Comparison of parameterization schemes for solving the discrete material optimization problem of composite structures

P. Duysinx\textsuperscript{1}, M. Guillermo\textsuperscript{1}, T. Gao\textsuperscript{1,2}
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**INTRODUCTION**

- Development of new renewable energy systems: high performance materials (strength, durability...)

- Sustainability of transportation systems: lightweight solutions

- Revived interest in composite structures.
  - Optimization of composites to take the best of their performances
INTRODUCTION

- SS Great Britain (1843)
  - First ship to be built with an iron hull
  - Parts designed and fixed together according to the available technology at the time (wood technology)
  - If you don’t know it is metal you would think it is wood!
INTRODUCTION

- Boeing 787
  - Composite structure
  - BUT...
  - True potential is not fully explored
  - Composite structures are almost direct replicas of metallic structures
- Some of the issues
  - Repair
  - Failure modes
  - Systems interaction
- Weight saving =< 5% in current use
INTRODUCTION

- Classical design problems of composite structures to be addressed:
  - Optimal layout of laminates over the structure
  - Through-the-thickness-optimization of composites: stacking sequence optimization

- General/global approach to address simultaneously optimal layout and stacking sequence ➔ Discrete Material Optimization approach (Lund & Stegmann, 2005)
Discrete Material Optimization approach

- Formulate the optimization problem as a ‘n’ materials selection problem
- Use an extended topology optimization approach to solve the problem in continuous variables
  - Interpolation /parameterization scheme of material properties
  - Continuous optimization problem with penalization of intermediate solutions

\[ C_i = \sum_{j=1}^{m} w_{ij} C^{(j)}_i \]

- \( 0 \leq w_{ij} \leq 1 \)
- \( \sum_{j=1}^{m} w_{ij} \leq 1 \)
- \( w_{ik} = 0 \) (\( k \neq j \)) when \( w_{ij} = 1 \)
- Topology optimization of laminate
  - Selection of ply orientation and layout of the laminate
  - Add one an existence (density variable) $\mu$ in $[0,1]$

$$c^l = (\mu_l)^q \left( \sum_{i=1}^{n_l} w_i^l c_i^l \right)$$

- Introduce a volume constraint of the foam or of the fiber material

$$\sum_{l=1}^{n_v} \mu_l V_l \leq \bar{V}$$
This work:

- To compare DMO, SFP and BCP
- To investigate the approach parameters such as the penalization
- To tailor a robust solution procedure based on the sequential convex programming
- To validate the work with applications
  - Academic examples
  - Larger scale problems with real-life characteristics
DMO (Stegmann and Lund, 2005)

- DMO4 interpolation scheme:
  - Extension of Thomsen (1992) and Sigmund & Torquato (2000) topology optimization schemes
  
  \[ E = E_1 + x^p (E_2 - E_1) \]

- Introduces one existence variable ([0,1]) per material

- Uses a power law (SIMP) penalization of intermediate densities

\[ w_{ij} = x_{ij}^p \prod_{\xi=1}^{m_v} (1 - x_{i\xi}^p) \quad \text{with} \quad 0 \leq x_{ij} \leq 1 \]
Shape Function with Penalization (SFP)  
Bruyneel (2011)

- SFP scheme makes use of the Lagrange polynomial interpolation of finite element shape functions
  - For $0^\circ$ /$90^\circ$ /$45^\circ$ /$-45^\circ$: four-node finite element

\[ w_1 = \frac{1}{4}(1-R)(1-S) \quad w_2 = \frac{1}{4}(1+R)(1-S) \]
\[ w_3 = \frac{1}{4}(1+R)(1+S) \quad w_4 = \frac{1}{4}(1-R)(1+S) \]

- Introduces a power penalization (SIMP)

\[ w_i^{SFP} = \left[ \frac{1}{4}(1 \pm R)(1 \pm S) \right]^p \]
Shape Function with Penalization (SFP)
Bruyneel (2011)

- SFP shape functions and penalization

\[ w_i^{SFP} = \left[ \frac{1}{4} (1 \pm R)(1 \pm S) \right]^p \]

- Two design (instead of 4) variables ranging in [-1,1]
- Extension to ‘n’ node finite elements is theoretically possible, but problem rapidly complex in practice
Bi-value coded parameterization
(G. Tong et al. 2011)

- Bi-value coding parameterization generalizes the SFP scheme
- Abandon the shape function idea, but keep the idea of coding the materials using bi-value variables (typically [-1,1])

\[
w_{ij} = \left[ \frac{1}{2^{m_v}} \cdot \prod_{k=1}^{m_v} \left( 1 + s_{jk} x_{ik} \right) \right]^p \text{ with } -1 \leq x_{ik} \leq 1 \text{ and } k = 1, \ldots, m_v
\]

- Number of design variable is \( m_v = \log_2 m \)
  - Possible to interpolate between \( 2^{(m_v-1)} \) to \( 2^{m_v} \) materials with \( m_v \) variables
- Introduction of a penalization scheme (here power law) to end-up with -1/1 values
Bi-value coded parameterization

- Visualization: for $m_v=2$ and $m_v=3$, the method recovers 4-node and brick (8-node) elements shape functions.

<table>
<thead>
<tr>
<th></th>
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<th>3</th>
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<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
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<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
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</table>

Table 1 $s_{jk}$ values ($m_v=2, m=4$)

(a) $m_v=2, m=4$

(b) $m_v=3, m=8$
Penalization schemes

- To come to a solution with one single material, one introduces a penalization schemes:
  - SIMP
    \[ f(\chi) = \chi^p \]
  - RAMP (Stolpe & Svanberg, 2001)
    \[ f(\chi) = \frac{\chi}{1 + p(1 - \chi)} \]
  - Halpin Tsai (Halpin-Tsai, 1969)
    \[ f(\chi) = \frac{r \chi}{(1 + r) - \chi} \]
  - Polynomial penalization (Zhu, 2009)
    \[ f(\chi) = \frac{\alpha - 1}{\alpha} \chi^p + \frac{1}{\alpha} \chi \]
Optimization Problem Formulation

- Compliance minimization under given load cases

\[
\min \{x_{ik}\} \quad (i=1,\ldots,n; k=1,\ldots,m_v)
\]

\[
C = F^T u
\]

subject to: \(\sum_{l=1}^{n_v} \mu_l V_l \leq V\)

- For pure laminate optimization, no resource (volume) constraint is generally necessary

- Sensitivity analysis

\[
\frac{\partial C}{\partial x_{ik}} = 2u^T \frac{\partial F}{\partial x_{ik}} - u^T \frac{\partial K}{\partial x_{ik}} u = -u^T \frac{\partial K}{\partial x_{ik}} u
\]

- Requires the derivatives of the weighting functions

\[
\frac{\partial K_i}{\partial x_{ik}} = \sum_{j=1}^{m} \frac{\partial w_{ij}}{\partial x_{ik}} K_i^{(j)}
\]
## Optimization Problem Formulation

- **Maximization of fundamental natural frequencies**

  \[
  \min_{\{x_{ik}\}} \quad \omega^2 \\
  \text{subject to:} \quad (K - \omega^2 M)u = 0 \\
  \quad -1 \leq x_{ik} \leq 1
  \]

  \[
  M_i = \sum_{j=1}^{m} v_{ij} M_{ij} \\
  v_{ij} = \left[ \frac{1}{2^m} \cdot \prod_{k=1}^{m_v} (1 + s_{ik} x_{ik}) \right]^{p_M}
  \]

- **Sensitivity analysis**

  \[
  \frac{\partial \omega^2}{\partial x_{ik}} = \frac{u_i^T \frac{\partial K_i}{\partial x_{ik}} u_i - \omega^2 u_i^T \frac{\partial M_i}{\partial x_{ik}} u_i}{u_i^T M u}
  \]

  - The derivatives can be either positive or negative (non-monotonic function)
Implementation

- Implementation
  - Analysis carried out in SAMCEF Composites
    - Laminate plate elements
    - Thick composite shells (8-node bricks)
  - Optimization
    - Boss Quattro Open Object Oriented platform for Optimization
Implementation & Solution procedure

Solution of optimization problem: Sequential Convex Programming

- Sequence of explicit subproblems
  - CONLIN (Fleury, 1989)
  - GCMMA (Bruyneel et al., 2002)
- General strategy with efficient capabilities in treating large scale Problems

Remarks:

- For CONLIN variables must be >0, so a change of variables is necessary. For instance:

\[ z_i = \frac{x_i + 1}{2} \]
Numerical applications: Square plate under vertical force

- Maximum in-plane compliance problem is solved by selecting the optimal orientation of the ply

Loads and boundary conditions

Table 4 Material properties

<table>
<thead>
<tr>
<th></th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$G_{xy}$</th>
<th>$v_{xy}$</th>
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<tr>
<td></td>
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Table 3 Orientations

<table>
<thead>
<tr>
<th>Number of material phases $(m)$</th>
<th>Number of design variables for each region $(m_v)$</th>
<th>Discrete orientation angle (°)</th>
</tr>
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<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>90/45/0/-45</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>80/60/40/20/0/-20/-40/-60/-80</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>90/75/60/45/30/15/0/-15/-30/-45/-60/-75</td>
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Numerical applications: Square plate under vertical force

Optimization results of the square plate under vertical force ($m=4$)

(a) DMO ($C=1.220 \times 10^{-4}$)  
$\nu=4$

(b) SFP ($C=1.182 \times 10^{-4}$)  
$\nu=2$

(c) BCP ($C=1.182 \times 10^{-4}$)  
$\nu=2$
Numerical applications: Square plate under vertical force

*Iteration histories of the weight for patch 16 (BCP m=4)*
Numerical applications: Square plate under vertical force

Influence of the penalization factor $p$ of the BCP scheme upon the optimization results
Numerical applications: Square plate under vertical force

- Topology optimization: void + laminate
- Volume constraint: $V < 11/16$

4 orientations
90/45/0/-45

18 orientations
90/80/70/60/50/40/30/20/10/0/
-10/-20/-30/-40/-50/-60/-70/-80
Numerical applications: Natural frequency maximization

- Maximum fundamental eigenfrequency of a square plate supported at its four corners

4 candidate orientations (90/45/0/−45)

Table 1 Material properties (MC1)

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Numerical applications: Natural frequency maximization

Optimization results of the square plate with maximum eigenfrequency
Volume composite material < 75%
Numerical application: long box

Fig. 10 Model of a 4-layer laminated beam

- 4 layers; element size = 4
- Orientation: 90/45/0/-45
  in 1-2 plane for each element axes
- Load case: line load at the tip

L = 200 mm D = 40 mm T = 1 mm

Element size 4x4 mm
Numerical application: long box

**Line force**
- Objective: minimize compliance
- $90^\circ / 45^\circ / 0^\circ / -45^\circ$

Layer 1 (inner ply)
Layer 2
Layer 3
Layer 4 (outer ply)
Numerical application: delta wing

- Revisit classic applications involving composite structures
  - Two load cases: Upside and downside pressure distributions
  - Weight of fuel

Numerical application: delta wing

- Definition of design variables

16 design composite panels
(0°, 45°, 90°, -45°)

12 metallic spars and webs
Numerical application: delta wing

- Optimal design (31 iterations)
Discrete Material Optimization is an interesting and alternative approach for laminate & composite structure optimization

- For optimal layout of composite laminates over the structure
- For stacking sequence of composite panels

Comparison of different interpolation schemes

- Pioneer work by Stegmann and Lund (2005)
  - Several interpolation schemes (DMO1...5)
- New approach by Bruyneel (2011) with the Shape Function with Parameterization (SFP)
  - Limited to four materials (0°/90°/-45°/45°) or three materials (0°/90°/(45°/-45°))
- Generalization using Bi-value Coding Parameterization (BCP) (Gao et al. 2012)
CONCLUSIONS

- SFP and Bi-value Coding Parameterization mitigate the dramatic increase of design variables of DMO approach.

- BCP formulation is suited for a quite efficient solution using sequential convex programming algorithms (15-30 iterations necessary).

- DMO is validated on academic applications.
ON GOING WORK and PERSPECTIVES

- Address simultaneously the in-plane and the stacking sequence problems

- Extend the scope of the approach
  - Displacements
  - Stress constraints (Tsai Wu, Puck, etc.)
  - Buckling constraint
  - Non linear analysis (non linear buckling)

- Extend the application of BCP/SFP parameterization schemes to larger problems involving industrial composite structures

- Develop pre / post CAE tools to ease the data introduction and the visualization of results
THANK YOU VERY MUCH FOR YOUR ATTENTION

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