Artificial Abelian gauge potentials induced by dipole-dipole interactions between Rydberg atoms

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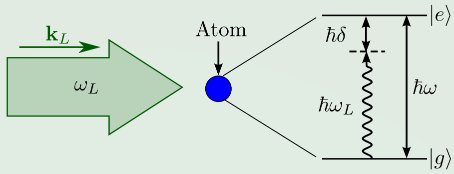
Introduction

Many works on artificial gauge potentials induced by atom-light interaction adopt a single-particle approach. The predicted potentials are then supposed to be valid for a system of many weakly interacting atoms. So far, the consequences of atom-atom interactions on the generation of artificial gauge fields has little been studied [1]. The aim of this work is to study the artificial gauge fields arising from the interaction of two Rydberg atoms driven by a common laser field [2].

Artificial gauge potentials without atom-atom interactions [3]



Ω : Rabi frequency
δ : detuning
$\delta = \omega_L - \omega$

Consider a single two-level atom interacting with a classical laser field.

- $E_\pm = \pm\hbar\sqrt{|\Omega|^2+\delta^2}$: eigenvalues of internal Hamiltonian ($\hat{H}_{2l}$)
  $|\chi_\pm(\Omega(\mathbf{r}),\delta(\mathbf{r}))\rangle$: eigenstates of $\hat{H}_{2l}$; depend parametrically on the atomic position $\mathbf{r}$
- Total hamiltonian : $\hat{H}_{1\mathrm{at}} = \hat{\mathbf{p}}^2/(2m)\otimes\hat{\mathbb{1}}^{\mathrm{int}} + \hat{\mathbb{1}}^{\mathrm{ext}}\otimes\hat{H}_{2l}$
- Global wave function in position representation :

$$\langle\mathbf{r}|\psi(t)\rangle = \sum_{j=\pm}\psi_j(\mathbf{r},t)|\chi_j(\mathbf{r})\rangle$$

- Adiabatic evolution of the internal state ($j=\pm$) :

$$\langle\mathbf{r}|\psi(0)\rangle = \psi_j(\mathbf{r},0)|\chi_j(\mathbf{r})\rangle \quad\Rightarrow\quad \langle\mathbf{r}|\psi(t)\rangle \approx \psi_j(\mathbf{r},t)|\chi_j(\mathbf{r})\rangle$$

$$i\hbar\partial_t|\psi(t)\rangle = \hat{H}_{1\mathrm{at}}|\psi(t)\rangle$$

$$i\hbar\frac{\partial}{\partial t}\psi_j(\mathbf{r},t) = \left[\frac{(\hat{\mathbf{p}}-\mathbf{A}^j)^2}{2m}+\phi^j+E_j\right]\psi_j(\mathbf{r},t) \qquad (1)$$

- Eq. (1) is formally equivalent to Schrödinger's equation for a particle of unit charge immersed in EM fields described by the artificial potentials $\mathbf{A}^j(\mathbf{r})$ and $\phi^j(\mathbf{r})$ given by [3] :

$$\mathbf{A}^\pm(\mathbf{r}) = i\hbar\langle\chi_\pm(\mathbf{r})|\boldsymbol{\nabla}_\mathbf{r}\chi_\pm(\mathbf{r})\rangle = -\langle\hat{\mathbf{p}}\rangle_{\chi_\pm}$$

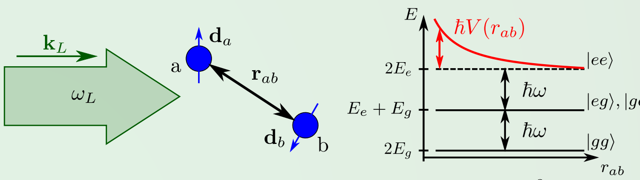
$$\phi^\pm(\mathbf{r}) = \hbar^2|\langle\chi_\mp(\mathbf{r})|\boldsymbol{\nabla}_\mathbf{r}\chi_\pm(\mathbf{r})\rangle|^2/2m = (\Delta\hat{\mathbf{p}})^2_{\chi_\pm}/2m$$

new formulation [2]

  - $(\hat{\mathbf{p}}-\mathbf{A}^\pm)^2/2m \Leftrightarrow$ kinetic energy of slow C.M. motion
  - $\phi^\pm$ originates from the quantum fluctuations of momentum
  - Ω and δ homogeneous $\Rightarrow \mathbf{A}^\pm$ homogeneous $\Rightarrow \mathbf{B}^\pm = 0$
- Classical electromagnetic field + many noninteracting atoms $\Rightarrow$ same artificial gauge potentials as in the single atom case

Two interacting Rydberg atoms

Consider a pair of Rydberg atoms driven by a common laser field [4] in the case of spatially uniform Ω and δ for which the artificial gauge fields vanish in the absence of atom-atom interactions



- Dipole-dipole interactions $\Rightarrow$ energy shift $\hbar V = \hbar C_3/r_{ab}^3$ of $|ee\rangle$
- Two-atom internal Hamiltonian :

$$\hat{H}_{\mathrm{d-d}} = \hat{H}_{2l,a}+\hat{H}_{2l,b}+\hbar V(r_{ab})|ee\rangle\langle ee|$$

- Two-atom internal eigenstates $|\chi_i(V(r_{ab}))\rangle$ ($i=0,1,\pm$) of energy E_i; depend parametrically on the atomic positions
- Definition of a crossover interatomic distance r_c :

Dipole-dipole Interaction energy $\qquad \hbar V(r_c) = \hbar\sqrt{|\Omega|^2+\delta^2} \qquad$ Atom-light Interaction energy

Artificial gauge potentials and fields with dipole-dipole interactions [2]

Adiabatic evolution of internal state $|\chi_i\rangle$ ($i=1,\pm$) $\Rightarrow$ equation for the two-atom spatial wave function $\psi_i(\mathbf{r}_a,\mathbf{r}_b,t)$ equivalent to Schrödinger's equation for two charged particles in EM fields

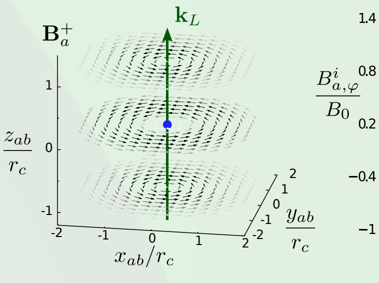
- Artificial gauge potentials (for atom $\alpha = a,b$, $\mathbf{e}_{\mathbf{k}_L} = \mathbf{k}_L/k_L$) :

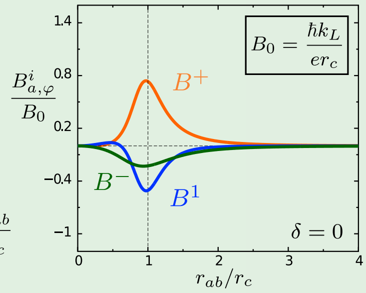
$$\mathbf{A}^i_\alpha = i\hbar\langle\chi_i|\boldsymbol{\nabla}_{\mathbf{r}_\alpha}\chi_i\rangle = A^i_\alpha(r_{ab})\,\mathbf{e}_{\mathbf{k}_L}$$

$$\phi^i_\alpha = \frac{\hbar^2}{2m_\alpha}\sum_{j\neq i}|\langle\chi_j|\boldsymbol{\nabla}_{\mathbf{r}_\alpha}\chi_i\rangle|^2$$

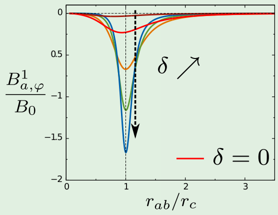
- Artificial magnetic fields experienced by atom a for $\mathbf{k}_L = k_L\mathbf{e}_z$, $\mathbf{r}_b = 0$ and spherical coordinates $\{r_{ab},\theta,\varphi\}$:

$$\mathbf{B}^i_a(\mathbf{r}_{ab}) = \boldsymbol{\nabla}_{\mathbf{r}_a}\times\mathbf{A}^i_a = \frac{dA^i_a}{dr_{ab}}\sin\theta\,\mathbf{e}_\varphi = B^i_{a,\varphi}(r_{ab})\,\mathbf{e}_\varphi$$





- For realistic Ω, δ, $\mathbf{k}_L$ and $V(r_{ab})$ [4], $r_c \approx 8\ \mu$m and $B_0 \approx 2$ mT
- $\mathbf{A}^i_a = \mathbf{A}^i_b \quad\Rightarrow\quad \mathbf{B}^i_a = -\mathbf{B}^i_b$
- $\delta\nearrow$ ($\delta\searrow$) $\Rightarrow |\mathbf{B}^i_\alpha|\nearrow$ ($\searrow$)
- For $\delta \gg 0$, peaks of intensity of $\mathbf{B}^i_\alpha$ located at $r_{ab} = r_c$ or $r_{ab} = r_c/\sqrt[3]{2}$

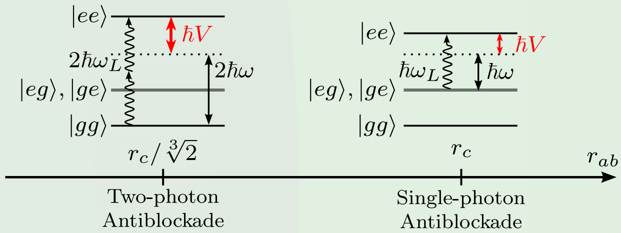


Physical interpretation

The location of the peaks of intensity of $|\mathbf{B}^i_\alpha|$ is related to transitions between bare states. For $\delta \gg 0$, transitions between bare states are possible only when $\hbar V(r_{ab})$ compensates for $\delta \Rightarrow$ **antiblockade** [5]

- Single-photon antiblockade : $\hbar\omega + \hbar V(r_{ab}) = \hbar\omega_L$
  $\Rightarrow$ transitions $|eg\rangle,|ge\rangle \leftrightarrow |ee\rangle$ resonant
- Two-photon antiblockade : $2\hbar\omega + \hbar V(r_{ab}) = 2\hbar\omega_L$
  $\Rightarrow$ transition $|gg\rangle \leftrightarrow |ee\rangle$ resonant

At interatomic distances where the antiblockade is effective, the internal eigenstates $|\chi_i(r_{ab})\rangle$ present strong nonuniform variations which induce large artificial magnetic fields.



Conclusion

We have shown that the combination of atom-atom and atom-field interactions in a uniform laser field gives rise to nonuniform artificial magnetic fields. These fields are maximum where atom-atom and atom-light interactions are of the same order of magnitude. See [2] for more details.

References

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