A POSSIBLE POLE IN THE \( \pi N \) VERTEX FUNCTION 
AND THE LOW ENERGY \( \pi N \) SCATTERING

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Using the \( \pi N \) phase shifts and inelasticities in the \( P_{11} \) and \( S_{11} \) partial waves, we have calculated the pion nucleon vertex function with one nucleon off-mass-shell through a dispersion approach (the kinematical situation is shown in Fig. 1). We can summarize the result as follows. (i) As a function of \( W (\equiv \pm \sqrt{s}) \) the vertex function \( \Gamma (W) \) has two poles in the unphysical region \( |W| < m_N + m_\pi \): at \( W_+ \sim 1030 \) MeV and \( W_- \sim -125 \) MeV respectively, whereas the dressed nucleon propagator \( S_F^p(W) \) has zeros at the same positions. (ii) The pole position \( W_+ \) is very much sensitive to input variations whereas \( W_- \) is pretty stable. Considering the nature of approximations we have made, the existence of the pole at \( W_- \) (and the corresponding zero of \( S_F^p(W) \) at \( W_- \)) will be definite. (iii) The physical amplitudes in the \( P_{11} \) and \( S_{11} \) may be written as

\[
\begin{align*}
\epsilon_{\pm} (W) & = - R_{\pm} \Gamma (\pm W) S_F^p (\pm W) \Gamma (\pm W) + \epsilon_{\text{irr}}^p (W) \\
(\pm \cdots P_{11}, - \cdots S_{11}) & \quad (W > 0) \quad (1)
\end{align*}
\]

In eq. (1) the first term which is the dressed (direct) nucleon pole term (\( R_{\pm} \) \((W) \) is some kinematical factor) has turned out to have pole(s) at \( W_- (W_+) \) with negative residue(s) (ghost(s) !). However, any such kind of pole has been shown to be cancelled exactly by the second term, viz. \( \epsilon_{\text{irr}}^p (W) \). Thus even if the vertex function does have a pole, we may not be able to conceive of its existence through physical scattering data.

Fortunately there appears one limiting situation where the contribution comes only from the dressed
A (+) (s = u = o, t) = \frac{G^2}{4\pi m_N} \left[ 1 - \Gamma^2(-m_N) J(-m_N) \right] = 1.07 \times \frac{G^2}{4\pi m_N}

with \quad J(W) \equiv \frac{S'_P(W)}{S_P(W)} , \quad S_P(W) \equiv \frac{1}{W-m_N}

(note that the above result is obtained not in the soft pion limit, so a deviation of \(O(m^2_W/m^2_N)\) is to be naturally expected). The fact that it is very close to Adler's value is due to the small value of \(\Gamma(-m_N)\), and the small \(\Gamma(-m_N)\) has resulted from the very existence of the pole at \(W_\perp\) which has absorbed a good part of the strength in \(\Gamma(W)\). Furthermore, the above result indicates that the effective \(\pi NN\) coupling to be used in low energy \(\pi N\) processes prefers pseudo-vector type.

As for the possible existence of the pole at \(W_\perp\) we can say nothing definite at present.

REFERENCES