^{--}) <$ 1970 MeV.

where $S = \langle T \rangle + \langle V_\chi \rangle$. The numerical values on the right hand side of Eq. (18) result from the quark masses given in Eqs. (5) and from the values of $\langle T \rangle$ and $\langle V_\chi \rangle$ exhibited in Table 2. One can see that the mass-induced breaking term is identical for N and Σ, as expected from simple SU(3) considerations. Its numerical value, 94.28 MeV, represents more than 1/2 of the total off-diagonal matrix element.

The masses of the physical states, the "mainly octet" N^* and the "mainly antidecuplet" N_5, result from the diagonalization of a 2×2 matrix in each case. Accordingly the nucleon solutions are

$$\begin{aligned} N^* &= N_8 \cos\theta_N - N_{\overline{10}} \sin\theta_N, \\ N_5 &= N_8 \sin\theta_N + N_{\overline{10}} \cos\theta_N, \end{aligned} \tag{19}$$

with the mixing angle defined by

$$\tan 2\theta_N = \frac{2V}{M(N_{\overline{10}}) - M(N_8)} \; . \tag{20}$$

The masses obtained from this mixing are 1451 MeV and 1801 MeV respectively and the mixing angle is $\theta_N = 35.34^0$, which means that the "mainly antidecuplet" state N_5 is 67 % $N_{\overline{10}}$ and 33 % N_8, and the "mainly octet" N^* the other way round. The latter is located in the Roper resonance mass region 1430 - 1470 MeV. However this is a $q^4\bar{q}$ state, i. e. it is different from the q^3 radially excited state obtained in Ref. [15] at 1493 MeV with the parameters (5) and assigned to the Roper resonance. A mixing of the q^3 and the $q^4\bar{q}$ states could possibly be a better description of reality. There is some experimental evidence that two resonances, instead of one, separated by about 100 MeV, and located around 1440 MeV, could consistently describe the $\pi - N$ and $\alpha - p$ scattering in this region [26], however. Thus the issue of the existence of more than one resonance with $J^P = 1/2^+$ in the 1430- 1500 MeV mass range remains unsettled. The "mainly antidecuplet" solution at 1801 MeV is far from the higher option of Ref. [24], at 1730 MeV, interpreted as the $Y = 1$ narrow resonance partner of Θ^+.

In a similar way we obtain two Σ resonances, the "mainly octet" one being at 1719 MeV and the "mainly antidecuplet" one at 2046 MeV. The octet-antidecuplet mixing angle is $\theta_\Sigma = -35.48^0$. The lower state is somewhat above the experimental mass range 1630 - 1690 MeV of the the $\Sigma(1660)$ resonance. As the higher mass region of Σ is less known experimentally, it would be difficult to make an assignement for the higher state.

The mixing angle θ_N and θ_Σ are nearly equal in absolute value, but they have opposite signs. The reason is that $M(N_{\overline{10}}) > M(N_8)$ while $M(\Sigma_{\overline{10}}) < M(\Sigma_8)$. Interestingly, each is close to the value of the ideal mixing angle $\theta_N = 35.26^0$ and $\theta_\Sigma = -35.26^0$. Only the relative strengths of decays and selection rules can discriminate between mixing schemes as well as between models [23,27]. This is a task for a future work.

6. Conclusions

In conclusion we have used a variational method, which provides upper bounds on the masses of all isomultiplets of the pentaquark antidecuplet. We calculated dynamically the masses of the pure exotic pentaquarks Θ^+ and Ξ^{--} and the masses of the other members of the antidecuplet. The model on which these calculations are based reproduces well the baryon spectrum, when baryons are described as q^3 systems. It assumes a flavour-spin structure for the hyperfine quark-quark interaction and its radial shape contains parameters which have been fitted not only to the ground state baryons, but also to a large number of excited states [15]. In particular this interaction places the Roper resonance, modeled as a q^3 system, below the lowest negative parity baryons, in agreement with the experiment. However the description of strong decays in this model is not satisfactory (see e. g. Ref. [28]). Besides the qq interaction a $q\bar{q}$ interaction is necessary to describe pentaquarks. Here we did not introduce it explicitly but relied on the conclusion of Ref. [16] that an attractive spin-spin interaction that operates only in the $q\bar{q}$ channel can lower the $q^4\bar{q}$ energy to accommodate the Θ^+. In this way we can explain the mass shift of -510 MeV necessary to reproduce the mass of Θ^+. It follows that this flavour-independent interaction equally lowers all the other members of the antidecuplet and of the octet.

But in the new light shed by the pentaquark studies, the usual practice of hadron spectroscopy is expected to change. There are hints that the wave functions of some excited states might contain $q^4\overline{q}$ components. These components, if obtained quantitatively, would perhaps better explain the widths and mass shifts in the baryon resonances [29]. In particular the mass of the Roper resonance may be further shifted up or down. In that case the model parametrization should be revised and more precise four- and five-body calculations should be performed. On the other hand a full experimental confirmation of the Θ^+ and of the Ξ^{--} resonances and more appropriate partial wave analysis of existing data would be of great help in understanding the structure of pentaquarks and of ordinary baryons.

Appendix

Here we give the form of one of the two independent flavour wave functions for each isomultiplet belonging to $\overline{10}_F$. It is the function where both pairs of quarks, 12 and 34, are in an antisymmetric state $\phi_{[11]}(q_a q_b) = (q_a q_b - q_b q_a)/\sqrt{2}$. By analogy with the q^3 system, we shall use the label ρ for all states which are antisymmetric under the permutation (12). For Θ this wave function is straightforward

$$\Phi^\rho(\Theta) = \phi_{[11]}(ud)\phi_{[11]}(ud)\overline{s} \; . \tag{21}$$

The $N_{\overline{10}}$ flavour wave function is obtained from that of Θ by applying the U-spin ladder operator U_- of SU(3). Its normalized form becomes

$$\Phi^\rho(N_{\overline{10}}) = \frac{1}{\sqrt{3}}\{[\phi_{[11]}(us)\phi_{[11]}(ud) + \phi_{[11]}(ud)\phi_{[11]}(us)]\overline{s} + \phi_{[11]}(ud)\phi_{[11]}(ud)\overline{d}\} \; . \tag{22}$$

Applying U_- again one obtains the wave function of $\Sigma_{\overline{10}}$ which is

$$\Phi^\rho(\Sigma_{\overline{10}}) = \frac{1}{\sqrt{3}}\{\phi_{[11]}(us)\phi_{[11]}(us)\overline{s} + [\phi_{[11]}(us)\phi_{[11]}(ud) + \phi_{[11]}(ud)\phi_{[11]}(us)]\overline{d}\} \; . \tag{23}$$

The wave function of Ξ^{--} is as simple as that of Θ but with another quark content of course

$$\Phi^\rho(\Xi^{--}) = \phi_{[11]}(ds)\phi_{[11]}(ds)\overline{u} \; . \tag{24}$$

In these functions the normal order of particles 1234 is understood. In each case one can get the other linear independent function in the flavour space, Φ^λ, with the quark pairs 12 and 34 in a symmetric state, $\phi_{[2]}(q_a q_b) = (q_a q_b + q_b q_a)/\sqrt{2}$ ($q_a \neq q_b$) or $\phi_{[2]}(q_a q_a) = q_a q_a$, by applying the permutation (23) to the above corresponding function (see e. g. [30]). For example we have

$$\Phi^\lambda(\Theta) = \sqrt{\frac{1}{3}}[\phi_{[2]}(uu)\phi_{[2]}(dd) + \phi_{[2]}(dd)\phi_{[2]}(uu) - \phi_{[2]}(ud)\phi_{[2]}(ud)]\bar{s}\ . \tag{25}$$

Both the Φ^ρ and Φ^λ functions are necessary in the calculation of the matrix elements of the hyperfine interaction.

In the same notation, the N_8 and Σ_8 the flavour octet wave functions, antisymmetric under the permutation (12) are

$$\Phi^\rho(N_8) = \frac{1}{\sqrt{6}}[\phi_{[11]}(us)\phi_{[11]}(ud) + \phi_{[11]}(ud)\phi_{[11]}(us)]\bar{s} - \sqrt{\frac{2}{3}}\phi_{[11]}(ud)\phi_{[11]}(ud)\bar{d}\ . \tag{26}$$

$$\Phi^\rho(\Sigma_8) = \sqrt{\frac{2}{3}}\phi_{[11]}(us)\phi_{[11]}(us)\bar{s} - \frac{1}{\sqrt{6}}[\phi_{[11]}(us)\phi_{[11]}(ud) + \phi_{[11]}(ud)\phi_{[11]}(us)]\bar{d}\ . \tag{27}$$

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