An eigenvector based approach to neutrino mixing

D. Aristizabal Sierra,[1,*] I. de Medeiros Varzielas,[2,†] and E. Houet[1,‡]

[1] *IFPA, Dep. AGO, Universite de Liege, Bat B5, Sart Tilman B-4000 Liege 1, Belgium*

[2] *Facultät für Physik, Technische Universität Dortmund D-44221 Dortmund, Germany*

(Dated: May 13, 2013)

We propose a model-independent analysis of the neutrino mass matrix through an expansion in terms of the eigenvectors defining the lepton mixing matrix, which we show can be parametrized as small perturbations of the tribimaximal mixing eigenvectors. This approach proves to be powerful and convenient for some aspects of lepton mixing, in particular when studying the sensitivity of the mass matrix elements to departures from their tribimaximal form. In terms of the eigenvector decomposition, the neutrino mass matrix can be understood as originating from a tribimaximal dominant structure with small departures determined by data. By implementing this approach to cases when the neutrino masses originate from different mechanisms, we show that the experimentally observed structure arises very naturally. We thus claim that the observed deviations from the tribimaximal mixing pattern might be interpreted as a possible hint of a "hybrid" nature of the neutrino mass matrix.

PACS numbers: 14.60.Lm, 12.60.Pq, 14.60.St

Keywords: Neutrino mass and mixing, Non-standard model neutrinos

There are two main approaches to describe lepton flavor mixing. One is based on assuming the mixing is governed by a fundamental organizing principle, such as a flavor symmetry, which dictates the structure of the lepton mixing pattern and might eventually account for quark mixing as well (see e.g. [1, 2]). The other, usually referred to as the anarchy approach, postulates that lepton mixing originates from a random distribution of unitary 3×3 matrices [3]. In either case these approaches are far from providing an ultimate solution to the lepton flavor puzzle. Before the striking measurements of θ_{13} [4, 5], even though global fits had hinted to a non-vanishing θ_{13} [7], lepton mixing was well described by the Tribimaximal mixing (TBM) pattern [6] defined by $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{13} = 0$. The TBM pattern was for almost a decade a paradigm since its regularity is very suggestive of an underlying principle at work. With a vanishing θ_{13} now excluded at more than 10σ [8] the situation has changed somewhat. The advent of experimental data proving non-vanishing θ_{13} and deviations of the best-fit-point values (BFPVs) of the other angles (particularly θ_{23}) from their TBM values [8, 11, 12] has greatly motivated the search for possible mechanisms yielding the required deviations from the TBM pattern, which almost without exception are induced by effective operators. In this way, flavor models unable to produce "large" deviations on θ_{13} from its TBM value have been ruled out, and often deviations on the TBM pattern must be sourced from next-to-leading order non-renormalizable operators, constraining model building.

Majorana neutrino masses can be incorporated in the standard model Lagrangian through the dimension five effective operator $\mathcal{O}_5 \sim LLHH$ [9], the type-I seesaw [10] being the most popular and simplest realization of this operator. Other realizations have been considered as pathways to neutrino masses but often the resulting neutrino mass matrix is solely sourced by a single set of lepton number violating parameters, e.g. in type-I seesaw the right-handed neutrino masses. However, given the multiple realizations of $\mathcal{O}_5$ a conceivable possibility is that in which the neutrino mass matrix involves several independent sets of lepton number breaking parameters, a situation we refer to generically as "hybrid neutrino masses", as would be the case e.g. in a scheme involving interplay between type-I and type-II seesaw [18]. Already with two contributions sourcing the neutrino mass matrix several scenarios for neutrino mixing arising from interplay between them can be envisaged.

In this article we start by using an expansion of the neutrino mass matrix in terms of the eigenvectors of the lepton mixing matrix as an alternative model-independent parametrization of the experimentally known values. Given that our parametrization is based on deviations from TBM values, there are some similarities to existing parametrizations such as [13]. We show the usefulness of the treatment based on eigenvectors by studying the constraints on the different mass matrix elements imposed by deviating from TBM, which become evident when using this approach due to the *TBM + deviations* structure the mass matrix exhibits. We proceed by harnessing this parametrization to analyze the different possibilities that arise with hybrid neutrino masses, and presenting a very appealing scenario where one of the contributions exhibits a purely TBM form that would be well motivated by a flavor symmetry, while the corresponding deviations, required by data

* daristizabal@ulg.ac.be

† ivo.de@udo.edu

‡ eric.houet@student.ulg.ac.be

are naturally accounted for by the other contribution. In this way we present a paradigm for neutrino masses that matches the qualitative features required by neutrino data and in which the deviations from TBM are interpreted as proof of the existence of hybrid neutrino masses.

In the flavor basis (where charged leptons are diagonal) the light neutrino mass matrix can be written as (henceforth we will denote matrices and vectors in boldface)

$$\boldsymbol{m_\nu} = \boldsymbol{U}^* \,\boldsymbol{\hat{m}_\nu}\, \boldsymbol{U}^\dagger \,, \tag{1}$$

where $\boldsymbol{\hat{m}_\nu} = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ and with $\boldsymbol{U}$ the lepton mixing matrix.

For any experimentally allowed point in parameter space, one can define the eigenvector $\boldsymbol{v_i}$ associated to the eigenvalue m_{ν_i} and thus

$$\boldsymbol{U} = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \,, \tag{2}$$

where each $\boldsymbol{v_i}$ is a column and the neutrino mass matrix can be expressed as the outer (tensor) product of the eigenvectors

$$\boldsymbol{m_\nu} = \sum_{i=1}^{3} m_{\nu_i}\, \boldsymbol{v_i} \otimes \boldsymbol{v_i} \,. \tag{3}$$

Consistency with data requires at least two eigenvectors to be present in the above decomposition, and we refer to those cases as "minimal". In the normal hierarchy a viable minimal setup involves $\boldsymbol{v_{2,3}}$, with $\boldsymbol{v_{1,2}}$ necessary in the inverted case.

We parametrize the mixing angles starting from TBM [1]:

$$\sin\theta_{12} = \sin\theta_{12}^{\mathrm{TBM}} - \epsilon_{12} = \frac{1}{\sqrt{3}} - \epsilon_{12} \,, \tag{4}$$

$$\sin\theta_{23} = \sin\theta_{23}^{\mathrm{TBM}} - \epsilon_{23} = \frac{1}{\sqrt{2}} - \epsilon_{23} \,, \tag{5}$$

$$\sin\theta_{13} = \sin\theta_{13}^{\mathrm{TBM}} + \epsilon_{13} = \epsilon_{13} \,. \tag{6}$$

This is useful as according to neutrino data [8, 11, 12], the ϵ_{ij} parameters are small: at the 3σ level, for the normal hierarchy data according to [8], we extract their ranges as

$$\epsilon_{12} \subset [-0.0309, 0.0577] \,, \tag{7}$$

$$\epsilon_{23} \subset [-0.117, 0.107] \,, \tag{8}$$

$$\epsilon_{13} \subset [0.130, 0.181] \,. \tag{9}$$

[1] Similar parametrizations have been discussed for the leptonic mixing matrix in [14, 15] and with special emphasis to a non-vanishing θ_{13} angle in [16, 17].

Using this parametrization the eigenvectors $\boldsymbol{v_i}$ can be expressed in terms of the ϵ_{ij} parameters. We write

$$\boldsymbol{v_i} = \boldsymbol{v_i}^{\mathrm{TBM}} + \boldsymbol{\varepsilon_i} \,, \tag{10}$$

with the TBM eigenvectors in $\boldsymbol{U}_{\mathrm{TBM}}$ given by

$$\{\boldsymbol{v_1}^{\mathrm{TBM}}, \boldsymbol{v_2}^{\mathrm{TBM}}, \boldsymbol{v_3}^{\mathrm{TBM}}\} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \,. \tag{11}$$

The perturbation vectors $\boldsymbol{\varepsilon_i}$ can be simplified by expanding the trigonometric functions entering in $\boldsymbol{U}$ up to second order in ϵ_{ij} [2]. By fixing $\delta = 0$ for illustration (this does not affect the main conclusions), they read

$$\boldsymbol{\varepsilon_1} = \begin{pmatrix} \epsilon_{12}/\sqrt{2} \\ \epsilon_{12}/\sqrt{2} - (\epsilon_{13} + \epsilon_{23})/\sqrt{3} \\ \epsilon_{12}/\sqrt{2} + (\epsilon_{13} + \epsilon_{23})/\sqrt{3} \end{pmatrix} \,, \tag{12}$$

$$\boldsymbol{\varepsilon_2} = \begin{pmatrix} -\epsilon_{12} \\ \epsilon_{12}/2 - \epsilon_{13}/\sqrt{6} + \sqrt{2}\epsilon_{23}/\sqrt{3} \\ \epsilon_{12}/2 + \epsilon_{13}/\sqrt{6} - \sqrt{2}\epsilon_{23}/\sqrt{3} \end{pmatrix} \,, \tag{13}$$

$$\boldsymbol{\varepsilon_3} = \begin{pmatrix} -\epsilon_{13} \\ \epsilon_{23} \\ \epsilon_{23} \end{pmatrix} \,. \tag{14}$$

With the eigenvectors written as perturbations of the $\boldsymbol{v_i}^{\mathrm{TBM}}$, the neutrino mass matrix can be conveniently interpreted as originating from a TBM structure with modifications that are fixed whenever a given point in the corresponding experimental data range is selected, that is to say

$$\boldsymbol{m_\nu} = \sum_{i=1}^{3} m_{\nu_i} \left[\left(\boldsymbol{v_i}^{\mathrm{TBM}} \otimes \boldsymbol{v_i}^{\mathrm{TBM}}\right) + \boldsymbol{\mathcal{V}_i} \right] \,, \tag{15}$$

with

$$\boldsymbol{\mathcal{V}_i} = \left[\left(\boldsymbol{v_i}^{\mathrm{TBM}} \otimes \boldsymbol{\varepsilon_i}\right) + \left(\boldsymbol{\varepsilon_i} \otimes \boldsymbol{v_i}^{\mathrm{TBM}}\right) + \left(\boldsymbol{\varepsilon_i} \otimes \boldsymbol{\varepsilon_i}\right) \right] \,. \tag{16}$$

It is clear that consistency with data at a certain confidence level requires some entries of the mass matrix to significantly deviate from the TBM structure. In order to quantify these deviations we calculated the different mass matrix elements through eqs. (15) and (16). In fig. 1 we display numerical results of the different mass matrix entries (normalized to the corresponding TBM entries), $R_{ij} = m_{\nu_{ij}}/m_{\nu_{ij}}^{\mathrm{TBM}}$, varying with θ_{13} and θ_{23} (the variation with θ_{12} is weaker, so we do not display it). We used the 3σ ranges of the angles for the normal hierarchical spectrum according to reference [8], with the remaining parameters fixed to their BFPVs and the lightest

[2] When compared with the exact expressions this approximation deviates at the permille level, and unitarity of U is guaranteed up to corrections of order ϵ^3.

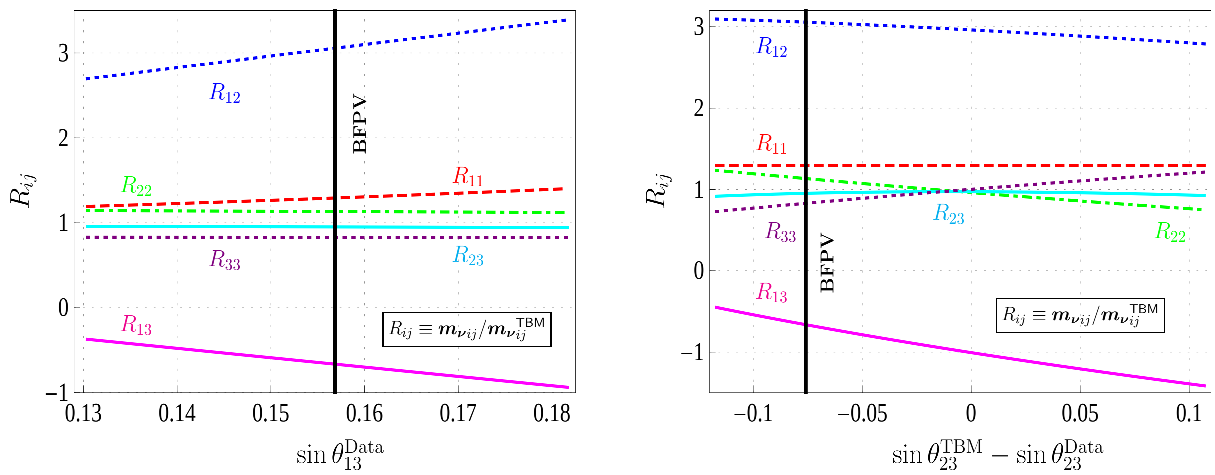

FIG. 1. *Sensitivity of the neutrino mass matrix entries, normalized to their TBM values, to departures from the TBM mixing pattern in the case of normal hierarchical light neutrino mass spectrum and for* $\delta = 0$*. Left-plot shows the dependence with* $\sin\theta_{13}$ *while the right-panel one with* $\sin\theta_{23}$ *(both angles taken in their* 3σ *experimental range). See the text for more details.*

neutrino mass set to 10^{-3} eV (the results are quite insensitive to this parameter). These results indicate clearly that deviating from $\theta_{13}^{\rm TBM} = 0$ requires $m_{\nu_{12}}$ and $m_{\nu_{13}}$ to have sizable departures from their TBM values together with small departures in the $m_{\nu_{11}}$ entry. It can be seen in fig. 1 that the other elements remain flat, not playing a relevant role. With θ_{23} the situation is different: deviating from $\theta_{23}^{\rm TBM} = \pi/4$ demands large deviations from the TBM structure in $m_{\nu_{13}}$ and $m_{\nu_{12}}$, but $m_{\nu_{22}}$ and $m_{\nu_{23}}$ need also to differ from their TBM values. For θ_{12} the results look similar in the sense that deviations from $\theta_{12}^{\rm TBM}$ are mostly determined by variations of $m_{\nu_{12}}$ and $m_{\nu_{13}}$ entries. Overall in the $\delta = 0$ case deviations of the neutrino mixing angles from their TBM values require neutrino mass matrices with sizable deviations from the TBM structure mainly in the $m_{\nu_{12}}$ and $m_{\nu_{13}}$ elements, and this conclusion holds independently of the neutrino mass spectrum.

Whether these deviations depend more strongly on deviations of θ_{13} or θ_{23} from their TBM values is something which can not be determined from Fig. 1. In order to see which of these TBM deviations dictate sizable deviations of $m_{\nu_{12,13}}$, in Fig. 2 we plot isocurves of both R_{12} and R_{13} showing their dependence with $\sin\theta_{13,23}$. It can be seen that although the deviations of $m_{\nu_{12}}$ ($m_{\nu_{13}}$) exhibit a slightly more pronounced dependence on θ_{13} (θ_{23}) they are clearly not strongly dominated by a single mixing angle. Thus, deviating from TBM either in θ_{13} or θ_{23} requires sizable deviations on both $m_{\nu_{12,13}}$. We emphasize that results in Fig. 1 as well as in Fig. 2 apply in the flavor basis, and that we obtained these conclusions following from the useful eigenvector decomposition approach in a model-independent way.

We now apply our formalism to hybrid neutrino masses (i.e. receiving contributions from physically distinct sources, such as different seesaw mechanisms). In the case with two sources with superindices A, B the effective light neutrino mass matrix reads [3]

$$\boldsymbol{m_\nu} = \boldsymbol{m_\nu^{(A)}} + \boldsymbol{m_\nu^{(B)}}\,. \tag{17}$$

From equation (1) we have

$$\hat{\boldsymbol{m}}_{\boldsymbol{\nu}} = \boldsymbol{U}^T\left(\boldsymbol{m_\nu^{(A)}} + \boldsymbol{m_\nu^{(B)}}\right)\boldsymbol{U}\,. \tag{18}$$

and in general situation $\boldsymbol{U}$ diagonalizes the sum but not the individual matrices $\boldsymbol{m_\nu^{(A,B)}}$. The contributions must add up to (15) but their individual structure needs not be determined by the eigenvectors of $\boldsymbol{U}$. The most general decomposition can be written as

$$\boldsymbol{m_\nu^{(X)}} = \sum_i \left[m_{\nu_i}^{(X)} \boldsymbol{v_i}^{\rm TBM} \otimes \boldsymbol{v_i}^{\rm TBM} + \delta m_{\nu_i}^{(X)} \boldsymbol{\mathcal{V}_i}\right]\,, \tag{19}$$

where $X = A, B$ and by definition, (18) requires $m_{\nu_i}^{(A)} + m_{\nu_i}^{(B)} = \delta m_{\nu_i}^{(A)} + \delta m_{\nu_i}^{(B)} = m_{\nu_i}$. We stress that any realization of hybrid neutrino masses can be defined according to the terms entering in each contribution. Consistency requires that when combining $\boldsymbol{m_\nu^{(A,B)}}$ through (17) the eigenvectors entering in the full mass matrix sum up to (10), or in other words that the matrix associated with the generation index i has at the end a structure like (15). Regardless of which eigenvectors appear in each individual contribution, the orthogonality relation $\boldsymbol{v_i}\cdot\boldsymbol{v_j} = \delta_{ij}$ guarantees the lepton mixing matrix diagonalizes the resulting $\boldsymbol{m_\nu}$ (due to $\boldsymbol{v_i}$ being approximate, this holds up to corrections at most of order $v_{kl}^{\rm TBM}\epsilon_{ij} \sim 10^{-1}$). Indeed, it is useful to apply the eigenvector decomposition to each contribution as it is made clear that the only

[3] It is straightforward to generalize to hybrid cases with more contributions.

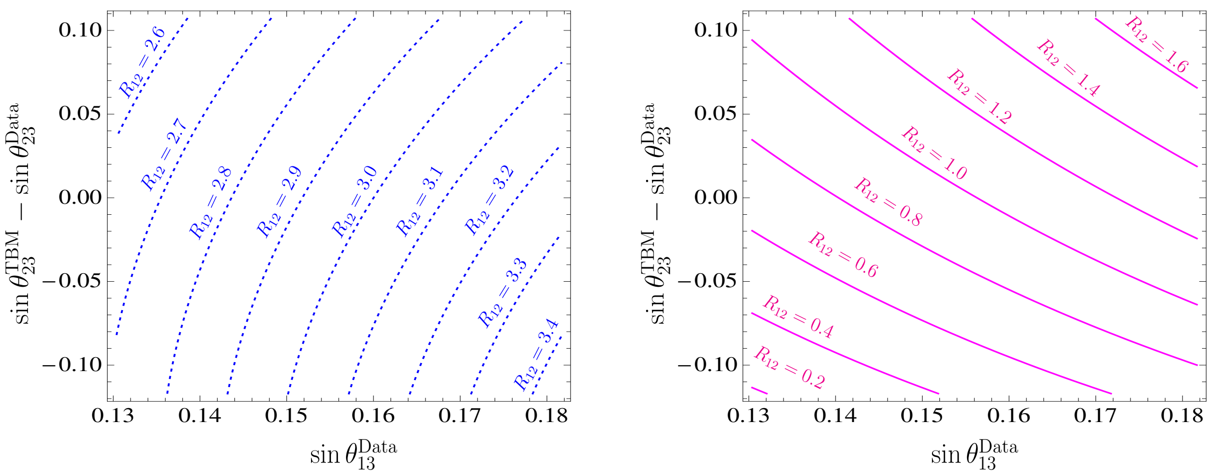

FIG. 2. *Contourplots showing the simultaneous dependence of* R_{12} *(left-hand side plot) and* R_{13} *(right-hand side plot) with* $\sin\theta_{13}$ *and* $\sin\theta_{23}$*. The plots have been obtained by fixing the remaining neutrino oscillation parameters as in Fig. 1.*

way for the eigenvectors building either $\boldsymbol{m_\nu^{(X)}}$ to appear in U unchanged is if they are already orthogonal, which in general will not be the case. For illustration, we consider a minimal setup (minimal in terms of the number of parameters defining the full neutrino mass matrix)

$$\boldsymbol{m_\nu^{(A)}} = m_{\nu_2} \boldsymbol{v_2}^{\rm TBM} \otimes \boldsymbol{v_2}^{\rm TBM} \,, \tag{20}$$

$$\boldsymbol{m_\nu^{(B)}} = m_{\nu_3} \boldsymbol{v_3} \otimes \boldsymbol{v_3} + m_{\nu_2} \boldsymbol{\mathcal{V}_2} \,. \tag{21}$$

In this case the vector $\boldsymbol{v_2}$ becomes "completed" through the combination of $\boldsymbol{m_\nu^{(A)}}$ and the second term in $\boldsymbol{m_\nu^{(B)}}$. This can be seen explicitly as it results in a particular case of (15). Minimal setups are appealing as there are only 6 defining parameters (with a phase in addition to the 3 ϵ_{ij}), so there is no room for arbitrariness on the parameter space in order to match the neutrino oscillation observables ($\Delta m_{21,32}$, θ_{ij} and δ). Although at the expense of introducing more parameters, going beyond minimal cases can lead to other possibilities. A classification according to the number of eigenvectors included in each mechanism can be done in analogy to the one shown in ref [19] for the exact TBM pattern.

Another appealing setup (minimal or not) is one where one of the structures (e.g. $\boldsymbol{m_\nu^{(A)}}$) is chosen to involve only TBM eigenvectors (as in (20)). In non-minimal scenarios of this type we can have the TBM pattern arise solely from one of the contributions while the other entirely accounts for the observed deviations. This corresponds to setting $\delta m_{\nu_i}^{(A)} = m_{\nu_i}^{(B)} = 0$ in (19):

$$\boldsymbol{m_\nu^{(A)}} = \sum_i m_{\nu_i} \boldsymbol{v_i}^{\rm TBM} \otimes \boldsymbol{v_i}^{\rm TBM} \,,$$
$$\boldsymbol{m_\nu^{(B)}} = \sum_i m_{\nu_i} \boldsymbol{\mathcal{V}_i} \,, \tag{22}$$

which is very suggestive that the experimentally observed small deviations from the TBM pattern may be interpreted as a hint that nature is described by hybrid neutrino masses. With the measured deviations from TBM such a scenario is extremely natural: in general we expect the eigenvectors associated with different mechanisms to not be orthogonal, and the smallness of the deviations would simply be due to a moderate hierarchy in the scales associated with each mechanism.

In conclusion, we parametrized the neutrino mixing angles by small perturbations of the TBM pattern, and expressed the eigenvectors of the neutrino mass matrix (in the flavor basis) in terms of a dominant TBM structure with small perturbations. To very good approximation the TBM deviations are simple eigenvectors depending linearly in the deviations of the mixing angles. This approach was used first to clarify which neutrino mass matrix elements are associated with the deviations of each angle. We then applied the same approach to "hybrid" neutrino mass matrices, where it conveniently describes how the eigenvectors from different sources of neutrino masses combine into the observed mixing. We identified some particularly appealing cases, starting with the minimal ones where a small number of parameters makes the scheme predictive and then considering cases where one of the sources had a mass matrix with the exact TBM form. We argued that given the data, the latter are extremely natural, and claim that neutrino mixing data might therefore be the first hint of the presence of several mechanisms generating neutrino masses in nature. Such a framework is a novel perspective for neutrino mixing which deserves further theoretical and phenomenological scrutiny.

Acknowledgments: The work of I.d.M.V. was supported by DFG grant PA 803/6-1 and partially through PTDC/FIS/098188/2008. DAS is supported by the Belgian FNRS agency through a "Chargé de recherche" contract.

[1] G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82**, 2701 (2010) [arXiv:1002.0211 [hep-ph]].

[2] S. F. King and C. Luhn, arXiv:1301.1340 [hep-ph].

[3] L. J. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000) [hep-ph/9911341]; A. de Gouvea and H. Murayama, arXiv:1204.1249 [hep-ph].

[4] F. P. An *et al.* [DAYA-BAY Collaboration], Phys. Rev. Lett. **108**, 171803 (2012) [arXiv:1203.1669 [hep-ex]].

[5] J. K. Ahn *et al.* [RENO Collaboration], Phys. Rev. Lett. **108**, 191802 (2012) [arXiv:1204.0626 [hep-ex]].

[6] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [hep-ph/0202074].

[7] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Phys. Rev. Lett. **101**, 141801 (2008) [arXiv:0806.2649 [hep-ph]].

[8] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **86**, 073012 (2012) [arXiv:1205.4018 [hep-ph]].

[9] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[10] P. Minkowski, *Phys. Lett.* B **67** 421 (1977); T. Yanagida, in *Proc. of Workshop on Unified Theory and Baryon number in the Universe*, eds. O. Sawada and A. Sugamoto, KEK, Tsukuba, (1979) p.95; M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds P. van Niewenhuizen and D. Z. Freedman (North Holland, Amsterdam 1980) p.315; P. Ramond, *Sanibel talk*, retroprinted as hep-ph/9809459; S. L. Glashow, in *Quarks and Leptons*, Cargèse lectures, eds M. Lévy, (Plenum, 1980, New York) p. 707; R. N. Mohapatra and G. Senjanović, *Phys. Rev. Lett.* **44**, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **22** (1980) 2227; Phys. Rev. D **25** (1982) 774.

[11] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP **1212**, 123 (2012) [arXiv:1209.3023 [hep-ph]].

[12] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, Phys. Rev. D **86**, 013012 (2012) [arXiv:1205.5254 [hep-ph]].

[13] S. F. King, Phys. Lett. B **659**, 244 (2008) [arXiv:0710.0530 [hep-ph]].

[14] S. Pakvasa, W. Rodejohann and T. J. Weiler, Phys. Rev. Lett. **100**, 111801 (2008) [arXiv:0711.0052 [hep-ph]].

[15] K. Kojima and H. Sawanaka, Phys. Lett. B **678**, 373 (2009) [arXiv:0901.4812 [hep-ph]].

[16] S. F. King, Phys. Lett. B **675**, 347 (2009) [arXiv:0903.3199 [hep-ph]].

[17] S. Morisi, K. M. Patel and E. Peinado, Phys. Rev. D **84**, 053002 (2011) [arXiv:1107.0696 [hep-ph]].

[18] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981); C. Wetterich, Nucl. Phys. B **187**, 343 (1981);

[19] D. Aristizabal Sierra, F. Bazzocchi and I. de Medeiros Varzielas, Nucl. Phys. B **858**, 196 (2012) [arXiv:1112.1843 [hep-ph]].