# Modeling and analysis of nonlinear systems applied to a guitar signal chain.

## ABAV, Sart-Tilman, Feb 21

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# Communication Outline

Introduction to the guitar instrumentation problem

Non-linear modelling
- IR measurement
- Model reconstruction

Non-linear loudspeaker simulation
- Objective evaluation of the model
- Subjective evaluation of the model

Conclusion

Section 1

# Introduction to the guitar instrumentation problem

# Goals of the thesis




FIGURE: A guitar chain example

FIGURE: Objective of the work

# Utility ?


FIGURE: Home studio ?

FIGURE: Home studio !

**Advantages of the simulation :**

- Wide variety of sounds and timbres.
- Weight and overcrowding.
- Easier comparison between two sounds.
- Cheaper.

**Disadvantages of the simulation :**

- Fidelity of the sound $\Rightarrow$ accuracy of the model.
- Psychologic :
  - An emulation still be an emulation.
  - Numerical sounds is cooler than analogical sound.

Section 2

# Non-linear modelling

# Non-linear modelisation : Black box approach

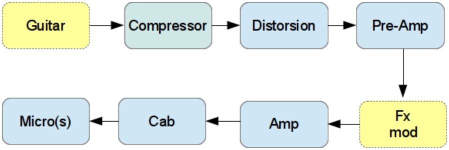



FIGURE: Guitar chain,in blue the non-linear elements

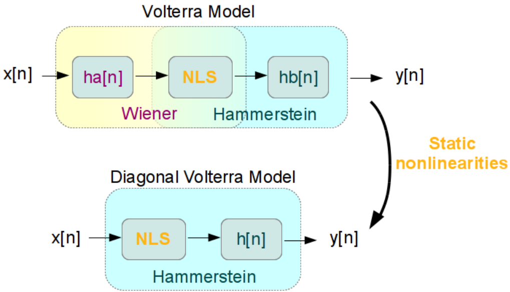



FIGURE: Nonlinears models

# Non-linear modelisation : Volterra Series

Relation between input x[n] and output y[n] for non-linear system

$$\left\{ \; y[n] = \sum_{i=1}^{M} \sum_{\tau_1=0}^{N} \dots \sum_{\tau_i=0}^{N} h_i[\tau_1, ..., \tau_i] \; x[n-\tau_1] ... \; x[n-\tau_i] \right. \quad (1)$$

Where $h_i[n]$ If the non-linearities are static, then $h_i[\tau_1, ..., \tau_i] = 0 \; \forall \tau_k \neq \tau_1$ with $k \in [1, i]$.

Ex $2^{th}$ Volterra kernel :

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0.9 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0.8 & 0 & \cdots & 0 \\ \vdots & \ddots & & & & \\ \cdots & & & & 0 & 0.1 \end{pmatrix}$$

Eq (1) becomes :

$$y[n] = x[n] \circledast h_1[n] + x[n]^2 \circledast h_2[n] + ... + x[n]^M \circledast h_M[n] \quad (2)$$

# Excitation signal

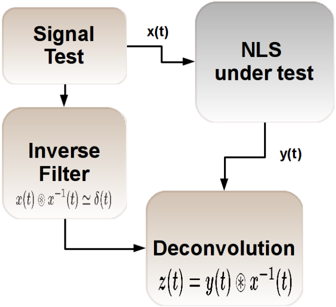



FIGURE: Impulse response measurement.

We choose :

$$x(t) = A\sin(\varphi(t)) \quad (3)$$

where the variable phase $\varphi(t)$ grows exponentialy. This property allows us to separate the differents orders of distorsion as (for example) :

$$x^3(t) = \frac{A^3}{4}\left[3\sin(\varphi(t)) - \sin(3\varphi(t))\right] \quad (4)$$

We can show that it exist a link between :

$$M.\varphi(t) \Leftrightarrow \varphi(t + \Delta_M) \quad (5)$$

So that

$$x^3(t) \circledast x^{-1}(t) \Rightarrow \delta(t) + \delta(t + \Delta_3) \quad (6)$$

# Deconvolved signal

$$
\begin{aligned}
z[n] &= y[n] \circledast x^{-1}[n] \\
&= \left( x[n] \circledast h_1[n] + x[n]^2 \circledast h_2[n] + \ldots + x[n]^M \circledast h_M[n] \right) \circledast x^{-1}[n]
\end{aligned} \quad (7)
$$

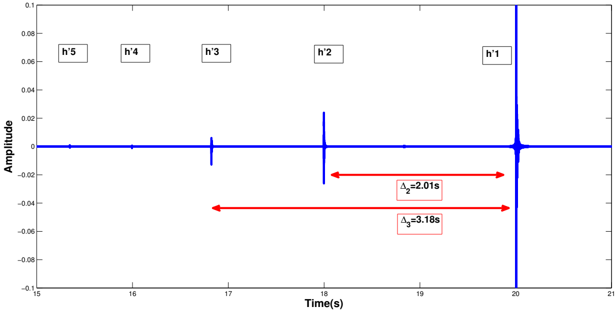



FIGURE: Impulse response measurement. z(t) the deconvolved signal

# Connection between Volterra kernels $h_i$ and measured kernels $m_i$

We have proved :

$$\mathcal{F}(h_i) \overset{link}{\Longleftrightarrow} \mathcal{F}(m_i)$$

So that, we obtain ($\forall \omega > 0$) :

$\mathcal{F}(h_i) \Longleftrightarrow \mathcal{F}(m_i)$

$$\begin{cases} H_1(w) = M_1(w) + 3e^{2jB} M_3(w) + 5e^{4jB} M_5(w) \\ H_2(w) = \frac{2}{A} \; je^{jB} [M_2(w) + 4e^{2jB} M_4(w)] \\ H_3(w) = \frac{-4}{A^2} \; e^{2jB} [M_3(w) + 5e^{2jB} M_5(w)] \\ H_4(w) = \frac{8}{A^3} \left(-je^{3jB}\right) M_4(w) \\ H_5(w) = \frac{16}{A^4} M_5(w) e^{4jB} \end{cases} \tag{8}$$

Where A takes into account of the amplitude of the emitted sine sweep and B depends on the parameters of the chosen sweep

Section 3

# Non-linear loudspeaker simulation

# Comparison between the real and simulated signal

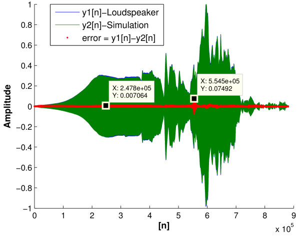
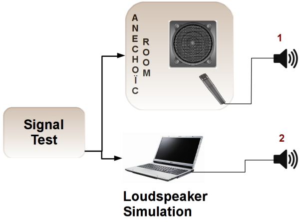



FIGURE: Simulation with 5 Volterra kernels



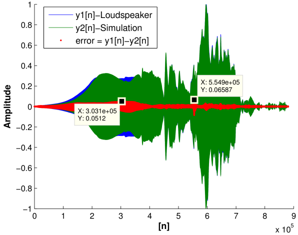




FIGURE: Simulation with 1 Volterra kernel

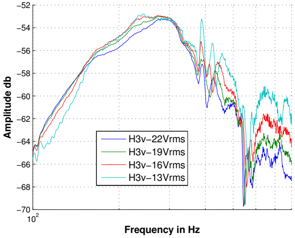
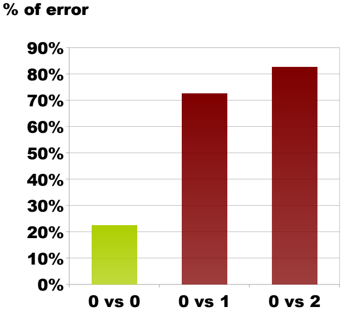



FIGURE: $H_3(\omega, Amplitude)$

# Blind test with real chords guitar



***Compare sample 0 to samples 0, 1, 2***




**Sample 0 : Real**

**Sample 1 : Simulated With 1 Volterra kernel**

**Sample 2 : Simulated With 5 Volterra kernels**

FIGURE: 82% of tested listener were not able to distinguish a simulation of type 2. 72% of tested listener were not able to distinguish a simulation of type 1 and 22% of tested listener hear a difference where there were none !

Section 4

# Conclusion

# Conclusion

## Achievement

- Non-linear loudspeaker modelisation based on Volterra series.
- Subjective and objective evaluation of the model.

## Future works

- Improve *simulation* fidelity.
- Solving the amplitude dependance of the model.
- Try the model on more non-linear system.

Thank you for your attention !