Comparison of parameterization schemes for solving the discrete material optimization problem of composite structures

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ECCOMAS COMPOSITES 2103 -
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INTRODUCTION

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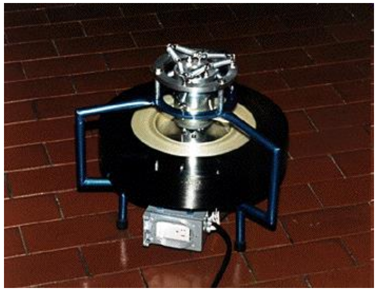
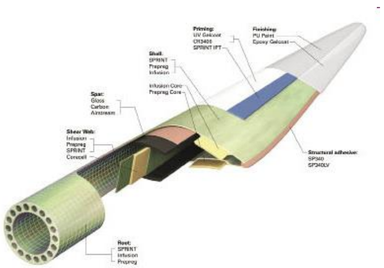
- Development of new renewable energy systems: high performance materials (strength, durability…)
- Sustainability of transportation systems: lightweight solutions

- Revived interest in composite structures.
  ➔ optimization of composites to take the best of their performances



INTRODUCTION

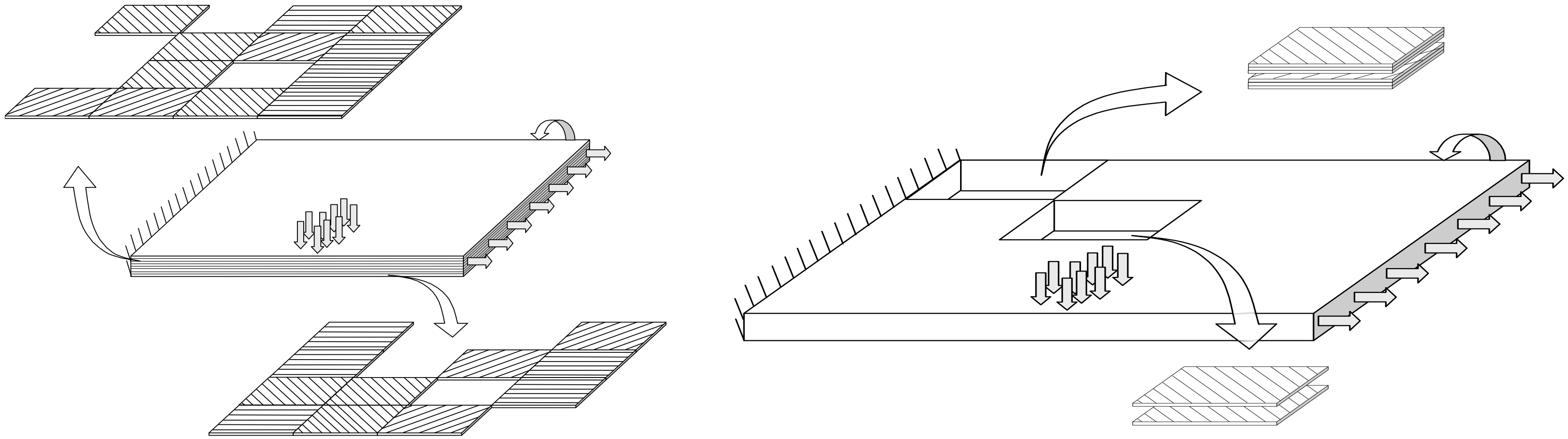
- SS Great Britain (1843)
  - First ship to be built with an iron hull
  - Parts designed and fixed together according to the available technology at the time (wood technology)
  - If you don't know it is metal you would think it is wood!

INTRODUCTION

- Boeing 787
  - Composite structure BUT…
  - True potential is not fully explored
  - Composite structures are almost direct replicas of metallic structures
  - Some of the issues
    - Repair
    - Failure modes
    - Systems interaction
  - Weight saving =< 5% in current use

INTRODUCTION

- Classical design problems of composite structures to be addressed:
  - Optimal layout of laminates over the structure
  - Through-the thickness-optimization of composites: stacking sequence optimization



- General/global approach to address simultaneously optimal layout and stacking sequence ➔ Discrete Material Optimization approach (Lund & Stegmann, 2005)

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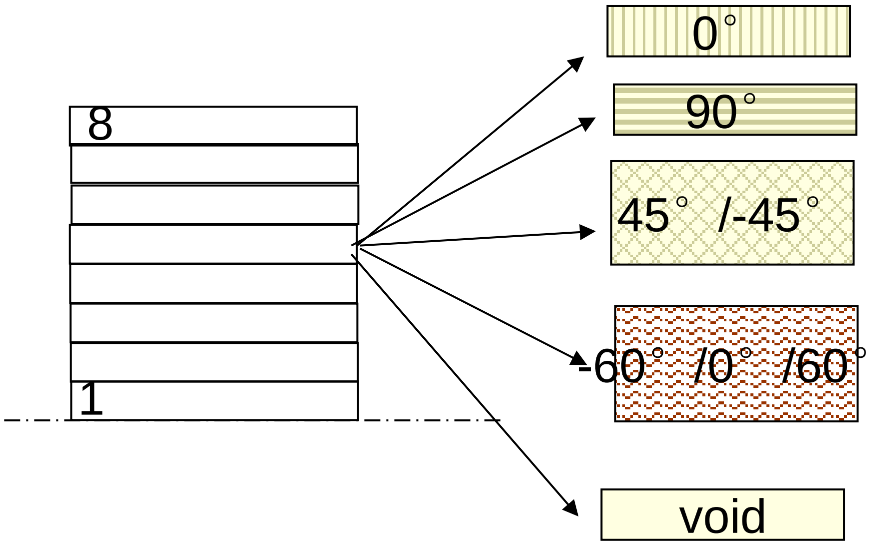
Discrete Material Optimization approach

- Formulate the optimization problem as a 'n' materials selection problem
- Use an extended topology optimization approach to solve the problem in continuous variables
  - Interpolation /parameterization scheme of material properties
  - Continuous optimization problem with penalization of intermediate solutions

$$\mathbf{C}_i = \sum_{j=1}^{m} w_{ij} \mathbf{C}_i^{(j)}$$

$$0 \le w_{ij} \le 1 \qquad \sum_{j=1}^{m} w_{ij} \le 1$$

$$w_{ik} = 0\left(k \neq j\right) \text{ when } w_{ij} = 1$$



Topology of Discrete Material Optimization

- Topology optimization of laminate
  - Selection of ply orientation and layout of the laminate
  - Add one an existence (density variable) μ in [0,1]

$$c^l = (\mu_l)^q \left(\sum_{i=1}^{n^l} w_i^l c_i^l \right)$$

  - Introduce a volume constraint of the foam or of the fiber material

$$\sum_{l=1}^{n_v} \mu_l V_l \le \overline{V}$$

INTRODUCTION

- This work:
  - To compare DMO, SFP and BCP
  - To investigate the approach parameters such as the penalization
  - To tailor a robust solution procedure based on the sequential convex programming
  - To validate the work with applications
    - Academic examples
    - Larger scale problems with real-life characteristics

DMO *(Stegmann and Lund, 2005)*

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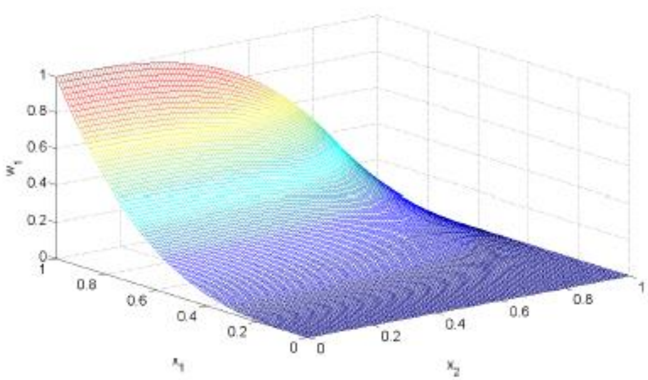
- DMO4 interpolation scheme:
  - Extension of Thomsen (1992) and Sigmund & Torquato (2000) topology optimization schemes

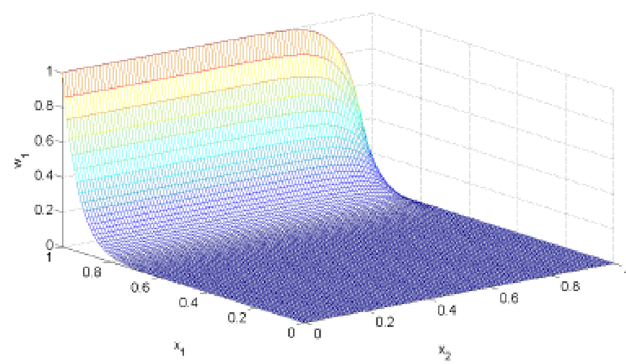
$$E = E_1 + x^p (E_2 - E_1)$$

  - Introduces one existence variable ([0,1]) per material
  - Uses a power law (SIMP) penalization of intermediate densities

$$w_{ij} = x_{ij}^p \prod_{\xi=1}^{m_v} \left(1 - x_{i\xi}^p\right) \quad \text{with} \quad 0 \leq x_{ij} \leq 1$$



w_1 with p=3



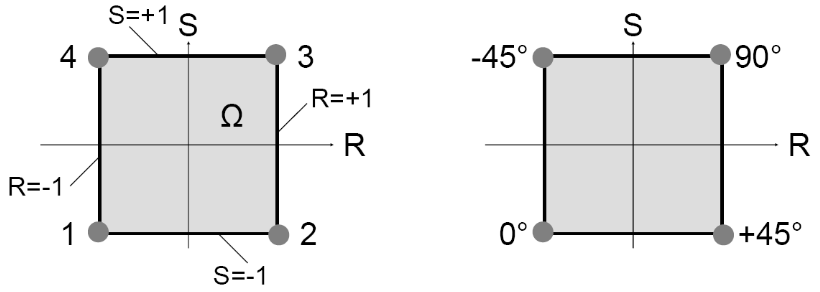
w_1 with p=15

Shape Function with Penalization (SFP)
Bruyneel (2011)

- SFP scheme makes use of the Lagrange polynomial interpolation of finite element shape functions
  - For 0° /90° /45° /-45° : four-node finite element

$$w_1 = \frac{1}{4}(1-R)(1-S) \quad w_2 = \frac{1}{4}(1+R)(1-S)$$

$$w_3 = \frac{1}{4}(1+R)(1+S) \quad w_4 = \frac{1}{4}(1-R)(1+S)$$

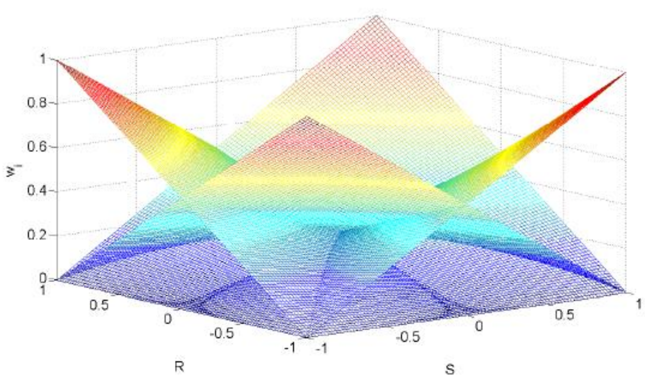


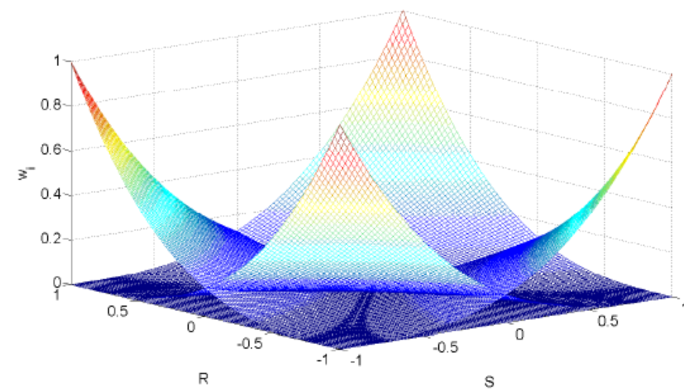
- Introduces a power penalization (SIMP)

$$w_i^{SFP} = \left[\frac{1}{4}(1\pm R)(1\pm S)\right]^p$$

Shape Function with Penalization (SFP)
Bruyneel (2011)

- SFP shape functions and penalization





$$w_i^{SFP} = \left[\frac{1}{4} \left(1 \pm R\right)\left(1 \pm S\right) \right]^p$$

- Two design (instead of 4) variables ranging in [-1,1]
- Extension to 'n' node finite elements is theoretically possible, but problem rapidly complex in practice

Bi-value coded parameterization (G. Tong et al. 2011)

- Bi-value coding parameterization generalizes the SFP scheme
- Abandon the shape function idea, but keep the idea of coding the materials using bi-value variables (typically [-1,1])

$$w_{ij} = \left[\frac{1}{2^{m_v}} \cdot \prod_{k=1}^{m_v} \left(1 + s_{jk} x_{ik} \right) \right]^p \text{ with } -1 \leq x_{ik} \leq 1 \text{ and } k = 1, \cdots, m_v$$

- Number of design variable is $m_v = \log_2 m$
  - Possible to interpolate between $2^{(mv-1)}$ to 2^{mv} materials with m_v variables
- Introduction of a penalization scheme (here power law) to end-up with -1/1 values

Bi-value coded parameterization

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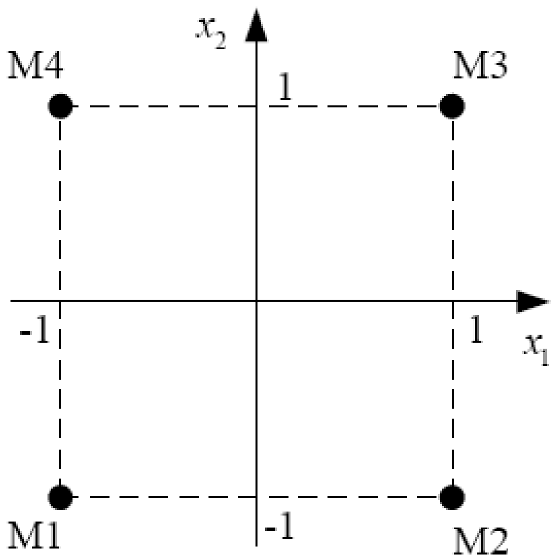
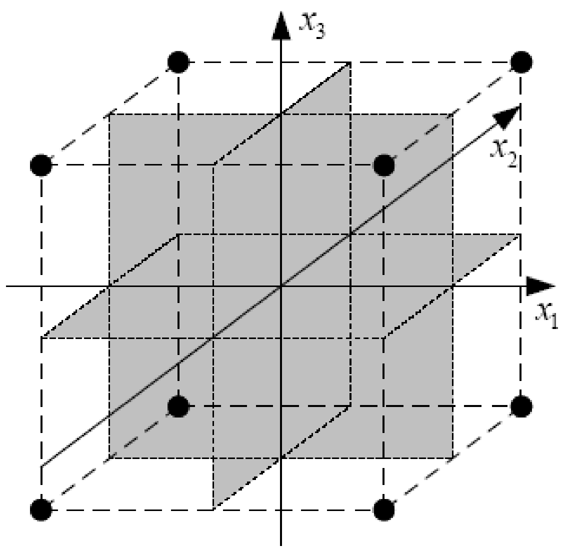
- Visualization: for m_v=2 and m_v=3, the method recovers 4-node and brick (8-node) elements shape functions.

Table 1 s_{jk} values (m_v=2, m=4)

k \ j	1	2	3	4
1	-1	1	1	-1
2	-1	-1	1	1



(a) m_v=2, m=4



(b) m_v=3, m=8

Penalization schemes

- To come to a solution with one single material, one introduces a penalization schemes:
  - SIMP

$$f(\chi) = \chi^p$$

  - RAMP (Stolpe & Svanberg, 2001)

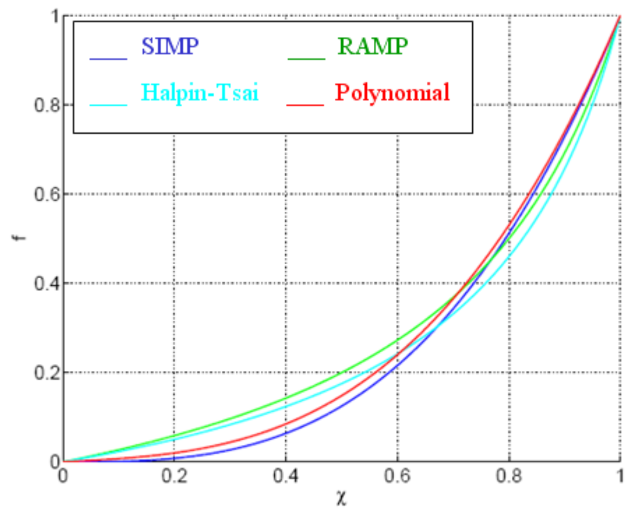
$$f(\chi) = \frac{\chi}{1 + p(1-\chi)}$$

  - Halpin Tsai (Halpin-Tsai, 1969)

$$f(\chi) = \frac{r\chi}{(1+r) - \chi}$$

  - Polynomial penalization (Zhu, 2009)

$$f(\chi) = \frac{\alpha - 1}{\alpha}\chi^p + \frac{1}{\alpha}\chi$$



Optimization Problem Formulation

- Compliance minimization under given load cases

$$\min_{\{x_{ik}\}\ (i=1,\cdots,n;\ k=1,\cdots,m_{\mathrm{v}})} \quad C = \mathbf{F}^{\mathrm{T}}\mathbf{u}$$

$$\text{subject to: } \left(\sum_{l=1}^{n_{\mathrm{v}}} \mu_l V_l \le \overline{V}\right)$$

  - For pure laminate optimization, no resource (volume) constraint is generally necessary

- Sensitivity analysis

$$\frac{\partial C}{\partial x_{ik}} = 2\mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{F}}{\partial x_{ik}} - \mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial x_{ik}}\mathbf{u} = -\mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial x_{ik}}\mathbf{u}$$

  - Requires the derivatives of the weighting functions

$$\frac{\partial \mathbf{K}_i}{\partial x_{ik}} = \sum_{j=1}^{m}\frac{\partial w_{ij}}{\partial x_{ik}}\mathbf{K}_i^{(j)}$$

Optimization Problem Formulation

- Maximization of fundamental natural frequencies

$$\min_{\{x_{ik}\}} \ \underset{(i=1,\cdots,n;\,k=1,\cdots,m_v)}{\min} \omega^2$$

$$\text{subject to: } \left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{u}=0$$

$$-1 \le x_{ik} \le 1$$

$$\mathbf{M}_i = \sum_{j=1}^{m} v_{ij}\mathbf{M}_i^{(j)} \qquad v_{ij} = \left[\frac{1}{2^{m_v}} \cdot \prod_{k=1}^{m_v}\left(1+s_{ik}x_{ik}\right)\right]^{p_M}$$

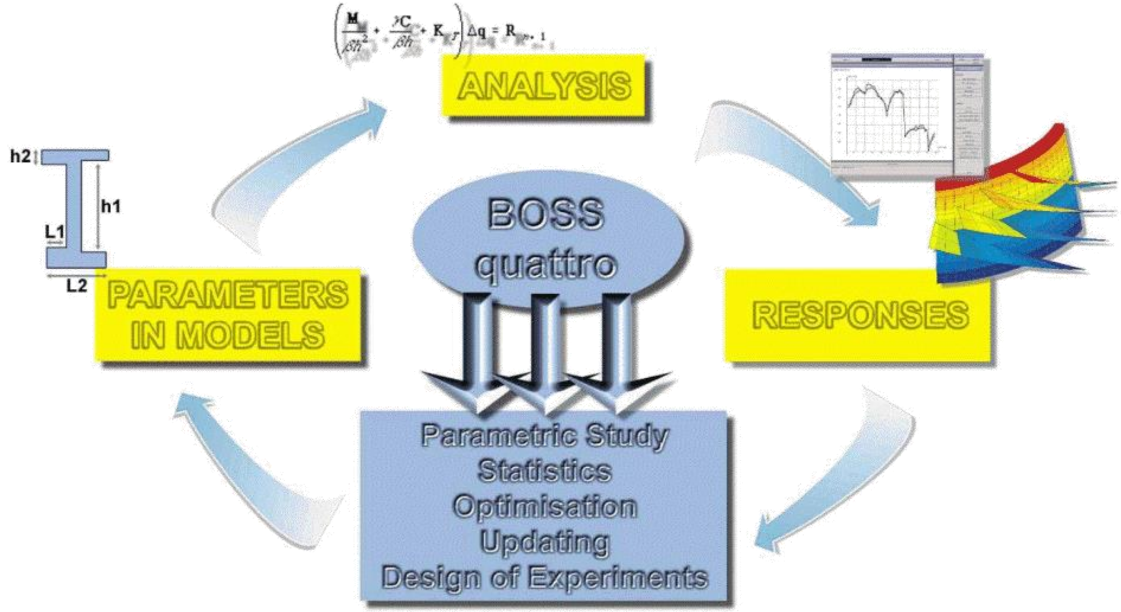
- Sensitivity analysis

$$\frac{\partial \omega^2}{\partial x_{ik}} = \frac{\mathbf{u}_i^{\mathrm{T}} \dfrac{\partial \mathbf{K}_i}{\partial x_{ik}} \mathbf{u}_i - \omega^2 \mathbf{u}_i^{\mathrm{T}} \dfrac{\partial \mathbf{M}_i}{\partial x_{ik}} \mathbf{u}_i}{\mathbf{u}^{\mathrm{T}}\mathbf{M}\mathbf{u}}$$

  - The derivatives can be either positive or negative (non monotonic function)

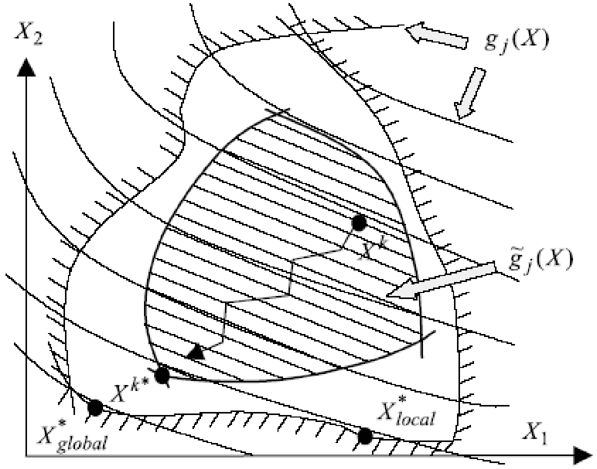
Implementation

- Implementation
  - Analysis carried out in SAMCEF Composites
    - Laminate plate elements
    - Thick composite shells (8-node bricks)
  - Optimization
    - Boss Quattro Open Object Oriented platform for Optimization



Implementation & Solution procedure

- Solution of optimization problem: Sequential Convex Programming
  - Sequence of explicit subproblems
    - CONLIN (Fleury, 1989)
    - GCMMA (Bruyneel et al., 2002)
  - General strategy with efficient capabilities in treating large scale Problems

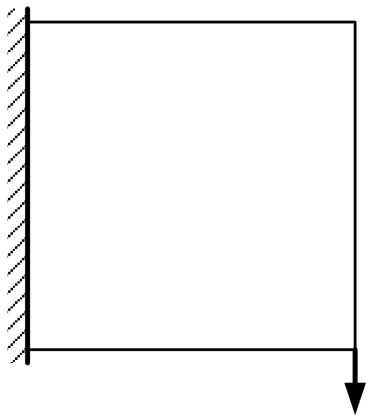


- Remarks:
  - For CONLIN variables must be >0, so a change of variables is necessary. For instance:

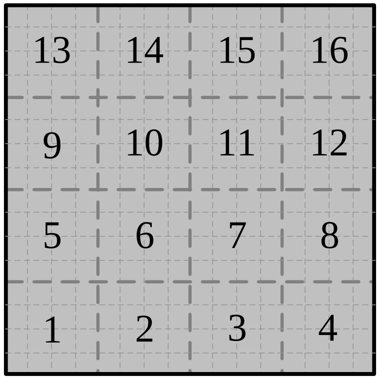
$$z_i = \frac{x_i + 1}{2}$$

Numerical applications: Square plate under vertical force

- Maximum in-plane compliance problem is solved by selecting the optimal orientation of the ply



Loads and boundary conditions



Design model with 4 × 4 patches

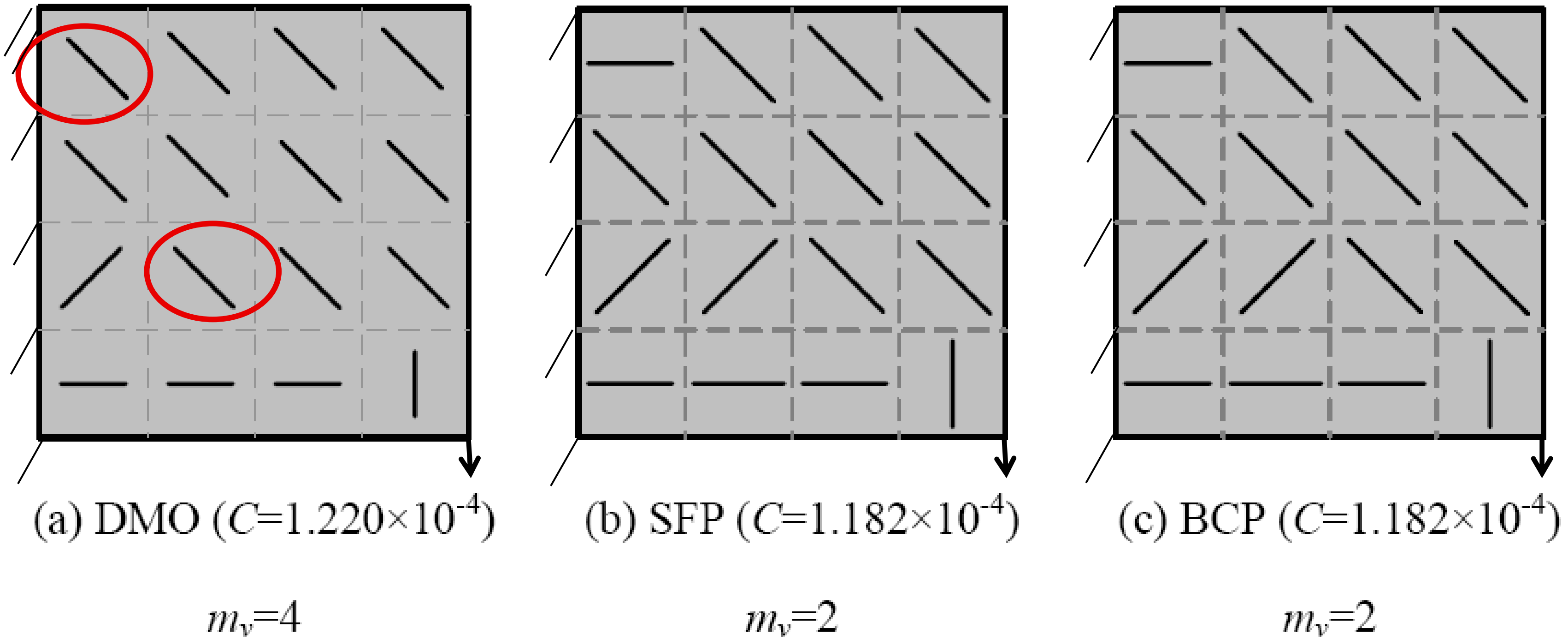
Table 4 Material properties

E_x	E_y	G_{xy}	v_{xy}
146.86GPa	10.62GPa	5.45GPa	0.33

Table 3 Orientations

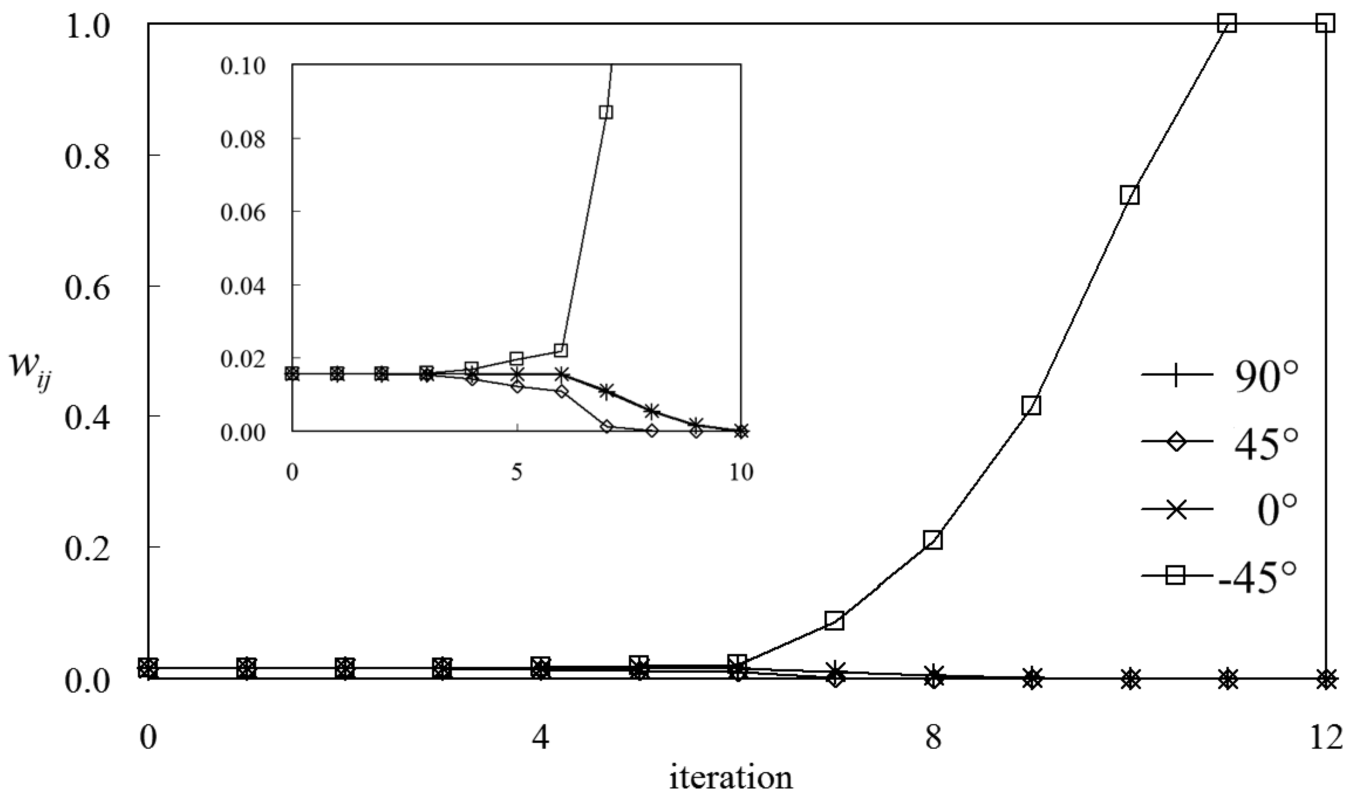
Number of material phases (m)	Number of design variables for each region (m_v)	Discrete orientation angle (°)
4	2	90/45/0/-45
9	4	80/60/40/20/0/-20/-40/-60/-80
12	4	90/75/60/45/30/15/0/-15/-30/-45/-60/-75

Numerical applications: Square plate under vertical force



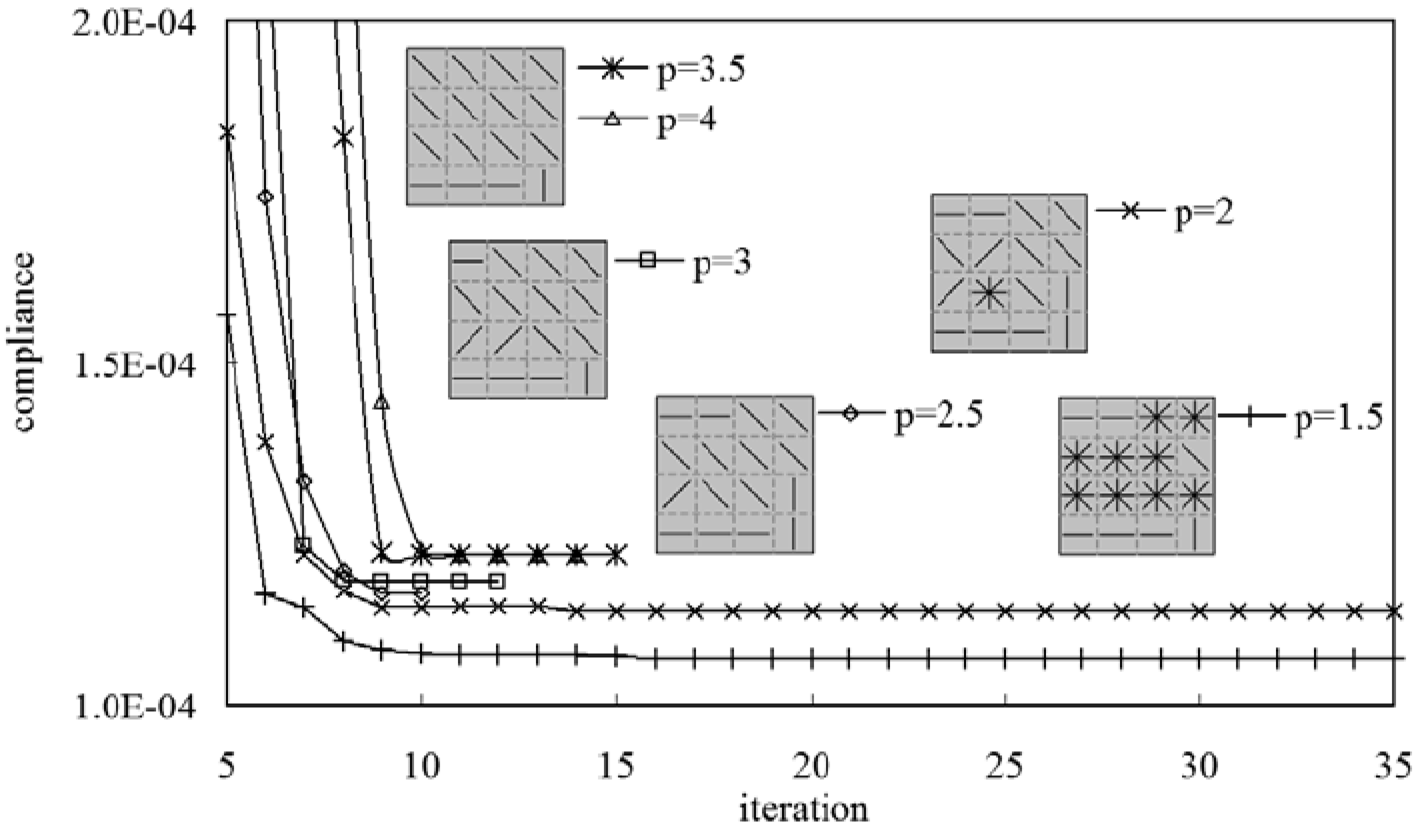
Optimization results of the square plate under vertical force (m=4)

Numerical applications: Square plate under vertical force



Iteration histories of the weight for patch 16 (BCP m=4)

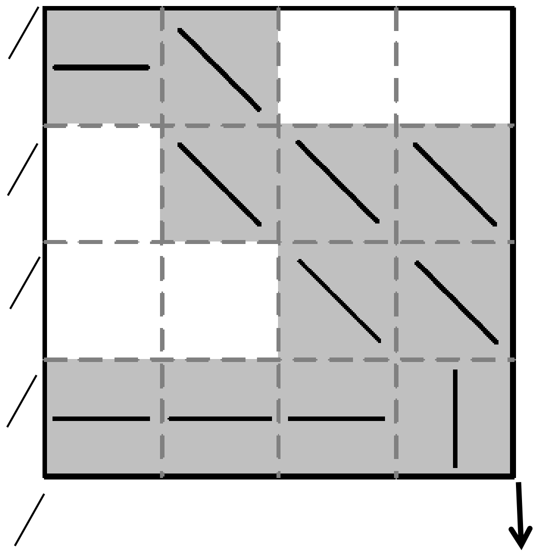
Numerical applications: Square plate under vertical force



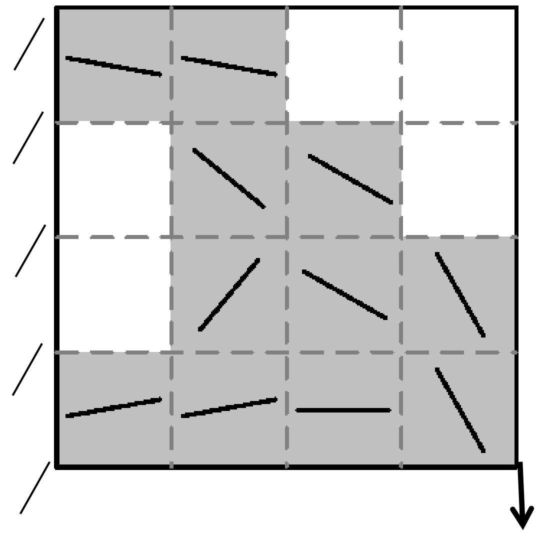
Influence of the penalization factor p of the BCP scheme upon the optimization results

Numerical applications: Square plate under vertical force

- Topology optimization: void + laminate
- Volume constraint: V < 11/16



4 orientations
90/45/0/-45



18 orientations
90/80/70/60/50/40/30/20/10/0/
-10/-20/-30/-40/-50/-60/-70/-80

Numerical applications: Natural frequency maximization

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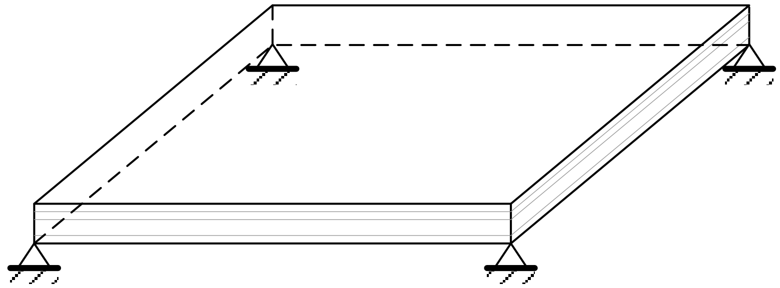
- Maximum fundamental eigenfrequency of a square plate supported at its four corners

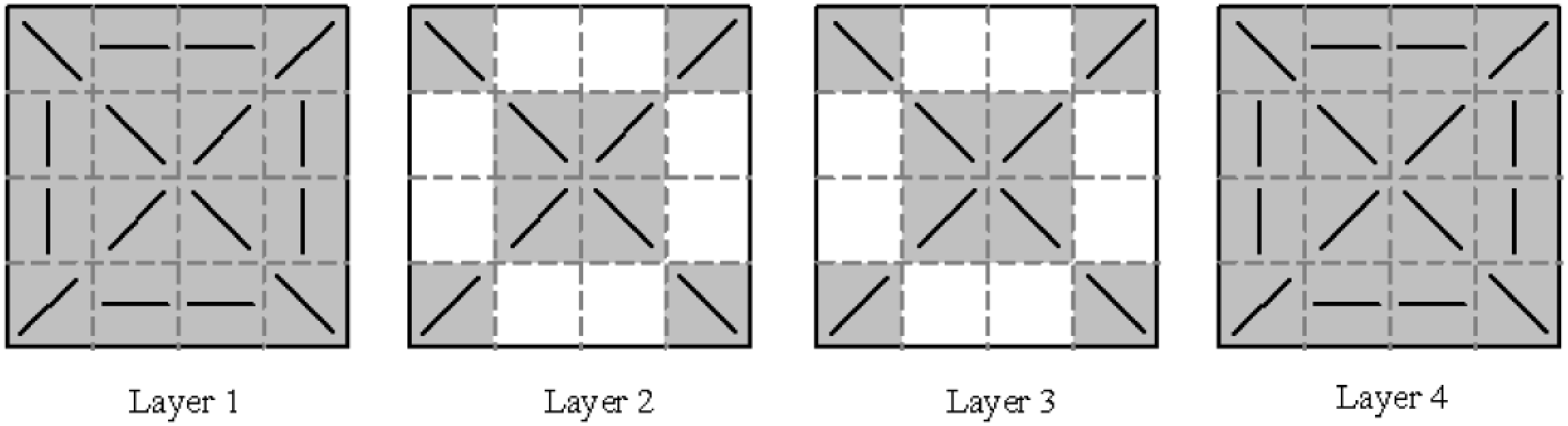


4 candidate orientations (90/45/0/-45)

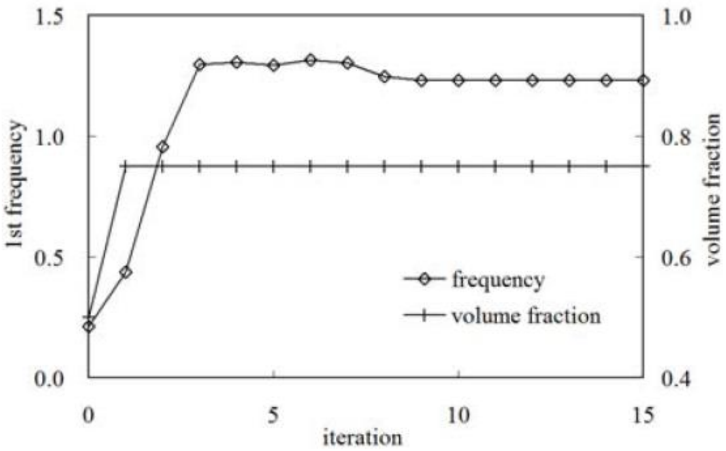
Table 1 Material properties (MC1)

E_x	E_y	G_{xy}	ν_{xy}
146.86GPa	10.62GPa	5.45GPa	0.33

Numerical applications: Natural frequency maximization



Optimization results of the square plate with maximum eigenfrequency
Volume composite material < 75%



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Numerical application: long box

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L=200 mm D=40 mm T=1 mm

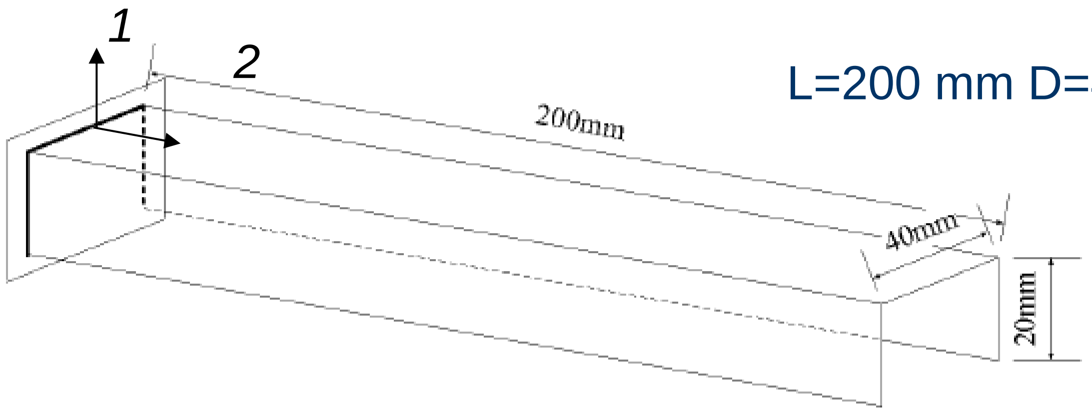
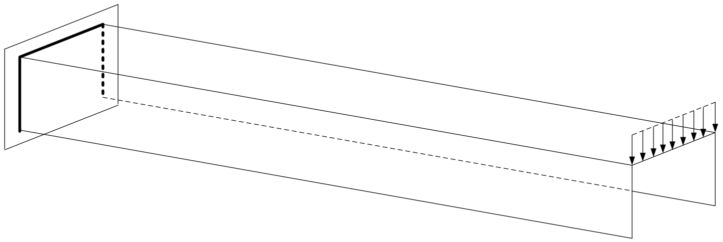


Fig.10 Model of a 4-layer laminated beam

Element size 4x4 mm

- 4 layers; element size=4
- Orientation: 90/45/0/-45
  in 1-2 plane for each element axes
- Load case: line load at the tip



Numerical application: long box

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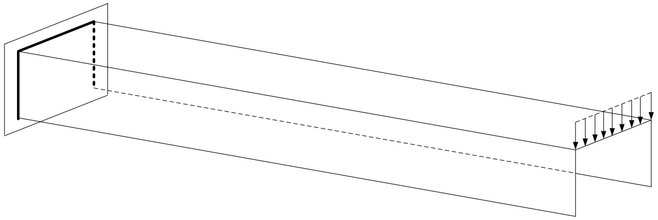
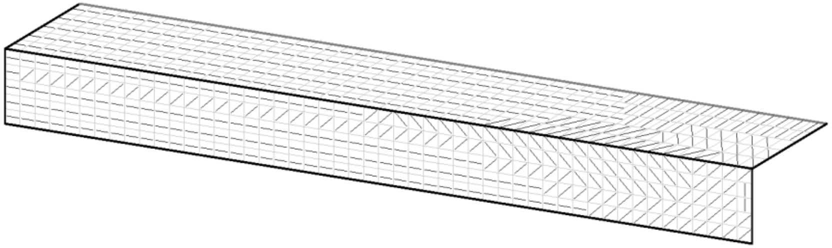
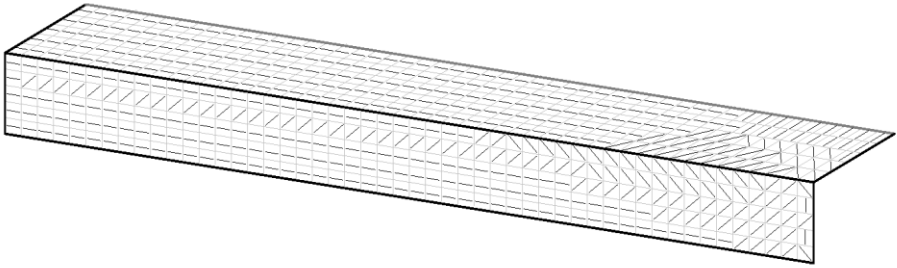
Line force

- Objective: minimize compliance
- 90° /45° /0° /-45°

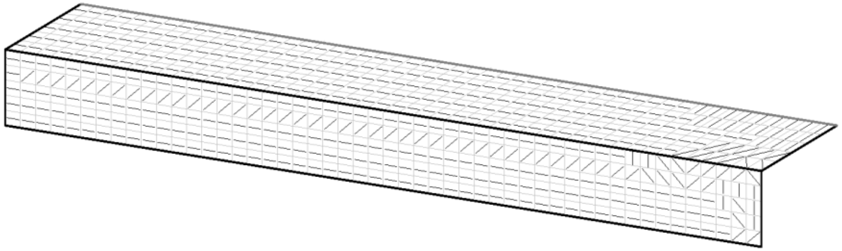




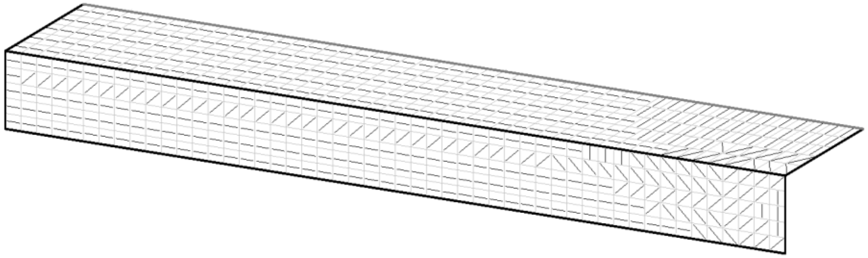
Layer 1 (inner ply)



Layer 2



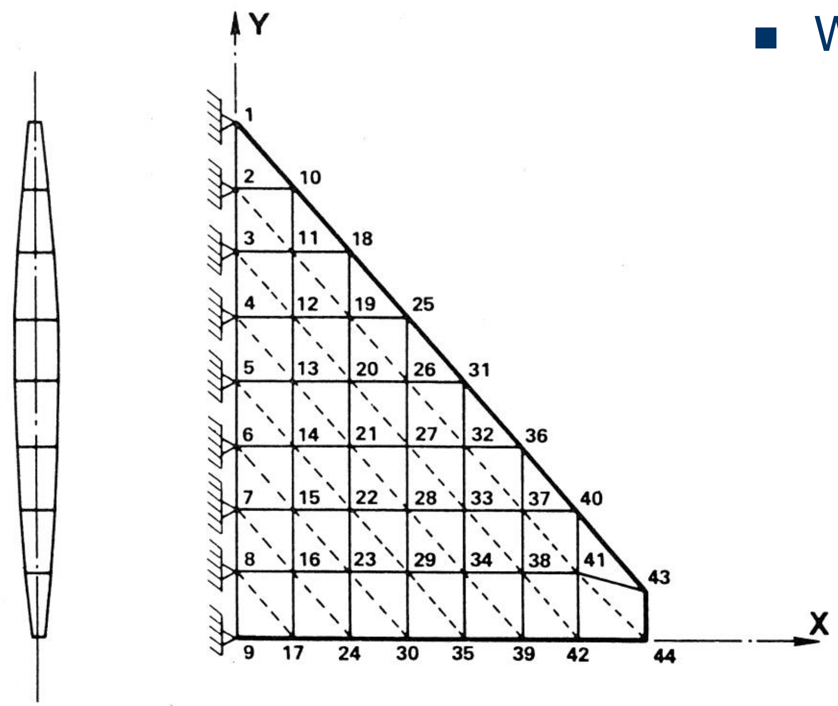
Layer 3

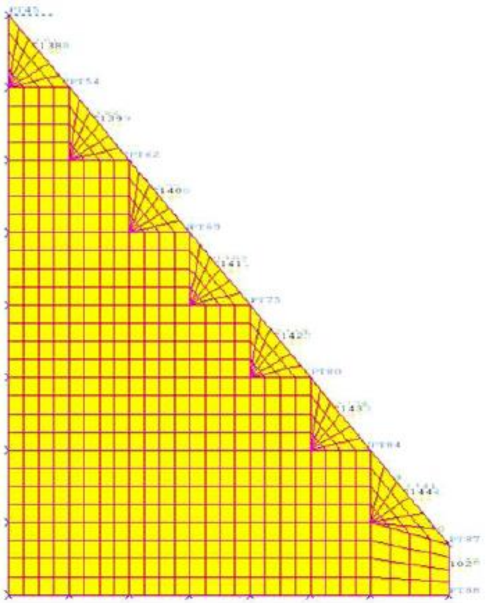


Layer 4 (outer ply)

Numerical application: delta wing

- Revisit classic applications involving composite structures
  - Two load cases: Upside and downside pressure distributions
  - Weight of fuel

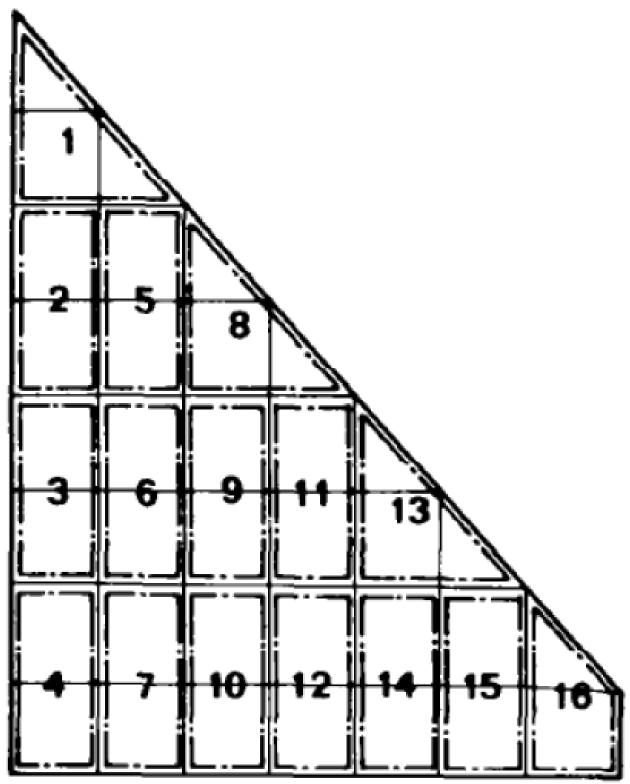




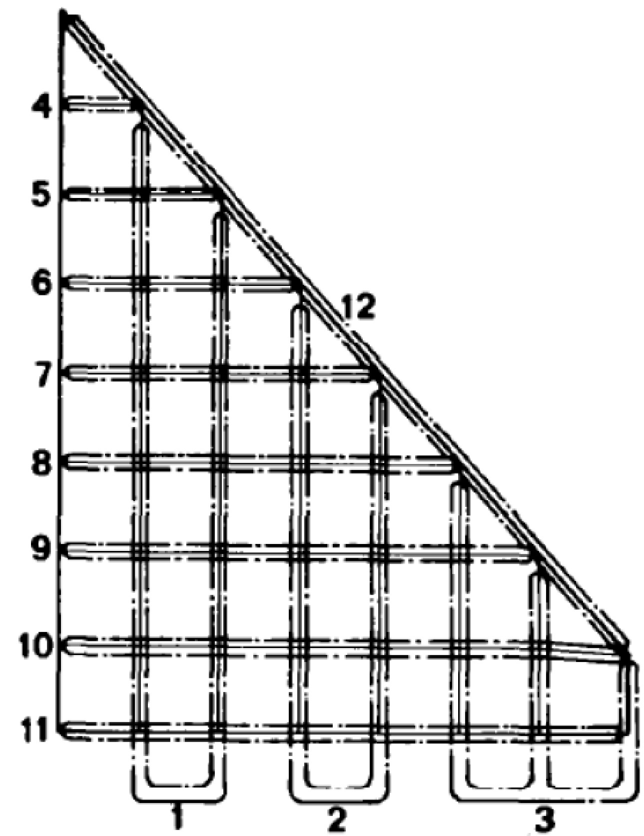
Schmidt and Fleury (1980) Dual and Approximation Concepts in Structural Synthesis. NASA Contractor Report 3226. Dec 1980.

Numerical application: delta wing

- Definition of design variables



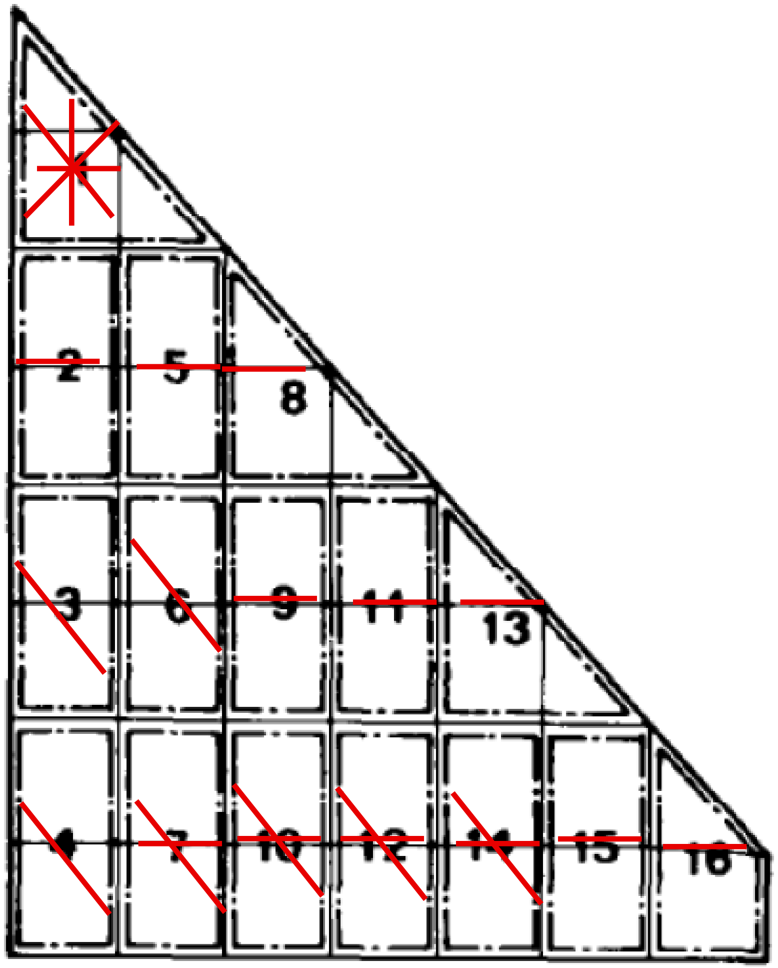
16 design composite panels
(0°, 45°, 90°, -45°)



12 metallic spars and webs

Numerical application: delta wing

- Optimal design (31 iterations)



CONCLUSIONS

- Discrete Material Optimization is an interesting and alternative approach for laminate & composite structure optimization
  - For optimal layout of composite laminates over the structure
  - For stacking sequence of composite panels
- Comparison of different interpolation schemes
  - Pioneer work by Stegmann and Lund (2005)
    - Several interpolation schemes (DMO1…5)
  - New approach by Bruyneel (2011) with the Shape Function with Parameterization (SFP)
    - Limited to four materials (0° /90° /-45° /45°) or three materials (0° /90° / (45° /-45°))
  - Generalization using Bi-value Coding Parameterization (BCP) (Gao et al. 2012)

CONCLUSIONS

- SFP and Bi-value Coding Parameterization mitigate the dramatic increase of design variables of DMO approach
- BCP formulation is suited for a quite efficient solution using sequential convex programming algorithms (15-30 iterations necessary)
- DMO is validated on academic applications

ON GOING WORK and PERSPECTIVES

- Address simultaneously the in-plane and the stacking sequence problems
- Extend the scope of the approach
  - Displacements
  - Stress constraints (Tsai Wu, Puck, etc.)
  - Buckling constraint
  - Non linear analysis (non linear buckling)
- Extend the application of BCP/SFP parameterization schemes to larger problems involving industrial composite structures
- Develop pre / post CAE tools to ease the data introduction and the visualization of results

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