

1 Experimental investigation of meandering jets in shallow reservoirs

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7 8 Abstract

9
10 Meandering flows in rectangular shallow reservoirs were experimentally investigated. The characteristic
11 frequency, the longitudinal wave length and the mean lateral extension of the meandering jet were
12 extracted from the first paired modes, obtained by a Proper Orthogonal Decomposition (POD) of the
13 surface velocity field measured by Large Scale PIV (LSPIV). The depth-normalised characteristic lengths
14 and the Strouhal number were then compared to the main dimensionless numbers characterizing the
15 experiments: Froude number, friction number and reservoir shape factor. The normalised wave length
16 and mean lateral extension of the meandering jet are neither correlated with the Froude number nor with
17 the reservoir shape factor; but a clear relationship is found with the friction number. Similarly, the
18 Strouhal number is found proportional to a negative power of the friction number. In contrast, the
19 Froude number and the reservoir shape factor enable to predict the occurrence of a meandering flow
20 pattern: meandering jets occur for Froude number greater than 0.21 and for a shape factor smaller than
21 6.2.

22 23 Keywords

24 Shallow reservoir, meandering jet, Proper Orthogonal Decomposition, Large-Scale PIV

25 1. INTRODUCTION

26 Shallow reservoirs are common in hydraulic engineering. They are used for water storage

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27 or sediment trapping. From an operational point of view, predicting accurately the amount and
28 the location of the sediment deposits is of high importance. The quality of this prediction is
29 strongly dependent on the detailed knowledge of the flow field developing within the
30 reservoir [1,2].

31 Recent studies emphasized the complexity of flows in rectangular shallow reservoirs [3-
32 5]. The jet developing at the entrance of the reservoir can either be straight from the inlet to
33 the outlet, or can impact one or several times the lateral walls. Using the shape factor,
34 $SF = L/\Delta B^{0.6}b^{0.4}$ (L the reservoir length, b the width of the inlet channel and ΔB the width of
35 the sudden expansion), Dufresne et al. [5] showed that for $SF < 6.2$ the flow patterns are
36 symmetric, for $SF > 6.8$ they are asymmetric, and for $6.2 < SF < 6.8$ both types of flow
37 patterns may be observed.

38 In his PhD Thesis at EPFL, Kantoush (2008) [6] revealed the existence of symmetric
39 flows with temporal and spatial periodical oscillations of the jet when $SF = 7.2$, $F > 0.1$
40 (F the Froude number) and $H/\Delta B < 0.2$ ($H/\Delta B$ the shallowness parameter and H the mean
41 water depth in the reservoir). He named these flows as “meandering”, but no quantitative
42 characterisation of the jets was made (neither their frequency nor their characteristics lengths)
43 to really confirm the meandering behaviour of these flows. More recently, Camnasio et al.
44 (2012) [7] performed visual observations of meandering jets, but again without quantitative
45 experimental characterisation, and showed that a 2D depth-averaged flow model can simulate
46 the occurrence of meandering jets.

47 The study of these meandering flows is of high relevance for reservoir management (*e.g.*,
48 for the prediction of reservoir trapping efficiency and of the spatial pattern of sediment
49 deposits ...). This “meandering” behaviour is responsible for the generation of large-scale
50 vortices on both sides of the jet, which transfer momentum from the jet towards the rest of the
51 reservoir and induce significant changes in the velocity distribution compared to a

52 configuration without meandering jet [8]. Using the numerical modelling WOLF 2D [3,9],
53 Peltier et al. [8] showed that these changes in the velocity distribution have a strong impact on
54 the sediment transport and deposition. Indeed, the meandering jet induces a larger spreading
55 of the sediments on both sides of the jet, which increases the reservoir trapping efficiency. A
56 two-way coupling may also be observed between the flow and the sediment transport, since
57 bathymetric changes due to sediment deposits may induce changes in the flow pattern [9].

58 The purpose of this paper is to present a first experimental characterization of these
59 meandering flows in short rectangular shallow reservoirs. The experimental setup is first
60 described and the key parameters of the problem are identified by a dimensional analysis.
61 Next, the influence of these parameters on the characteristics of the meandering flow
62 (frequency, longitudinal wave length and mean lateral extension of the jet) is analysed.
63 Finally the correlations found between the characteristics of the meandering jet and the other
64 parameters are discussed.

65 **2. MATERIAL AND METHODS**

66 **2.1. Experimental device**

67 The experiments were carried out at the laboratory of engineering hydraulics of the
68 University of Liege (ULg), Belgium as shown in Fig. 1. The experimental flume consists in a
69 10.40 m long and 0.98 m wide glass channel, in which blocks can be arranged to build
70 different geometries of rectangular reservoirs. The bottom of the flume is horizontal. The flow
71 enters the channel from a stilling basin through a porous screen in order to prevent
72 fluctuations in water level and to facilitate the establishment of a fully developed velocity
73 profile. The flow is then contracted to the width of the inlet channel, b , through a converging
74 section. The inlet channel is 2.00 m long and has straight parallel walls. At the entrance of the
75 reservoir, the flow suddenly expands to the width of the reservoir, $B=b+2\times\Delta B$. At the exit of

76 the reservoir, the flow suddenly contracts to the outlet channel width, which is the same as in
77 the inlet channel. The outlet channel is 1.50 m long and it ends with a tailgate and a control
78 weir. All the surfaces are made of glass, except the bottom of the flume (PVC) and the
79 converging section (metallic sheets).

80 The discharge, Q , was measured with an electromagnetic flowmeter (uncertainty of
81 0.025 L/s) mounted on the pipe connecting the downstream tank to the upstream tank. The
82 discharge was regulated to ensure the temporal stability of the supply. This regulation was
83 ensured through a pressure sensor mounted on the pump and an overflow system, which
84 enabled to keep constant the head at the entrance of the pump (constant water level in the
85 downstream tank).

86 The water depth was measured using an ultrasonic probe and the surface velocity as well
87 as the vortex dynamics was measured by LSPIV [10,11]. The uncertainty on the water depth
88 was estimated to 1% of the mean value. The uncertainty on the mean velocity was estimated
89 to 5%.

90 **2.2. Dimensional analysis**

91 Meandering jets in rectangular shallow reservoirs can be described based on 14
92 independent parameters (Tab. 1) and the number of dimensions of such a problem is 3
93 (Length = H , Time H^3/Q , Mass = ρH^3). As a result, the number of Π -parameters is equal to
94 $14-3 = 11$ (see Tab. 1).

95 The Π -parameters in Tab. 1 are classified into three groups: geometry, hydraulics and
96 fluid. For the Π -parameters related to the hydraulics and the fluid, the choice of H for
97 expressing the length-scale, instead of the width b or ΔB , is driven by the observations of
98 Dracos et al. [12], who showed that the behaviour of a plane turbulent jet in a bounded fluid
99 layer is better explained when the dimensional analysis is based on the depth of the fluid layer

100 rather than the width of the orifice. Nevertheless, for the geometry, we combined the Π -
 101 parameters 1-3 in order to come up with standard non-dimensional parameters in the literature
 102 for characterizing the reservoir geometry ($\Delta B/L, b/\Delta B$).

103 Focusing on the characteristics of the meandering jet, the parameters Π_6, Π_7 and Π_8 can
 104 be expressed as a function of all other Π -parameters:

$$\left(\text{St}, \frac{\Lambda_x}{H}, \frac{\Lambda_y}{H} \right) = \mathbb{F} \left(\frac{H}{\Delta B}, \frac{\Delta B}{L}, \frac{b}{\Delta B}, \frac{\varepsilon}{H}, S_o, R, F, W \right) \quad (1)$$

105 The friction coefficient, λ , as deduced from a friction formula such as Colebrook-White,
 106 is a function of the roughness, ε , and of the Reynolds number R . Consequently, we assumed
 107 that the influence of ε and R is lumped into the friction coefficient λ .

108 Using the friction coefficient and the shallowness parameter $H/\Delta B$, a friction number can
 109 be obtained for flows in shallow reservoirs [5], similarly to the expression introduced by Chu
 110 et al. [13]:

$$S = \frac{\lambda \Delta B}{8H} \quad (2)$$

111 As a result, Eq. 1 reduces to:

$$\left(\text{St}, \frac{\Lambda_x}{H}, \frac{\Lambda_y}{H} \right) = \mathbb{F} \left(\frac{\Delta B}{L}, \frac{b}{\Delta B}, S_o, S, F, W \right) \quad (3)$$

112 Since most real-world reservoirs configurations have no significant slope, the present
 113 experimental study focuses on reservoirs with a horizontal bottom and therefore the influence
 114 of S_o can be neglected. In addition, using the shape factor, $SF = \Pi_3^{-0.4} / \Pi_1 = L/\Delta B^{0.6} b^{0.4}$ as
 115 defined by Dufresne et al. [5], Eq. 3 can be simplified as follows:

$$\left(\text{St}, \frac{\Lambda_x}{H}, \frac{\Lambda_y}{H} \right) = \mathbb{F}(SF, S, F, W) \quad (4)$$

116 In the present experiments, the free surface deformations remain low (1 or 2 mm). As a
117 consequence, the Weber number, W , is expected to have little influence on the development
118 of the flow. The only way of verification would be to compare results from models of
119 different scales, which was not possible here. As a consequence, we decided to neglect Weber
120 number in our analysis and Eq. 1 finally reduces to:

$$\left(\text{St}, \frac{\Lambda_x}{H}, \frac{\Lambda_y}{H} \right) = \mathbb{F}(\text{SF}, \text{S}, \text{F}) \quad (5)$$

121 **2.3. Experimental data set**

122 Different hydraulic and geometric configurations were considered to enable the
123 observation of a wide range of meandering flows (Tab. 2). Two inlet channel widths and five
124 lengths of reservoir were tested. Three different crest heights were used in order to obtain
125 various water depths for a given discharge. For each combination of geometry and crest
126 height, at least five different discharges were used and all experiments were repeated 2 to 3
127 times. The resulting Froude, friction and Reynolds numbers of the whole set of experiments
128 are summarized in Tab. 2. The Froude numbers lay between 0.08 and 0.53. The Friction
129 number is between 0.01 (non-frictional regime, see Chu et al. [14]) and 0.24 (frictional
130 regime), which indicates that various types of coherent structures are expected in the
131 experiments and therefore various behaviours of the jet. The Reynolds number is between
132 7,200 and 65,700, which emphasizes that the jet can be considered as turbulent. Given these
133 Reynolds numbers, the flows are hydro-dynamically smooth. This justifies the necessity of
134 considering both the Reynolds number and the roughness for estimating the friction number
135 as noticed in section 2.2. Among all the tested geometric and hydraulic conditions, 50 distinct
136 configurations out of 80 led to a meandering flow.

137 For thirty nine of these meandering jets (see Tab. 3 in appendix), the instantaneous
138 surface velocity field was measured by LSPIV and a Proper Orthogonal Decomposition

139 (POD) analysis [15] was performed on these fields to extract the frequency, the wave length
140 and the mean lateral extension of the meandering jet. The process to extract these
141 characteristics is schematized in Fig. 2. For each experiment, seeds of 2 mm of mean diameter
142 were first placed on the surface of the flow and a field of 1 m \times 1 m, containing the entrance
143 of the reservoir, was video recorded at a rate of 25 Hz during at least 7'30'' using a
144 commercial video-camera (Canon© HD-HG20). After extraction from the video, correction
145 and orthorectification of the images to be processed [16], one pixel was equal to a square of 1
146 mm side. Using a homemade LSPIV code based on the work of Hauet [16], the surface
147 velocity fields were worked out on a square mesh of 1 cm \times 1cm.

148 The POD analysis was then performed on the surface velocity fields in order to identify
149 the most energetic structures characterizing the meandering flow.

- 150 • The POD analysis emphasized the existence of pairs of modes with similar patterns:
151 the temporal modes are similar, but phase-shifted in time, while the spatial modes are
152 shifted in space (Fig. 2). This indicates that both modes in a pair are directly related
153 to the same coherent structure [17].
- 154 • By definition of the POD [15], the first paired modes corresponding to the meandering
155 jet are the most energetic ones (modes 1 & 2) or are among the most energetic modes
156 (modes 2 & 3 or modes 3 & 4), the first modes representing in the latter case a very
157 energetic slow motion of the jet.

158 A 1D Fourier analysis was finally performed on the identified modes for extracting the
159 characteristic frequency, f_{max} , of the most energetic oscillations and the characteristic lengths
160 of the jet in the longitudinal direction (Λ_x , wave length of the meander) and in the lateral
161 direction (Λ_y , mean lateral extension of the structures in the jet). For extracting the
162 characteristic frequency, the Fourier analysis was performed on the temporal modes. The

163 characteristic lengths were identified using a Fourier analysis made on the vorticity field
164 deduced from the spatial modes (Fig. 2):

- 165 • Λ_x is the inverse of the wave number corresponding to the maximum of the spectrum
166 of the longitudinal distribution of the vorticity field within an interval of five
167 centimetres on both sides of the reservoir centreline,
- 168 • Λ_y is the mean of the inverses of the wave numbers corresponding to the maximums of
169 the spectrum of the distribution of vorticity in each cross-section.

170 3. RESULTS

171 3.1. Normalised characteristic lengths

172 The normalised characteristic lengths Λ_x/H and Λ_y/H are plotted against \mathbf{S} , \mathbf{F} and \mathbf{SF} in
173 Fig. 3. There is no simple relationship between the normalised characteristic lengths and the
174 Froude number nor the shape factor (Fig. 3a). In contrast, the dependence of the normalised
175 characteristic lengths on the friction number shows a distinctive linear distribution for Λ_x/H
176 on the whole range of \mathbf{S} and a power-law distribution for Λ_y/H (Fig. 3b). Based on a total
177 least square fitting (variables on both abscesses are considered as random) [18]; these
178 distributions can be approximated as follows:

$$\frac{\Lambda_x}{H} = 145 \times \mathbf{S} + 2.4 \quad (\mathbf{R}^2 = 0.85) \quad (6)$$

$$\frac{\Lambda_y}{H} = 20 \times \mathbf{S}^{0.45} \quad (\mathbf{R}^2 = 0.62) \quad (7)$$

179 \mathbf{R}^2 being the linear standard coefficient of determination calculated with \mathbf{S} and Λ_x/H for Eq. 6
180 and calculated with $\log(\mathbf{S})$ and $\log(\Lambda_y/H)$ for Eq. 7.

181 Multiplying Eq. 6 by H and using Eq. 2 indicate that the longitudinal characteristic length
182 (*i.e.* the meandering wave-length) is mostly proportional to the product $\lambda \times \Delta B$ for high values
183 of \mathbf{S} and to the water depth H for relatively low values of \mathbf{S} .

184 Eq. 7 highlights the existence of an attenuation of the growth rate of Λ_y/H with increasing
185 \mathbf{S} . Moreover, multiplying Eq. 7 by H and using Eq. 2 reveal that the mean lateral extension of
186 the jet is almost proportional to the square root of $\lambda \times \Delta B \times H$. The dependency to the water
187 depth suggests that a vertical confinement operates on the jet. This is consistent with the
188 observations of Chu et al. [14], which reveal that, for small \mathbf{S} values (*i.e.* H is high relative to
189 the dimension of the experiment), the lateral spreading of the jet is mainly driven by a large
190 scale turbulence (non-frictional regime) with a characteristic length scale proportional to
191 $\lambda \times \Delta B$, whereas for high values of \mathbf{S} (*i.e.* H is small), the lateral spreading of the jet is driven
192 by the bottom generated turbulence, with a characteristic length-scale proportional to H .

193 When considering the ratio of Λ_x and Λ_y as a function of \mathbf{S} (Fig. 4), no correlation can be
194 found when $\mathbf{S} < 0.07$, which corresponds to the upper limit of the non-frictional regime [14].
195 In contrast for $\mathbf{S} > 0.07$, a linear relationship can be found and follows Eq. 8.

$$\frac{\Lambda_x}{\Lambda_y} = 6.83 \times \mathbf{S} + 1.61 \quad (R^2 = 0.65) \quad (8)$$

196 R^2 being the linear standard coefficient of determination calculated with \mathbf{S} and Λ_x/Λ_y .

197 **3.2. Strouhal number**

198 The Strouhal number, which is representative of the characteristic frequency f_{max} , is
199 plotted against F and SF in Fig. 5a. No clear correlation is found between St , F and SF . Data
200 are too scattered to identify a clear tendency. In contrast, a clear dependency is found between
201 St and \mathbf{S} (Fig. 5b). Indeed, the Strouhal number can be expressed as a function of the friction
202 number as follows:

$$St = 0.004 \times \mathbf{S}^{-0.776} \quad (R^2 = 0.66) \quad (9)$$

203 R^2 being the linear standard coefficient of determination calculated with $\log(\mathbf{S})$ and $\log(St)$.

204 Eq. 9 indicates that the vortex shedding within the jet is attenuated by the vertical
205 confinement resulting from lower water depths. The meandering of the flow is also weakened
206 by an increase in the friction effects.

207 **4. DISCUSSION**

208 In the previous section, we showed that the normalised characteristics of the meandering
209 flows are mainly related to the friction number and they are neither directly affected by the
210 shape factor nor by the Froude number (Tab. 2).

211 To further explore the role of F and SF , the present data (Tab. 2) and the data of [4,5,6]
212 were plotted altogether in Fig. 6. Four types of flow can be distinguished:

- 213 1. Meandering flows
- 214 2. Instable flows: the flow was alternatively meandering or symmetric/asymmetric
215 during the same experiment
- 216 3. Symmetric flows: the jet was straight all along the experiment
- 217 4. Asymmetric flows: the jet impacts one or several times the lateral wall of the
218 reservoir

219 The distribution of the data points in Fig. 6 reveals that the Froude number F and the slope
220 factor SF enable to clearly define the respective domains of occurrence of each type of flow.
221 The meandering flows are defined for $F > 0.21$ and $SF < 6.2$. The symmetric flows are
222 defined for $F < 0.21$ and $SF < 6.2$ (horizontal line in Fig. 6), while the asymmetric flows exist
223 for $SF > 6.2-8.1$ with no restriction on the Froude number value. The instable regime is
224 observed for $6.2 < SF < 8.1$, especially $F > 0.21$.

225 **5. CONCLUSION**

226 The present paper investigates meandering flows in shallow rectangular reservoirs. Fifty
227 meandering jets were identified amongst a data set of 80 jets in shallow rectangular reservoirs.

228 For thirty nine of these meandering jets, the instantaneous surface velocity field was measured
229 and a POD analysis was performed on these fields to extract their main characteristics
230 (frequency, wave length and mean lateral extension of the meandering jet).

231 The characteristic lengths of the meandering jet were normalised by the mean water depth
232 in the reservoir and the frequency was written in the form of a Strouhal number. They were
233 then compared to the shape factor of the reservoir, to the Froude number and to the friction
234 number evaluated at the reservoir inlet.

235 No correlation is found between the shape factor, the Froude number and the
236 characteristic parameters of the meandering jet, but the comparison of the present data-set
237 with past experiments emphasizes that the shape factor defines the upper-limit of existence of
238 the symmetrical flows ($SF \leq 6.2$) and the Froude number enables to identify the limit, above
239 which the jet meanders ($F > 0.2$).

240 Nevertheless, correlations are found between the friction number and the characteristic
241 parameters of the meandering jet. The depth-normalised wave length of the meander varies
242 linearly with the friction number, which indicates that the wave length of the meander grows
243 with the reservoir width and/or friction coefficient. The depth-normalised mean lateral
244 extension of the jet is almost proportional to the square root of the friction number. It results
245 that the mean lateral extension is affected by the water depth; a vertical confinement of the jet
246 occurs and affects the lateral spreading of the jet at high values of S (*i.e.* at low water depth).
247 Finally, the Strouhal number is proportional to a negative power of the friction number, which
248 emphasizes a damping of the meandering for increasing friction or decreasing water depth.

249 Future research should confirm whether the present findings remain valid in the case of a
250 rough bottom, as encountered in more realistic configurations.

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255 7. REFERENCES

- 256 1. Dufresne M, Dewals BJ, Ercicum S, Archambeau P, Piroton M (2010) Experimental investigation
257 of flow pattern and sediment deposition in rectangular shallow reservoirs. *International Journal of*
258 *Sediment Research* 25 (3):258-270
- 259 2. Dufresne M, Dewals B, Ercicum S, Archambeau P, Piroton M (2012) Flow patterns and sediment
260 deposition in rectangular shallow reservoirs. *Water and Environment Journal*
- 261 3. Dewals BJ, Kantoush SA, Ercicum S, Piroton M, Schleiss AJ (2008) Experimental and numerical
262 analysis of flow instabilities in rectangular shallow basins. *Environmental Fluid Mechanics* 8 (1):31-
263 54
- 264 4. Camnasio E, Orsi E, Schleiss AJ (2011) Experimental study of velocity fields in rectangular shallow
265 reservoirs. *Journal of Hydraulic Research* 49 (3):352-358
- 266 5. Dufresne M, Dewals BJ, Ercicum S, Archambeau P, Piroton M (2010) Classification of flow
267 patterns in rectangular shallow reservoirs. *Journal of Hydraulic Research* 48 (2):197-204
- 268 6. Kantoush SA (2008) Experimental study on the influence of the geometry of shallow reservoirs on
269 flow patterns and sedimentation by suspended sediments. Ph. D. thesis, No 4048, EPFL, Lausanne
- 270 7. Camnasio E, Piroton M, Ercicum S, Dewals B Experimental and numerical investigation of a
271 meandering jet in shallow rectangular reservoirs under different hydraulic conditions. In: 3rd
272 International Symposium on Shallow Flows, Iowa City, USA, June 4-6 2012.
- 273 8. Peltier Y, Ercicum S, Archambeau P, Piroton M, Dewals B (2013) Experimental and numerical
274 investigation of meandering jets in shallow reservoir: potential impacts on deposit patterns. Paper
275 presented at the THESIS 2013, Two-phase modelling for Sediment dynamics in Geophysical Flows,
276 Chatou, France, 10-12 June
- 277 9. Camnasio E, Ercicum S, Orsi E, Piroton M, Schleiss AJ, Dewals B (2013) Coupling between flow
278 and sediment deposition in rectangular shallow reservoirs. *Journal of Hydraulic Research*.
279 doi:10.1080/00221686.2013.805311
- 280 10. Hauet A, Kruger A, Krajewski WF, Bradley A, Muste M, Creutin JD, Wilson M (2008)
281 Experimental system for real-time discharge estimation using an image-based method. *Journal of*
282 *Hydrologic Engineering* 13 (2):105-110. doi:10.1061/(ASCE)1084-0699(2008)13:2(105)
- 283 11. Kantoush SA, Schleiss AJ (2009) Large-scale piv surface flow measurements in shallow basins
284 with different geometries. *Journal of Visualization* 12 (4):361-373
- 285 12. Dracos T, Giger M, Jirka GH (1992) Plane turbulent jets in a bounded fluid layer. *Journal of Fluid*
286 *Mechanics* 241:587-614
- 287 13. Chu VH, Khayat RE, Wu JH (1983) Stability of turbulent shear flows in shallow channel. In: 20th
288 IAHR Congr., Sep. 5-9, Moscow, USSR, 1983. Delft, The Netherlands, IAHR, pp 128-133

- 289 14. Chu VH, Liu F, Altai W (2004) Friction and confinement effects on a shallow recirculating flow.
290 Journal of Environmental Engineering and Science 3 (5):463-475. doi:10.1139/S04-034
- 291 15. Holmes P, Lumley JL, Berkooz G, Rowley CW (2012) Turbulence, Coherent Structures,
292 Dynamical Systems and Symmetry. Second Edition. Cambridge Monographs on Mechanics, 2nd edn.
293 Cambridge University Press
- 294 16. Hauet A (2009) Discharge estimate and velocity measurement in river using Large-Scale Particle
295 Image Velocimetry; Estimation de débit et mesure de vitesse en rivière par Large-Scale Particle image
296 Velocimetry. La Houille Blanche (1):80-85. doi:10.1051/lhb.2009009
- 297 17. Rempfer D, Fasel HF (1994) Evolution of three-dimensional coherent structures in a flat-plate
298 boundary layer. Journal of Fluid Mechanics 260:351-375
- 299 18. Van Huffel S (2004) Total Least Squares and Errors-in-Variables Modeling: Bridging the Gap
300 between Statistics, Computational Mathematics and Engineering. K.U.Leuven, Dept. of Electrical
301 Engineering (ESAT), Research Group SCD (SISTA)
302

303 **8. APPENDIX**

304 In Tab. 3, the geometric and hydraulic conditions of the thirty nine meandering flows are
305 summarized
306

Tab. 1 Dimensional analysis: variables and Π -parameters

Types	Variables		Π -parameters	
Geometry	Length of the reservoir	L	$\Pi_1 = H / L \times \Pi_2^{-1} = \Delta B / L$	
	Width of the lateral expansion	ΔB	$\Pi_2 = H / \Delta B$	
	Width of the inlet and outlets channels	b	$\Pi_3 = b / H \times \Pi_2 = b / \Delta B$	
	Slope of the reservoir	S_o	$\Pi_4 = S_o$	
Hydraulics	Discharge	Q	Variable used for expressing the dimensions of the problem	
	Velocity at the inlet	V	Dependent Variable: $V = Q / (bH)$ ⁽¹⁾	
	Depth considered as constant in the reservoir	H	Variable used for expressing the dimensions of the problem	
	Roughness	ε	$\Pi_5 = \varepsilon / H$	
	Characteristic frequency of the jet	f	$\Pi_6 = fH^2 / Vb \times \Pi_3 \Pi_2^{-1} = fH / V = St$	
	Characteristic longitudinal length of the jet	Λ_x	$\Pi_7 = \Lambda_x / H$	
	Characteristic lateral length of the jet	Λ_y	$\Pi_8 = \Lambda_y / H$	
Gravity acceleration	g	$\Pi_9 = \sqrt{Q^2 / (gH^5)} \times \Pi_3^{-2} \Pi_2^2 = F$		
Fluid	Volume mass	ρ	Variable used for expressing the dimensions of the problem	
	Dynamic viscosity	μ	$\Pi_{10} = \rho V b / \mu \times 4R / b = R$ ⁽³⁾	
	Surface tension	σ	$\Pi_{11} = \rho V^2 b^2 / (\sigma H) \times \Pi_3^{-2} \Pi_2^2 = \rho V^2 H / \sigma = W$	

309 ⁽¹⁾ This dependent variable is used to express in a known formulation some Π -parameters.

310 ⁽²⁾ The Strouhal number is a non-dimensional number used for describing oscillating flow mechanism.

311 ⁽³⁾ In Π_{10} , $R=bH/(2H+b)$ corresponds to the hydraulic radius in the inlet channel.

312

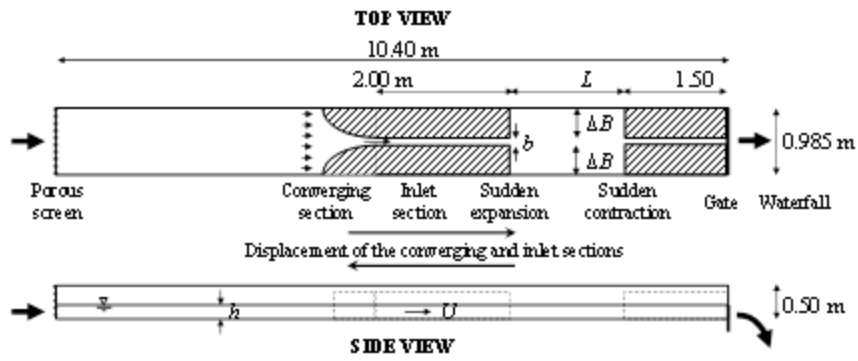
313 **Tab. 2 Geometry and hydraulic conditions of the data-set for the present experiments**

Geometry				Hydraulic conditions				
L (m)	ΔB (m)	b (m)	SF	H (cm)	Q (L/s)	F	S	$R \times 10^2$
1.6	0.45	0.08	7.10	[1.7 - 7.3]	[0.13 - 2]	[0.09 - 0.45]	[0.02 - 0.11]	[78 - 562]
1.4	0.45	0.08	6.19	[1.0 - 6.0]	[0.13 - 1.5]	[0.09 - 0.51]	[0.02 - 0.22]	[80 - 500]
1.2	0.45	0.08	5.30	[1.2 - 7.7]	[0.13 - 2.5]	[0.08 - 0.47]	[0.02 - 0.19]	[72 - 657]
1	0.45	0.08	4.42	[1.2 - 5.8]	[0.13 - 1.5]	[0.10 - 0.47]	[0.02 - 0.18]	[83 - 546]
1	0.46	0.06	4.89	[1.4 - 6.8]	[0.13 - 1.5]	[0.13 - 0.46]	[0.02 - 0.15]	[84 - 445]
0.7	0.45	0.08	3.1	[0.9 - 5.3]	[0.13 - 1.5]	[0.11 - 0.53]	[0.02 - 0.24]	[86 - 593]

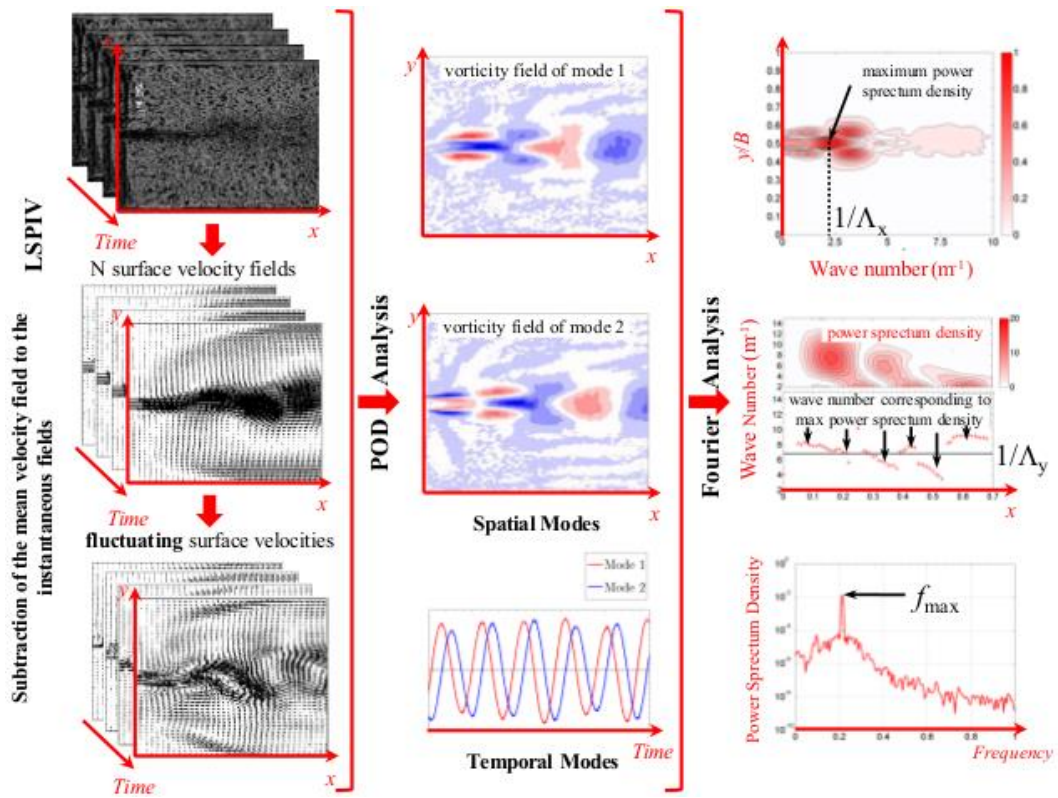
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Tab. 3 Detailed geometric and hydraulic conditions of the thirty nine meandering flows

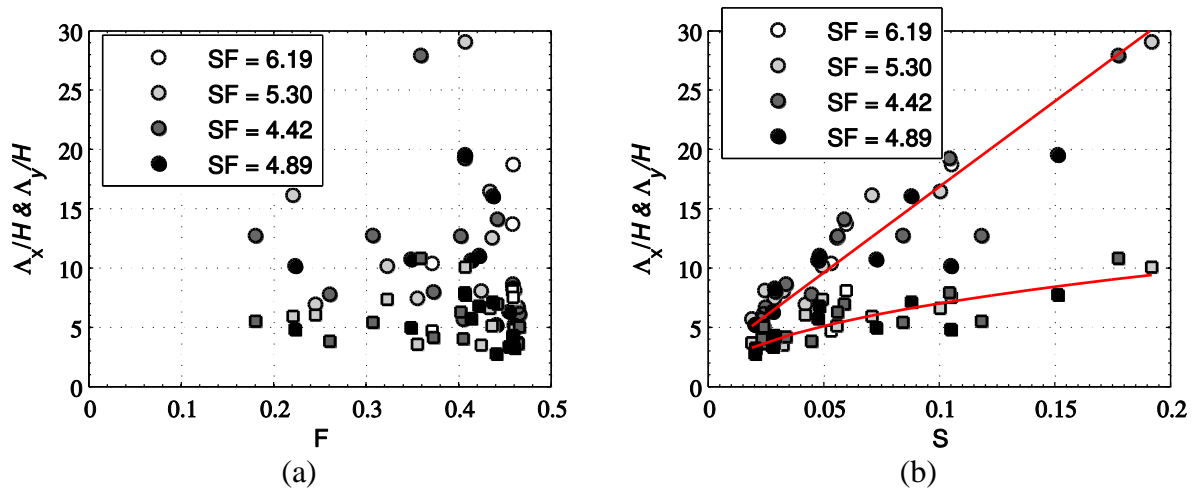
Geometry				Hydraulic conditions				
L (m)	ΔB (m)	b (m)	SF	H (cm)	Q (L/s)	F	S	$R \times 10^2$
1.4	0.45	0.08	6.19	2.69	0.51	0.46	0.06	151
1.4	0.45	0.08	6.19	1.74	0.26	0.46	0.11	92
1.4	0.45	0.08	6.19	3.07	0.50	0.37	0.05	141
1.2	0.45	0.08	5.30	5.46	1.47	0.46	0.02	310
1.2	0.45	0.08	5.30	6.75	2.04	0.46	0.02	379
1.2	0.45	0.08	5.30	2.87	0.53	0.44	0.06	154
1.2	0.45	0.08	5.30	1.83	0.27	0.43	0.10	92
1.2	0.45	0.08	5.30	1.15	0.13	0.41	0.19	48
1.2	0.45	0.08	5.30	4.43	0.99	0.42	0.03	235
1.2	0.45	0.08	5.30	3.35	0.50	0.32	0.05	134
1.2	0.45	0.08	5.30	2.74	0.25	0.22	0.07	74
1.2	0.45	0.08	5.30	4.02	0.50	0.25	0.04	123
1.2	0.45	0.08	5.30	5.08	1.02	0.36	0.03	224
1.0	0.45	0.08	4.42	1.80	0.25	0.41	0.10	84
1.0	0.45	0.08	4.42	2.74	0.50	0.44	0.06	148
1.0	0.45	0.08	4.42	5.56	1.53	0.47	0.02	320
1.0	0.45	0.08	4.42	1.25	0.13	0.36	0.18	47
1.0	0.45	0.08	4.42	1.95	0.12	0.18	0.12	41
1.0	0.45	0.08	4.42	2.24	0.26	0.31	0.08	82
1.0	0.45	0.08	4.42	2.90	0.50	0.40	0.06	144
1.0	0.45	0.08	4.42	4.23	1.00	0.46	0.03	242
1.0	0.45	0.08	4.42	5.40	1.46	0.46	0.02	310
1.0	0.45	0.08	4.42	5.84	1.43	0.40	0.02	290
1.0	0.45	0.08	4.42	4.96	1.03	0.37	0.03	230
1.0	0.45	0.08	4.42	3.78	0.48	0.26	0.04	123
1.0	0.45	0.08	4.42	3.27	0.24	0.16	0.06	66
1.0	0.46	0.06	4.89	3.39	0.50	0.42	0.05	154
1.0	0.46	0.06	4.89	2.10	0.25	0.44	0.09	97
1.0	0.46	0.06	4.89	1.41	0.13	0.41	0.15	58
1.0	0.46	0.06	4.89	5.19	1.01	0.45	0.03	246
1.0	0.46	0.06	4.89	2.12	0.13	0.22	0.11	50
1.0	0.46	0.06	4.89	2.55	0.27	0.35	0.07	95
1.0	0.46	0.06	4.89	3.44	0.50	0.41	0.05	153
1.0	0.46	0.06	4.89	5.06	0.98	0.46	0.03	243
1.0	0.46	0.06	4.89	6.69	1.50	0.46	0.02	308
1.0	0.46	0.06	4.89	6.84	1.48	0.44	0.02	301
1.0	0.46	0.06	4.89	5.59	1.00	0.40	0.03	232
1.0	0.46	0.06	4.89	4.04	0.51	0.33	0.04	144
1.0	0.46	0.06	4.89	3.24	0.25	0.22	0.06	78



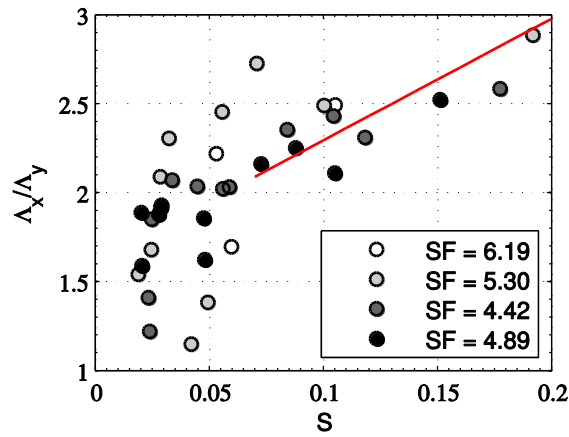
320 Fig. 1 Sketch of the experimental setup (adapted from Dufresne et al. [5]).



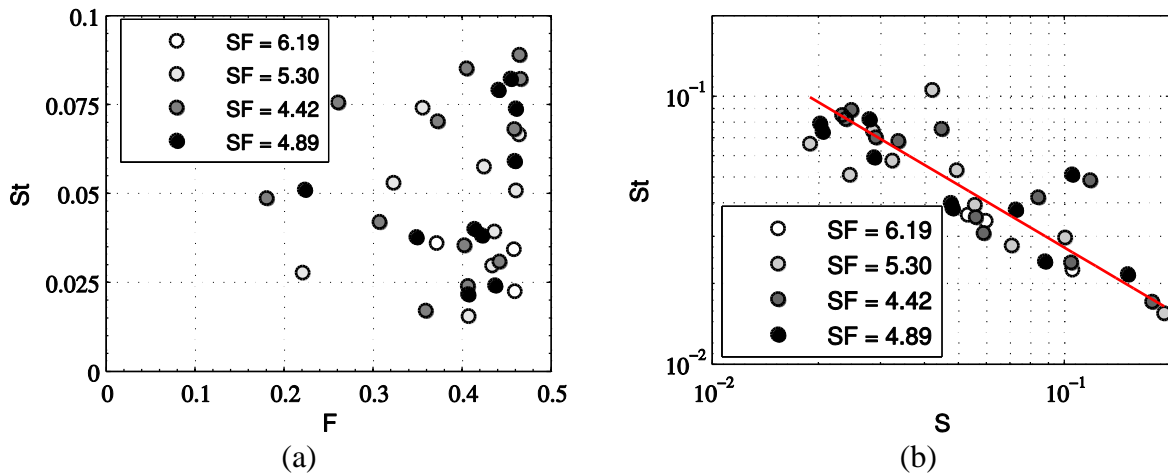
323 Fig. 2 Principles for extracting the characteristic lengths and frequency of the meandering jet



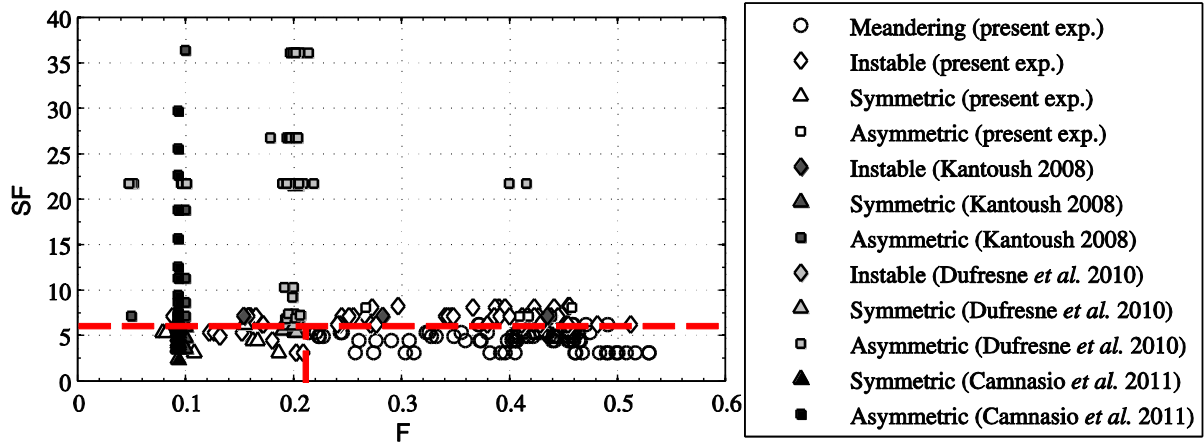
325 **Fig. 3 (a)** Characteristics lengths Λ_x/H (\square) and Λ_y/H (\circ) plotted with respect to F , for various SF .
 326 **(b)** Λ_x/H (\square) and Λ_y/H (\circ) as a function of S . S follows Eq. 6 and Eq. 7.
 327



328 **Fig. 4** Ratio of Λ_x and Λ_y plotted with respect to S , for various SF . Λ_x/Λ_y follows Eq. 8.
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 330



331 **Fig. 5 (a)** Strouhal number, St , plotted with respect to the Froude number F for various shape
 332 factor, SF . **(b)** St plotted relative to the friction number S . St follows Eq. 9.
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Fig. 6 Representation of the different flow regimes in shallow rectangular reservoirs as a function of SF and F . The horizontal dashed line reference $SF = 6.2$ and the vertical dashed line $F = 0.21$.