
Coupling Lattice Boltzmann 3D flow simulation in porous media with X-ray microtomographic imaging

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Liège at the heart of Europe



Within 2 European cross-border regions :

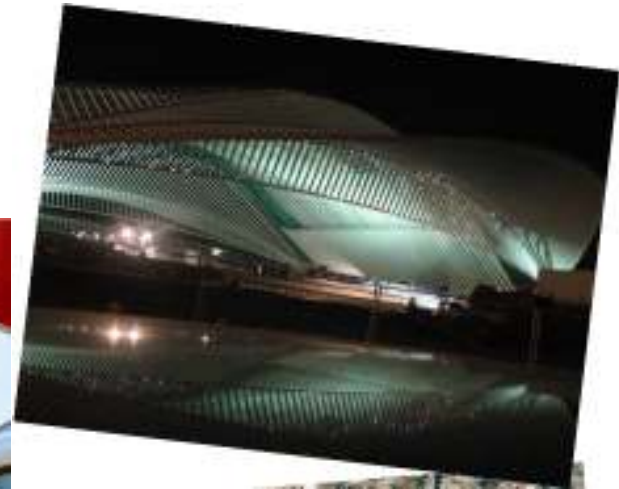
Meuse-Rhine Euregio

- 3 countries (BE, NL, DE)
- 4 regions

University of the Greater Region :

- 4 countries (BE, DE, LU, FR)
- 7 universities
- 115 000 students
- 6 000 professors

Liège: an attracting city



Important centre for Transport & Logistics:

- 2nd river port in Europe
- 8th freight airport in Europe

A collaborative work at LGC (*)

Chemical Reaction Engineering

D. Toye

P. Marchot

Multiphase systems

Separation technologies

M. Crine

A. Léonard

**Chemical processes and
sustainable development**

Flow through porous media

- **A great number of unit operations, e.g.**
 - Fixed bed catalytic reactors
 - Fixed bed adsorbers
 - Fibrous or granular filtration systems
 - Packed column absorbers
 - Distillation columns
 -
- **Complex (random) porous media whose geometry and texture directly affect fluid flow hydrodynamics.**
- **How to deal with this complexity ?**

Two objectives:

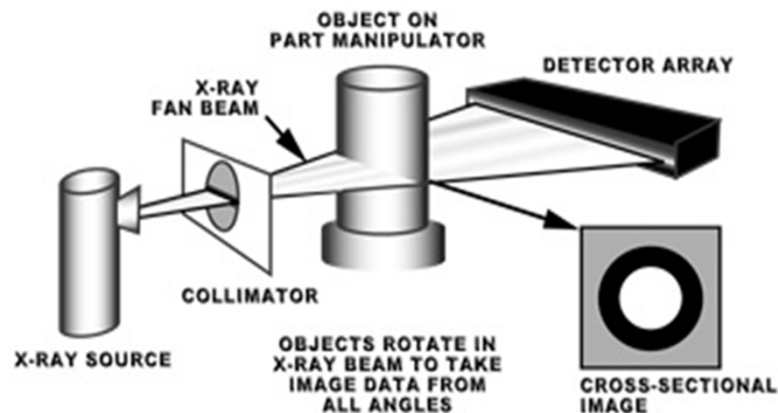
- 1. To get a detailed description of porous media geometry**
 - ↪ X-ray (micro)tomographic imaging
- 2. To develop a flow simulation model able to deal with the complex geometry of porous media, in terms of boundary conditions**
 - ↪ Lattice Boltzmann Methods

X-ray tomography principles

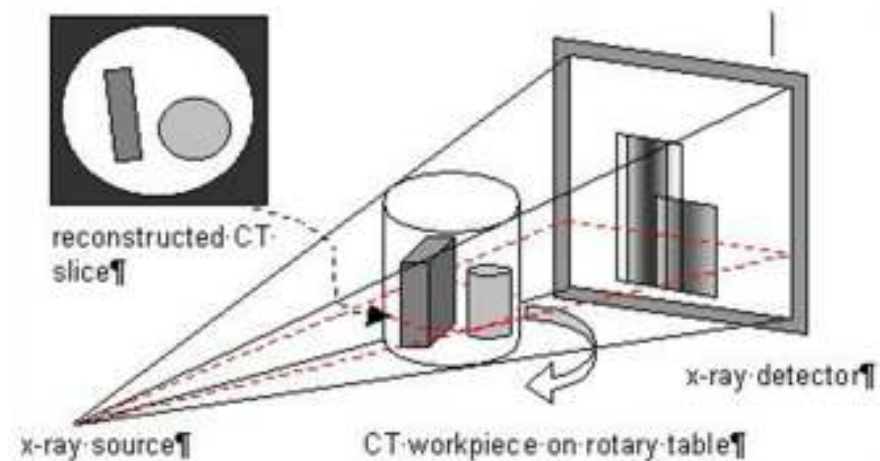
■ Objective

- Irradiating an object with X-ray under numerous angles, using a fan beam or a cone beam configuration, to get 2D or 3D images of phase saturation distribution

Fan beam



Cone beam



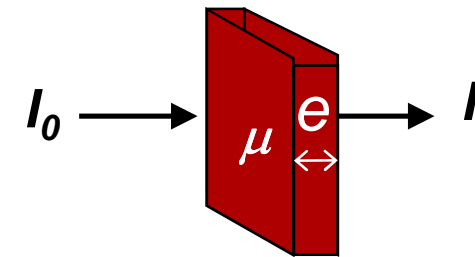
X-ray tomography principles

■ Theory

- Visualization technique based on X-ray transmission:

Beer-Lambert law :

$$\frac{dI}{I} = -\mu de$$



- μ : attenuation coefficient
 - $\mu = \text{const.}$: homogeneous material
 - $\mu = \text{fct.}(e)$: heterogeneous material

$$\frac{I}{I_0} = \exp(-\mu e)$$
$$\frac{I}{I_0} = \exp\left(-\int_e \mu(e) de\right)$$

- ↳ X-ray intensity attenuation along one line depends on local attenuation coefficients values encountered along that line

X-ray tomography principles

■ Values of the attenuation coefficient μ depends on:

□ Material properties

- the density ρ
- the atomic number Z
- the atomic mass A

$$\mu(E) = a \rho \frac{1}{E^n} \frac{Z^m}{A}$$

□ X-ray energy E

■ *The most important factors are:*

- The **density** (gas-solid ; gas-liquid): ρ^1
- The **atomic number** $\sim Z^2$ (at low energies, $Z/A \sim \text{Const.}$)

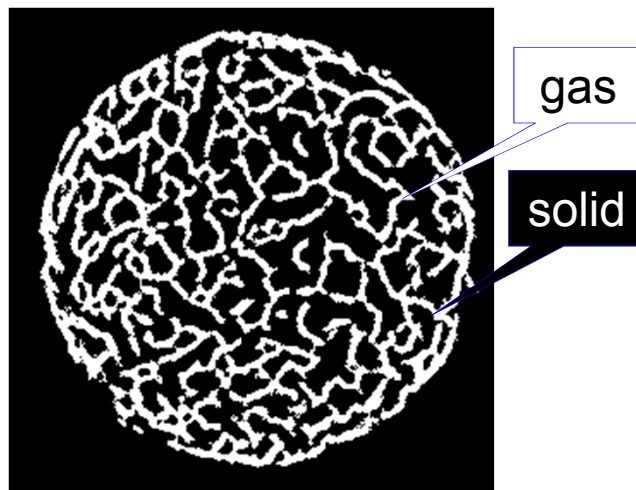
n and m depends on X-ray energy range

- At “low” energies:
 $m = 3.8$; $n = 3.2$
- At high energies
 $m = 1$; $n = 1$

X-ray tomography applications

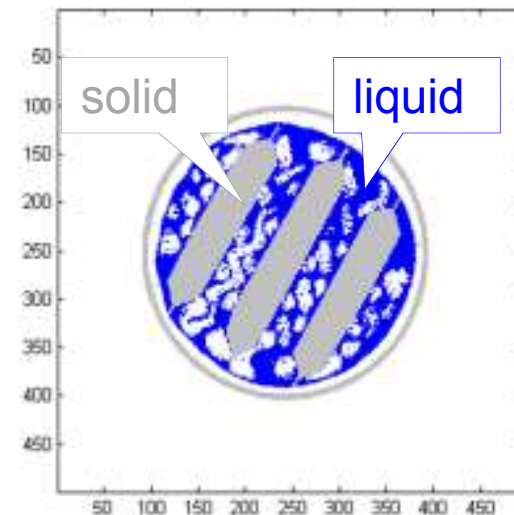
- X-ray tomography allows differentiating elements with strong differences in density, e.g.:

Gas and solid in porous material (deer antlers)



Léonard et al.(2007).
Journal of Microscopy-Oxford
225(Pt 3), 258-263

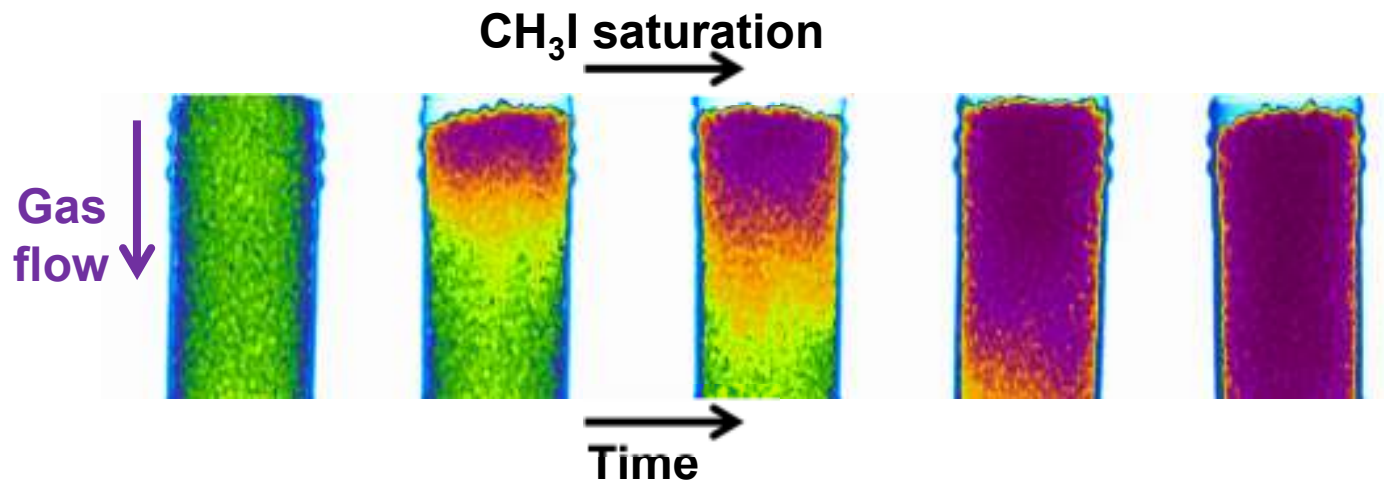
Gas and liquid in gas-liquid contactors



Aferka et al.(2010).
Canadian Journal of Chemical Engineering
88(4), 611-617

X-ray tomography applications

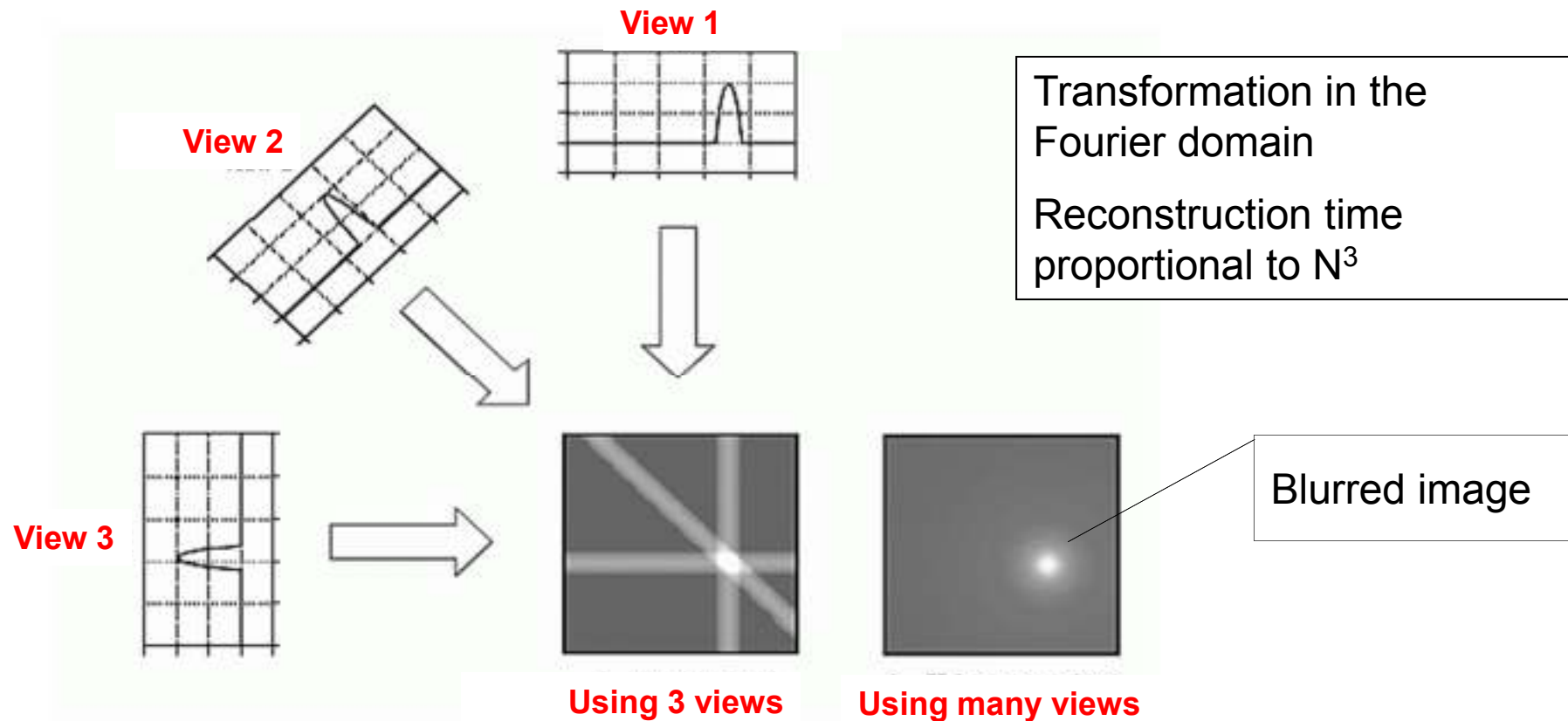
- X-ray tomography also allows differentiating elements according their atomic numbers Z , e.g.:
 - Adsorption of methyl iodide (CH_3I ; $Z_I = 53$) on activated carbon ($Z_C = 6$) in a gas mask filter (coll. Royal Military Academy)



Verdin, et al. (2010). 7th European Congress of Chemical Engineering (ECCE7), Prague

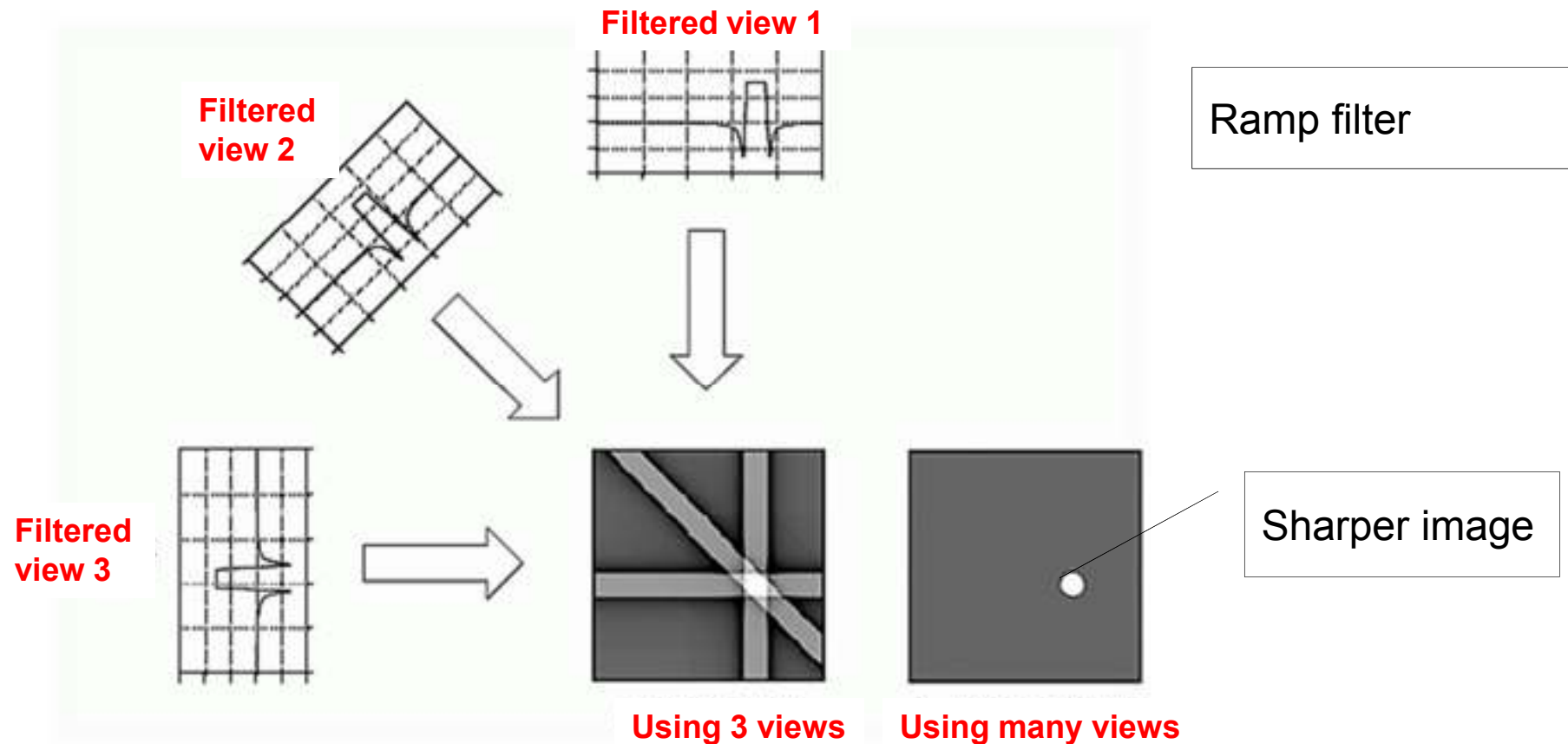
Tomographic image reconstruction

■ Back projection algorithm



Tomographic image reconstruction

- Filtered back projection algorithm



Microtomographic equipments

- **High resolution (a few microns) visualization of small size (a few centimeters) objects**
- **Several commercial microtomographs, e.g. :**
 - High resolution tomograph Skyscan 1172 (Skyscan, BE)

- **X-ray ray source**

Microfocus

Voltage : 20-100 kV

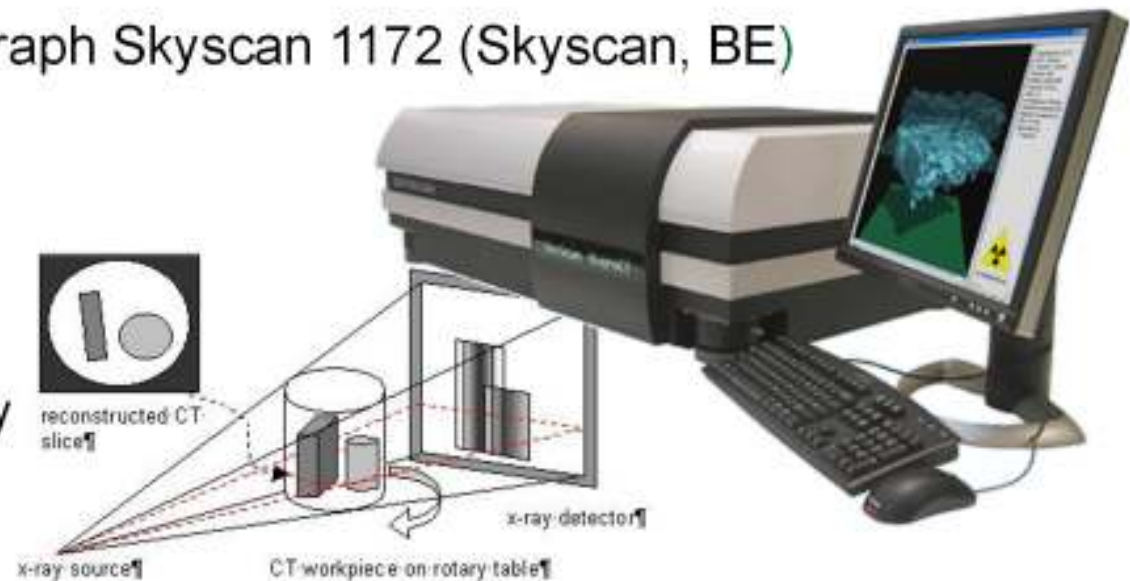
Curent : 0-250 mA

Cone beam geometry

- **Detector**

High resolution CCD camera (10 Mpixels)

Pixel size: from 2 to 35 μm



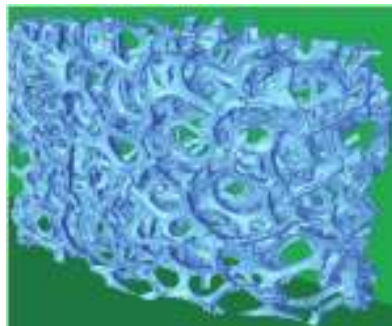
Microtomographic equipments

■ A wide range of spatial resolutions and X-ray energies

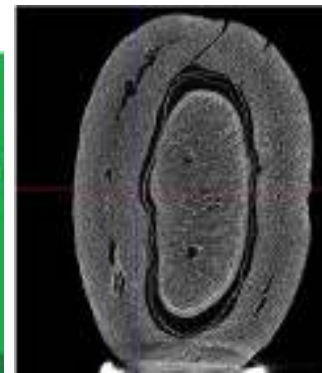
- Microtomograph 2 – 40 μm 30 – 200 keV
- Synchrotron
microtomograph $\cong 0.3 \mu\text{m}$ 6 – 150 keV
- Nanotomograph $\cong 100 \text{ nm}$ 20-80 keV

■ A lot of applications

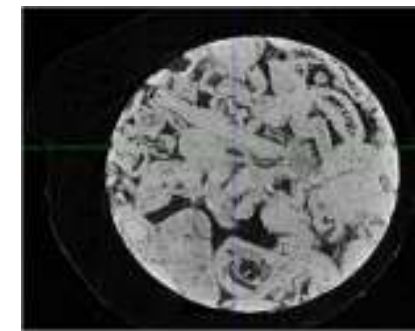
- Material sciences
- Electronics
- Biomedical
- Agrofood
- Geology
-



Nickel foam



Coffee bean



Rock

Skyscan, Kontich, BE
<http://www.skyscan.be>

Macrotomographic equipments

- **Macrotomography or 'industrial' tomography aims at visualizing much larger objects, e.g. unit operations.**

- **No commercial standard equipment**

- An example : the high energy tomographic facility available at the LGC
- Main applications : visualizing hydrodynamics in distillation and gas-liquid absorption columns

e.g. ; Aferka, et al. (2010)

Chemical Engineering Science,
65(1), 511-516.

X-ray ray source

Focal spot : 0.8 mm

Voltage : 30-420 kV

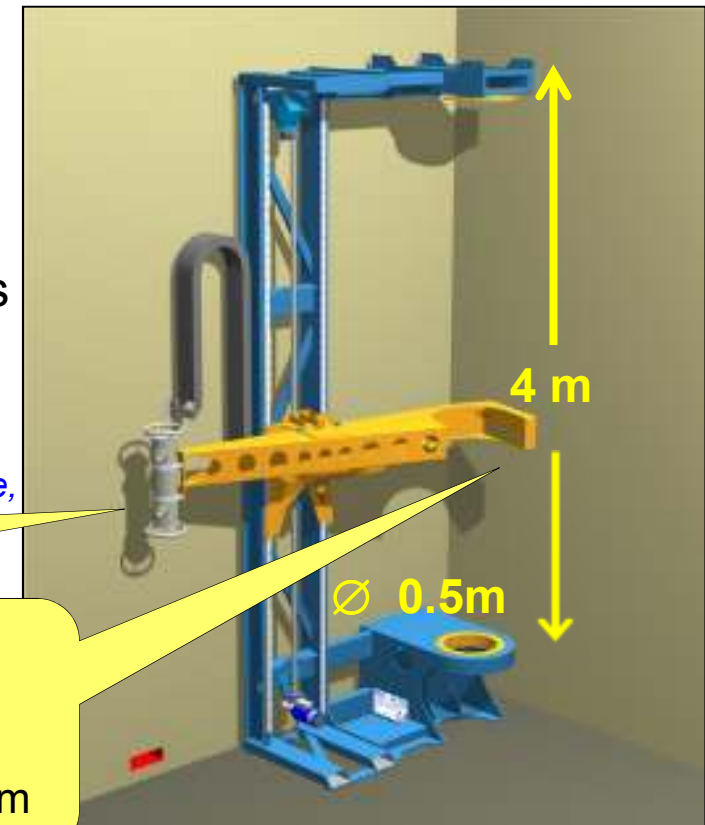
Curent : 2-8 mA

Fan beam geometry (40°)

Linear detector

1280 photodiodes

Pixel size: ~ 0.3 mm

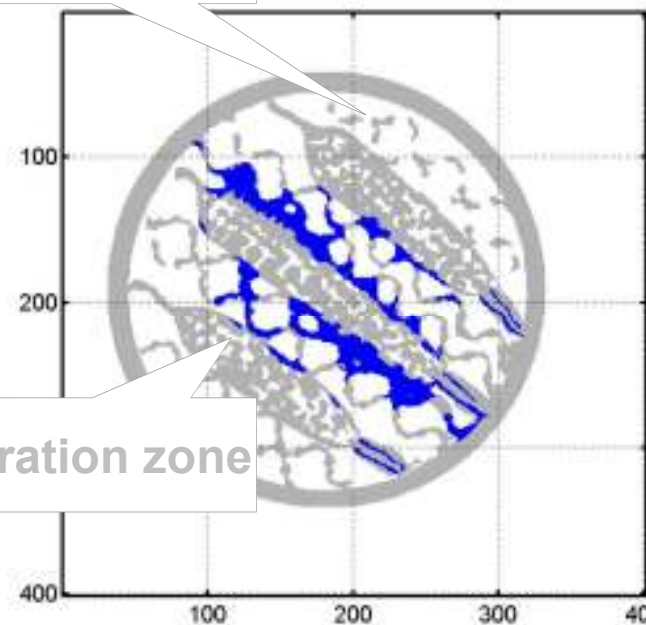


A few macrotomographic images

- Gas-liquid distribution in a catalytic distillation Packing (Katapak™ SP12, Sulzer Chemtech, CH)

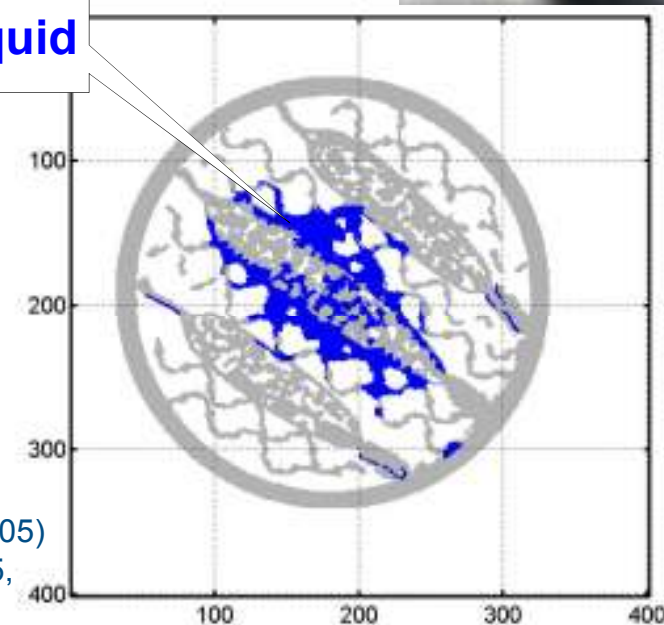


Catalytic zone



Separation zone

Liquid

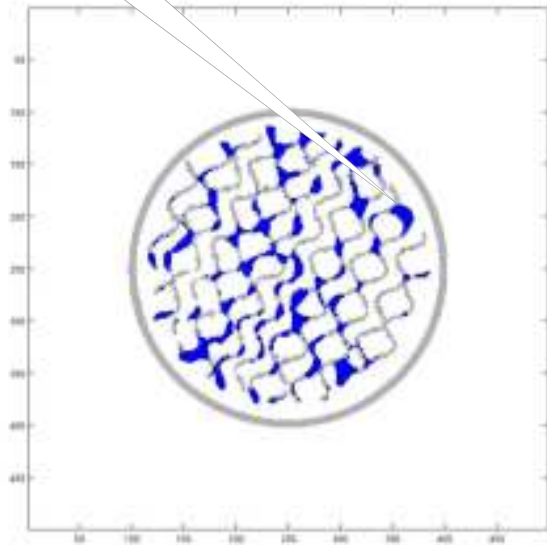


Aferka, et al. (2005)
CHEMCON 2005,
New Delhi

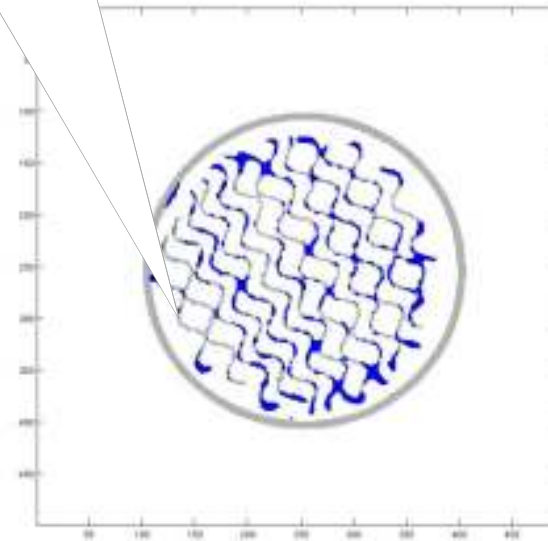
A few macrotomographic images

- Gas-liquid distribution in a structured packing
(Mellapak Plus™ 752Y , Sulzer Chemtech, CH)

Liquid



Corrugated sheet



(Macro)tomographic applications

- Large scale high energies tomographic facilities available, a.o., at:
 - *Helmholtz-Zentrum Dresden-Rossendorf (DE)*
(X-ray)
 - *University of Texas at Austin (USA)*
(X-ray)
 - *IIT Delhi (IN)*
(gamma-ray)
 - *IFP-EN (FR)*
(gamma-ray)
 - *Missouri University of Science and Technology*
(gamma ray)
 - *University of Washington St Louis (USA)*
(gamma ray)

Image analysis: a first example

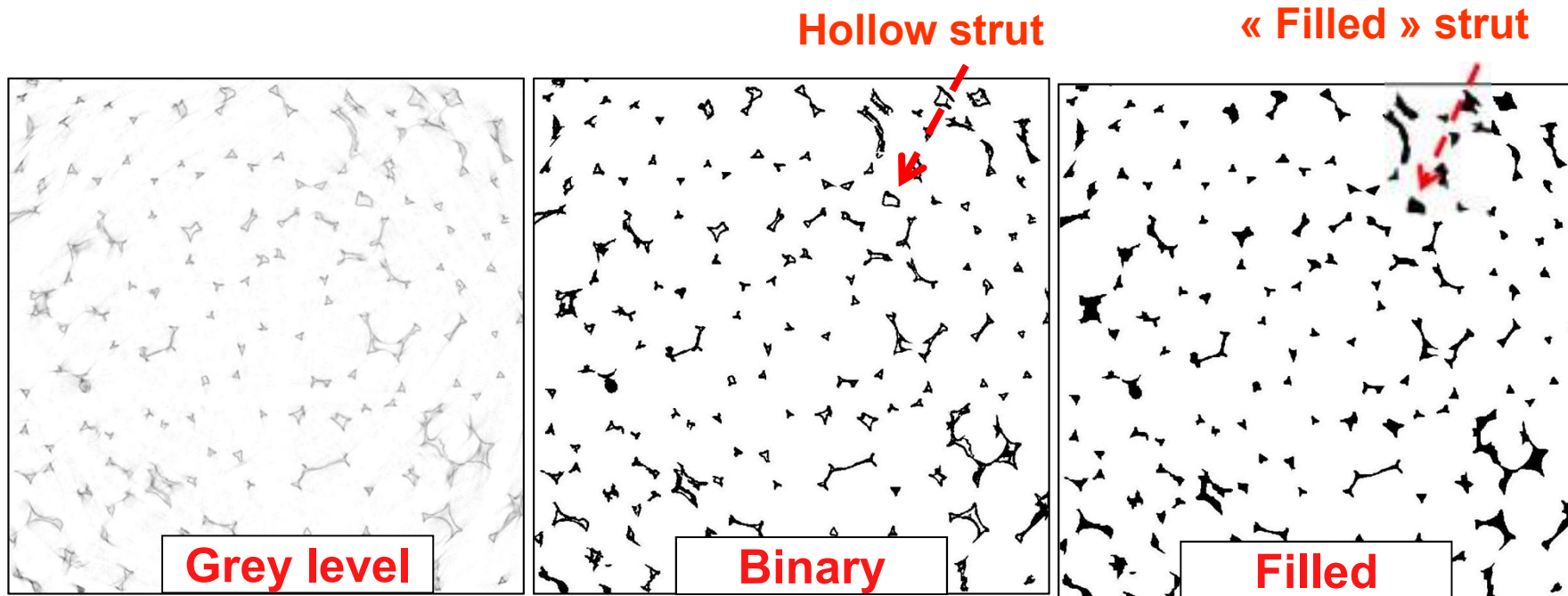
■ Structure visualization of a open cell metallic foam

(RCM-NCX-1116 of RECEMAT® International, NL)

- Material: Nickel – Chromium deposited
onto open-cell polyurethane foam
- Mean pore diameter: 1.4 mm
- Grade number: 11 – 16 ppi
- Specific area: 1000 m²/m²
- Porosity: ~ 95 %
- Density: ~ 0.6 g/cm³



2D images processing



**Thresholding
+
Binarization
+
Erosion**

**Internal
« porosity »
filling**

3D images processing

■ 3D imaging

- ❑ 3D images are obtained by stacking a series of 2D cross sections
- ❑ The grey level image is obtained by adding shadow effects on the binary image

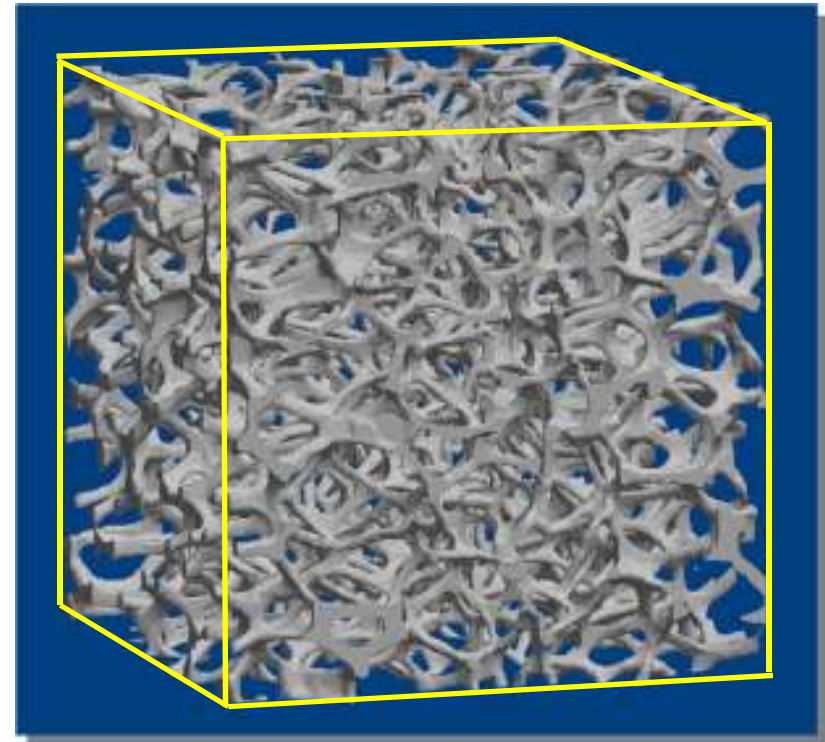


Image analysis: a second example

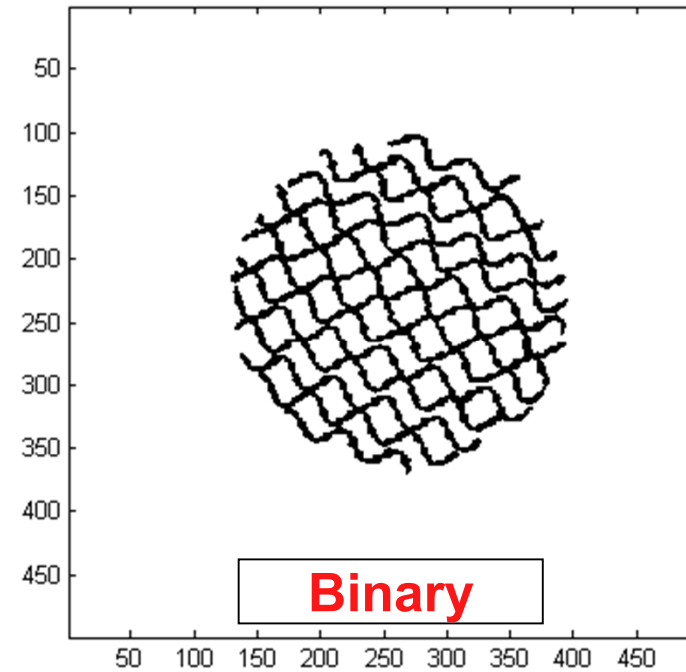
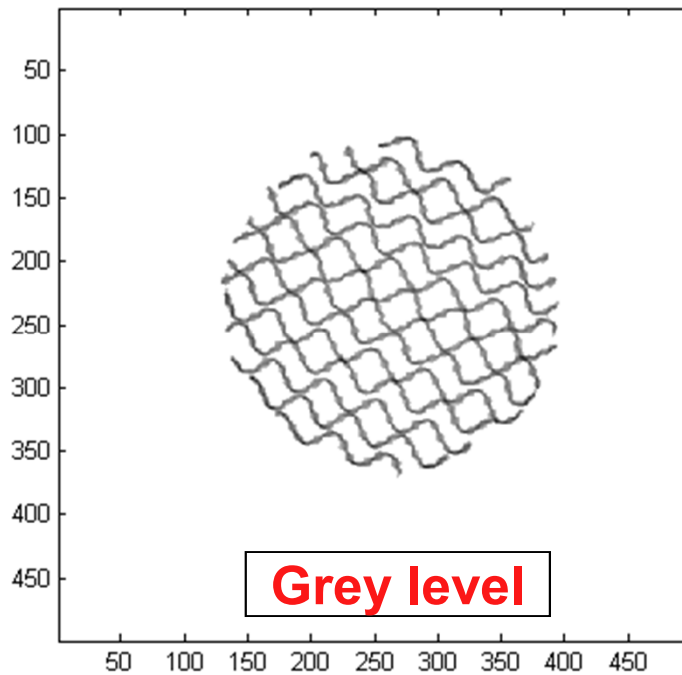
■ Visualization of a structured packing used in absorption columns

(Mellapak Plus™ 752Y of Sulzer Chemtech, CH)

- Material: Embossed, perforated corrugated sheets of stainless steel
- Element height : 0.2 m
- Element diam. : 0.09 m
- Specific surface area : 510 m^{-1}
- Void fraction : 97.5 %



2D images processing



Thresholding
+
Binarization

Two objectives

1. To get a detailed description of porous media geometry
A 3D binary (gas-solid) matrix

↪ X-ray (micro)tomographic imaging

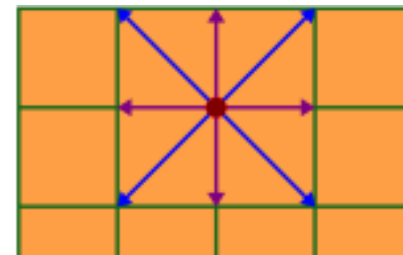
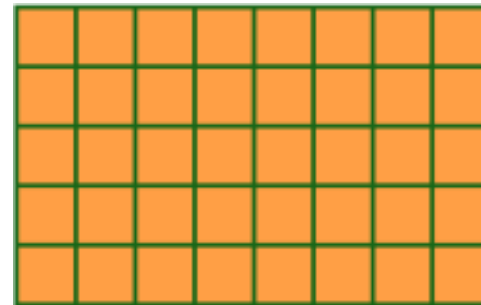
2. To develop a flow simulation model able to deal with the complex geometry of porous media, in terms of boundary conditions

↪ Lattice Boltzmann Methods

Lattice Boltzmann Models

- Fluid flow is represented by the movement of fictitious particles allowed to **move** and **collide** on a lattice.

- A triple discretization
 - Space → Lattice
 - Velocity or momentum
 - Time



Lattice Boltzmann Models

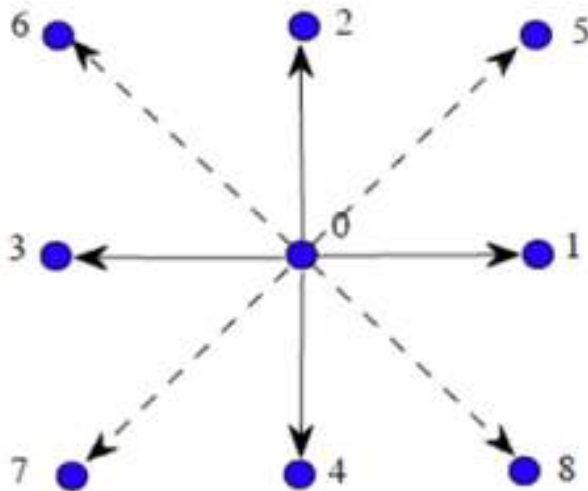
- The particles jump from one lattice node to the next, according to their (discrete) velocity.
 - *This is the **propagation phase**.*
- Then, the particles collide and get a new velocity.
 - *This is the **collision phase**.*
- Rules governing the collisions are designed such that the time-average motion satisfies mass and momentum conservation

Adequate lattices

- A limited number of lattices are able to cope with this constraint, e.g. :

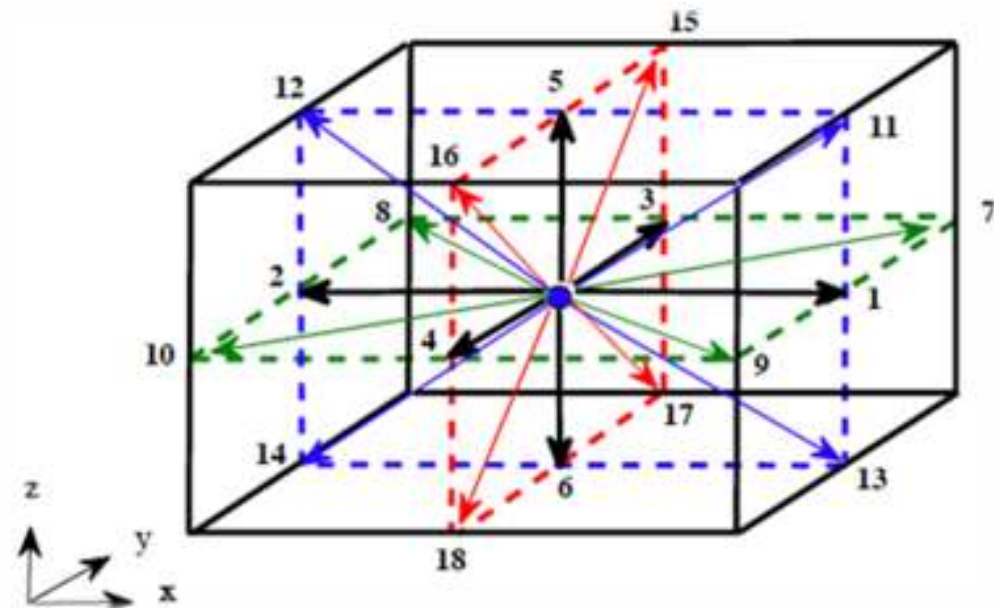
2D : D2Q9

2 dimensions, **9** velocities



3D : D3Q19

3 dimensions, **19** velocities



Kinetic equation of evolution

- Particle occupation within the lattice is described by a series of particle density distribution functions f_k
- The evolution of these functions is described by a discrete form of the Boltzmann equation

$$\tilde{f}_k(x, t) = f_k(x, t) + \Omega_k(x, t), \quad k = 0, \dots, q-1,$$

q : number of directions

- Ω_k is the collision operator : it determines the way velocities are modified after each collision

Collision operator


- Different ways to determine the collision operator.
- They all describe a relaxation process towards equilibrium
 - **SRT**: single relaxation time approximation
Bhatnager-Gross-Kook (BGK) collision model

$$\Omega_k^{BGK}(f) = -\frac{1}{\tau}(f_k - f_k^{(eq)}).$$

- **MRT**: multiple relaxation times approximation

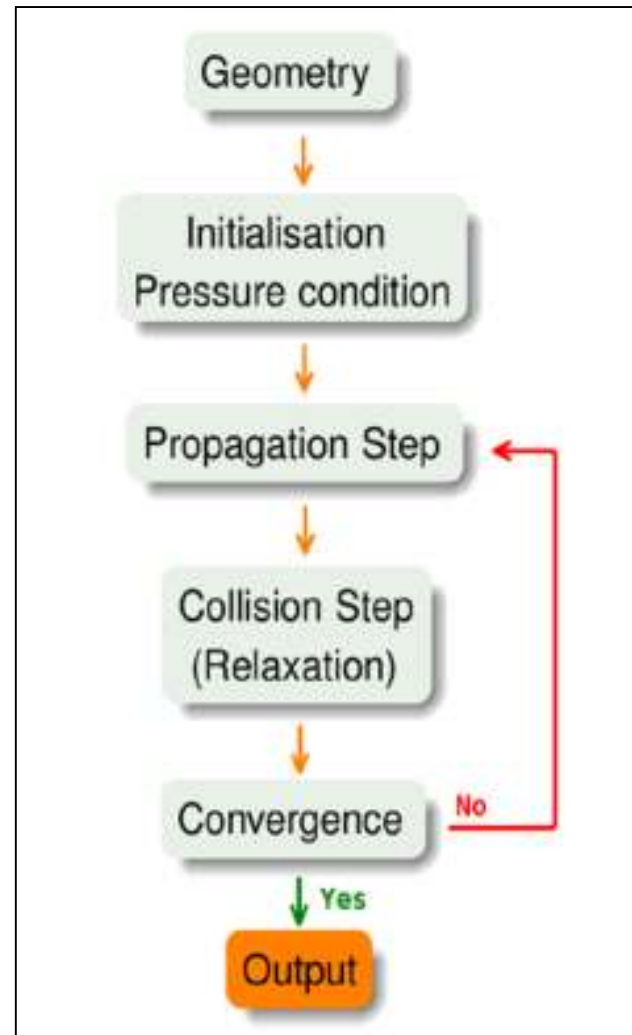
$$\Omega_k^{MRT}(x, t) = -\mathbf{M}^{-1}\hat{\mathbf{R}} \left[\mathbf{m}(x, t) - \mathbf{m}^{(eq)}(x, t) \right].$$

Simulation algorithm

- 3D tomographic binary image (matrix) 

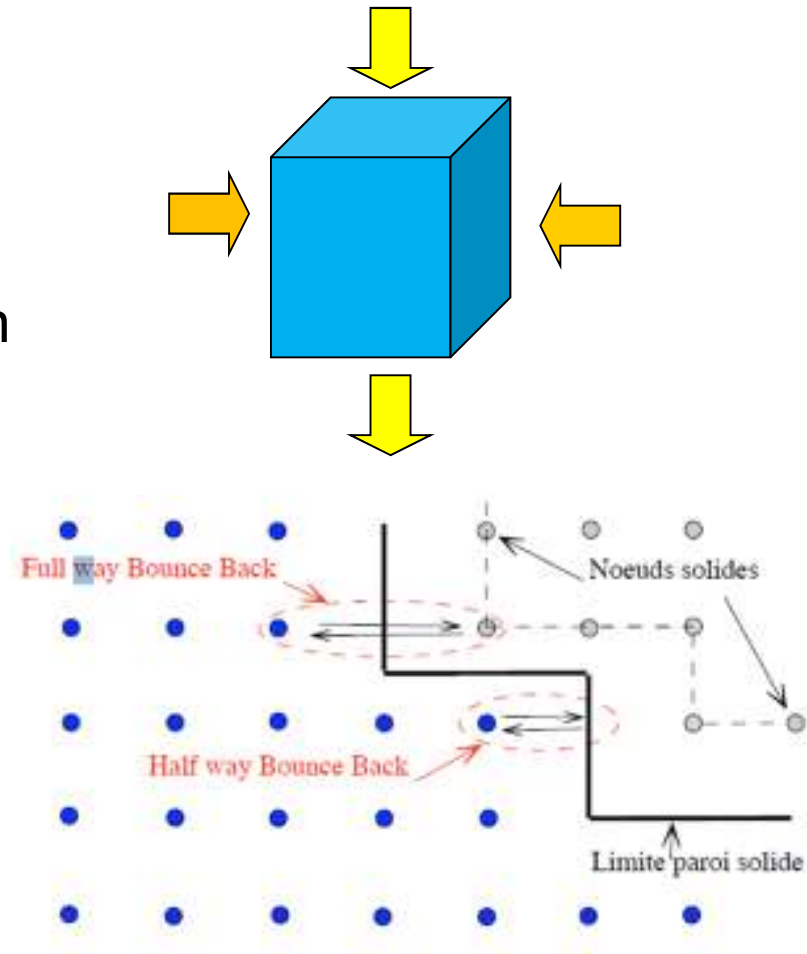
- $f_k(x + e_k \delta_t, t + \delta_t) = \tilde{f}_k(x, t), \quad k = 0, \dots, q-1$

- $\tilde{f}_k(x, t) = f_k(x, t) + \Omega_k(x, t), \quad k = 0, \dots, q-1.$



Boundary conditions

- Pressure gradient between the entrance and exit faces
- Periodic boundary conditions on faces parallel to the flow direction
- Bounce back conditions (no slip) at the fluid-solid interface



Lattice units conversion

- LB simulations are expressed in lattice units
- The comparison with real systems implies to convert LB units into real (physical) units.
- The conversion is rather tricky and different ways have been proposed.
- A guideline : dimensionless variables, e.g., Reynolds number are conserved.

$$Re_{phys.} = \frac{u_{phys} d}{\nu_{phys}} = Re_{LB} = \frac{u_{LB} d_{LB}}{\nu_{LB}}$$

«Physical» world

«LB» world

ν_{LB} represents the kinematic viscosity in the LB domain.

It is fixed according to the value of the relaxation parameter τ

A few examples of flow simulation

- Through a metallic foam
- Through an activated carbon fixed bed
- Through a fiber filter
- Through a structured packing

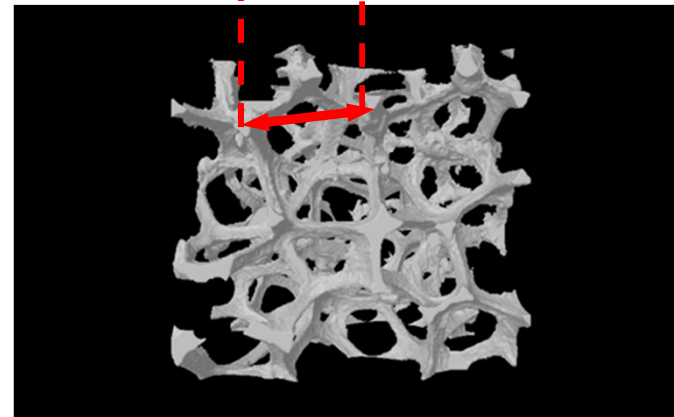
Flow through a metallic foam

■ Open cell metallic foam

(RCM-NCX-1116 of RECEMAT[®] International, NL)

- Material: Nickel – Chromium
- Mean pore diameter: 1.4 mm
- Grade number: 11 – 16 ppi → Pore aperture ~ 1.6 – 2.3 mm
- Specific area: 1000 m²/m²
- Porosity: ~ 95 %
- Density: ~ 0.6 g/cm³

Beugre et al, 2009,
Journal of Computational & Applied Mechanics

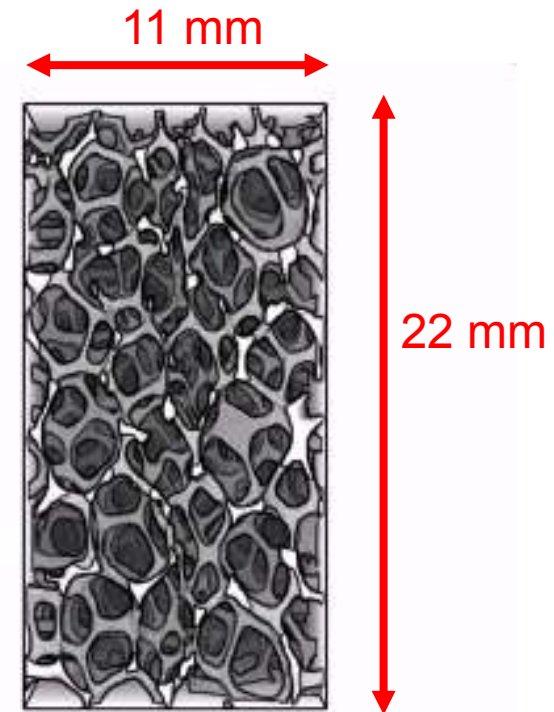
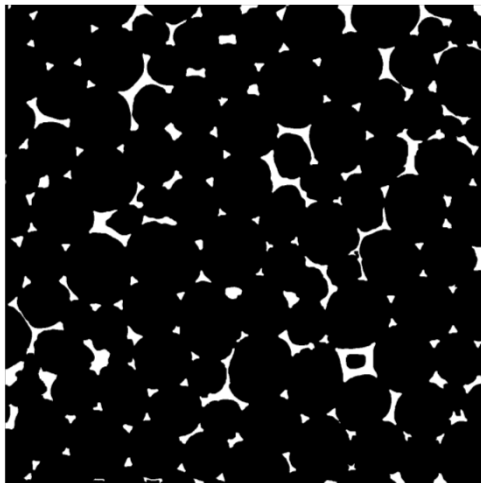


3D imaging of the porous texture

- Sample size

- $176 \times 176 \times 352$ voxels $\sim 11 \times 11 \times 22$ mm
- 1 voxel $\sim 64 \times 64 \times 64 \mu\text{m}^3$
- Porosity: $\sim 95\%$

2D image



By stacking a series
of 2D images

2D Flow visualization

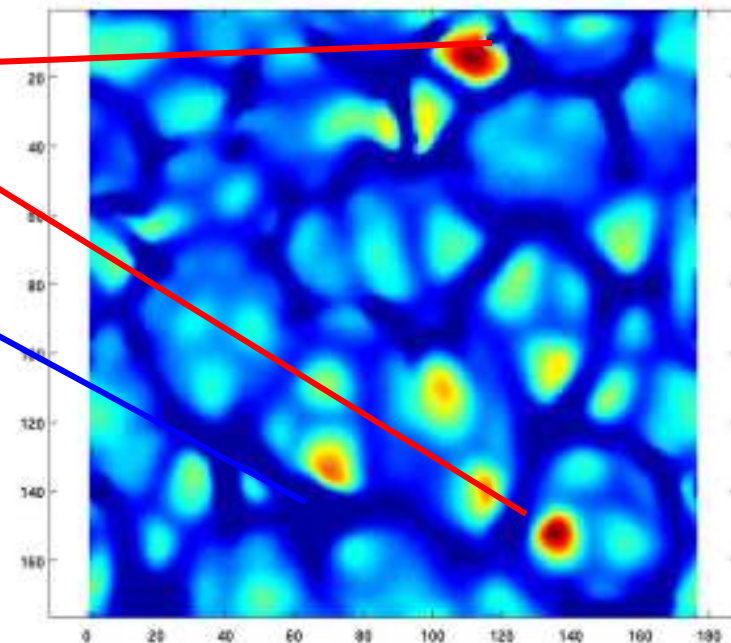
- Velocity distribution in a plane perpendicular to the flow direction.

Moving along the flow direction : 54 images

Preferential flow paths

« Strut »

- Simulation conditions
 - D3Q19
 - SRT (BGK) collision mode
 - B.C. : - Pressure gradient
 - Periodic boundary cond.
 - Full bounce back (no slip)



3D Flow visualization

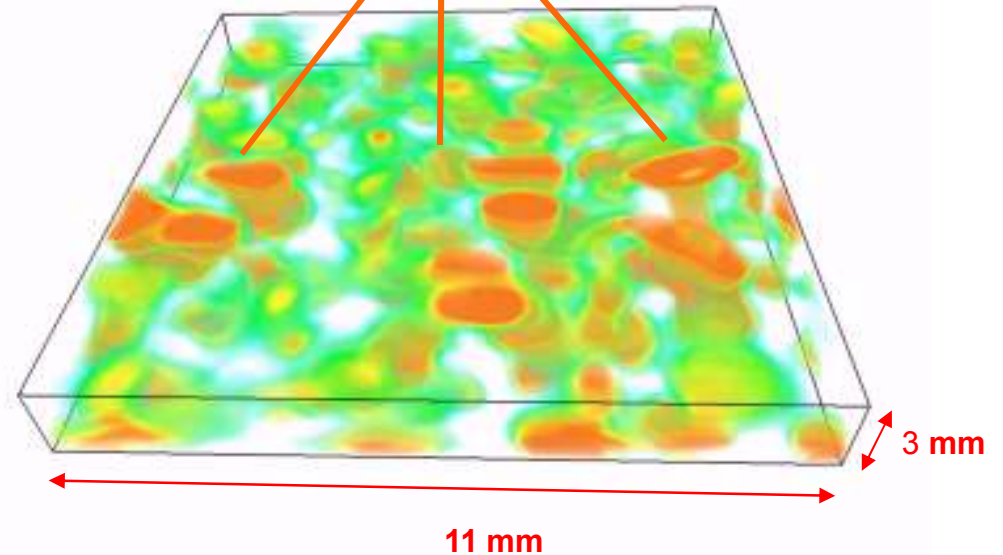
■ Simulation conditions

- **D3Q19**
- **SRT** (BGK) collision mode
- **B.C.** : - Pressure gradient
 - Periodic boundary cond.
 - Full bounce back (no slip)

Preferential flow paths

Solid : white zones

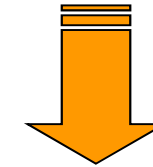
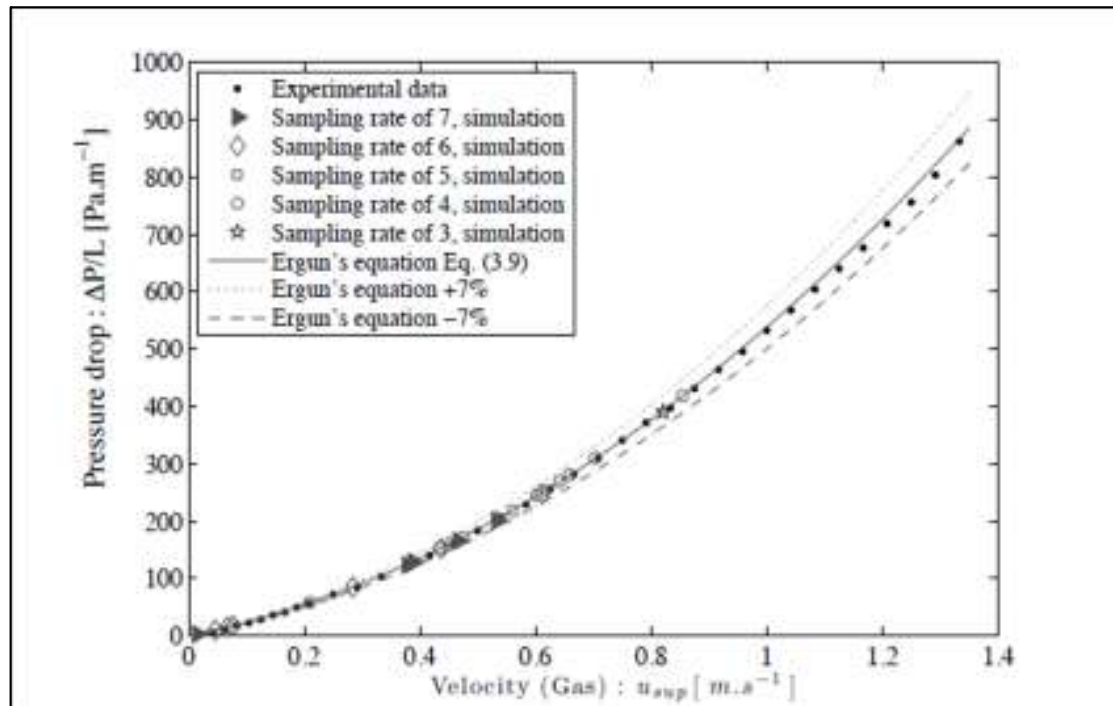
Orange: preferential flow paths



Pressure drop

$$\frac{\Delta P}{L} = \frac{\rho}{dp} \frac{(1 - \varepsilon)}{\varepsilon^3} \left[A \frac{(1 - \varepsilon)}{Re} + B \right] \cdot u_{\text{sup}}^2$$

Ergun equation fitted
on LB numerical
simulations



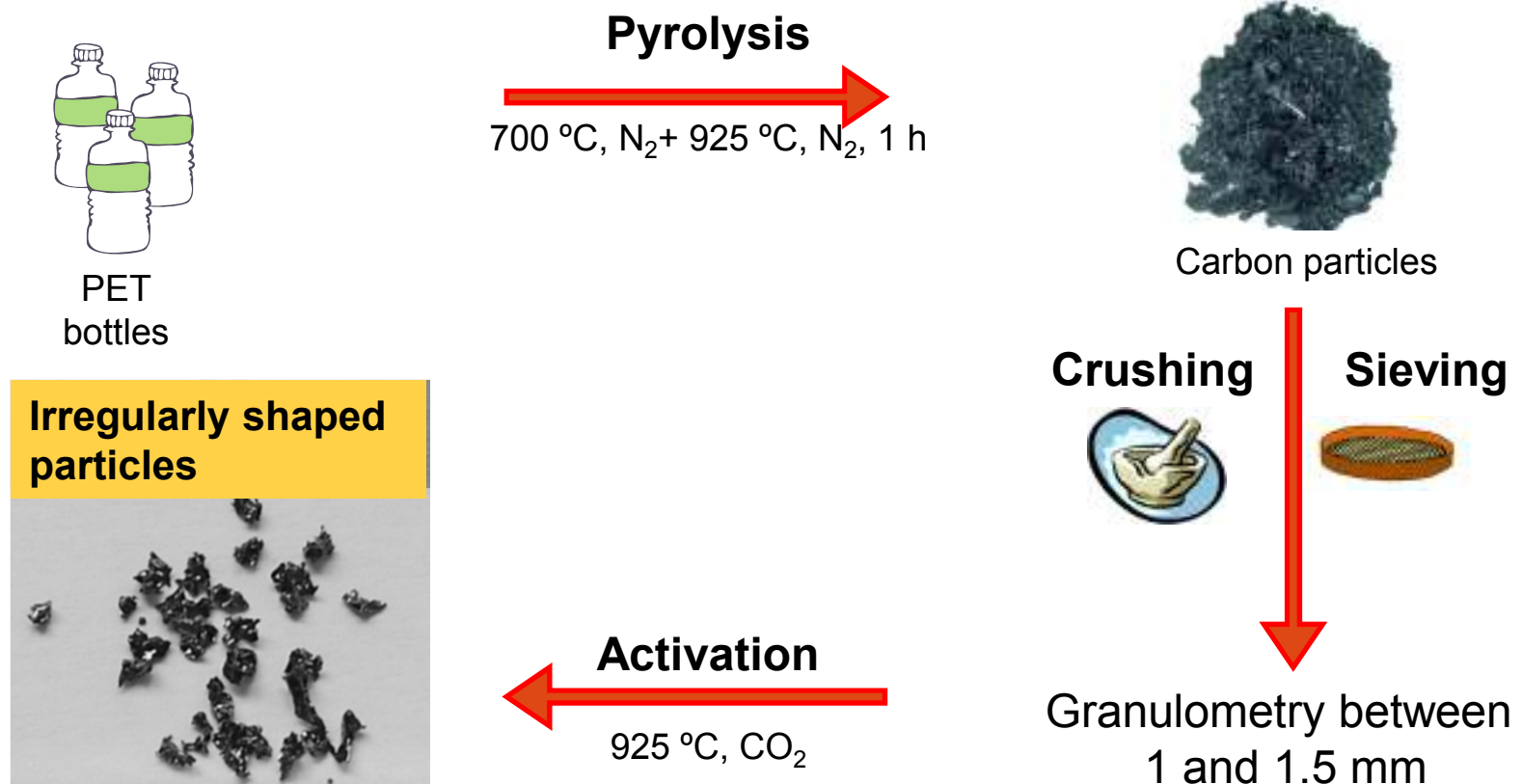
$$A = 77.9$$
$$B = 0.77$$

A few examples of flow simulation

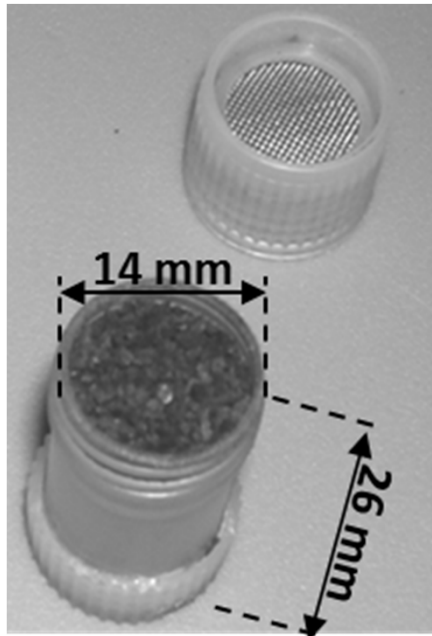
- Through a metallic foam
- Through an activated carbon fixed bed
- Through a fiber filter
- Through a structured packing

Flow through activated carbon

- Activated carbon granules synthesised from recycled PET bottles



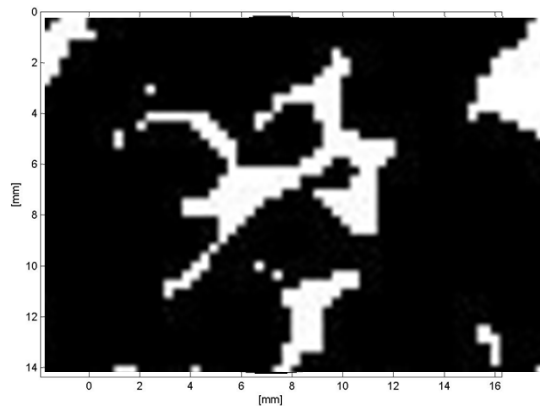
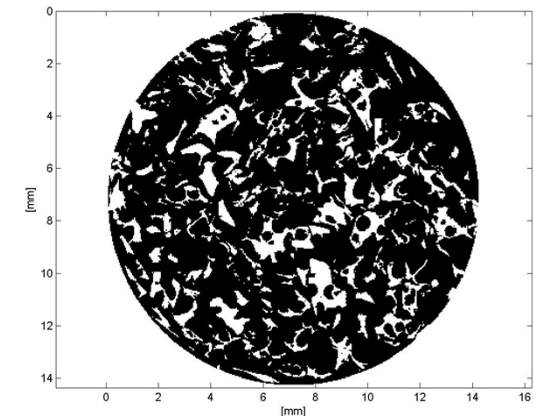
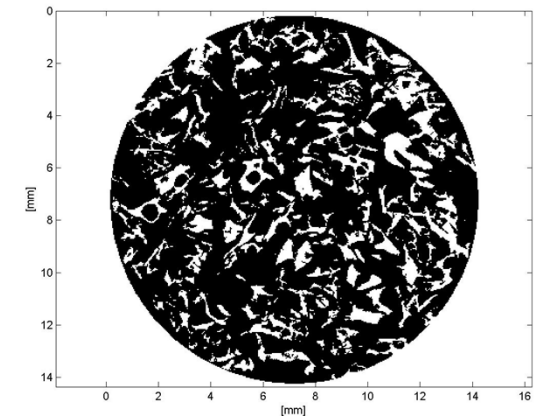
2D imaging of the porous texture



Images of cross-sections at three different depths separated by 2 mm

Bed porosity : $\sim 78\%$

Solid : white zones



- Sample size
 - Diam. : 14 mm
 - 1 pixel $\sim 40 \times 40 \mu\text{m}^2$

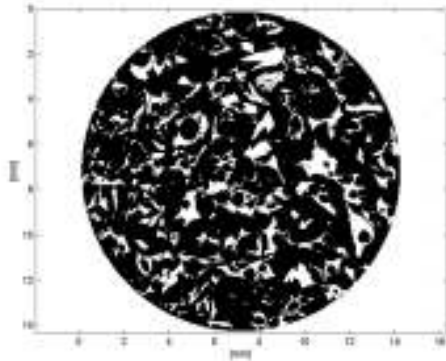
Verdin et al, 2010
FOA10, 10th International Conference on Fundamentals of adsorption

3D imaging of the porous texture

1 voxel $\sim 10 \times 10 \times 10 \mu\text{m}^3$

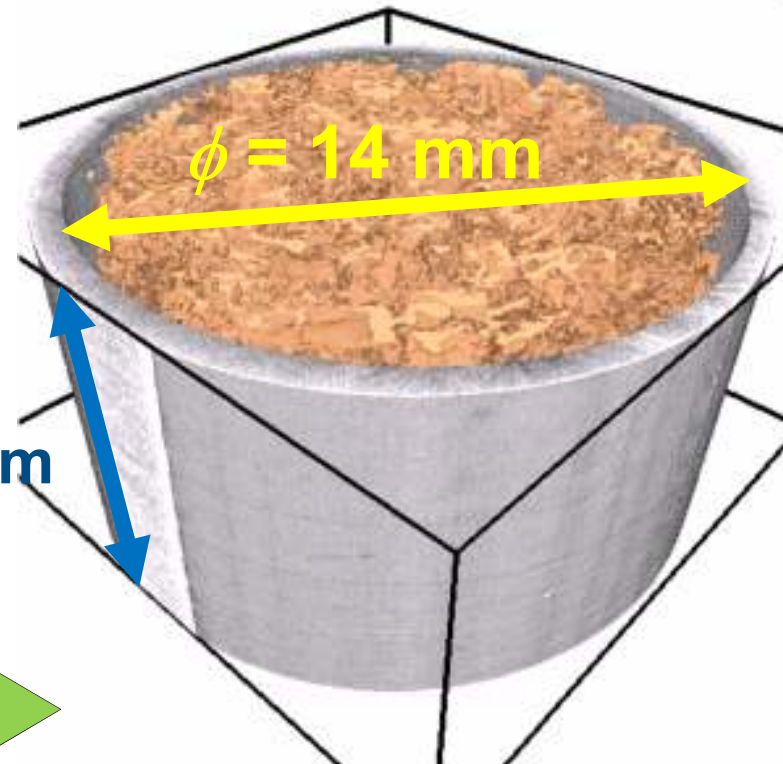
CA: Brown zone

2D Image



$H = 8 \text{ mm}$

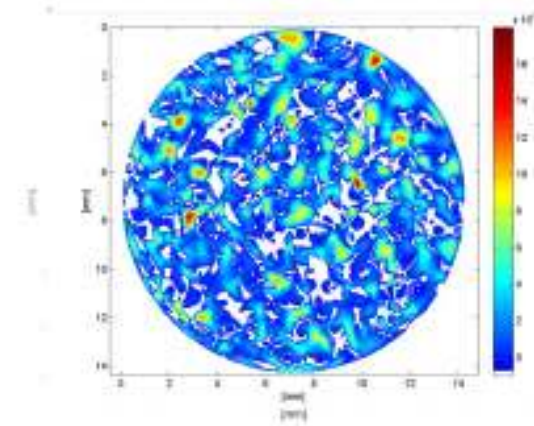
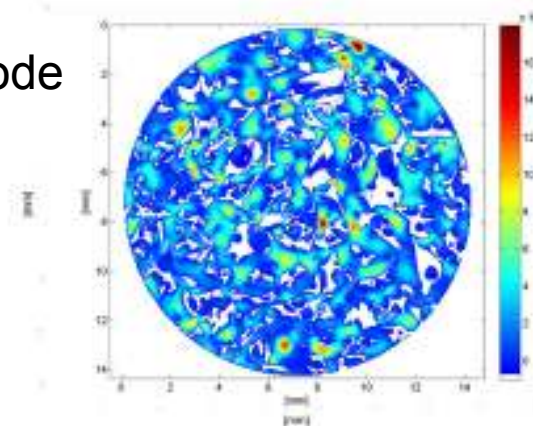
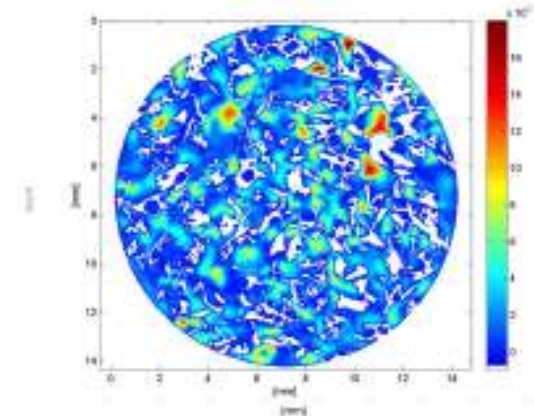
**By stacking a series
of 2D images**



Flow visualization

- Simulation conditions
 - **B.C.** : - Pressure gradient
 - Periodic boundary conditions
 - Full bounce back (no slip)
 - **D3Q19**
 - **SRT (BGK) collision mode**
- Sample size
 - Diam. : 14 mm
 - 1 pixel $\sim 40 \times 40 \mu\text{m}^2$

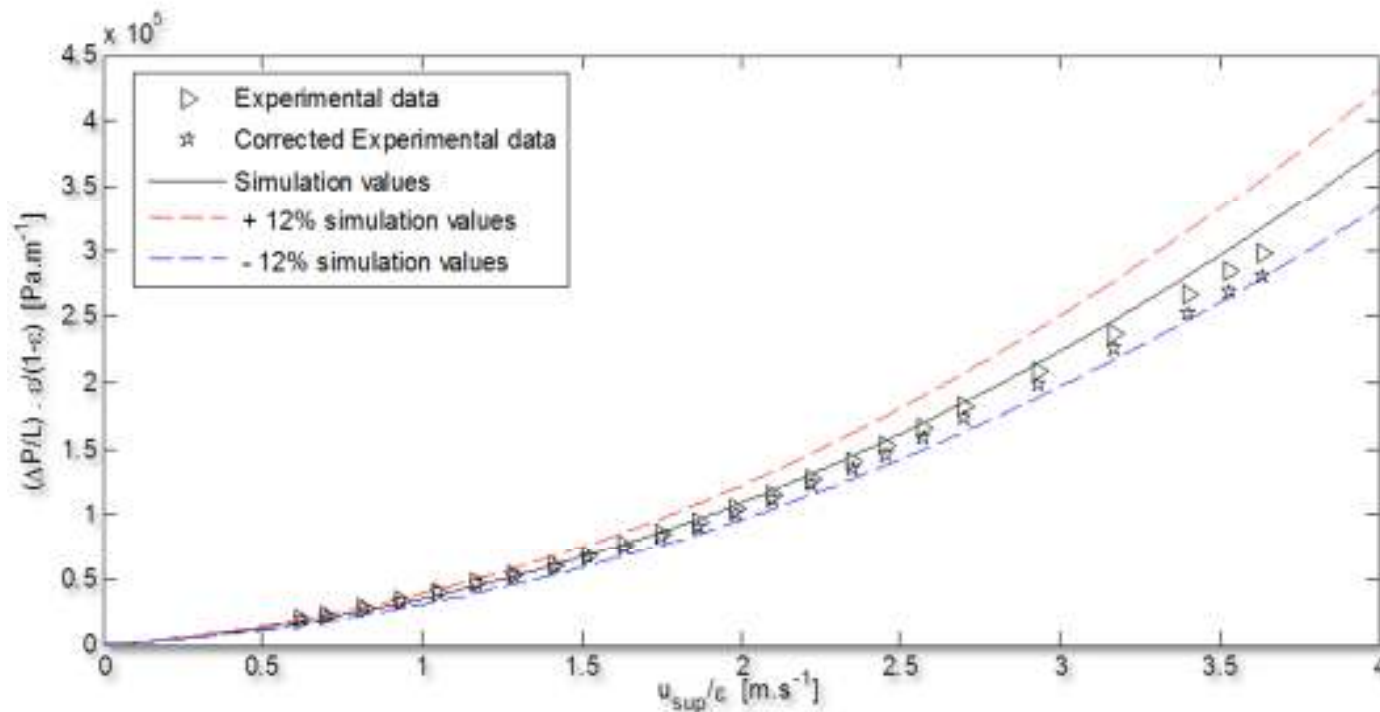
Solid : white zones



Pressure drop

$$\frac{\Delta P}{L} = \frac{\rho}{dp} \frac{(1 - \varepsilon)}{\varepsilon^3} \left[A \frac{(1 - \varepsilon)}{Re} + B \right] \cdot u_{\text{sup}}^2$$

Ergun equation fitted
on LB numerical
simulations



$$A = 4887$$
$$B = 25$$

A few examples of flow simulation

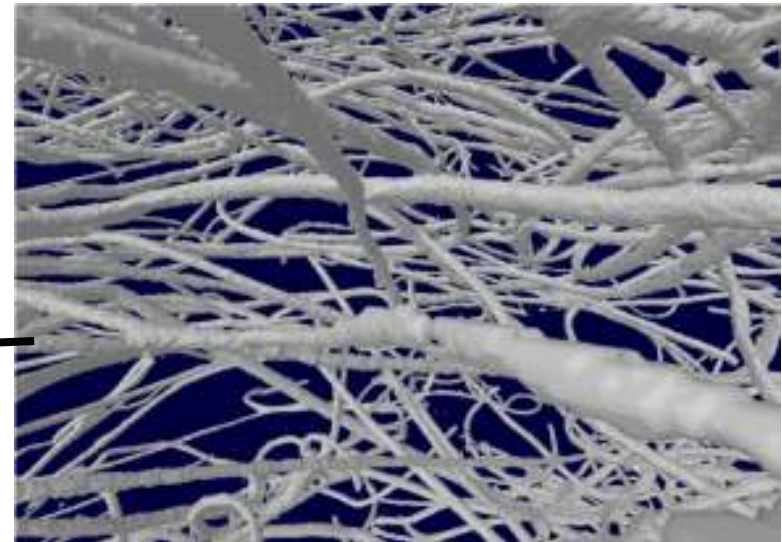
- Through a metallic foam
- Through an activated carbon fixed bed
- Through a fiber filter
- Through a structured packing

Flow through a fibre filter

- High permeability fibre filtration media (PST 290 of ACS Filter Mechelen, BE).
 - ❑ Material: Polyester
 - ❑ Grade number: G3 (EN 779)
 - ❑ Fiber diameter: $\sim 20 \mu\text{m}$
 - ❑ Porosity: $\sim 98 \%$

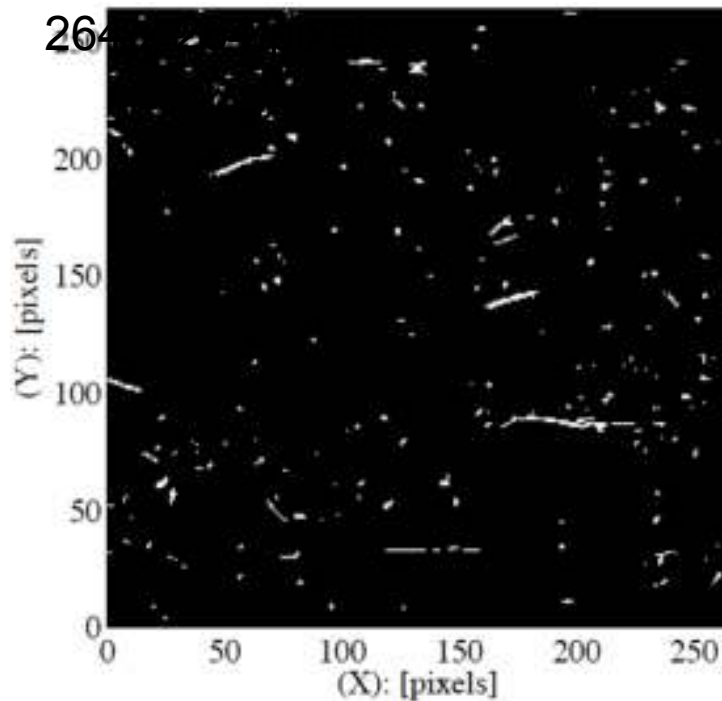


3D tomographic view

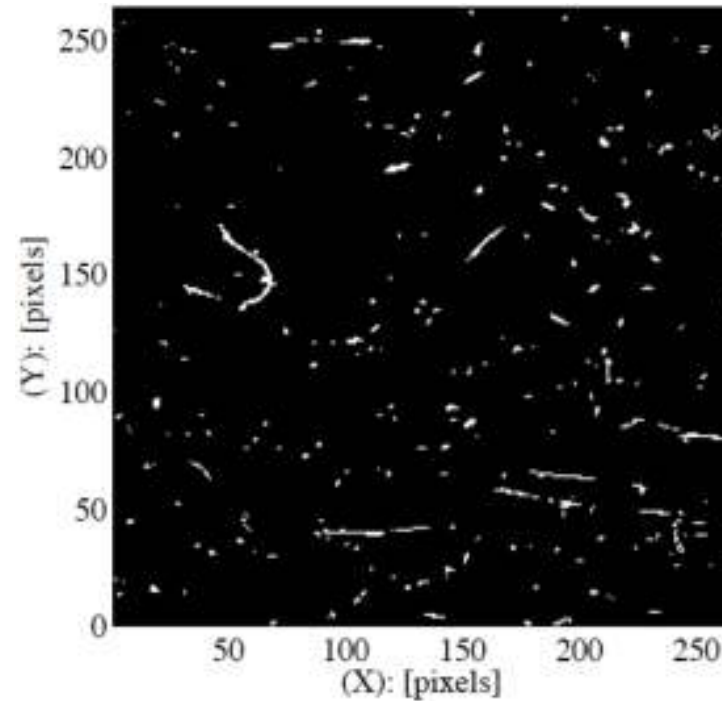


2D imaging of the porous texture

- Sample size
 - 358 x 358 x 200 voxels
 - 1 voxel $\sim 20 \times 20 \times 20 \mu\text{m}^3$



$Z = 20 \mu\text{m}$



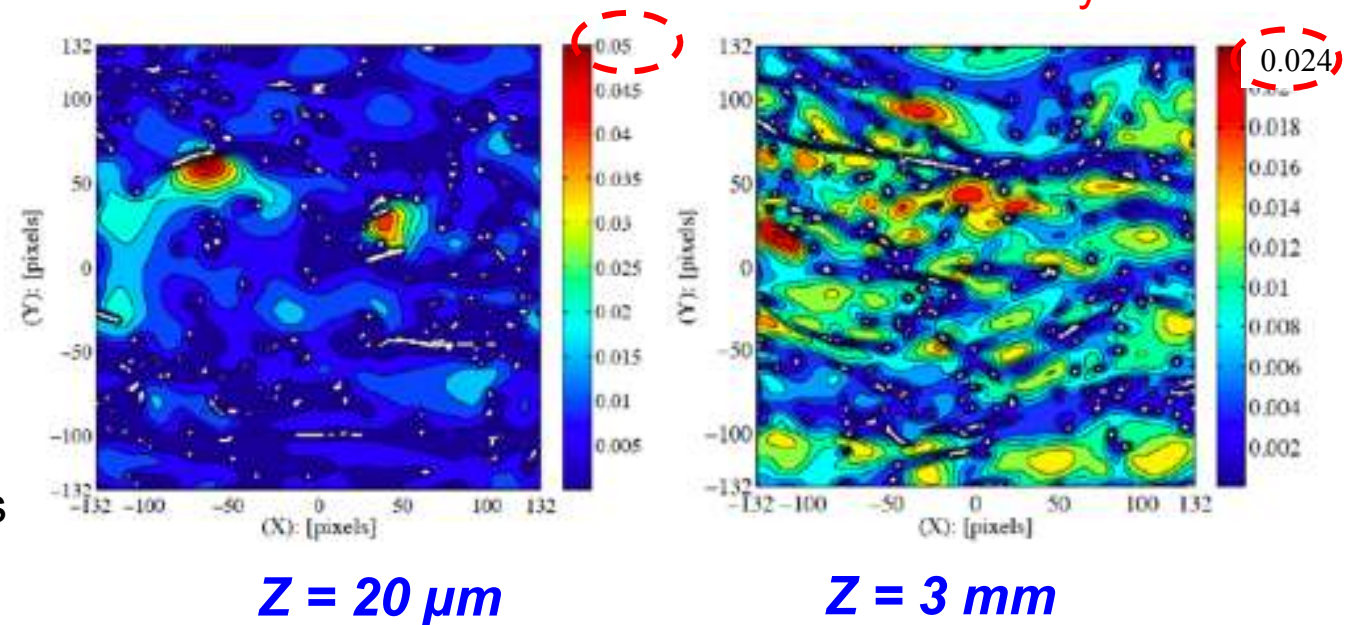
$Z = 1 \text{ mm}$

Flow visualization

- Flow distribution within a series of cross-sections

$Re \sim 1$

- From the inlet of the gas flow ($Z = 20 \mu\text{m}$) towards the outlet ($\rightarrow Z = 3 \text{ mm}$) of the filter

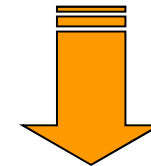
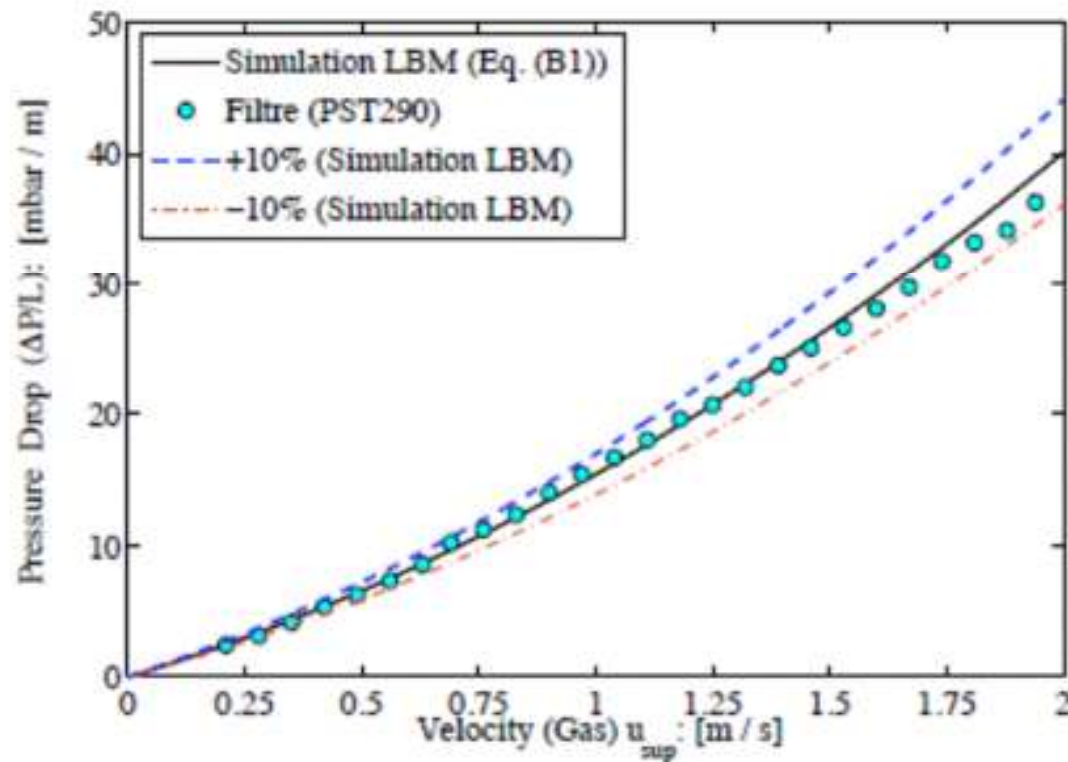


- Spreading of the gas flow within the filter cross-section
- Decrease of the maximum velocity

Pressure drop

$$\frac{\Delta P}{L} = \frac{\rho}{dp} \frac{(1 - \varepsilon)}{\varepsilon^3} \left[A \frac{(1 - \varepsilon)}{Re} + B \right] \cdot u_{sup}^2$$

Ergun equation fitted
on LB numerical
simulations



$$A = 46.2$$
$$B = 0.35$$

Permeability of porous media

- Computed permeability

$$K = \frac{\varepsilon^3 d_p^2}{A (1 - \varepsilon)^2}$$

	ε (-)	d_p (m)	A (-)	K (m ²)	
Metallic foam	0.95	1.4 10 ⁻³	77.9	5.38 10⁻⁸	Max. permeability
Activated carbon	0.78	1.25 10 ⁻³	4887	3.07 10⁻⁹	Min. permeability
Fibre filter	0.98	6.5 10 ⁻⁴	46.2	2.02 10⁻⁸	Less permeable than foam

A few examples of flow simulation

- Through a metallic foam
- Through an activated carbon fixed bed
- Through a fiber filter
- Through a structured packing

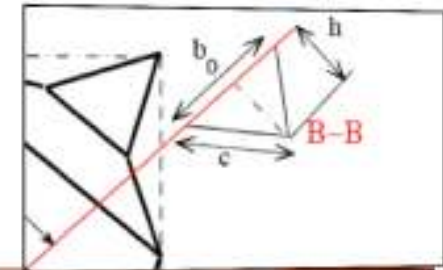
Flow through a structured packing

- **Flow between two corrugated sheets of a structured packing**

- **Structured packing**

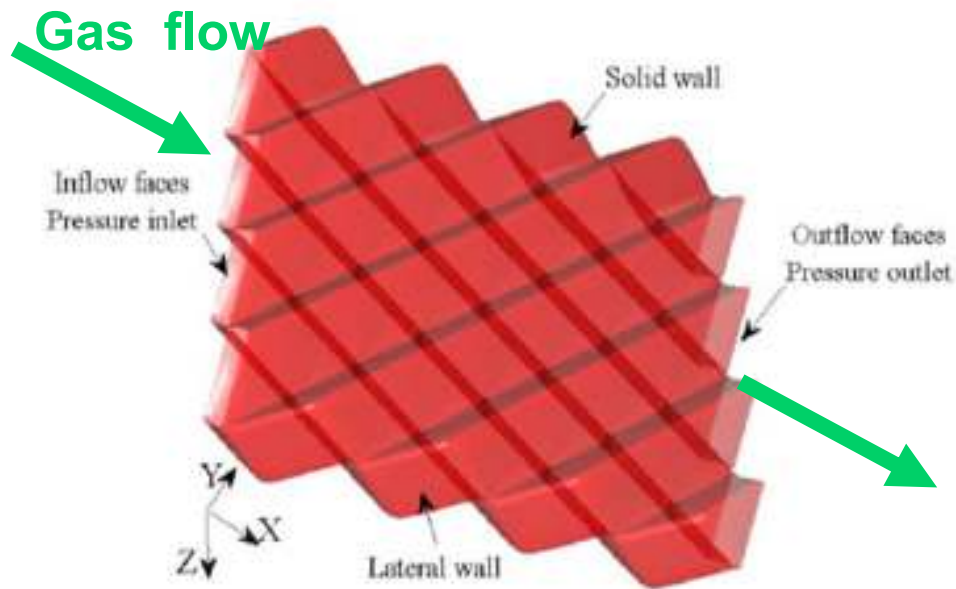
Sulzer Plastic Mellapak™ 250 Y, Sulzer ChemTech, CH

- Material: Polyethylene
- Hydraulic diam. : 2 cm
- Corrugation geometry
 - Triangular channel base : $b_0 = 26\text{mm}$
 - Triangular channel height : $h = 13\text{mm}$
- Specific area: $250\text{ m}^2/\text{m}^2$
- Porosity: $\sim 85\%$

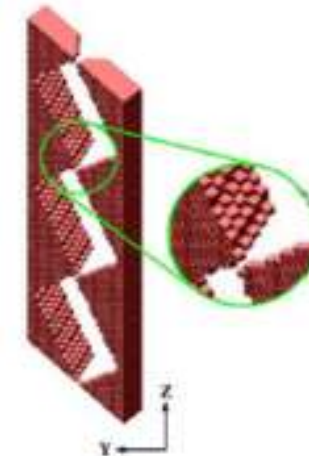


3D geometry modelling

- Two corrugated sheets of Mellapak 250 Y containing four triangular channels each (without perforation)

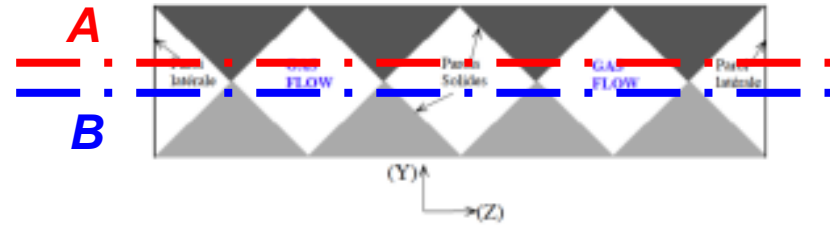
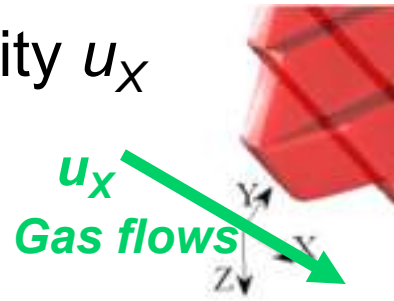


Details

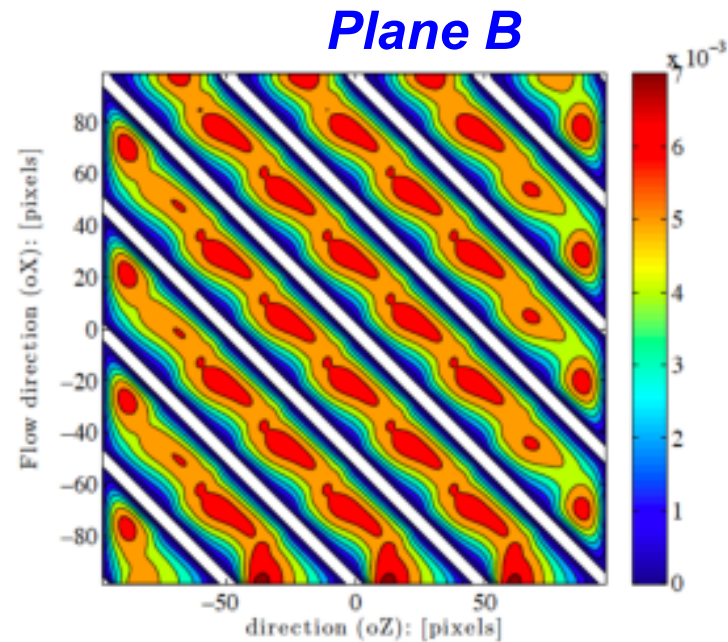
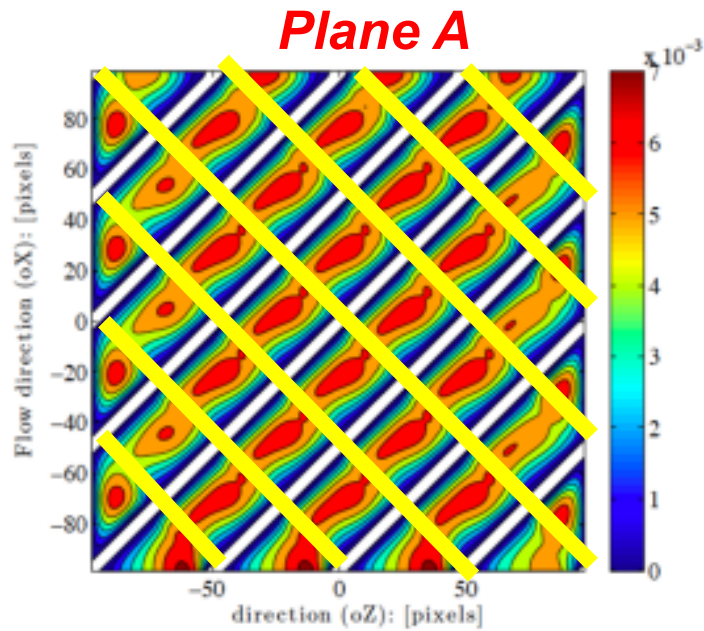


Flow visualization at low Reynolds

- Axial velocity u_x



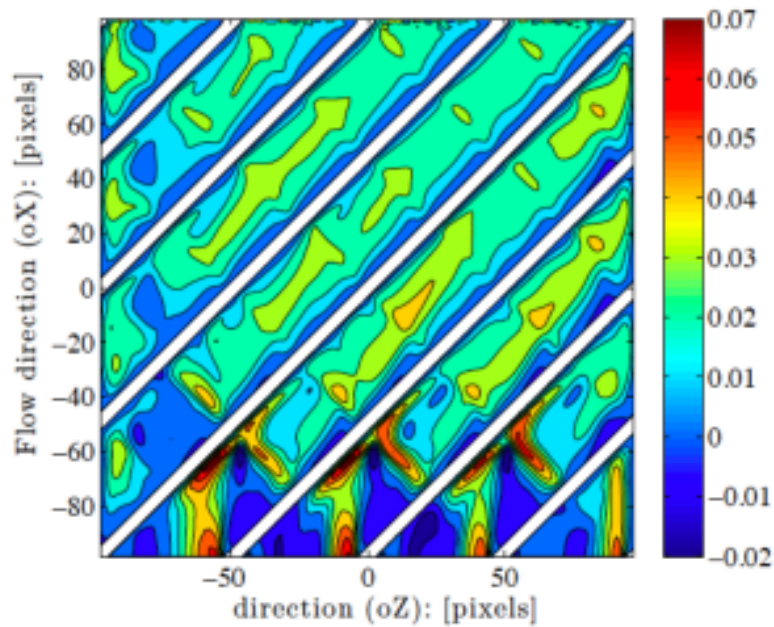
$Re = 20$ (SRT-LBE)



Flow visualization

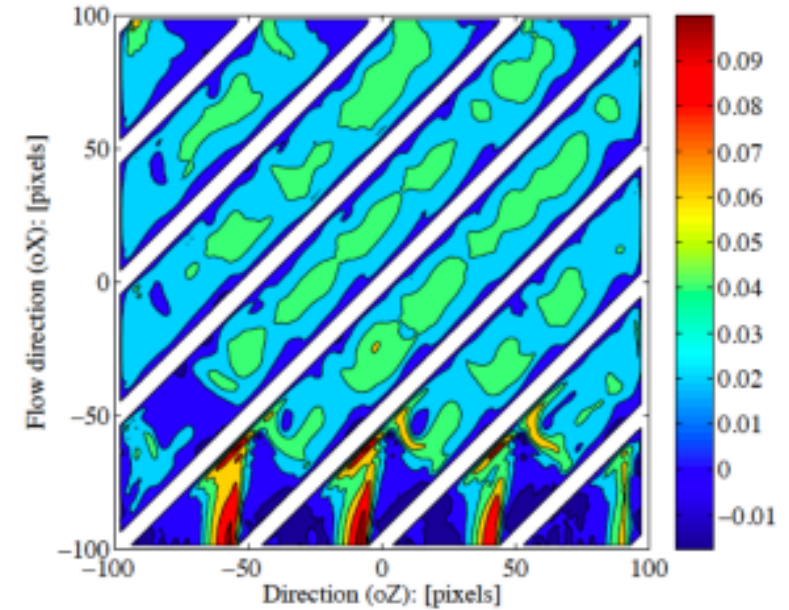
At higher Reynolds numbers : $Re > 100$

Plane A



$Re = 310$

Plane A



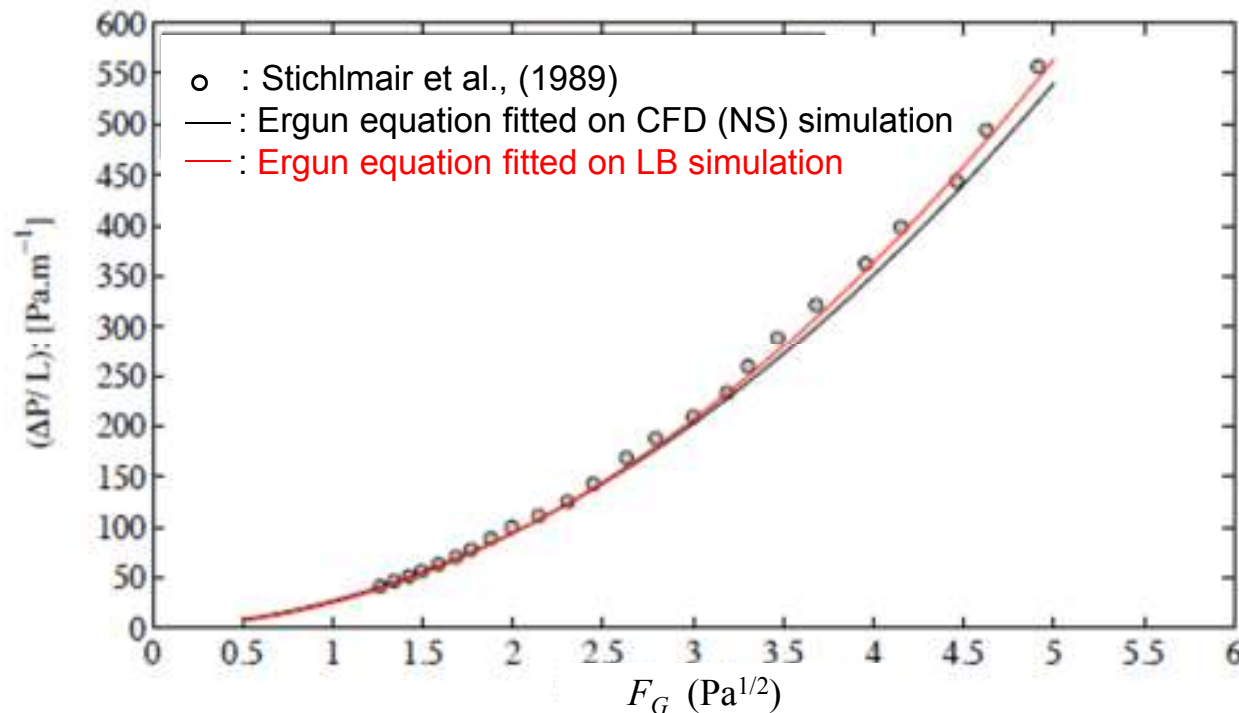
$Re = 5040$

MRT-LBE

Pressure drop

$$\frac{\Delta P}{L} = \frac{\rho}{dp} \frac{(1 - \varepsilon)}{\varepsilon^3} \left[A \frac{(1 - \varepsilon)}{Re} + B \right] \cdot u_{\text{sup}}^2$$

Ergun equation fitted
on LB and CFD
numerical simulations



LB **A = 89**
sim. **B = 0.54**

Stichl. **A = 80**
et al. **B = 0.37**

Beugre, D., et al.. (2011),
Chem. Eng. Sci.

Two objectives

1. To get a detailed description of porous media geometry
A 3D binary (gas-solid) matrix

↪ X-ray (micro)tomographic imaging

2. To develop a flow simulation model able to deal with the complex geometry of porous media, in terms of boundary conditions
A simulation model validated for various complex porous media

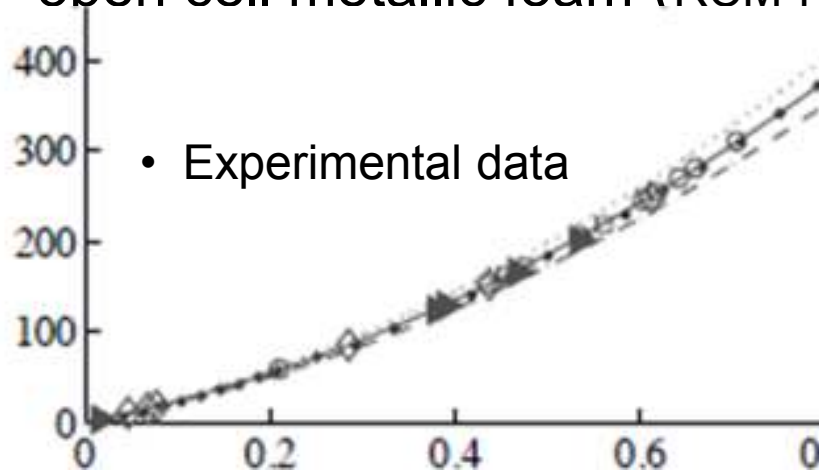
↪ **A few challenges to take up**

CPU and memory requirements

- The numerical resolution of LB equation is simple and straightforward.
- It however requires large execution time and massive memory access
- Two main solutions to reduce these drawbacks:
 - To reduce the sample size: **image undersampling**
 - To **parallelize** the computing architecture

Image undersampling

- Undersampling consists in sampling a reduced number of voxels from the original image
 - *The sampling rate is defined as the ratio between the total and reduced numbers of voxels, along one direction in the space*
- This modification of the gas-solid interface does not affect significantly the calculated values of pressure drop through an open cell metallic foam (RCM-NCX-1116)



- ▶ Sampling rate of 7 voxel size : **224 μm**
- ◇ Sampling rate of 6
- Sampling rate of 5
- Sampling rate of 4
- ☆ Sampling rate of 3 voxel size : **96 μm**

The pore diameter is around 1.4 mm

Image undersampling

- *This small modification can probably be explained by the absence of modification of the porous media morphology, e.g., its porosity* **× 32**

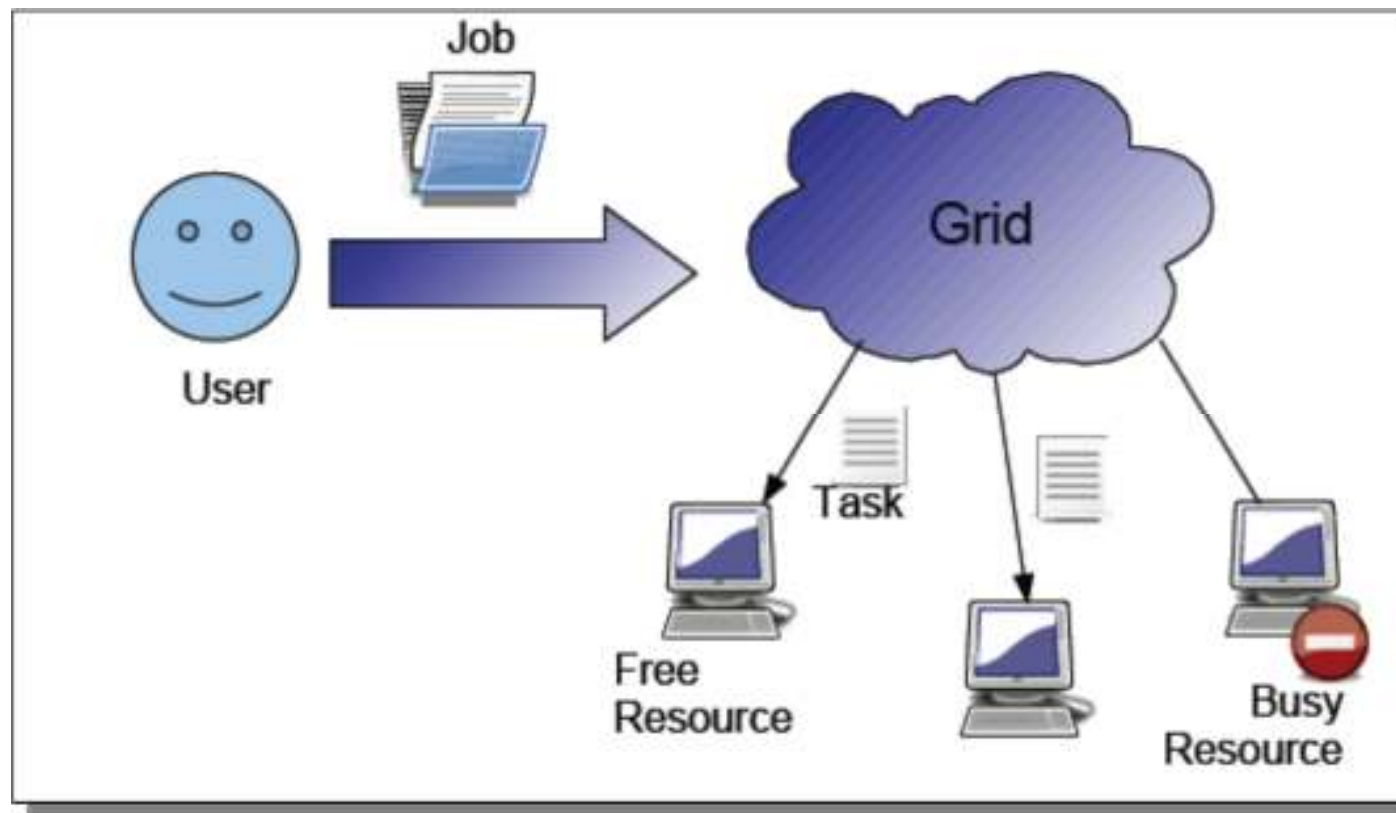
Number of voxels	~ 2 10⁵						6.4 10⁶
	58 ³	67 ³	80 ³	100 ³	134 ³	200 ³	400 ³
Sampling rate	7	6	5	4	3	2	1
Resolution (μm)	224	192	160	128	96	64	32
Porosity (%)	93.38	93.41	93.41	93.43	93.42	93.43	93.45

Parallel computing

- An LB simulation can hardly be executed on a single PC:
 - 10 gigabytes of memory and more than 10000 time steps are typically required
 - On a computer with a Pentium IV 3GHz processor, equipped with enough memory, it would take around 2 years to complete the simulation's execution.
- The LB simulation can easily be executed in a distributed way
 - Using a supercomputer equipped with a great number of identical processors and a large amount of memory
 - Using a cluster of numerous desktop computers interconnected on a grid

Parallel computing

- LB codes are easily adapted to parallelization or task sharing among several computers on a grid



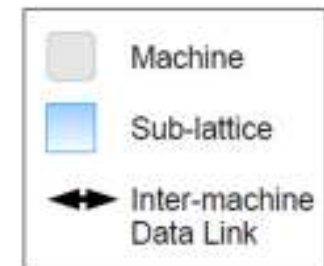
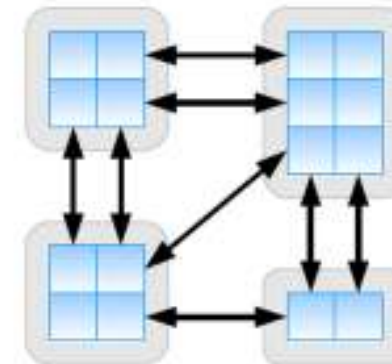
Parallel computing

- Lattice is decomposed into sub-lattices
- Sub-lattices can be deployed on different machines
- Data link between sub-lattices



Illustration:

- a 2D lattice decomposed into
- 16 sub-lattices



Main challenges

■ Distribution on a single (multiprocessors) machine

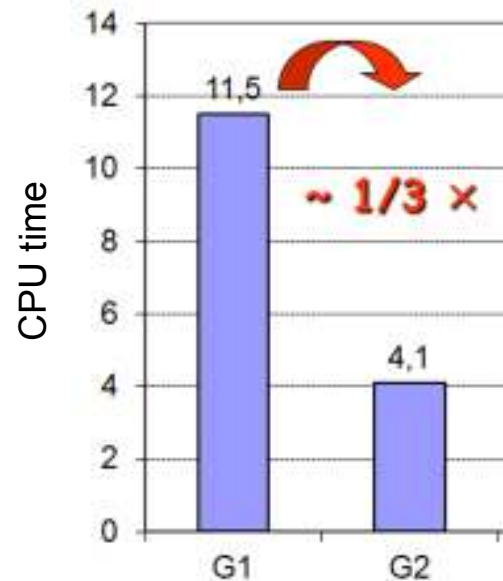
□ G1: 2 Xeon dual-core

(4 cores), 2.0 Ghz, 4 Mo cache, each with 16 GB of RAM

□ G2: 6 Xeon dual-core

(12 cores), 2.0 Ghz, 4 Mo cache, each with 16 GB of RAM

× 3



Speedup :

$$\text{CPU time}_1 / \text{CPU time}_2 = 11,5 / 4,1 \approx 2,80$$

Efficiency :

$$\text{Speedup} / \text{Processor number} = 2,8 / 3 = 0,93 \sim 1$$

Efficiency almost equal to 1

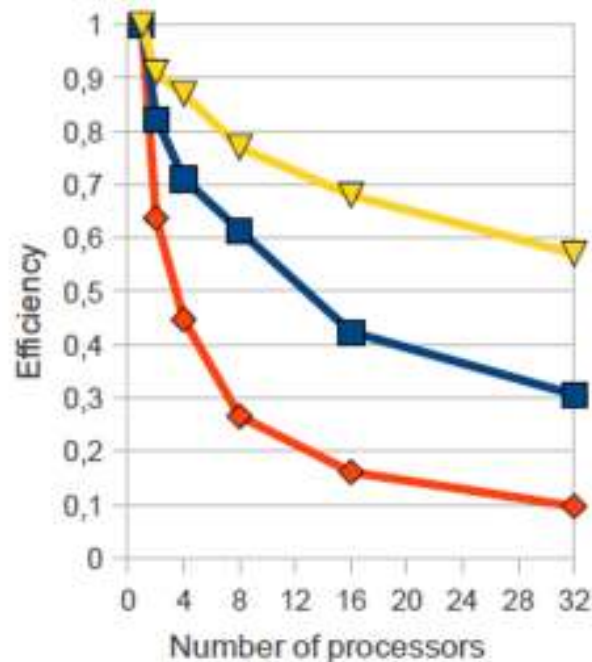
No time lost in data exchange

Beugre D. et al., 2009,
Journal of Computationam &
Applied Mathematics

Marchot et a., 2007,
ECCE6; Copenhagen

Main challenges

- Cluster of Pentium Celeron 2.4 Ghz processors interconnected by a switched-ethernet 100 Mbits network
 - Celeron 2.40 Ghz, 128 ko cache, each with 512 MB of RAM



200 times steps on :

- ▽ 64-MRT : 64×64×64 sites lattice and a MRT-LB model
- 64-SRT : 64×64×64 sites lattice and a SRT-LB model
- ▲ 32-SRT : 32×32×32 sites lattice and a SRT-LB model

Efficiency markedly smaller
Depends on sample size et
model complexity

Dethier G.. et al., 2011
Procedia Computer Science

Possible improvements

■ Load balancing

The efficiency of parallel computing in heterogeneous distributed systems (computers of different computational power and memory size), depends strongly on proper load balancing among the computers.

- *A priori : static load balancing*
depends on processor
parameters, memory available,...
- *During computation : dynamic load balancing*
depends ...

LaboGrid
a code
developed
at
LGC

■ Computing architecture

- *Locality* : data associated to addresses that are close in memory address space should be accessed at close moments in time

■ Network latency and bandwidth

Two phase flow LB models

- LB models provide are promising tools to describe complex fluid flow in porous media.
- Existing models are based on SRT (BGK) equations applied to the two fluids:

$$\widetilde{f_k^\alpha(x, t)} = f_k^\alpha(x, t) + \Omega_k^\alpha(x, t) \quad \text{Phase } \alpha$$

$$\widetilde{f_k^\beta(x, t)} = f_k^\beta(x, t) + \Omega_k^\beta(x, t) \quad \text{Phase } \beta$$

- All these models rely on one or several modifications of the BGK collision operator Ω_k

Two phase flow collision operators

- Introduction of 2 relaxation parameters

$$\Omega_k^\alpha = -\frac{1}{\tau_\alpha} \left(f_k^\alpha - f_k^{\alpha(eq)} \right) \quad \tau_\alpha : \text{relaxation parameter for phase } \alpha$$

$$\Omega_k^\beta = -\frac{1}{\tau_\beta} \left(f_k^\beta - f_k^{\beta(eq)} \right) \quad \tau_\beta : \text{relaxation parameter for phase } \beta$$

- Modification of distribution functions at equilibrium to account
- for fluid-solid and fluid-fluid interactions .
- Up to now, surface properties (surface tension and wetting) have been included.

