Bayesian method for predicting and modelling winter wheat biomass

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Abstract

The objectives of this paper are threefold. The first objective is to propose to use an Improved Particle Filtering (IPF) based on minimizing Kullback-Leibler divergence for crop models' predictions. The performances of the proposed technique are compared with those of the conventional Particle Filtering (PF) for improving nonlinear crop model predictions. The main novelty of this task is to develop a Bayesian algorithm for nonlinear and non-Gaussian state and parameter estimation with better proposal distribution. The second objective is to investigate the effects of practical challenges on the performances of state estimation algorithms PF and IPF. Such practical challenges include (i) the effect of measurement noise on the estimation performances and (ii) the number of states and parameters to be estimated. The third objective is to use the state estimation techniques PF and IPF for updating prediction of nonlinear crop model in order to predict winter wheat biomass. PF and IPF are applied at a dynamic crop model with the aim to predict a state variable, namely the winter wheat biomass, and to estimate several model parameters. Furthermore, the effect of measurement noise (e.g., different signal-to-noise ratios) on the performances of PF and IPF is investigated. The results of the comparative studies show that the IPF provides a significant improvement over the PF because, unlike the PF which depends on the choice of sampling distribution used to estimate the posterior distribution, the IPF yields an optimum choice of the sampling distribution, which also accounts for the observed data.

Keywords: Crop model; Bayesian methods; Biomass.

1. Introduction

Dynamic crop models such as EPIC [1], SALUS [2], and STICS [3] are non-linear models that describe the growth and development of a crop interacting with environmental factors (soil and climate) and agricultural practices (crop species, tillage type, fertilizer amount…). They are developed to predict crop yield and quality or to optimize the farming practices in order to satisfy agricultural objectives, as the reduction of nitrogen lixiviation. More recently, crop models are used to simulate the effects of climate changes on the agricultural production. Nevertheless, the prediction errors of these models may be important due to uncertainties in the estimates of initial
values of the states, in input data, in the parameters, and in the equations. The measurements needed to run the model are sometimes not numerous, whereas the field spatial variability and the climatic temporal fluctuations over the field may be high. The degree of accuracy is therefore difficult to estimate, apart from numerous repetitions of measurements. For these reasons, the problem of state/parameter estimation represents a key issue in such nonlinear and non-Gaussian crop models including a large number of parameters, while measurement noise exists in the data.

For example, it is useful to predict the evolution of variables, such as the biomass and the grain protein content during the crop lifecycle. State estimation techniques can be of a great value to solve that problem since they have the potential to estimate simultaneously the variables and several parameters. As an example, involved parameters are the radiation use efficiency, the maximal value of the ratio of intercepted to incident radiation, the coefficient of extinction of radiation, the maximal value of LAI. Several estimation techniques, such as Particle filtering [4] method has been developed and utilized in many applications. PF methods approximate the posterior probability distribution by a set of weighted samples, called particles. Since real world problems usually involve high dimensional random variables with complex uncertainty, the nonparametric and sample-based estimation of uncertainty has thus become quite popular to capture and represent the complex distribution in nonlinear and non-Gaussian models [5]. PF methods offer a number of significant advantages over other conventional methods. However, since they use the prior distribution as the importance distribution [6], the latest data observation is not considered and not taken into account when evaluating the weights of the particles. While the importance sampling distribution has computational advantages, it can cause filtering divergence. In cases where the likelihood distribution is too narrow compared to the prior distribution, few particles will have significant weights. Hence, a better proposal distribution that takes the latest observation data into account is needed. In other words, new adaptive methods that incorporate better feedback and smoothing in the selection or deletion of particles and their weights need to be investigated. The objectives of this paper are twofold. The first objective is to develop an improved Particle filtering (IPF) for improving nonlinear and non-Gaussian crop model predictions. In case of standard PF, the latest observation is not considered for the evaluation of the weights of the particles as the importance function is taken to be equal to the prior density function. This choice of importance sampling function simplifies the computation but can cause filtering divergence. In cases where the likelihood function is too narrow compared to the prior distribution, very few particles will have significant weights. Hence, a better proposal distribution that takes the latest observation into account is needed. The main novelty of this task is to develop new Bayesian algorithm for nonlinear and non-Gaussian state and parameter estimation with better proposal distribution based on minimizing Kullback-Leibler divergence.

The second objective is to investigate the effects of practical challenges on the performances of state estimation algorithms PF and IPF. Such practical challenges include (i) the
effect of measurement noise on the estimation performances and (ii) the number of states and parameters to be estimated.

The third objective is to apply the proposed state estimation techniques PF and IPF for predicting and modeling biomass and grain protein content. We present an application of PF and IPF to a dynamic crop model with the aim to predict a single state variable, namely winter wheat biomass.

The rest of the paper is organized as follows. In Section II, a description of proposed improved particle filtering for nonlinear crop model predictions and modeling is presented. Then, in Section III, the performances of the proposed new improved particle filtering are evaluated and compared to the standard particle filtering through the application cases. Finally, some concluding remarks are presented in Section IV.

2. Improved Particle Filtering Description

The choice of optimal proposal function is one of the most critical design issues in importance sampling schemes. In [7], the optimal proposal distribution \( \hat{p}(z_k | z_{0:k-1}, y_{0:k}) \) is obtained by minimizing the variance of the importance weights given the states \( z_{0:k-1} \) and the observations data \( y_{0:k} \). This selection has also been studied by other researchers. However, this optimal choice suffers from one major drawback. The particles are sampled from the prior density \( p(z_k | z_{0:k-1}) \) and the integral over the new state need to be computed. In the general case, closed form analytic expression of the posterior distribution of the state is untractable [8]. Therefore, the distribution \( p(z_k | z_{0:k-1}) \) is the most popular choice of proposal distribution. One of its advantages is its simplicity in sampling from the prior functions \( p(z_k | z_{0:k-1}) \) and the evaluation of weights \( l_k^{(i)} \) (as presented in the previous section). However, the latest observation is not considered for the computation of the weights of the particles as the importance density is taken to be equal to the prior density ([8]). The transition prior \( p(z_k | z_{0:k-1}) \) does not take into account the current observation data \( y_k \), and many particles can be wasted in low likelihood areas. This choice of importance sampling function simplifies the computational complexity but can cause filtering divergence [8]). In cases where the likelihood density is too narrow as compared to the prior function, very few particles will have considerable weights. Next, we present an overview of KLD-based improved particle filter.

a. Improved Particle Filter based on KLD minimization

As mentioned above, the distribution of interest for the state takes the form of a marginal posterior distribution \( p(z_k | y_{0:k}) \). The proposed extended Bayesian sampling algorithm (also named as improved particle filtering, IPF) is proposed for approximating intractable integrals arising in Bayesian statistics. By using a separable approximating distribution \( q(z_k) = p(z_k | z_{0:k-1}, y_{0:k}) = \Pi_i p(z_{i,k}) \) to lower bound the marginal likelihood, an analytical approximation to the posterior probability \( p(z_k | y_{0:k}) \) is provided by minimizing the Kullback-Leibler divergence (KLD):
\begin{equation}
D_{KL}(q||p) = \int q(z_s) \log \frac{q(z_s)}{p(z_s|z_{a_{t-1}}, y_{a_{t}})} dz_s
\end{equation}

Where, 
\begin{equation}
\hat{q}(z_s) = \Pi_{i} \hat{q}(z_{s}^{i} | z_{a_{t-1}}, y_{a_{t}}) = \hat{q}(z_{s}^1) \hat{q}(\mu_i) \hat{q}(\lambda_i)
\end{equation}

Minimizing the KLD subject to the constraint \( \int q(z_s) dz_s = \Pi \int q(z_s^i) dz_s^i = 1 \), the Lagrange multiplier scheme is used to yield the following approximate distribution,
\begin{equation}
\hat{q}(z_s^i) \propto \exp \left[ E(\log(p(y_{0:k}, z_s))_{\mu_{i}, \lambda_{i}}) \right]
\end{equation}

where \( E(\log(p(y_{0:k}, z_s))_{\mu_{i}, \lambda_{i}}) \) denotes the expectation operator relative to the distribution \( \hat{q}(z_s^i) \). Therefore, these dependent parameters can be jointly and iteratively updated. Taking into account the separable approximate distribution \( \hat{q}(z_s^i) \) at time \( k - 1 \), the posterior distribution \( p(z_s|y_{0:k}) \) is sequentially approximated according to the following scheme:
\begin{equation}
\hat{p}(z_s|y_{0:k}) \propto p(y_{0:k}|z_s^1) p(z_s^1, \lambda_s) q_p(\mu_s)
\end{equation}

where \( q_p(\mu_s) = \int p(\mu_s | \mu_{i-1}, \lambda_{i}) \hat{q}(\mu_{i-1}) d\mu_{i-1} \)

Hence, the particles \( \{z_{0:k}^{(i)}\}_{i=0}^{N} \) are sampled according to the following optimal function:
\begin{equation}
\hat{q}(z_s^i) = \int N(z_s^i|\mu_s, \lambda_s) p(\mu_s, \lambda_s) p(z_s^i|z_s^{i-1}) \hat{q}(\mu_{i-1}) d\mu_s d\lambda_s
\end{equation}

The recursive estimate of the importance weights can be derived as follows:
\begin{equation}
f_k^{(i)} = f_k^{(i-1)} p(y_{0:k}|z_{0:k}^{(i)}) p(z_{0:k}^{(i)}|z_{0:k}^{(i-1)}) \hat{q}(z_{0:k}^{(i)}|y_{0:k})
\end{equation}

Equation (6) provides a mechanism to sequentially update the importance weights, given an appropriate choice of proposal distribution, \( \hat{q}(z_{0:k}^{(i)}|y_{0:k}) \). Then, the estimate of the augmented state \( \hat{z}_s \) can be approximated by a Monte Carlo scheme as follows:
\begin{equation}
\hat{z}_s = \sum_{i=0}^{N} f_k^{(i)} z_{s}^{(i)}
\end{equation}

2. Simulation Results Analysis

2.1. A dynamic model simulating wheat biomass

2.1.1. The overall formalism
In this section, we describe a simple dynamic crop model that will be used to compare the performances of PF and IPF. The crop model has a single state variable representing above-ground winter-wheat biomass. This state variable is simulated on a daily basis in function of the daily temperature and the daily incoming radiation according to the classical method presented in ([9]). The biomass at time \( k+1 \) is linearly related to the biomass at time \( k \) as follows:

\[
Biom_{k+1} = Biom_k + E_b E_{r_{\text{max}}} \left(1 - e^{-K_{LAI_k}}\right) PAR_k + w_k,
\]

where \( k \) is the day number since sowing, \( Biom_k \) is the true above-ground plant biomass on day \( k \), \( PAR_k \) is the incoming photosynthetically active radiation on day \( k \), \( LAI_k \) is the leaf-area index on day \( k \) and \( w_k \) is a random term representing the model error. The crop biomass at sowing is set equal to zero: \( Biom_0 = 0 \). \( LAI_k \) is calculated in function of the cumulative degree-days (over a basis of 0°C) from sowing until day \( k \), noted \( T_c \), as follows ([10]):

\[
LAI_k = L_{\text{max}} \left(1 + \frac{1}{e^{A(T_c - T_{s1})} - e^{-B(T_c - T_{s2})}}\right),
\]

where the parameter \( T_{s2} \) is set equal to \( \frac{1}{B \log(1 + e^{AT_{s1}})} \) in order to have \( LAI_1 = 0 \).

The model includes two input variables \( X_k = [T_c, PAR_k] \) and seven parameters \((E_b, E_{r_{\text{max}}}, K, L_{\text{max}}, A, B, T_{s1})\). \( E_b \) is the radiation use efficiency which expresses the biomass produced per unit of intercepted radiation, \( E_{r_{\text{max}}} \) is the maximal value of the ratio of intercepted to incident radiation, \( K \) is the coefficient of extinction of radiation, \( L_{\text{max}} \) is the maximal value of \( LAI \), \( T_{s1} \) defines a temperature threshold, and \( A \) and \( B \) are two additional parameters. At this stage, the parameter values are assumed to be known and obtained from ([10]).

We suppose that \( N \) measurements of biomass, \( y_1, y_2, y_3, ..., y_N \), are made at different times before harvest on the site-year of interest. In practice, values of \( y_k \) can be derived from plant samples or from remote-sensing data. We assume that each measurement \( y_k \) is related to the biomass \( Biom_k \) by

\[
y_k = Biom_k + v_k
\]

where \( v_k \) is a random term representing measurement errors. In the next section we show how such measurements can be used to improve the accuracy of biomass predictions.

### 2.1.2. Numerical application

#### 2.1.2.1. Estimation of the biomass

Based on the equation (9), the Biomass is estimated at each date of measurement using both IPF and PF algorithms (Fig. 1). Table 1 illustrates the Root Mean Square Error (RMSE) using the
two algorithms PF and IPF. Fig. 1 and Table 1 show that IPF outperforms PF, these advantages of the IPF are due to the fact it provides an optimum choice of the sampling distribution used to approximate the posterior density function, which also accounts for the observed data.

![Graph showing estimation of state variable Biomass (g/m2) versus N (days) using PF and IPF techniques.]

**Fig. 1.** Estimation of state variable Biomass (g/m2) versus N (days) using PF and IPF techniques.

<table>
<thead>
<tr>
<th>Technique</th>
<th>ERROR</th>
<th>Biomass (Kg/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF</td>
<td>6.328</td>
<td></td>
</tr>
<tr>
<td>IPF</td>
<td>3.743</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.** ERROR of estimated Biomass

2.1.2.2. *Estimation of the biomass and of several parameters*

The model (9) assumes that the parameters are fixed and/or have been determined previously. However, the model involves several parameters that are usually not exactly known, or that have to be estimated. Estimating these parameters to completely define the model usually requires several experiment setups, which can be expensive and challenging in practice. Hence, in a second step, we propose to use PF and IPF to simplify the task of modeling compared to the conventional experimental intensive methods. Let’s thus consider some of the parameters that
have to be estimated to improve the simulations, by example: $E_b$ is the radiation use efficiency which expresses the biomass produced per unit of intercepted radiation, $E_{\text{max}}$ is the maximal value of the ratio of intercepted to incident radiation, $K$ is the coefficient of extinction of radiation, $L_{\text{max}}$ is the maximal value of LAI, $T_{st}$ defines a temperature threshold, and $A$ and $B$ parameters. To estimate these parameters, the following equations that describe their evolution are also needed:

$$
E_{b,k} = E_{b,k-1} + \gamma_{k-1}^j, \ E_{\text{max},k} = E_{\text{max},k-1} + \gamma_{k-1}^2, \ K_k = K_{k-1} + \gamma_{k-1}^3, \ L_{\text{max},k} = L_{\text{max},k-1} + \gamma_{k-1}^4
$$

$$
A_k = A_{k-1} + \gamma_{k-1}^5, \ B_k = B_{k-1} + \gamma_{k-1}^6, \ T_{st,k} = T_{st,k-1} + \gamma_{k-1}^7
$$

where, $\gamma_{j=1,...,7}$ is a process Gaussian noise with zero mean and known variance $\sigma_\gamma^2$. Combining (11), (10) and (9), one obtains:

The biomass at time $k+1$ is linearly related to the biomass at time $k$ as follows:

$$
Biom_{k+1} = Biom_k + E_{b,k} E_{\text{max},k} \left(1 - e^{-K_k \text{LAI}_k}\right) \text{PAR}_k + w_k,
$$

and the leaf-area index $\text{LAI}_k$ on day $k$ is given by:

$$
\text{LAI}_k = L_{\text{max},k} \left(1 + e^{-A_k \left(T_k - T_{st,k}\right)} - e^{-B_k \left(T_k - T_{z2,k}\right)}\right),
$$

where the parameter $T_{z2,k}$ is set equal to

$$
\frac{1}{B_k \log(1 + e^{A_k z_k})}
$$

in order to have $\text{LAI}_1 = 0$.

Combining (12) and (13), one obtains:

$$
f_1 : \ Biom_k = Biom_{k-1} + E_{b,k-1} E_{\text{max},k-1} \left(1 - e^{-K_{k-1} \text{LAI}_{k-1}}\right) \text{PAR}_{k-1} + w_{k-1},
$$

$$
f_2 : \ E_{b,k} = E_{b,k-1} + \gamma_{k-1}^j,
$$

$$
f_3 : \ E_{\text{max},k} = E_{\text{max},k-1} + \gamma_{k-1}^2,
$$

$$
f_4 : \ K_k = K_{k-1} + \gamma_{k-1}^3,
$$

$$
f_5 : \ L_{\text{max},k} = L_{\text{max},k-1} + \gamma_{k-1}^4,
$$

$$
f_6 : \ A_k = A_{k-1} + \gamma_{k-1}^5,
$$

$$
f_7 : \ B_k = B_{k-1} + \gamma_{k-1}^6,
$$

$$
f_8 : \ T_{st,k} = T_{st,k-1} + \gamma_{k-1}^7
$$

where $f_{k=1,...,8}$ are some nonlinear functions and where $w$ and $\gamma_{j=1,...,7}$ are respectively the measurement and process noise vector, which quantify randomness at both levels. In other words, we are forming the augmented state: $z_k = [x_k \theta_k]^T$ which is the vector that we wish to estimate. It can be given by a 8 by 1 matrix:

$$
x_k \rightarrow Biom_k, \ x_k(2,:) \rightarrow E_{b,k}, \ x_k(3,:) \rightarrow E_{\text{max},k}, \ x_k(4,:) \rightarrow K_k
$$

$$
x_k(5,:) \rightarrow L_{\text{max},k}, \ x_k(6,:) \rightarrow A_k, \ x_k(7,:) \rightarrow B_k, \ x_k(8,:) \rightarrow T_{st,k}
$$

The idea here is that, if a dynamic model structure is available, the model parameters can be estimated using one of state estimation technique, PF and IPF. To characterize the ability of the different approaches to estimate both the states and the parameters at same time, we have
chosen true parameter values and then tested each technique to see how well it could retrieve these true parameter values given the data. It was thus possible to calculate the quality of the estimated parameters and the predictive quality of the adjusted model for each method.

It can be seen from the results presented in Table 1 and Table 2 that the IPF outperforms PF (i.e., provides smaller ERROR for the state variables). These results confirm those obtained in the first comparative study, where only the state variables are estimated. The advantages of the IPF over the PF can also be seen through its abilities to estimate the model parameters. The results also show that the number of estimated parameters affect the estimation accuracy of the estimated state variables. In other words, for all estimation techniques, the estimation ERROR of Biomass increases from the first comparative study (where only the state variables are estimated) to case 1 (where seven parameters, $E_b$, $E_{\text{max}}$, $K$, $L_{\text{max}}$, $T_{s1}$, $A$ and $B$, are estimated).

In order to investigate the performance of the PF and IPF estimation algorithms versus the number of states and parameters to be estimated. Tables 1 and 2 compare the estimated of the crop model parameters using the two techniques PF and IPF for the different number of states and parameters to be estimated. For example, for the PF estimation technique, the estimation error of the Biomass $\text{Biom}_{k}$, increases from the first comparative study (states and parameters to be estimated = 2) to case (where the number of states and parameters to be estimated = 8). For example, the ERRORs obtained using PF the Biomass $\text{Biom}_{k}$ where the number of states and parameters to be estimated = 2 and = 8 are 6.346, and 6.768, respectively, which increase as the number of states and parameters to be estimated increases (refer to Table 1). This observation is valid for IPF technique (refer to Table 2).

<table>
<thead>
<tr>
<th>True parameter set</th>
<th>$E_b$</th>
<th>$E_{\text{max}}$</th>
<th>$K$</th>
<th>$L_{\text{max}}$</th>
<th>$T_{s1}$</th>
<th>$A$</th>
<th>$B$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.2</td>
<td>1200</td>
<td>0.0032</td>
<td>0.0024</td>
<td>6.346</td>
</tr>
<tr>
<td>Number of states and parameters to be estimated=2 ($z_k = [\text{Biom}<em>k E</em>{b,k} ]^T$)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>6.394</td>
</tr>
<tr>
<td>$z_k = [\text{Biom}<em>k E</em>{b,k} E_{\text{max},k} ]^T$</td>
<td>1</td>
<td>0.48</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>6.414</td>
</tr>
<tr>
<td>$z_k = [\text{Biom}<em>k E</em>{b,k} E_{\text{max},k} K_k ]^T$</td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>6.459</td>
</tr>
<tr>
<td>$z_k = [\text{Biom}<em>k E</em>{b,k} E_{\text{max},k} K_k L_{\text{max},k} ]^T$</td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.18</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>6.564</td>
</tr>
<tr>
<td>$z_k = [\text{Biom}<em>k E</em>{b,k} E_{\text{max},k} K_k L_{\text{max},k} T_{s1,k} ]^T$</td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.175</td>
<td>1198</td>
<td>--</td>
<td>--</td>
<td>6.621</td>
</tr>
<tr>
<td>$z_k = [\text{Biom}<em>k E</em>{b,k} E_{\text{max},k} K_k L_{\text{max},k} T_{s1,k} A_k ]^T$</td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.178</td>
<td>1197</td>
<td>0.00318</td>
<td>--</td>
<td>6.768</td>
</tr>
<tr>
<td>$z_k = [\text{Biom}<em>k E</em>{b,k} E_{\text{max},k} K_k L_{\text{max},k} T_{s1,k} A_k B_k ]^T$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>6.768</td>
</tr>
</tbody>
</table>
Table 1. PF estimations of the values of the crop model parameters versus the number of states and parameters to be estimated

<table>
<thead>
<tr>
<th>True parameter set</th>
<th>$E_b$</th>
<th>$E_{\text{max}}$</th>
<th>$K$</th>
<th>$L_{\text{max}}$</th>
<th>$T_s$</th>
<th>$A$</th>
<th>$B$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.2</td>
<td>1200</td>
<td>1196</td>
<td>0.00172</td>
<td>0.0022</td>
<td>3.573</td>
</tr>
</tbody>
</table>

Number of states and parameters to be estimated $= 2$ ($z_k = [\text{Biom}_k, E_{b,k}]^T$)

1. $z_k = [\text{Biom}_k, E_{b,k}, E_{\text{max},k}]^T$

1. $z_k = [\text{Biom}_k, E_{b,k}, E_{\text{max},k}, K_k]^T$

1. $z_k = [\text{Biom}_k, E_{b,k}, E_{\text{max},k}, K_k, L_{\text{max},k}]^T$

Table 2. IPF estimations of the values of the crop model parameters versus the number of states and parameters to be estimated

<table>
<thead>
<tr>
<th>True parameter set</th>
<th>$E_b$</th>
<th>$E_{\text{max}}$</th>
<th>$K$</th>
<th>$L_{\text{max}}$</th>
<th>$T_s$</th>
<th>$A$</th>
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Number of states and parameters to be estimated $= 2$ ($z_k = [\text{Biom}_k, E_{b,k}]^T$)

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1. $z_k = [\text{Biom}_k, E_{b,k}, E_{\text{max},k}, K_k, L_{\text{max},k}]^T$

2.1.2.3. Presence of a noise in the data

Here, we assume that a Gaussian noise is added to the time profiles of Biomass. In order to show the performance of the PF and IPF estimation algorithms in the presence of measurement noise, four different measurements noise values, $10^{-4}$, $10^{-3}$, $10^{-2}$ and $10^{-1}$, are considered. The final estimated values of the crop model parameters are summarized in Tables 3 and 4. The simulation results of estimating the states Biomass using PF and IPF when the variances noise vary in $\{10^{-4}, 10^{-1}\}$ are shown in Tables 3 and 4. In other words, for the PF estimation technique, the estimation ERROR of the Biomass $\text{Biom}_k$, increases from the first comparative study (noise variance $= 10^{-4}$) to case (where the noise variance $= 10^{-1}$). For example, the ERRORs obtained using PF for Biomass where the noise variance $= 10^{-4}$ and $= 10^{-1}$.
are 6.248, and 6.674, respectively, which increase as the noise variance increases (refer to Table 3). This observation is valid for the IPF algorithm (refer to Table 4).

### Table 3. PF estimations of the values of the crop model parameters versus noisy measurement variances

<table>
<thead>
<tr>
<th>True parameter set</th>
<th>$E_b$</th>
<th>$E_{i_{max}}$</th>
<th>$K$</th>
<th>$L_{max}$</th>
<th>$T_{sl}$</th>
<th>$A$</th>
<th>$B$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.2</td>
<td>1200</td>
<td>0.0032</td>
<td>0.0024</td>
<td></td>
<td>6.248</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-4}</td>
<td>0.99</td>
<td>0.479</td>
<td>0.519</td>
<td>6.19</td>
<td>1199</td>
<td>0.00319</td>
<td>0.00238</td>
<td>6.314</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-3}</td>
<td>0.98</td>
<td>0.475</td>
<td>0.518</td>
<td>6.18</td>
<td>1198</td>
<td>0.00318</td>
<td>0.00236</td>
<td>6.453</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-2}</td>
<td>0.97</td>
<td>0.469</td>
<td>0.517</td>
<td>6.16</td>
<td>1197</td>
<td>0.00315</td>
<td>0.00231</td>
<td>6.674</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-1}</td>
<td>0.95</td>
<td>0.46</td>
<td>0.515</td>
<td>6.14</td>
<td>1195</td>
<td>0.00312</td>
<td>0.00223</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4. IPF estimations of the values of the crop model parameters versus noisy measurement variances

<table>
<thead>
<tr>
<th>True parameter set</th>
<th>$E_b$</th>
<th>$E_{i_{max}}$</th>
<th>$K$</th>
<th>$L_{max}$</th>
<th>$T_{sl}$</th>
<th>$A$</th>
<th>$B$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.2</td>
<td>1200</td>
<td>0.0032</td>
<td>0.0024</td>
<td></td>
<td>3.641</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-4}</td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.2</td>
<td>1200</td>
<td>0.0032</td>
<td>0.0024</td>
<td>3.683</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-3}</td>
<td>1</td>
<td>0.48</td>
<td>0.52</td>
<td>6.2</td>
<td>1200</td>
<td>0.0032</td>
<td>0.0024</td>
<td>3.724</td>
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<tr>
<td>PF estimates: noisy measurement variance= 10^{-2}</td>
<td>0.986</td>
<td>0.475</td>
<td>0.5185</td>
<td>6.18</td>
<td>1198</td>
<td>0.00318</td>
<td>0.00236</td>
<td>3.815</td>
</tr>
<tr>
<td>PF estimates: noisy measurement variance= 10^{-1}</td>
<td>0.95</td>
<td>0.471</td>
<td>0.5179</td>
<td>6.173</td>
<td>1197</td>
<td>0.00316</td>
<td>0.00231</td>
<td></td>
</tr>
</tbody>
</table>

### CONCLUSIONS

In this paper, we applied the state estimation techniques for crop model predictions and modeling. In the comparative study, we presented an application of PF and IPF to a linear dynamic crop model predicting only one state variable, namely winter wheat biomass and
estimating several model parameters. In addition to comparing the performances of the state estimation techniques; Particle Filter (PF), and improved Particle Filter (IPF), the effect of number of estimated model parameters on the accuracy and convergence of these techniques are also assessed.

The results of the comparative studies show that the IPF provides a significant improvement over the PF because, unlike the PF which depends on the choice of sampling distribution used to estimate the posterior distribution, the IPF yields an optimum choice of the sampling distribution, which also accounts for the observed data. We have investigated the effects of practical challenges on the performances of Particle Filter (PF), and improved Particle Filter (IPF). The comparative analysis is conducted to study the effects of two practical challenges (measurement noise, and the number of states and parameters to be estimated) on the estimation performances of PF, and IPF. To study the effect of measurement noise on the estimation performances, several measurement noise contributions (e.g., different signal-to-noise ratios) are considered. Then, the estimation performances of PF and IPF are compared for different noise levels. Similarly, to investigate the effect of the number of states and parameters to be estimated on the estimation performances of PF and IPF, the estimation performance is analysed for different numbers of estimated states and parameters. The performance of the proposed method is evaluated on a synthetic example in terms of estimation accuracy, and root mean square error.

REFERENCES


