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★**The best approximation method in computational mechanics. (English summary)**
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This book is an introduction to functional analysis and to mathematical analysis of computer modeling algorithms. A goal of the book is to present the more important and useful functional analysis concepts that may serve the computer modeler in various fields to analyze the validity and the accuracy of the approximations he may do.

The first four chapters present summarized topics in functional analysis, integration theory, Hilbert space and generalized series, and linear operators. Chapter 2 reviews the basic theory of Lebesgue integration needed to develop the generalized Fourier series theory and develops the theory fundamental to converging sequences. Many numerical methods deal with $L_2$ convergence which only guarantees convergence in measure.

But in engineering problems in general, additional hypotheses are available, such as continuity, piecewise continuity, etc., which can lead to other types of convergence. Chapter 3 deals with these additional hypotheses along with the Hilbert space environment, generalized Fourier series and finite-dimensional vector space representations of piecewise continuous functions defined over the problem domain, which are implicitly used in many numerical methods. The best approximation in the vector representations is introduced, as well as a computer program. The linear operator theory is covered in Chapter 4 with examples in engineering and with the principle of superposition.

Chapter 5 is devoted to the mathematical development of the best approximation method; in this chapter, an inner product and norm (or a weighted inner product) are used which enable the engineer to approximate engineering problems by developing a generalized Fourier series. The resulting approximation is the “best” approximation in that a least-squares $(L_2)$ error is minimized simultaneously for both fitting the problem’s boundary conditions and satisfying the linear operator relationship (the governing equations) over the problem’s domain (both space and time). Some considerations concerning the appropriate choice of the basis functions (global basis elements, spline basis functions, mixed basis function) which may be more successful in reducing approximation error are developed. Detailed examples are included to illustrate the inner products employed in the method and to demonstrate the progression of steps used in the development of the associated computer program.

Applications of the best approximation method are developed in Chapters 6, 7, and 8. Extension of the best approximation to a computer program for the approximation of boundary value problems of the two-dimensional Laplace and Poisson equations is contained in Chapter 6. This chapter focuses on the topics of weighting factor selection and modeling sensitivity, effects of additional basis functions on computational accuracy, and the effects on modeling results due to the addition of collocation points. Thirteen simple but detailed example problems are used to illustrate the approximate results obtained by the method when applied to practical problems involving partial differential equations in transport-type problems. Chapter 7 explains the use of analytic functions for approximating two-dimensional transport problems involving the Laplace or Poisson equations, demonstrating how the choice of basis functions influences the computational effort. The two-dimensional Laplace equation (or Poisson equation) is examined with respect to using the best approximation method where the set of basis functions are analytic functions, i.e. an approximation function is developed as the sum of complex variable analytic functions (complex variable boundary element method). The later sections of this chapter are devoted to some other families of basis functions and their application to potential problems. A FORTRAN computer program for the best approximation method is included as a final section of this chapter. Several families of linear operator problems are reviewed in Chapter 8.

Reviewed by *P. Rochus* (Liège)