Pion-nucleon vertex function with one nucleon off shell

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The pion-nucleon vertex function with an off-mass-shell nucleon is obtained through sideways dispersion relations with the $P_{11}$ and $S_{11}$ pion-nucleon phase shifts as only input. Contrary to the recent calculation of Nutt and Shakin, we find that the proper and improper vertex functions behave quite differently, indicating the importance of the nucleon propagator dressing. In particular the proper vertex function is found to have two poles in the unphysical region.

[NUCLEAR REACTIONS pion-nucleon vertex functions, sideways dispersion relations, nucleon self-energy corrections.]

Recently, with applications to pion-nucleon physics in mind, Nutt and Shakin\(^1\) (hereafter referred as NS) performed a model calculation of the $\pi$NN vertex function in which one of the nucleon legs is put off the mass shell. Starting with a nonlinear integral equation for a completely off-shell vertex function and after numerous approximations (neglect of negative energy nucleon spinors, use of a separable form for the input $\pi N$ amplitude, etc.), their solution was found to be dominated by the $P_{11}$ Roper (1470 MeV) resonance. Their result also indicated the relative unimportance of the correction due to the nucleon self-energy, which they termed $\Delta^p$.

In view of the nature of the approximations made in the NS model, we think it worthwhile to reexamine the problem from a different methodological point of view. We make use of sideways dispersion relations together with a spectral representation of the single nucleon propagator, as in the work of Ida.\(^2\) This makes approximations of the NS type unnecessary.

To start with, we introduce the improper $\pi$NN vertex function of Bincer,\(^3\) $K(W)$, by\(^4\)

$$S^{-1}_F(q) (0 | \phi (0) | \bar{p} \alpha, q \alpha; \text{in}) = i \Lambda(Q) K(W) \Lambda(Q) K(-W) G \gamma_5 \sigma_{\mu \nu} (\bar{p}, \alpha),$$  \hspace{1cm} (1)

where $Q = p + q, Q = Q \gamma^\mu, W = (Q^2)^{1/2}; \psi$ is the nucleon interpolating field, $\Lambda(Q) = (W \pm Q)/2W$ the projector for a positive (negative) energy nucleon state,\(^5\) and $S_F(q) = 1/(q^2 - m^2)$ is the bare (Feynman) propagator for the nucleon with $m$ the nucleon mass. $K(W)$ represents the one nucleon off-shell vertex function normalized as

$$K(m) = 1,$$

so that $G$ is identified as the renormalized $\pi$NN coupling constant.

In a similar manner the proper vertex function $\Gamma(W)$ may be introduced with the following replacements in Eq. (1):

$$K(W) \rightarrow \Gamma(W), \quad S_F(q) \rightarrow S_F^+(q),$$

where $S_F^+(q)$ is the fully dressed nucleon propagator. Then we may write

$$\Gamma(W) = K(W)/J(W)$$  \hspace{1cm} (2)

with

$$J(W) = S_F^+(W)/S_F(W),$$  \hspace{1cm} (3)

from which it follows that [noting $J(m) = 1$]

$$\Gamma(m) = 1.$$  \hspace{1cm} (4)

As far as their analytic structure is concerned, $K(W), S_F^+(W)$, and hence $\Gamma(W)$ all have branch cuts starting at $W = (m \pm \mu)$ ($\mu$ is the pion mass). It is worthwhile to note that the vertex function $\Gamma$ of NS ($\Gamma = 1 + \Gamma_1 + \Gamma_2$) can be identified with our $\Gamma(W)$, while $1 + \Gamma_1$, which is their first approximation to $\Gamma$, corresponds to our $K(W)$ when a correct renormalization procedure is carried out in the NS scheme.

The discontinuity relation for $K(W)$ reads\(^3\)

$$\text{Im} K(z W) = f_s(W) + \sigma_s(W) \theta(W - m - 2\mu)$$

$$\hspace{1cm} (W > m + \mu)$$  \hspace{1cm} (4)

where $f_s(W)$ is the $\pi N$ amplitude for $P_{11}$ (positive sign) and $S_1$ (negative sign) which takes the form

$$f_s(W) = \frac{\eta_s(W) \exp[2i \delta_s(W)] - 1}{2i}.$$
with $\delta_{\gamma}(W)$ and $\eta_{\gamma}(W)$ the phase shift and inelasticity ($+\text{ for } P_{11}$, $-\text{ for } S_{11}$), respectively. In addition $\sigma_{\gamma}(W)$ in (4) is the contribution from higher mass (multiparticle) intermediate states.

\[
K(W) = P(W) \exp \left\{ \frac{W - m}{\pi} \int_{m+i\epsilon}^{\infty} \frac{\delta_{\gamma}(W^\prime)}{(W^\prime - m)(W^\prime - W - i\epsilon)} \right\}.
\]

In the absence of inelastic contributions, $K(W)$ could be written, through sideways dispersion relations, as a solution to the homogeneous Omnès-Maskellishvili equation:

\[\rho_{\gamma}^{\eta}(M) = \frac{3C_{\eta}^2}{32\pi^2} \frac{((M + m)^2 - \mu^2)^{1/2}}{(M + m)^2M^2} \times |K(\pm M)|^2.
\]

Because of the effective phase in Eq. (6) mentioned earlier some part of the multiparticle (inelastic) contribution is effectively included in the above spectral functions. Thus once we get $K(W)$ from Eqs. (5) and (6), $J(W)$ and then $\Gamma(W)$ follow from Eqs. (7) and (2).

Phase shifts (and inelasticity parameters) are taken from (i) an analytic expression \superscript{7} up to $W - 1300$ MeV, (ii) the most recent amplitude analysis by Pietarinen\superscript{8} for the intermediate region ($1300 \text{ MeV} < W < 4000$ MeV), and (iii) a Regge model\superscript{9} for higher energies where no data are available. Owing to the once-subtracted form of the integral in Eq. (5), no artificial damping of the phase shifts to zero is necessary for $W - \infty$ [our $\delta_{\gamma}(W)$ approaches $\pi/2$ in that limit]. Such damped phases would make $K(W)$ constant ($>0$) asymptotically.\superscript{10} The improper vertex function $K(W)$ thus obtained is shown in Figs. 1 and 2 (in solid lines). Like the NS model, its structure is governed by the resonances $P_{11}$ ($1470$ and $1780$) for $W > m + \mu$ and $S_{11}$ ($1520$ and $1700$) for $W < -(m + \mu)$. It may be worth pointing out that there is a third bump in Im$K(W)$ above $W = 2000$ MeV. Its origin is not known to us; perhaps it may be associated with a possible new resonance coming out of the Pietarinen analysis.

With regard to the value of $K(W)$ below the physical threshold we find

$K(-m) = 1.03$

consistent with the soft pion result: $K(-m) = 1/g_{A}(0)$,\superscript{11} where $g_{A}(t)$ is the nucleon axial-vector form factor. We note in passing that it is important to keep $\delta_{\gamma}(W)$ in the evaluation of $K(W<0)$ as its omission leads to $K(-m) = 2.83$. e.g. On the other hand, a neglect of $\delta_{\gamma}(W)$ for $K(W>0)$ makes only a small change, of the order of $10\%$. The
contribution from the resonances $P_{11}$ (1470) and $S_{11}$ (1520 and 1700) to the spectral functions at not too high energies. A correct inclusion of the multiparticle contributions $\delta \rho_{\nu}(M)$ would only increase the magnitude of $\rho_{\nu}(M)$ at higher energies; hence the possible existence of zeros in $J(W)$ would be further strengthened.\textsuperscript{13}

However, one might ask how reliable the effective phase in (6) is for the accurate evaluation of $K(W)$ and hence of $\rho_{\nu}^{\text{eff}}(W)$ throughout those important inelastic resonance regions. A definite answer may only be given after solving for $K(W)$ with an inclusion of $\sigma_{\nu}(W)$ in (4). We are planning such a calculation, but for the time being we shall be content with a couple of sensitivity tests through varying $\rho_{\nu}^{\text{eff}}(M)$. First we use a different set of phase shifts and inelasticities to calculate them. For this purpose we have used the CERN theoretical phases\textsuperscript{14} for $1300 < W < 1900$ (MeV) matched smoothly to both the lower and higher energy parts mentioned before. Its main differences from the Pietarinen phases are as follows: (i) On the average it is less inelastic above the main resonances and (ii) the $S_{11}$ (1700) resonance is less eminent. The gross feature of $J(W)$ thus obtained is not much different from our former result except for a change in the locations of the zeros: $W_-, -1025$ MeV; $W_-, -140$ MeV. In the second test, using the Pietarinen phases we take $C \rho_{\nu}^{\text{eff}}(M)$ in place of $\rho_{\nu}^{\text{eff}}(M)$ in Eq. (7) and vary the constant $C$. Already for $C = 0.5$, $W_+$ has been lost and in its place appears a zero of $\text{Re} J(W)$ above but not far from threshold. For $C = 0.1$, even $\text{Re} J(W)$ becomes nonzero for all $W > m + \mu$ but $W_+$ still stays at $-700$ MeV.\textsuperscript{15} It is quite unlikely that we have overestimated our spectral functions by an order of magnitude. Thus at least one zero, $W_+$, will remain. On the other hand some attempt in putting the multiparticle effects [through $\sigma_{\nu}(W)$ and $\delta \rho_{\nu}(M)$] into $\rho_{\nu}(M)$ is certainly needed, together with an accurate phase shift determination in order to make a definite statement about $W_+$. Incidentally, there is another argument\textsuperscript{16} for the existence of zero(s) in $J(W)$ from a somewhat different line of reasoning.

In Figs. 1 and 2 $\Gamma(W)$ is plotted in dashed lines. Owing primarily to the zeros in $J(W)$, it looks considerably different from $K(W)$, contrary to the NS result, indicating the importance of their $\Gamma_2$ correction for the nucleon self-energy. In fact, in the NS model, nucleon propagators are always put on the mass shell. Together with the multiplicative renormalization procedure adopted there, this approximation might eventually end up with small corrections to $\Gamma$ from $\Gamma_2$, hence with $K(W) \approx \Gamma(W)$.

Clearly, $\Gamma(W)$ acquires poles at the zeros of
In contrast to $K(W)$ the proper vertex $\Gamma(W)$ stays quite small and negative throughout all the physical region. It seems that the poles in the unphysical region simply have absorbed the strength of $\Gamma(W)$.

Now one might ask if one could see the effect of these poles of $\Gamma(W)$ in some physical processes. For $nN$ scattering in the $P_{11}$ and $S_{11}$ partial waves the amplitudes take the form:

$$f_{\text{tot}}(W) = -R(W)\Gamma(W)S_{\text{n}}(W)\Gamma(W) + f_{\text{irr}}(W)$$

where the first term is the fully dressed nucleon pole contribution with $R(W)$ a kinematical factor and the second term the one nucleon irreducible amplitude. Obviously the first term gets poles at $W_1$. However, the residues of $\Gamma(W)S_{\text{n}}(W)\Gamma(W)$ there can be shown to be negative, that is, they are ghostlike. Fortunately, it was shown that those poles in the dressed nucleon pole term are canceled exactly by the poles in $f_{\text{irr}}(W)$, so that there is no inconsistent behavior in the $nN$ scattering amplitude [it is not difficult to show

that $f_{\text{irr}}(W)$ is unitary by itself; $\arg \Gamma(W) = \arg f_{\text{irr}}(W) \pmod{\pi}$, etc. for $|W| < m + \mu$, suggesting the existence of such poles in $f_{\text{irr}}(W)$]. This, however, makes it impossible to see the poles in pion-nucleon scattering. So if these poles are really there in $\Gamma(W)$, we are forced to look for a possibility of observing them in other processes involving pions e.g., in pion-nucleus systems.

Finally, it may be worth mentioning that the well-known unrealistic behavior of the $S$-wave $nN$ amplitude, which is calculated from the "bare" nucleon pole term in the $\gamma_5(PS)$ theory, especially near threshold is found to be greatly improved by the "dressed" one. In fact, the improvement turns out to come from the existence of the $\Gamma(W)$ pole at $W_c$. A more detailed and extended version of the present paper will be reported elsewhere.

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4. Our normalization convention is $\bar{\psi}_0 \psi_0 = \frac{2m}{2\sqrt{3}}$.
5. Note that $f(W)$ as a function of $W$ may generally be written as $f = \lambda W + \Lambda W$.
6. We prefer this choice over that of Goldberger and Treiman for the same reason as found in T. N. Pham and T. N. Truong, Phys. Rev. D 14, 185 (1976).
10. We have checked that in fact the Goldberger-Treiman phases $\delta_{\text{GT}}(W)$ constructed from our $\delta_{\text{GT}}(W)$ and $\eta_{\text{GT}}(W)$ become very small at high energies and the resulting $K(W)$ indeed approaches some positive constant.
13. Our spectral functions give the renormalization constant $Z_{\gamma}^n$ as

$$Z_{\gamma}^n = \lim_{M \to \infty} \int \frac{\alpha(M) + \rho(M)}{\pi} dM.$$