Decaying neutralino dark matter in anomalous $U(1)_H$ models

D. Aristizabal Sierra  
*INFN, Laboratori Nazionali di Frascati, C.P. 13, I00044 Frascati, Italy*

D. Restrepo  
*Instituto de Física, Universidad de Antioquia, A.A.1226, Medellín, Colombia*

Oscar Zapata  
*Escuela de Ingeniería de Antioquia, A.A.7516, Medellín, Colombia*

Abstract

In supersymmetric models extended with an anomalous $U(1)_H$ different R-parity violating couplings can yield an unstable neutralino. We show that in this context astrophysical and cosmological constraints on neutralino decaying dark matter forbid bilinear R-parity breaking neutralino decays and lead to a class of purely trilinear R-parity violating scenarios in which the neutralino is stable on cosmological scales. We have found that among the resulting models some of them become suitable to explain the observed anomalies in cosmic-ray electron/positron fluxes.
I. INTRODUCTION

Recent measurements of high energy cosmic rays reported by different collaborations have attracted a great deal of attention as, in contrast to what is expected from spallation of primary cosmic rays on the interstellar medium, the electron/positron flux exhibits intriguing features. The PAMELA collaboration saw a rise in the ratio of positron to electron-plus-positron fluxes at energies 10-100 GeV [1]. The ATIC experiment reported the discovery of a peak in the total electron-plus-positron flux at energies 600-700 GeV [2] and more recently the Fermi LAT [3] collaboration reported an excess on the total electron-plus-positron flux in the same energy range as ATIC but less pronounced [4]. These findings in addition to those published by the HESS [5], HEAT [6], and PPB-BETS [7] experiments might be indicating the presence of a nearby source of electrons and positrons. Possible sources can have an astrophysical origin e.g. nearby pulsars [8] but more interesting they can be related with either dark matter (DM) annihilation [9, 10] or decay [10, 11, 12, 13, 14, 15, 16, 17]. In particular, decaying DM scenarios are quite appealing as, in contrast to models relying on DM annihilation, they are readily reconcilable with the observed electron-positron excess [12].

In R-parity breaking models the LSP is unstable and its lifetime is determined by supersymmetric parameters and R-parity breaking couplings. Depending on their values, the phenomenological implications of a decaying LSP can range from collider physics up to cosmology and astrophysics. In the later case the possibility of a long–lived, but not absolutely stable LSP, leads to decaying DM scenarios as was shown in Refs. [18, 19, 20, 21, 22] and indeed they have been recently reconsidered as a pathway to explain the observed anomalies in cosmic–ray electron/positron fluxes [13, 14, 15, 16]. So far, most of the analyses have been carried out by ad hoc selections of particular sets of R-parity violating couplings and/or by assuming tiny couplings. Thus, it will be desirable to build a general framework for supersymmetric decaying DM in which the allowed couplings and their relative sizes arise from generic considerations rather than from ad hoc choices as was done in [16, 17, 19]. This is the purpose of this work.

In supersymmetric models extended to include an anomalous horizontal $U(1)_H$ symmetry à la Froggatt-Nielsen (FN) [23], the standard model particles and their superpartners do not carry a R-parity quantum number and instead carry a horizontal charge ($H$–charge).
For a review see [24]. In addition, these kinds of models involve new heavy FN fields and, in the simplest realizations, an electroweak singlet superfield $\Phi$ of $H$–charge $-1$. R-parity conserving as well as R-parity violating $SU(3) \times SU(2) \times U(1)_{Y} \times U(1)_{H}$ invariant effective terms arise once below the FN fields scale ($M$) the heavy degrees of freedom are integrated out. These terms involve factors of the type $(\Phi/M)^n$, where $n$ is fixed by the horizontal charges of the fields involved and determines whether a particular term can or cannot be present in the superpotential. The holomorphy of the superpotential forbids all the terms for which $n < 0$ and although they will be generated after $U(1)_{H}$ symmetry breaking (triggered by the vacuum expectation value of the scalar component of $\Phi$, $\langle \phi \rangle$) via the Kähler potential [25] these terms are in general much more suppressed than those for which $n \geq 0$. Terms with fractional $n$ are also forbidden and in contrast to those with $n < 0$ there is no mechanism through which they can be generated. Finally, once $U(1)_{H}$ is broken the terms with positive $n$ yield Yukawa couplings determined—up to order one factors—by $\theta^n = (\langle \phi \rangle/M)^n$. The standard model fermion Yukawa couplings typically arise from terms of this kind. Correspondingly, supersymmetric models based on an $U(1)_{H}$ Abelian factor are completely specified in terms of the $H$–charges.

In the case of supersymmetric models based on an anomalous $U(1)_{H}$ flavor symmetry the quark masses, the quark mixing angles, the charged lepton masses, and the conditions of anomaly cancellation constrain the possible $H$–charge assignments [26, 27]. Since the number of constraints is always smaller than the number of $H$–charges some of them are necessarily unconstrained and apart from theoretical upper bounds on their values [28] they can be regarded as free parameters that should be determined by additional phenomenological input. For this purpose neutrino experimental data has been used resulting in models in which neutrino masses are explained [24, 29, 30, 31, 32, 33]. Here we adopt another approach by assuming a decaying neutralino as a dark matter candidate. We will show that astrophysical and cosmological observations exclude the possibility of having neutralino decays induced by bilinear R-parity violating couplings and that this in turn lead to a variety of purely trilinear R-parity breaking scenarios among which we found models that feature a single trilinear R-parity breaking coupling (minimal trilinear R-parity violating models) and that turn out to be suitable to explain the reported anomalies in cosmic-ray electron/positron fluxes in either models with TeV-ish supersymmetric mass spectra or split supersymmetry.

The rest of this paper is organized as follows: In Sec. II we will describe the possible
models that arise as a consequence of the constraints imposed by astrophysical and cosmological observations on a decaying neutralino as dark matter. We will focus on the resulting minimal trilinear R-parity violating models and discuss some realizations coming from specific $H$–charge assignments. In Sec. IIIA we will show that current data on cosmic-ray electron/positron fluxes are well described by these type of models. Finally in sec. IV we will summarize and present our conclusions.

II. MINIMAL R-PARITY VIOLATING MODEL

The most general supersymmetric version of the standard model has a renormalizable superpotential given by

$$W = \mu_\alpha \hat{L}_\alpha \hat{H}_u + h_{ij}^u \hat{H}_u \hat{Q}_i \hat{d}_j + \lambda_{\alpha \beta k} \hat{L}_\alpha \hat{L}_\beta \hat{E}_k + \lambda'_{\alpha jk} \hat{L}_\alpha \hat{Q}_j \hat{d}_k + \lambda''_{ijk} \hat{u}_i \hat{d}_j \hat{d}_k,$$  \hspace{1cm} (1)

where Latin indices $i,j,k,\ldots$ run over the fermion generations whereas Greek indices $\alpha,\beta,\ldots$ run from 0 up to 3. In the notation we are using $\hat{L}_0 = \hat{H}_d$ and the fermion Yukawa couplings are given by $h_{ij}^u = \lambda_{0ij}$ and $h_{ij}^d = \lambda'_{0ij}$. Bilinear couplings $\mu_i$ as well as the trilinear parameters $\lambda_{ijk}$ and $\lambda'_{ijk}$ break lepton number whereas baryon number is broken by the couplings $\lambda''_{ijk}$. When extending a supersymmetric model with a $U(1)_H$ Abelian factor, the size of all the parameters entering in the superpotential arises as a consequence of $U(1)_H$ breaking. In particular, the lepton and baryon number couplings are well suppressed or can even be absent without the need of $R$–parity \cite{24,29,30,31,32,33,34,35}

These kinds of frameworks are string inspired in the sense that the anomalous $U(1)_H$ symmetry may be a remnant of a string model \cite{24,34} implying that the natural scale of the FN fields $M$ can be identified with $M_P$ and that anomaly cancellations can proceed through the Green-Schwarz mechanism \cite{36}. Below the string scale, the terms in the superpotential [Eq. (1)] as well as the Kähler potential are generated after $U(1)_H$ breaking induced by $\langle \phi \rangle$ and as we already discussed may be vanishing or suppressed depending on the $H$–charge assignments of the different fields involved which in string models are always constrained to be not too large. Accordingly, in what follows we will constrain the $H$–charges to satisfy the condition $|H(f_i)| < 10$ that as highlighted in Refs. \cite{24,34} leads to a complete consistent supersymmetric flavor model.

Before proceeding we will fix our notation: Following Ref. \cite{30} we will denote a field and
<table>
<thead>
<tr>
<th>$Q_{23}$</th>
<th>$d_{13}$</th>
<th>$d_{23}$</th>
<th>$u_{13}$</th>
<th>$u_{23}$</th>
<th>$L_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>$-Q_{13}$</td>
<td>0</td>
<td>8</td>
<td>$-Q_{13}$</td>
</tr>
</tbody>
</table>

**TABLE I:** Standard model fields $H$–charges differences. Here $L_{i3} = L_{i3} + l_{i3}$

its $H$–charge with the same symbol, i.e. $H(f_i) = f_i$, $H$–charge differences as $H(f_i - f_j) = f_{ij}$ [37], bilinear $H$–charges as $n_\alpha = L_\alpha + H_\alpha$, and trilinear $H$–charges according to $n_\lambda_{ijk}$ with the index determined by the corresponding trilinear coupling, that is to say the index can be given by $\lambda_{ijk}$, $\lambda^{'}_{ijk}$, or $\lambda^{''}_{ijk}$. We fix $\theta = \langle \phi \rangle / M \simeq 0.22$ [31, 38] and $H(\phi) = -1$ without loss of generality. Furthermore we parametrize $\tan \beta = x \times -\frac{3}{3}$ such that it ranges from 90 to 1 for $x$ running from 0 to 3 (see Ref. [30] for more details).

As already stressed any coupling in the superpotential is determined up to order 1 factors by its $H$–charge. Thus, any bilinear or trilinear couplings $\mu_\alpha$ and $\lambda_T$ must be given by [24, 27]

$$
\mu_\alpha \sim \begin{cases} 
M_p \theta^{n_\alpha} & n_\alpha \geq 0 \\
\frac{m_3}{2} \theta^{|n_\alpha|} & n_\alpha < 0 \\
0 & n_\alpha \text{ fractional}
\end{cases}
$$

$$
\lambda_T \sim \begin{cases} 
\theta^{n_\lambda} & n_\lambda \geq 0 \\
(m_3/2/M_p) \theta^{|n_\lambda|} & n_\lambda < 0 \\
0 & n_\lambda \text{ fractional}
\end{cases}
$$

(2)

The individual $H$–charges in turn are determined by a set of phenomenological and theoretical conditions which can be listed as follows:

- Eight phenomenological constraints arising from six quark and lepton mass ratios plus two quark mixing angles:

$$
m_u : m_c : m_t \simeq \theta^8 : \theta^4 : 1, \\
m_d : m_s : m_b \simeq \theta^4 : \theta^2 : 1, \\
m_e : m_\mu : m_\tau \simeq \theta^5 : \theta^2 : 1,
$$

$$
V_{us} \simeq \theta, \\
V_{cb} \simeq \theta^2.
$$

(3)

Once imposed they give rise to the constraints (see Ref. [30] and references therein)

$$
Q_{13} = 3, -3, \\
L_{23} = L_{23} + l_{23} = 2, -2
$$

(4)

and those given in Table I. According to Ref. [24] the negative values in Eq. (4) do not yield correct quark mass matrices and therefore we will not consider them.
- Two additional phenomenological constraints corresponding to the absolute value of the third generation fermion masses, \( m_t \approx \langle H_u \rangle \) and \( m_b \approx m_\tau \).

- Three theoretical restrictions resulting from anomaly cancellation through the Green-Schwarz mechanism, namely, two Green-Schwarz mixed linear anomaly cancellation conditions, with canonical gauge unification \( g_3^2 = g_2^2 = (5/3)g_1^2 \), and the mixed quadratic anomaly vanishing on its own [32].

Given the above set of conditions 13, out of 17 \( H \)-charges are constrained and can be expressed in terms of the remaining 4 that we choose to be the lepton number violating bilinear \( H \)-charges \( n_i \) and \( x \). When doing so, in addition to the constraint \( n_0 = -1 \), the expressions for the standard model field \( H \)-charges shown in Table II result [30] [51]. Note that \( n_0 = -1 \) implies, according to Eq. (2), \( \mu_0 \sim m_{3/2} \theta \) thus yielding a solution to the \( \mu \) problem [40].

As can be seen from Table III the \( H \)-charges \( n_i \) and \( x \) act as free parameters and their possible values should be fixed by additional experimental constraints. Mostly motivated by the fact that \( R \)-parity breaking models provide a consistent framework for neutrino masses and mixings [24, 29, 30, 32], so far in models based on a single \( U(1)_H \) Abelian symmetry the \( n_i \) charges have been fixed by using neutrino experimental data. Here, as already mentioned, we argue that another approach can be followed by requiring a long-lived, but not absolutely stable, neutralino. Astrophysical and cosmological observations require the neutralino decay lifetime to be much more larger than the age of the Universe [18, 19, 20, 41] which is completely consistent with the value required to explain the recent reported anomalies in cosmic-ray electron/positron fluxes (\( \tau_\chi \gtrsim 10^{26} \) sec) through decaying DM [12]. Certainly true, such a long-lived neutralino will be possible only if the couplings governing its decays are sufficiently small.

If neutralino decays are induced by bilinear \( R \)-parity violating couplings, the constraint on \( \tau_\chi \) will enforce the ratio \( \mu_i/\mu_0 \) to be below \( \sim 10^{-23} \) [22]. Whether such a bound can be satisfied will depend upon the values of the \( n_i \) charges that once fixed will determine the fermion \( H \)-charges and a viable model will result if the condition \( |f_i| < 10 \) can be satisfied as we already discussed. Consider the case \( n_i < 0 \): The constraint \( \mu_i/\mu_0 \) implies, according to Eq. (2), \( \mu_i/\mu_0 \sim \theta^{n_i-1} \sim \theta^{34} \) and thus \( n_i = -35 \). With these values and from the setup of equations in Table III we have found that in this case the largest fermion \( H \)-charge is a
\[
Q_3 = \frac{-3x(x+10) + (x+4)n_1 + (x+7)n_2 + (x+9)n_3 - 67}{15(x+7)}
\]
\[
L_3 = \frac{2(x+1)(3x+22) - (2x+23)n_1 - 2(x+7)n_2 + (13x+97)n_3}{15(x+7)}
\]

\[
L_2 = L_3 + n_2 - n_3
\]
\[
L_1 = L_3 + n_1 - n_3
\]
\[
H_u = n_3 - L_3
\]
\[
H_d = -1 - H_u
\]
\[
u_3 = -Q_3 - H_u
\]
\[
d_3 = -Q_3 - H_d + x
\]
\[
l_3 = -L_3 - H_d + x
\]
\[
Q_1 = 3 + Q_3
\]
\[
Q_2 = 2 + Q_3
\]
\[
u_1 = 5 + u_3
\]
\[
u_2 = 2 + u_3
\]
\[
d_1 = 1 + d_3
\]
\[
d_2 = d_3
\]
\[
l_1 = 5 - n_1 + n_3 + l_3
\]
\[
l_2 = 2 - n_2 + n_3 + l_3
\]

**TABLE II:** Standard model fields \(H\)--charges in terms of the bilinear \(H\)--charges \(n_i\) and \(x\)

fraction close to 21 in clear disagreement with the condition \(|f_i| < 10\). In the case \(n_i \geq 0\) the suppression has to be much more stronger implying larger values for \(n_i\) and correspondingly larger values for \(|f_i|\). Consequently, consistency with astrophysical and cosmological data excludes the possibility of neutralino decays induced by bilinear \(R\)--parity breaking couplings and therefore fix the \(n_i\) charges to be fractional.

Once the \(n_i\) charges are chosen to be fractional we are left with a purely trilinear \(R\)--parity violating framework in which the order of magnitude of the couplings is constrained by \(n_i\). Unavoidably, fractional \(n_i\) charges imply vanishing \(\lambda_{ijj}\) and \(\lambda'\) [see Eqs. (A8) and (A9) in the Appendix]. The other couplings \([\lambda_{ijk} (i \neq j \neq k)\) and \(\lambda''\)] can vanish or not depending
on the values of the $n_i$ charges which are arbitrary as long as they satisfy the constraint $|f_i| < 10$. This freedom allows one to define a set of models which we now discuss in turn:

(i) Models in which the fractional $n_i$ charges are such that all the trilinear $R$–parity violating couplings are forbidden as well, and the MSSM is obtained \[31, 33\]. This can be achieved for example by fixing $n_1 = -3/2$, $n_2 = -5/2$ and $n_3 = -5/2$ \[31\].

(ii) Models with only a single nonvanishing $\lambda_{ijk}$ and vanishing $\lambda''$ couplings. Let us discuss this in more detail. An integer $n_{\lambda_{ijk}}$ will imply a nonvanishing $\lambda_{ijk}$ coupling. The $H$–charges of the remaining couplings ($n_{\lambda''}$, $n_{\lambda_{jki}}$, $n_{\lambda_{ikj}}$) can be determined from Eq. (A4), which can be rewritten as

$$n_i = n_{\lambda_{ijk}} - x - L_{k3} - 1 - n_{jk} \quad (i \neq j \neq k). \tag{5}$$

From this expression the sum of $n_i$ charges ($N = n_i + n_j + n_k$), that according to Eq. (A10) determine the $n_{\lambda''}$, become

$$N = n_{\lambda_{ijk}} - x - L_{k3} - 1 + 2n_k, \tag{6}$$

and from Eqs. (5) and (A4) the other trilinear charges ($n_{\lambda_{jki}}$ and $n_{\lambda_{ikj}}$) can be expressed as

$$n_{\lambda_{jki}} = -n_{\lambda_{ijk}} + L_{i3} + L_{k3} + 2x + 2 + 2n_j, \tag{7}$$

$$n_{\lambda_{ikj}} = n_{\lambda_{ijk}} - L_{k3} + L_{j3} - 2n_{jk}. \tag{8}$$

Thus, from the set up of Eqs. (6), (7) and (8) it can be seen that as long as $n_j$, $n_k$ and $n_{jk}$ are not half-integers an integer $n_{\lambda_{ijk}}$ charge enforces $N$, $n_{\lambda_{jki}}$, and $n_{\lambda_{ikj}}$ to be fractional which implies that all the $\lambda''$ as well as any other $\lambda$ coupling different from $\lambda_{ijk}$ vanishes. Consequently, in models in which there is an integer trilinear charge $n_{\lambda}$ and the charges $n_j$, $n_k$ and $n_{jk}$ are not half-integers only a single $\lambda$ coupling is allowed.

(iii) Models in which a single $\lambda_{ijk}$ and all the $\lambda''$ are nonvanishing. As in the previous case $n_{\lambda_{ijk}}$ must be an integer and in addition the corresponding $n_k$ must be a half-integer as to guarantee an integer $N$ (see Eq. (6)). Moreover, $x$ and the resulting $N$ should conspire to yield a set of integer $n_{\lambda''}$ charges (see Eq. (A10)).
(iv) Models with nonvanishing $\lambda_{ijk}$ and $\lambda_{jki}$. These models result once the $n\lambda_{ijk}$ is an integer and $n_j$ a half-integer. As can be seen from Eq. (7) in this case $n\lambda_{jki}$ turn out to be an integer allowing the $\lambda_{jki}$ as required. Nonvanishing $\lambda_{ijk}$ and $\lambda_{ikj}$ are also possible but never the three couplings simultaneously.

(v) Along similar lines as those followed in (ii), it can be shown that models including only $B$-violating couplings can be properly defined once any $n\lambda'_{ijk}$ becomes an integer and the bilinear charges $n_j, n_k$ and $n_j + n_k$ turn out to be not half-integers.

All the models described above have phenomenological implications. For instance, the family of models discussed in (v) might lead to a neutralino decaying hadronically whereas those discussed in (ii) and (iv) share the property of a leptonically decaying neutralino. These phenomenological aspects can have interesting consequences for collider experiments but here we will not deal with them. Instead, in light of the recent data on cosmic-ray electron/positron fluxes, we will analyze the phenomenology of a neutralino decaying DM in the minimal trilinear $R$–parity violating models outlined in (ii).

A few additional comments regarding these models are necessary. Bilinear $R$–parity violating couplings are always induced through RGE running of the trilinear breaking parameters [42]:

$$
\mu_i = \frac{\mu_0}{16\pi^2} \left[ \lambda_{ijk} (\hat{h}^*_e)_{jk} + 3\lambda'_{ijk} (\hat{h}^*_d)_{jk} \right] \ln \left( \frac{M_X}{M_S} \right), \tag{9}
$$

where $M_X$ is the scale that defines the purely trilinear model and $M_S$ is the scale of the supersymmetric scalars. At first sight these parameters could render the minimal trilinear $R$–parity violating models valid only when the corresponding trilinear couplings are sufficiently small so to guarantee that the astrophysical and cosmological bounds on the neutralino lifetime are satisfied. However, since the $\lambda'$ and $\lambda_{ijj}$ couplings are always vanishing and the contributions of the allowed $\lambda_{ijk}$ ($i \neq j \neq k$) require nondiagonal $h^*_e_{jk}$ which are forbidden by $U(1)_H$, in these kind of models no bilinear parameters can be induced at all.

### III. DECAYING NEUTRALINO DARK MATTER

In this section we will study neutralino decays in the context of the minimal $R$–parity violating models that were defined in the previous section. The lifetime of a mainly gaugino
TABLE III: Set of bilinear $H$–charges consistent with the trilinear $H$–charge choice $n_\lambda = -10$.

| $x$ | $n_1$ | $n_2$ | $n_3$ | $|f_i|$ |
|-----|-------|-------|-------|--------|
| $\lambda_{231}$ | 1 | $7/3$ | $-19/3$ | $-25/3$ | $< 7$ |
| $\lambda_{123}$ | 1 | $-10/3$ | $-19/3$ | $7/3$ | $< 6$ |
| $\lambda_{132}$ | 1 | $-5/3$ | $17/3$ | $-20/3$ | $< 7$ |

According to this expression the viability of a neutralino decaying DM will depend, for a few TeV neutralino mass, on the slepton mass spectrum and the size of the corresponding $\lambda$ coupling that will be determined by the choices $n_\lambda < 0$ or $n_\lambda \geq 0$. These choices are to some extent not arbitrary as they must satisfy the condition $|f_i| < 10$: Given a value for $n_\lambda$, the $n_j$ and $n_k$ charges can be fixed through Eq. (5) and for a particular $x$ the different $|f_i|$ charges can be calculated. Tables [III] and [IV] show some examples.

In the case $n_\lambda < 0$, due to the strong suppression induced by the factor $m_{3/2}/M_P$, a coupling $\lambda$ as small as $10^{-23}$ is possible if $n_\lambda = -10$ and accordingly even with a not so heavy slepton the constraint $\tau_\chi = 10^{26}$ sec can be satisfied. In the case $n_\lambda \geq 0$ such a small $R$–parity breaking coupling will require a value for $n_\lambda$ irreconcilable with the limit $|f_i| < 10$. Thus, in this case $\lambda$ will be larger and the correct neutralino lifetime will result, if possible, only from an additional suppression given by a superheavy slepton as those featured by split supersymmetry [43]. Figure [I] shows the values of $\lambda$ (arising from different $n_\lambda$ choices) and $M_S$ consistent with $\tau_\chi = 10^{26}$ sec In the solid line (lower left corner) $n_\lambda = -10, \ldots, -1$. In the dashed line $n_\lambda > 10$ and the resulting $|f_i|$ charges are inconsistent with the requirement $|f_i| < 10$ thus ruling out the possibility of a decaying neutralino dark matter in the range $M_S = 10^7 - 10^{12}$ GeV. Finally, in the solid line (upper right corner) $n_\lambda = 6, \ldots, 10$. The values below 6 will require a slepton mass above $10^{13}$ GeV that leads to a gluino lifetime exceeding the age of the Universe [44] and therefore are excluded.

Some words are in order concerning the minimal trilinear $R$–parity violating models with superheavy sleptons. In split supersymmetry the scalar masses, apart from the Higgs boson, are well above the electroweak scale, $M_S \lesssim 10^{13}$ GeV. Assuming $M_S = 10^{13}$ GeV it can...
TABLE IV: Set of bilinear $H$–charges consistent with the trilinear $H$–charge choice $n_{\lambda} = 7$.

| $\lambda_{231}$ | $n_{1}$ | $n_{2}$ | $n_{3}$ | $|f_{i}|$ |
|----------------|--------|--------|--------|--------|
| $\lambda_{123}$ | 1      | $-2/3$ | $-1/3$ | $-1/3$ | $< 6$ |
| $\lambda_{132}$ | 1      | $5/3$  | $5/3$  | $-5/3$ | $< 5$ |

FIG. 1: $\lambda$ coupling as a function of the slepton mass for a neutralino lifetime of $10^{26}$ sec. The solid lines (blue) correspond to values of $\lambda$ and $M_{S}$ well given by a minimal trilinear $R$–parity breaking model whereas those in the range of the dashed line (red) are not consistent with the limits on $H$–charges (see text for details).

be seen from Fig. 1 that the correct neutralino lifetime requires couplings of order $10^{-5}$. According to Eq. (9) couplings of this size will lead to $\mu_{i}/\mu_{0} \sim 10^{-12}$ in sharp disagreement with the bounds from astrophysical and cosmological data [22]. However, in contrast to previous analysis of neutralino decaying DM within split supersymmetry [15, 21], in this context the RGE running of the trilinear $R$–parity breaking parameters do not induce any $R$–parity breaking bilinear coupling.

A. PAMELA, Fermi and ATIC anomalies

In this section we will show that the PAMELA, ATIC and Fermi LAT data can be well accounted for by a decaying neutralino DM in the context of minimal trilinear $R$–
FIG. 2: Ratio of positron to electron-plus-positron (left panel) and total electron-plus-positron (right panel) fluxes arising from a long–lived neutralino decaying through trilinear $R$–parity breaking couplings (see text for details).

parity violating models. In order to fit the electron-positron fluxes we fix the trilinear $R$–parity breaking coupling and the neutralino mass and lifetime according to $\lambda = 3.2 \times 10^{-23}$, $m_\chi = 2038$ GeV and $\tau_\chi = 1.3 \times 10^{26}$ sec Note that such a coupling can arise from $n_\lambda = -10$. We generate a supersymmetric spectrum with SuSpect \cite{45} by choosing the benchmark point $F$ defined in Ref. \cite{13}. In the resulting spectrum the neutralino becomes mainly wino and the scalar masses have a size of $M_S \sim 10^4$ GeV. For cosmic rays propagation, we followed Ref. \cite{46} whereas for DM we used the spherically symmetric Navarro, Frenk, and White \cite{47} profile and the propagation model MED introduced in Ref. \cite{48}. The electron and positron energy spectra were generated using PYTHIA \cite{49}.

Figure 2 shows the ratio of positron to electron-plus-positron and the total electron-plus-positron fluxes originated from neutralino decays induce by the trilinear $R$–parity violating couplings $\lambda_{231}$, $\lambda_{132}$ and $\lambda_{123}$. Decays induced by $\lambda_{231}$ always involve hard electrons and positrons and therefore are well suited to explain ATIC data as can be seen in Fig. 2. In contrast, the decays through the couplings $\lambda_{132}$ and $\lambda_{123}$ involve either final state muons or taus and thus electron and positrons with lower energies as those required to explain Fermi LAT and PAMELA measurements. Once the neutralino mass is fixed its lifetime will depend only on the ratio $M_S^2/\lambda$ [see Eq. (10)] and of course will not change as long as this ratio remains constant. Moreover, the effect of $M_S$ on dark matter relic density is completely negligible for $M_S > 10^4$ GeV \cite{44,50}. Accordingly, the results in Fig. 2 also hold in the case
of superheavy sleptons and large couplings.

IV. CONCLUSIONS

We have studied decaying neutralinos as DM candidates in the context of supersymmetric models extended with an anomalous $U(1)_R$ flavor symmetry. We have shown that theoretical motivated limits on the standard model field $H$–charges in addition to astrophysical and cosmological constraints on neutralino decaying DM forbid the decays induced by bilinear $R$–parity breaking couplings and allow one to define a set of purely trilinear $R$–parity violating models in which the neutralino can be stable on cosmological scales. Among all these scenarios we have found a class of models (minimal $R$–parity violating models) in which a single $R$–parity and lepton number breaking coupling $\lambda_{ijk} (i \neq j \neq k)$ give rise to leptonic neutralino decays. In these schemes, for a few TeV neutralino mass, a decaying lifetime of $\tau_\chi \sim 10^{26}$ sec can be readily achieved for a variety of $H$–charge assignments and slepton masses ranging from few TeV up to the typical scales of split supersymmetry. Moreover, we have shown that these minimal $R$–parity violating models (depending on the trilinear $R$–parity breaking parameter defining the model itself) provide an explanation to the observed anomalies in the electron-positron fluxes reported by PAMELA, ATIC, and Fermi LAT.

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APPENDIX A: $H$–CHARGES OF THE DIFFERENT COUPLINGS

In this Appendix we give expressions for the standard model Yukawa couplings as well as for the lepton and baryon number couplings appearing in the superpotential. The quark Yukawa couplings can be written as

$$h_{ij}^u \sim \theta^{Q_{i3} + u_{j3}} \quad \quad h_{ij}^d \sim \theta^{Q_{i3} + d_{j3} + x}, \quad (A1)$$
while the charged lepton Yukawa couplings we have

\[ n_{h_{ij}} = L_{i3} + l_{j3} + x \Rightarrow h_{ij}^l \sim \begin{cases} 
\theta^{L_{i3} + l_{j3} + x} & L_{i3} + l_{j3} + x \geq 0 \\
\frac{m_{3/2} \theta}{\Lambda_p}^{L_{i3} + l_{j3} + x} & L_{i3} + l_{j3} + x < 0 \\
0 & L_{i3} + l_{j3} + x \text{ fractional.}
\end{cases} \quad (A2) \]

Here \( n_{h_{ij}} \) denotes the \( H \)-charge of the gauge invariant term with coupling \( h_{ij} \). From the expressions in tab. II these \( H \)-charges can be rewritten in terms of \( n_i \) and \( x \) as follows:

\[ n_{h_{ij}} = L_{i3} + l_{j3} + x = n_i + n_j + L_{j3} + x, \quad (A3) \]

where \( n_{ij} = n_i - n_j \). For the lepton number and \( R \)-parity breaking couplings one can proceed along similar lines, that is to say from the \( H \)-charges of the standard model fields involved in each case, and according to Table II the following relations can be derived:

\[ n_{\lambda_{ijk}} = L_i + L_j + l_k = n_i + n_j + L_{k3} + 1 + x, \quad (A4) \]

and

\[ n_{\lambda'_{ijk}} = L_i + Q_j + d_k = n_i - n_0 + n_{h_{ij}^d}. \quad (A5) \]

Explicitly, the quark Yukawa couplings matrices can be written—up to order 1 factors—as

\[ h^u \sim \begin{pmatrix} \theta^8 & \theta^5 & \theta^3 \\ \theta^7 & \theta^4 & \theta^2 \\ \theta^5 & \theta^2 & 1 \end{pmatrix}, \quad h^d \sim \theta^x \begin{pmatrix} \theta^4 & \theta^3 & \theta^3 \\ \theta^3 & \theta^2 & \theta^2 \\ \theta & 1 & 1 \end{pmatrix}, \quad (A6) \]

and the charged lepton \( H \)-charges as

\[ n_{h_{ij}^l} = \begin{bmatrix} x + 5 & x + n_1 - n_2 + 2 & x + n_1 - n_3 \\
x - n_1 + n_2 + 5 & x + 2 & x + n_2 - n_3 \\
x - n_1 + n_3 + 5 & x - n_2 + n_3 + 2 & x \end{bmatrix}. \quad (A7) \]
The trilinear lepton number violating couplings $H$–charges are given by

\[
\begin{pmatrix}
  n_{\lambda_{211}} & n_{\lambda_{212}} & n_{\lambda_{213}} \\
  n_{\lambda_{311}} & n_{\lambda_{312}} & n_{\lambda_{313}} \\
  n_{\lambda_{231}} & n_{\lambda_{232}} & n_{\lambda_{233}}
\end{pmatrix}
= 
\begin{pmatrix}
  x + n_2 + 6 & x + n_1 + 3 & x + n_1 + n_2 - n_3 + 1 \\
  x + n_3 + 6 & x + n_1 - n_2 + n_3 + 3 & x + n_1 + 1 \\
  x - n_1 + n_2 + n_3 + 6 & x + n_3 + 3 & x + n_2 + 1
\end{pmatrix}
\]  
(A8)

and

\[
n_{\lambda'_{ijk}} = 
\begin{pmatrix}
  x + n_i + 5 & x + n_i + 4 & x + n_i + 4 \\
  x + n_i + 4 & x + n_i + 3 & x + n_i + 3 \\
  x + n_i + 2 & x + n_i + 1 & x + n_i + 1
\end{pmatrix},
\]  
(A9)

Finally for the baryon number breaking couplings we found

\[
\begin{pmatrix}
  n_{\lambda''_{121}} & n_{\lambda''_{221}} & n_{\lambda''_{321}} \\
  n_{\lambda''_{131}} & n_{\lambda''_{231}} & n_{\lambda''_{331}} \\
  n_{\lambda''_{123}} & n_{\lambda''_{223}} & n_{\lambda''_{323}}
\end{pmatrix}
= 
\begin{pmatrix}
  \frac{1}{3} (3x + N + 17) & \frac{1}{3} (3x + N + 8) & \frac{1}{3} (3x + N + 2) \\
  \frac{1}{3} (3x + N + 17) & \frac{1}{3} (3x + N + 8) & \frac{1}{3} (3x + N + 2) \\
  \frac{1}{3} (3x + N + 14) & \frac{1}{3} (3x + N + 5) & \frac{1}{3} (3x + N - 1)
\end{pmatrix},
\]  
(A10)

where we have defined $N = n_1 + n_2 + n_3$.


[51] We have fixed a global sign misprint on $Q_3$ in Ref. 30.