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★**Identification problems of wave phenomena.**
Theory and numerics.
Inverse and Ill-posed Problems Series.
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This book introduces and analyzes a wide class of inverse problems related to wave propagation, which arise in mathematical physics, and especially in geophysics, electrodynamics, acoustics, electromagnetoelasticity, and ecology. The authors give their book a basic introductory character in order to make it appealing not only to mathematicians, but also to physicists, geophysicists and engineers: the main theoretical results (existence, uniqueness, conditional stability) are proved mainly in the case of one-dimensional inverse problems and sometimes in the case of multidimensional ones when related to simple domains.

One of the most important aspects in the theory of inverse problems is the analysis of experimental data via numerical algorithms. As a rule, inverse problems are essentially nonlinear and for the most part, intrinsically unstable. In addition, they usually admit no solution for arbitrary data, and, what is more, the data can be measured only approximately.

This book is focused on inverse problems for hyperbolic equations (IPHEs) for the following reasons. First, the great majority of IPHEs can be reduced to the Volterra integral (operator) equations, thus giving the possibility of extending to IPHEs the theoretical results and the corresponding numerical methods from the theory of Volterra equations. In particular, the Picard and Carathéodory successive approximations developed for Volterra equations can also be applied to IPHEs. Secondly, many numerical methods, such as the inversion of finite-difference schemes, linearization, optimization, and many others, were developed for and tested on IPHEs. Finally, it is well known that in some special cases the direct problems for elliptic and parabolic equations can be reduced to those for hyperbolic ones. Therefore, the technique worked out for IPHEs may also be useful when studying inverse problems for elliptic or parabolic equations.

The book is divided into two parts: the first is concerned with the usual identification problems related to hyperbolic (second-order) differential equations, while the second is devoted to extending the previous problems to the case of integro-differential equations. Chapter 1 describes several results from the theory of direct and inverse problems for hyperbolic equations which are necessary for understanding the remaining part of the book. A broad class of inverse problems for hyperbolic equations and systems is shown to reduce to Volterra operator equations of the first or second kind with Volterra and boundedly Lipschitz continuous kernels. Chapter 2 shows that the above properties ensure the local well-posedness of the inverse problem as well as its global well-posedness when suitable neighborhoods of the exact solutions are considered. The methods developed in this chapter are applied to a few identification problems related to Maxwell's equations in Chapter 3 and are used to prove theorems analogous to those from Chapter 2 but for more general cases. They are also used to justify the method of linearization in Section 4.1 and a regularization method in Chapter 6. The discrete analog of what is illustrated in Chapter 2 is used for estimating the convergence rate of the method of inversion of finite-difference schemes in Section 8.1 as well as of the methods related to the Carathéodory and Picard successive approximations.

Chapter 4 deals with the Newton-Kantorovich method. In Section 4.1, the linearization method for the class of nonlinear Volterra operator equations is justified. In Section 4.2, the linearization method (together with the projection method and regularization) is applied to some multidimensional inverse problem for the wave equation. The Newton-Kantorovich method and its application to the inverse problem for Maxwell's equations is described in Section 4.3. Chapter 5 is devoted to the well-known Gel′fand-Levitan method. Section 5.1 gives a short scheme of the dynamic version of the Gel′fand-Levitan scheme. In Section 5.2, the same ideas are applied to a multidimensional inverse problem and the multidimensional analog of the Gel′fand-Levitan equation is deduced. In Sections 5.3–5.7, the discrete version of the Gel′fand-Levitan method is considered when both the equation and the data are replaced by their finite-difference analogs.

The Tikhonov regularization method for solving ill-posed problems is presented in Chapter 6. Section 6.2 describes and justifies a general method for regularizing hyperbolic identification problems. This method is applied in Chapter 14 to identification problems for integro-differential equations. Chapter 7 is devoted to the method of optimal control and is confined to the discrete inverse problem and to investigating only one (but very important) question—the uniqueness of the stationary point of the cost function. In Section 7.2, a discrete inverse problem is investigated. Section 7.3 proposes a special representation of the solution to the discrete direct problem which is used in Section 7.4 for proving the uniqueness of the stationary point. Chapter 8 presents the inversion of finite-difference schemes. In Section 8.1, the method of inversion of finite-difference schemes is applied to the one-dimensional inverse problem for the acoustic equation and the convergence of the nonlinear Volterra operator equation is then proved. Section 8.2 compares the method of Carathéodory successive approximations (which can be treated as the continuous version of the method of inversion of finite-difference schemes) and Picard successive approximations. Chapter 9 is devoted to the analysis of some of the most known strongly ill-posed problems, such as the Dirichlet problem for the Laplace equation and the non-characteristic problem for the heat equation. Although they are not at all identification problems, the authors nevertheless decided to deal with them at least in the model situation related to a strip, because they are the "heart" and the "starting point" of any study concerning inverse problems. An idea of the difficulties related to such problems is given without making use of hard Sobolev spaces and sophisticated functional analysis. Chapter 10 generalizes the identification problems, so far related to linear hyperbolic equations, allowing the equation under consideration to be semilinear. Chapter 11 considers a model identification problem related to a linear, first-order partial integro-differential scalar equation in the strip $(0, T) \times \mathbf{R}$. An inverse hyperbolic integro-differential problem arising in geophysics, a model identification problem related to hyperbolic linear integro-differential equations, is considered in Chapter 12. For this purpose, the simplest geometric situation, i.e. the strip $(0, T) \times \mathbf{R}$, is chosen instead of a bounded cylinder with a smooth lateral surface, in order to keep on a low level the technicalities usually connected with such problems. Chapter 13 deals with the integro-differential identification problems related to the one-dimensional wave equation, more exactly, with the problem of identifying a coefficient depending only on the space variable in the differential part assuming that the relaxation kernel in the integro-differential one is known. In other words, an identification problem related to a viscoelastic model body is considered, which is assumed to be one-dimensional and to occupy an entire line. Under these assumptions, results analogous to those proved in Chapter 1 for this more general situation are obtained. The detailed analysis given here allows the reader to understand how to perform the passage from the differential identification problem to the integro-differential one. Chapter 14 applies the regularization procedure due to Lavrent′ev to solve linear integro-differential inverse equations arising from geophysics. Some existence and uniqueness theorems are proved and the solution of the regularized inverse problem is shown to converge to the solution of the exact inverse problem when the regularization parameters tend (accordingly) to zero and additional information is affected by a known error. The optimization method is applied to the regularized inverse problem and the properties of the cost function and its gradient are investigated. As a result, the convergence of the conjugate gradient method is demonstrated. Chapter 15 introduces firstly an identification problem related to a semilinear hyperbolic equation containing an unknown nonlinearity term. The equation considered is a particular hyperbolic equation proposed by several authors to describe, under a suitable approximation, the nonlinear transverse vibration of an elastic string. Chapter 16 applies an optimization procedure, namely, the conjugate gradient method, to inverse problems in electromagnetoelasticity. This last chapter is devoted to some examples concerning severely ill-posed problems and to introducing some simple techniques for solving them.

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