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Biorthogonal wavelet approximation for the coupling of FEM-BEM. (English summary)
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The present paper is devoted to the multiscale matrix compression for the coupling of finite element methods (FEM) and boundary element methods (BEM). Multiscale methods are applied to this BEM-FEM coupling to solve an exterior Dirichlet boundary value problem for the two-dimensional Poisson equation. Adapting biorthogonal wavelet matrix compression to the boundary terms with N degrees of freedom, the resulting compression strategy fits the optimal convergence rate of the coupling Galerkin methods, while the number of nonzero entries in the corresponding stiffness matrices is considerably smaller than N. It is observed, theoretically as well as in practical computations, that these biorthogonal wavelet bases combined with suitable compression techniques improve the efficiency of the discretization to the boundary integral operators. The accuracy of the FEM-BEM discretization is not deteriorated by the fast wavelet algorithm.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.