fusion cross section with the Siwek-Wilczyńska and Wilczyński potential[10] is given. As these authors found, their potential should be renormalized by a factor $N<1$ for achieving good agreement with the experiment. In our case too, a slight reduction of the attraction would be necessary.

In conclusion we consider that the small polarization effect we have obtained at $s_0>0$ reinforces the validity of the proximity concept as introduced in Ref. 3 for the nucleus-nucleus interaction. For $s_0<0$ we obtain potential shapes very different from those of Ref. 3. As Huizenga *et al.*[14] have pointed out recently, the fusion excitation function of systems around $A_1\sim 40$ and $A_2\sim 120$ is sensitive to the form of the nuclear potential at negative separation distances and experimental fusion results at higher energies might possibly distinguish between different potentials.

A stimulating discussion with J. Randrup is gratefully acknowledged.

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Nucleon-Nucleus Potential at Low and Intermediate Energy in a Dirac-Hartree Model

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We calculate the average nucleon-nucleus potential from the Dirac-Hartree model, extended to positive energy. The sole input is a one-boson–exchange nucleon-nucleon interaction which reproduces ground-state properties. We obtain fair agreement with empirical values. Between 170 and 400 MeV, the calculated potential is repulsive in the nuclear interior but still attractive at the surface. This shape is related to the scalar and vector nature of the exchanged bosons.

In a series of pioneering papers,[1-3] Green, Miller, and co-workers constructed a relativistic self-consistent (Dirac-Hartree) meson field theory of nuclear ground states, in conjunction with one-boson–exchange nucleon-nucleon interactions. They obtained good agreement between theoretical and experimental total binding energies, single-particle energies, and radial charge distributions. The interest of this fundamental approach to ground-state properties has been confirmed by the recent work of Brockmann and Weise.[4-6] Green[7] had pointed out that this relativistic self-consistent model could also be used to calculate the real part of the optical-model potential. This is performed in the present Letter. To our knowledge, this is the first calculation of the optical-model potential that uses as sole input a one-boson–exchange interaction. The latter accounts for the ground-state properties.[6] We show that it also yields an optical-model potential which is in good agreement with empirical potentials up to intermediate energies.

Between 170 and 400 MeV, the calculated potential turns out to be repulsive in the nuclear interior but still attractive at the nuclear surface. In the present relativistic Dirac-Hartree model this shape originates from the fact that the attractive part of the nucleon-nucleon interaction is due to the exchange of scalar bosons, while the repulsive component is due to vector bosons. We note that the Dirac-Hartree method can also be used for antiprotons, for which elastic-scattering experiments are being planned.

In the optical model, one assumes that the wave function of the nucleon-nucleus system can be approximated by a Slater determinant. In first order in the strength of the nucleon-nucleon interaction, the average nucleon-nucleus potential is then given by the Hartree-Fock approximation.[8] Miller and Green,[2] and more recently Brockmann and Weise,[4-6] applied the relativistic version of the Hartree approximation to the description of nuclear ground states. These authors first eliminate the meson degrees of freedom from the original Lagrangian; the relativistic Hartree approximation then yields a Dirac equation for the wave function of each nucleon. We denote by $G(r)$ the large components and by $F(r)$ the small components of the relativistic single-particle scattering wave function. Following Walecka[9] and Brockmann and Weise,[4-6] we assume that the main contribution to the free nucleon-nucleon interaction arises from the exchange of a neutral scalar meson (σ) and of a neutral vector meson (ω). The Dirac equation for $F(r)$ and $G(r)$ then involves only two potentials. The first one, $U_s(r)$, is an attractive potential associated with σ exchange; it behaves as a scalar under a Lorentz transformation. The second one, $U_0(r)$, is a repulsive potential that arises from ω exchange, and it is the fourth component of a quadrivector.

By eliminating the small components $F(r)$ from the original Dirac equation, one obtains for the quantity

$$g(r) = [2m + \epsilon + U_s(r) - U_0(r)]^{-1/2}\, G(r) \tag{1}$$

the following Schrödinger-like equation:

$$g''(r) + \{k^2 - 2m[U_e(r) + U_{\mathrm{s.o.}}(r)] - l(l+1)r^{-2}\}\, g(r) = 0. \tag{2}$$

This result is easily obtained from Eq. (57) of Ref. 6. The quantity k is the relativistic asymptotic momentum, with

$$k^2 = 2m\epsilon + \epsilon^2, \tag{3}$$

where $\epsilon + m$ is the total energy. Since $g(r)$ is proportional to $G(r)$ for large r, its asymptotic behavior yields the correct phase shift. The quantity $U_{\mathrm{s.o.}}(r)$ denotes the spin-orbit potential,[4] while $U_e(r)$ is the central part of the Schrödinger-equivalent nucleon-nucleus average potential. Here, the expression Schrödinger equivalent refers to the fact that Eq. (2) yields exactly the same scattering phase shift as the original Dirac equation which contains $U_s(r)$ and $U_0(r)$: Equation (2) is not a nonrelativistic limit. Omitting for simplicity the Coulomb field and negligible derivative terms,[6] the quantity $U_e(r)$ can be written as the sum of two contributions: $U_e(r) = U_e{}^s(r) + U_e{}^0(r)$, with

$$U_e{}^s(r) = U_s(r) + (2m)^{-1}\, U_s{}^2(r), \tag{4}$$

$$U_e{}^0(r) = U_0(r) - (2m)^{-1}\, U_0{}^2(r) + m^{-1} U_0(r)\epsilon. \tag{5}$$

In the present work, we use the potentials $U_s(r)$ and $U_0(r)$ which have been computed by Brockmann.[6] This author used as input the contribution of the mesons σ and ω to the one-boson-exchange potential nucleon-nucleon interaction of Erkelenz, Holinde, and Machleidt.[10] The ground-state properties of ^{16}O and ^{40}Ca calculated by Brockmann and Weise[4,6] in the Hartree approximation are in good agreement with experiment. As expected on general grounds, these authors find that $U_s(r)$ is attractive $[U_s(0) \approx -440$ MeV$]$, while $U_0(r)$ is repulsive $[U_0(0) \approx 336$ MeV$]$.

The analysis of elastic-scattering data mainly determines the volume integral $J = \int d^3r\, U_e(r)$. In Fig. 1, we compare empirical values of this quantity with the result computed from Eqs. (4) and

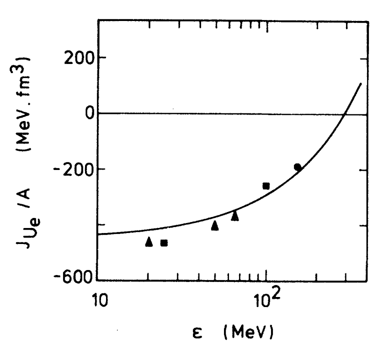

FIG. 1. The full curve represents the energy dependence of the volume integral per nucleon of the real part of the optical-model potential $U_e(r)$ calculated from the Dirac-Hartree model in the case of ^{16}O. The emperical values have been taken from Ref. 18 (triangles), Ref. 19 (squares), and Ref. 20 (circle), respectively.

(5), in the case of ^{16}O. We see that the Dirac-Hartree model that accounts for ground-state properties also works well for scattering states. Similar agreement is obtained in the case of ^{40}Ca.[14] It is not trivial since the energy dependence of the average field plays a crucial role in this comparison. This energy dependence is contained in the last term on the right-hand side of Eq. (5) and originates from the vector component of the relativistic field.

In Fig. 2, we plot the calculated potential well at $\epsilon = 0$ MeV and at $\epsilon = 170$ MeV. We note that at the latter energy $U_e(r)$ is still attractive at the nuclear surface, while $U_e(0) = 0$. This "wine-bottle–bottom" shape is responsible for the fact that the volume integral of the average field changes sign only at 280 MeV, i.e., at a energy higher than $U_e(0)$. Experimental evidence for the occurrence of this unorthodox shape was found by Elton,[15] who claimed that elastic-scattering data at 180 MeV require the existence of an attractive potential valley at the nuclear surface. Some theoretical support for this wine-bottle–bottom shape also emerges from a Bethe-Brueckner calculation of the optical-model potential based on the phenomenological Reid hard-core interaction.[16]

We now turn to the origin of this wine-bottle–bottom shape in the present model. Humphreys[17] had suggested that the range of the repulsive vector potential $U_0(r)$ is somewhat smaller than the range of the attractive scalar potential $U_s(r)$, because the mass of the ω meson (783 MeV) is larger than that (550 MeV) of the σ meson. This would give rise to a wine-bottle–bottom shape in-

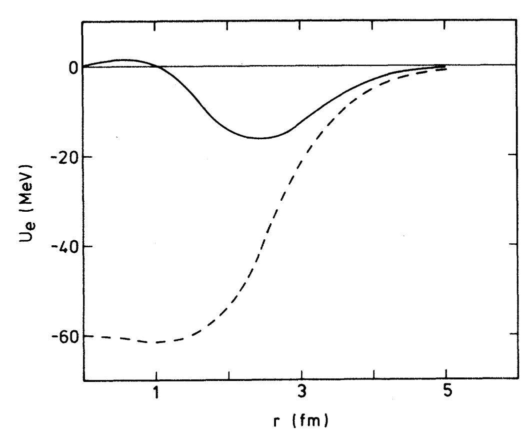

FIG. 2. Radial dependence of the calculated $U_e(r)$ at $\epsilon = 0$ MeV (dashes) and at $\epsilon = 170$ MeV (full curve), in the case of ^{16}O.

deed. However, it turns out that the σ-ω mass difference only affects the tail of the potentials U_0 and U_s: While the root-mean-square radius of $U_s(r)$ is larger than that of $U_0(r)$, the half-depth radii of these two potentials are essentially equal. The results of Brockmann[6] show that the potentials $U_s(r)$ and $U_0(r)$ are practically proportional to one another for $r < 2$ fm. This can be understood from the work of Myers.[18] More specifically, the following approximation is accurate[6]:

$$U_s(r) \approx -1.33\,U_0(r) \quad \text{for } r < 2 \text{ fm}. \tag{6}$$

Since the wine-bottle–bottom shape is already well pronounced for $r < 2$ fm, we conclude from relation (6) that its main origin does not lie in the difference in shape between U_s and U_0, i.e., in the σ-ω mass difference. We now show that the unorthodox shape of the potential at intermediate energy is due to the terms proportional to $U_s^2(r)$ and to $U_0^2(r)$ in Eqs. (4) and (5). We first note that their signs depend upon the scalar or vector nature of the exchanged boson. Let us write the effective central potential in the form

$$U_e(r) = [U_e(r)]_{\epsilon=0} + m^{-1}U_0(r)\epsilon, \tag{7}$$

$$[U_e(r)]_{\epsilon=0} = [U_s(r) + U_0(r)] \times \{1 + (2m)^{-1}[U_s(r) - U_0(r)]\}. \tag{8}$$

The numbers given above indicate that the factor contained in the curly brackets on the right-hand side of Eq. (8) increases from approximately 0.6 in the nuclear interior to 1 outside the target nucleus. The range of the attractive energy-independent part $[U_e(r)]_{\epsilon=0}$ of $U_e(r)$ is thus larger than that of the energy-dependent repulsive component $U_0(r)$. This is the main origin of the wine-bottle–bottom shape at intermediate energy. The quadratic terms U_s^2 and U_0^2 in Eqs. (4) and (5) are typical of a relativistic approach and of the scalar and vector nature of the mesons σ and ω, respectively. In the present model, the shape displayed in Fig. 2 is thus a direct consequence of the boson-exchange nature of the nucleon-nucleon force. The value of the energy at which $U_e(0)$ changes sign depends on the boson masses and coupling strengths. These are not accurately known. However, the discussion given above shows that the occurrence of a wine-bottle–bottom shape is quite independent of these parameters.

In conclusion, we urge experimentalists to perform accurate measurements of elastic proton scattering cross sections and of analyzing powers in the 150–400-MeV domain where data are presently badly missing. It is in this energy range that detailed information can be obtained on the balance between the contributions to the optical-model potential of the attractive and repulsive components of the nucleon-nucleon interaction. The Dirac-Hartree model could be tested in some detail, since it makes definite predictions on the spin-orbit potential.[4] The latter depends only very weakly on energy and can thus essentially be taken over from Refs. 4 and 6. It will be advisable to perform the analysis of these data with a relativistic optical-model equation, which is best adapted to the Dirac-Hartree theoretical model. Such relativistic wave equations have already been used by Arnold, Clark, and Mercer,[19] in conjunction with phenomenological potentials wells $U_0(r)$ and $U_e(r)$. These were fitted to the available data which correspond to energies larger than 500 MeV.

After the submission of the first version of the present Letter, a paper by Arnold and Clark[20] appeared in which a relativistic optical-model potential is constructed at low energy. These authors also use a σ- and ω-exchange interaction. However, they adopt a folding prescription rather than a self-consistent theory; hence, they use densities as input. Moreover, they do not discuss the value and the shape of the potential at intermediate energy.

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Anticorrelations in Light Scattered by Nonspherical Particles

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We describe both the theory and first experimental realization of a new light-scattering method for studying nonspherical particles in dilute solution. It is based on measuring the cross correlation of signals from two spatially separated detectors each of which receives light from the same *small number* of scatterers. Possible reasons for observed differences between experiment and theory are discussed.

We report an experiment which essentially observes the rotational Brownian motion of a succession of *single*, nonspherical macromolecules. A dilute solution of tobacco mosaic virus (TMV), a rod-shaped particle of length about 300 nm and diameter of about 15 nm, was illuminated by las-

