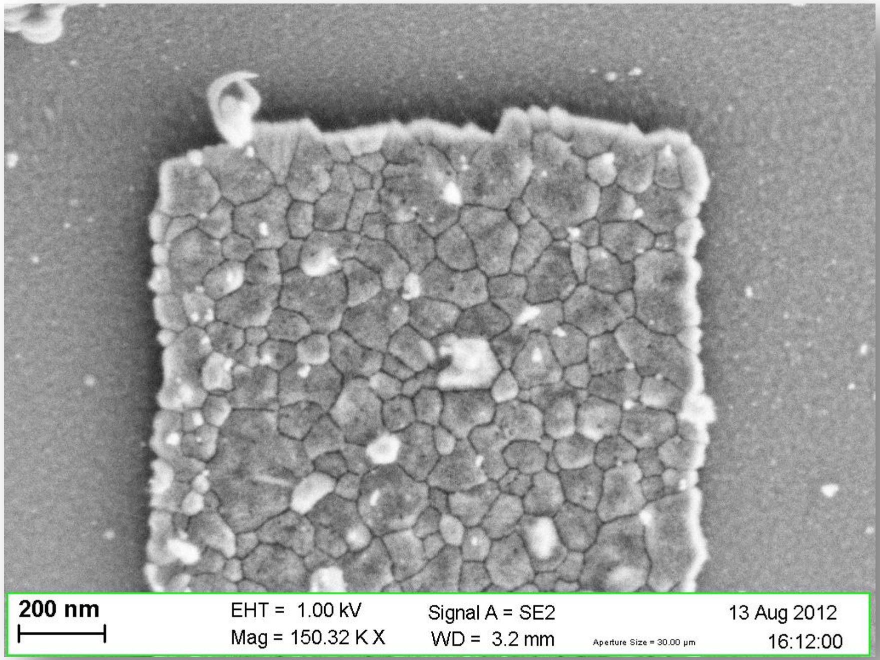
Probabilistic model for MEMS micro-beam resonance frequency



Lucas Vincent
Wu Ling
Arnst Maarten
Golinval Jean-Claude
Paquay Stéphane
Noels Ludovic

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MEMS

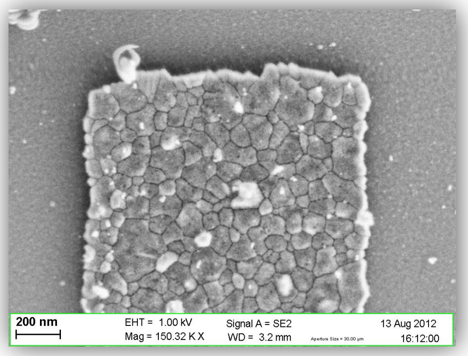
- MEMS
  - MicroElectroMechanical systems
  - Application in a wide variety of fields
    - Automobile industry
    - Aeronautics
    - Medecine
    - Telecom
    - …

Common denominator: microscopic scale.

➢ How does it affect the material properties ?

The problem

- A macroscopic property of a MEMS can exhibit a scatter
  - Due to the fabrication process
  - Due to uncertainties of the material
  - …



→ The objective of this work is to estimate this scatter

- Characteristic of the model:
  - Clamped microbeam
  - Macroscopic property of interest: first mode eigenfrequency
    - For a MEMS gyroscope for example

- In our model, the uncertainties come from the material:
  - Polysilicon is anisotropic
  - Polysilicon is polycrystalline

→ **Each grain has a random orientation**

- Considering each grain to compute the macro quantity is too heavy

↳ **3-scale procedure**

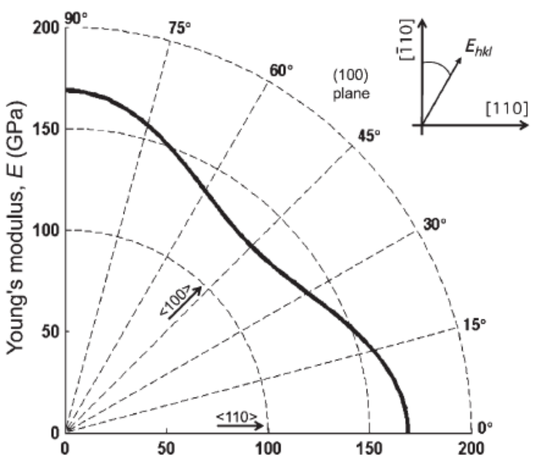
A Silicon crystal

- What is the Young's Modulus of Silicon ? [1]

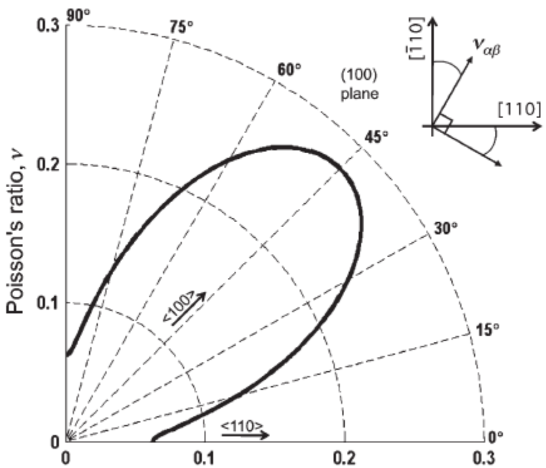
  Based on Hall, with x, y, z aligned with $[100], [010]$ and $[001]$, we have:

E_x [GPa]	ν_{yz}	G_{yz}[GPa]	c_{11}[GPa]	c_{12}[GPa]	c_{44}[GPa]
130	0,28	79,6	165,6	63,9	79,5

- Maximum value of E in <111> direction: $188\ GPa$



Young's modulus of Silicon, (100) plane

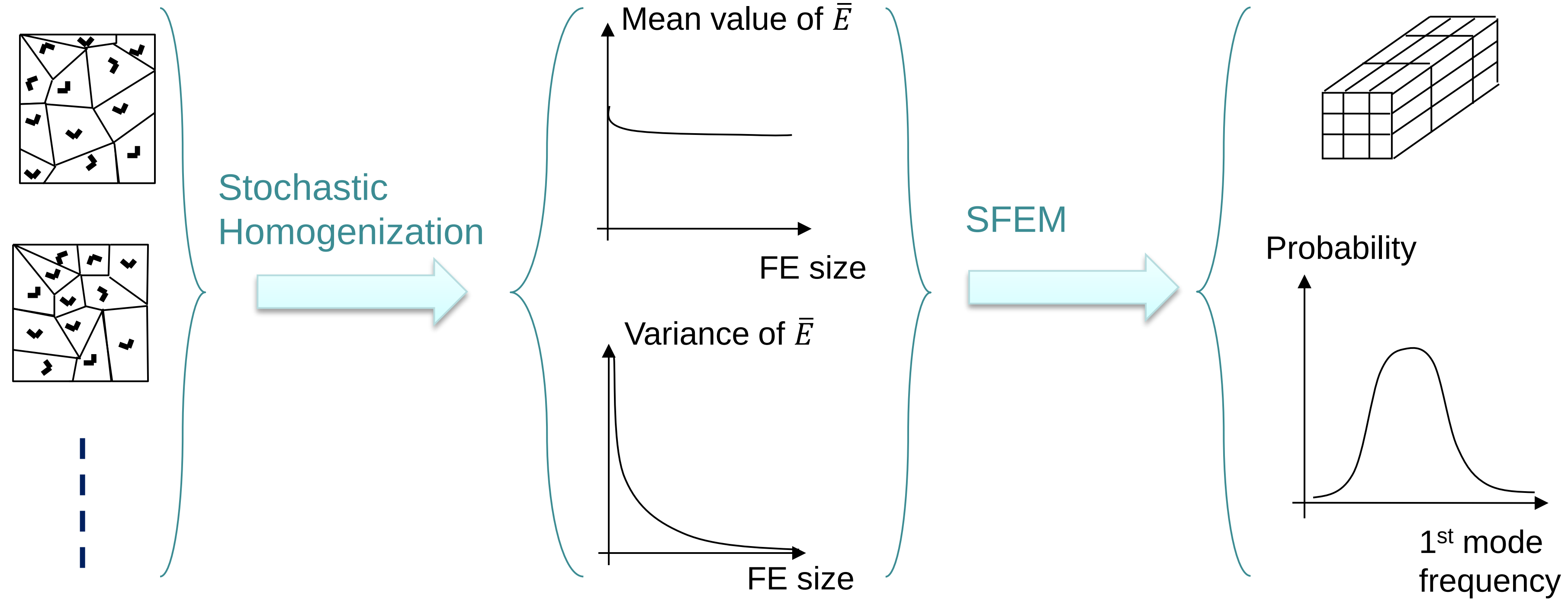


Poisson's ratio of Silicon, (100) plane

[1] "*What is the Young's Modulus of Silicon*", M.A. Hopcroft, W.D. Nix, T.W. Kenny, Journal of microelectromechanical systems, april 2010, p.229-238

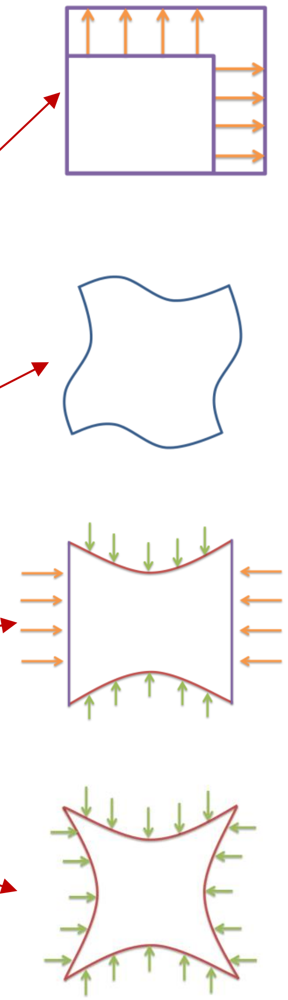
A 3-scale procedure

Grain-scale or micro-scale	Meso-scale	Macro-scale
➢ Samples of the microstructure (volume elements) are generated ➢ Each grain has a random orientation	➢ Intermediate scale ➢ The distribution of the material property $\mathbb{P}(C)$ is defined	➢ Uncertainty quantification of the macro-scale quantity ➢ E.g. the first mode frequency $\mathbb{P}(f_1)$



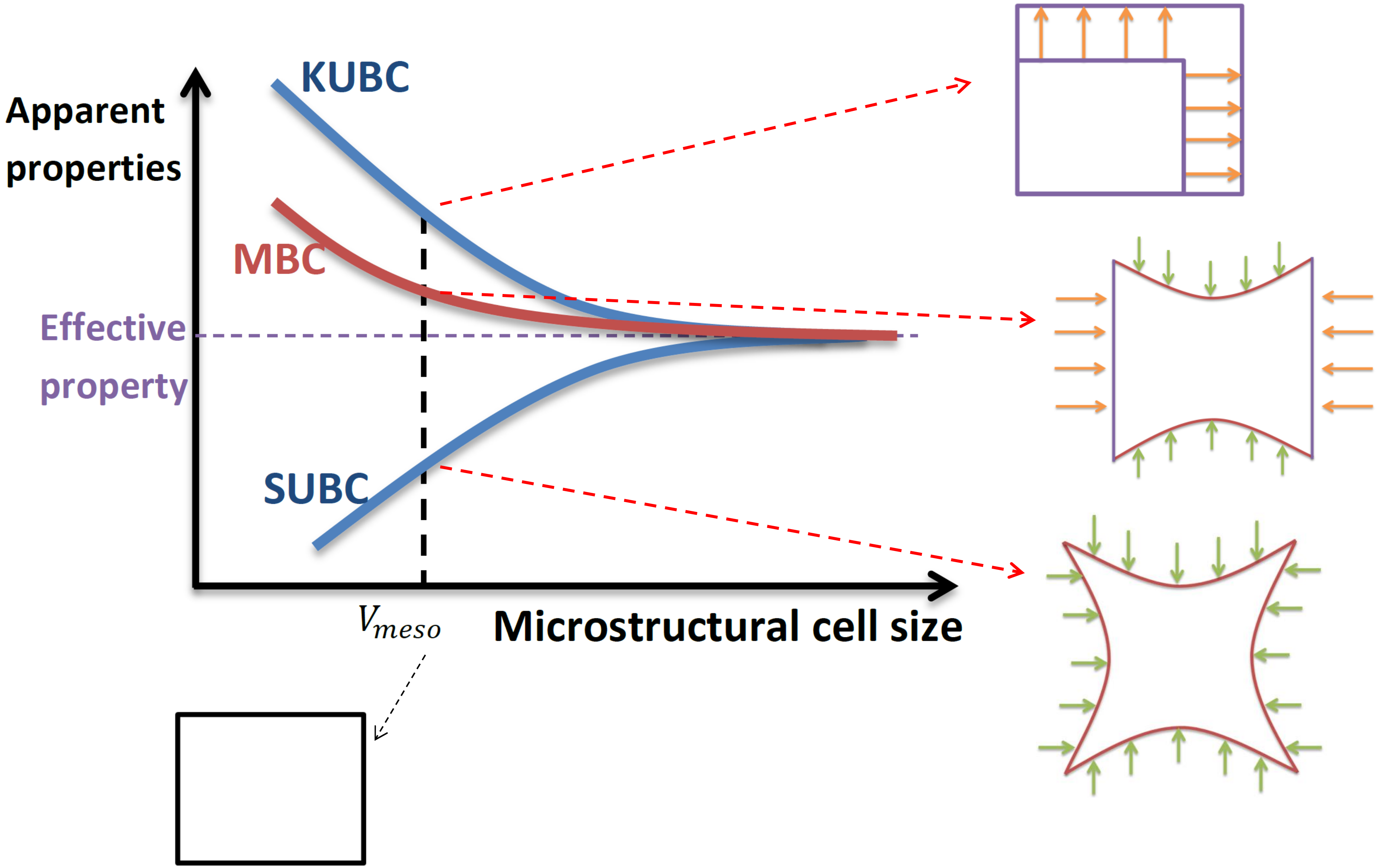
RVE & SVE

- **Representative Volume Element (RVE)**
  - 1 elasticity tensor : C^{eff}
- **Statistical Volume Element (SVE)**
  - A range of elasticity tensor
    - Depends on boundary conditions
      - Kinematic Uniform BC (KUBC)
      - Periodic BC (PBC)
      - Mixed BC (MBC)
      - Static Uniform BC (SUBC)

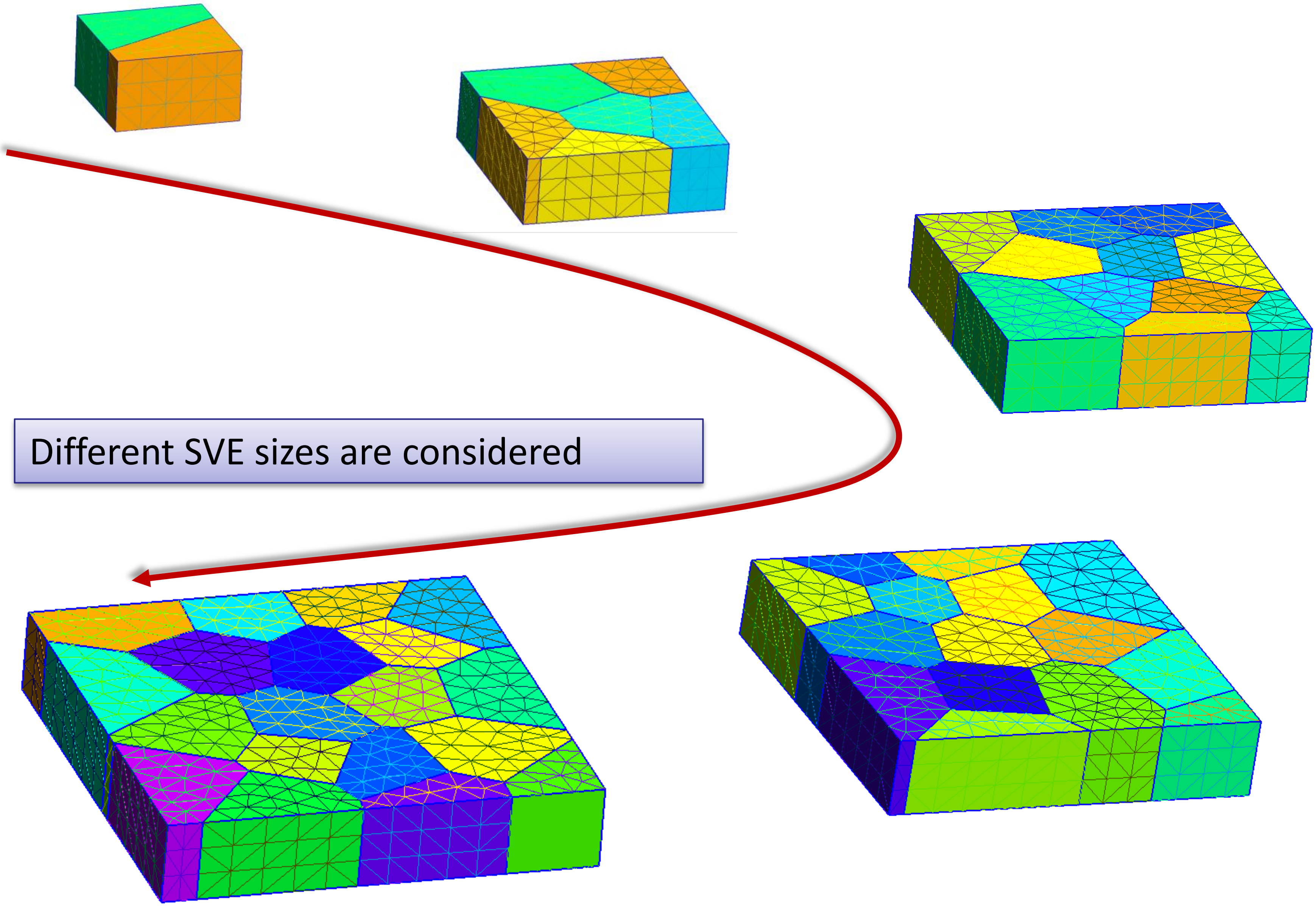


Note : $A > B \ iff \ A - B \ is \ positive - definite \ (Loewner \ ordering)$

Different boundary conditions



SVE of different sizes



Different SVE sizes are considered

Elasticity tensor computation

- When no access to the finite element stiffness matrix $\boldsymbol{K}$:

$$\boldsymbol{C}_{BC} = \underset{\boldsymbol{C}^{Lower}<\boldsymbol{C}<\boldsymbol{C}^{Upper}}{\arg\min} \| < \boldsymbol{\sigma} >_{BC} - \boldsymbol{C} < \boldsymbol{\epsilon} >_{BC} \|$$

  – The bounds can be :
    » Voight & Reuss bounds
    » KUBC & SUBC
    » In Between (Huet partition theorem [1])

    → Each are used in [3] and [4]

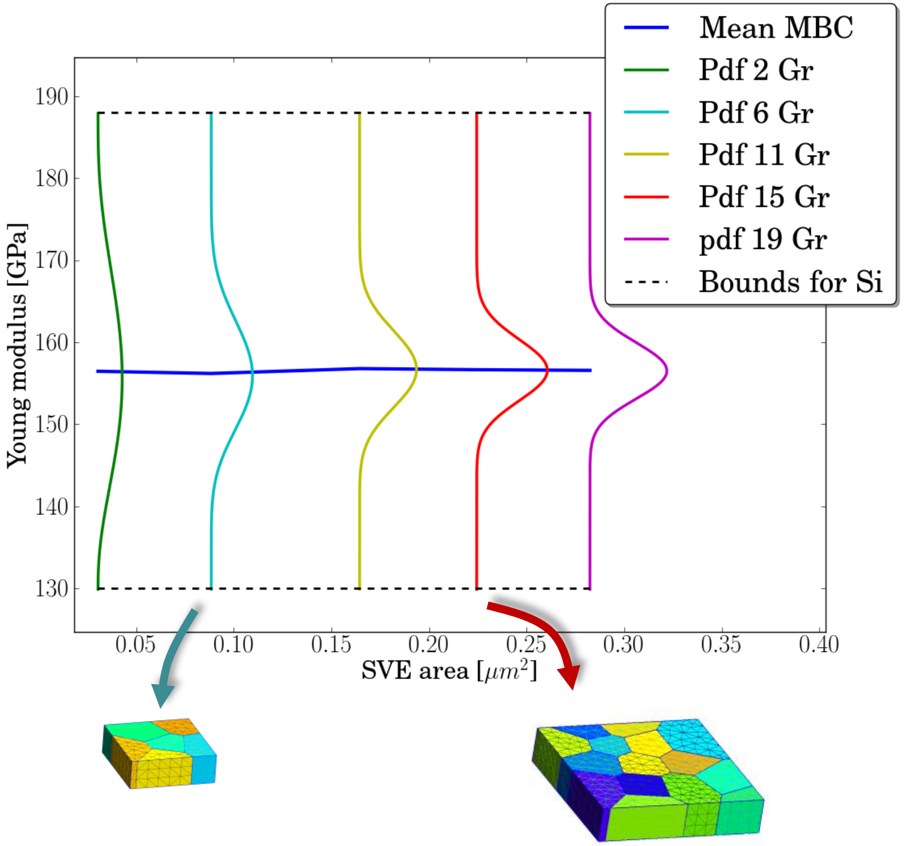
  – E.g., this can happen with concrete experimental samples (see [2])

- When access to the finite element stiffness matrix $\boldsymbol{K}$:
  – The elasticity tensor can be directly computed from the stiffness matrix
  – This is computational homogenization [5]
  – K^* being a rewritten version of K depending on the boundary condition:

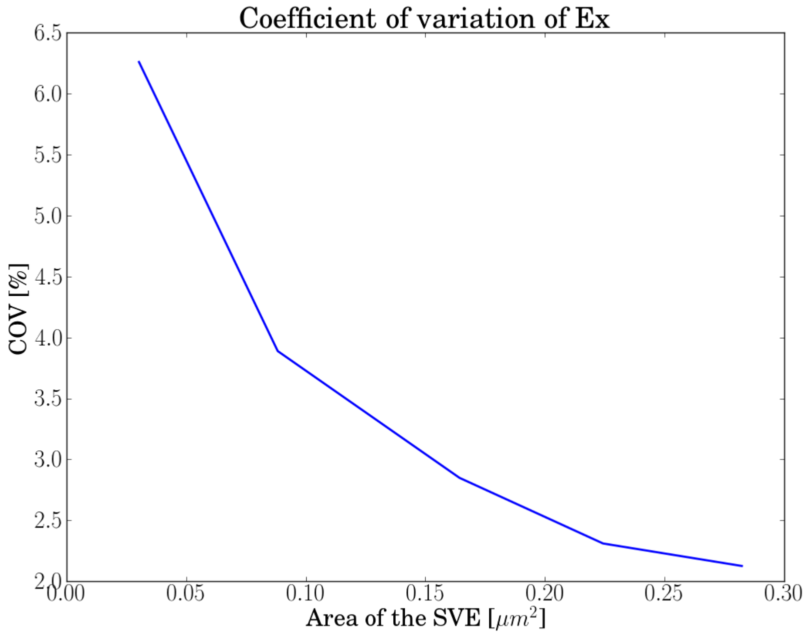
$$\mathbb{C}_{BC} = \frac{1}{V_0} \sum_i \sum_j \boldsymbol{X}_{(i)} \boldsymbol{K}^*_{(ij)} \boldsymbol{X}_{(j)}$$

[2] "*Application of variational concepts to size effects in elastic heterogenous bodies*", Huet C.,J.Mech.Phys.Solids, 1990
[3] "*A probabilistic model for bounded elasticity tensor random fields with application to polycrystalline microstructures* ", Guilleminot J., Noshadravan A., Soize C., Ghanem R.G., Elsevier, 2011
[4] "*Validation of a probabilistic model for mesoscale elasticity tensor of random polycrystals*", Noshadravan A., Ghanem R., Guilleminot J., Atodaria I, International journal for uncertainty quantification, 2011
[5] "*Computational homogenization: implementation and extension*", Kouznetsova V., Eindhoven University of Technology, 2010

From micro to meso: results



- The bounds do not depend on the SVE size
- However → The bigger the SVE, the less likely ℂ is close to them



- The coefficient of variation is defined as :

$$COV = \frac{\sqrt{Variance}}{mean} \cdot 100$$

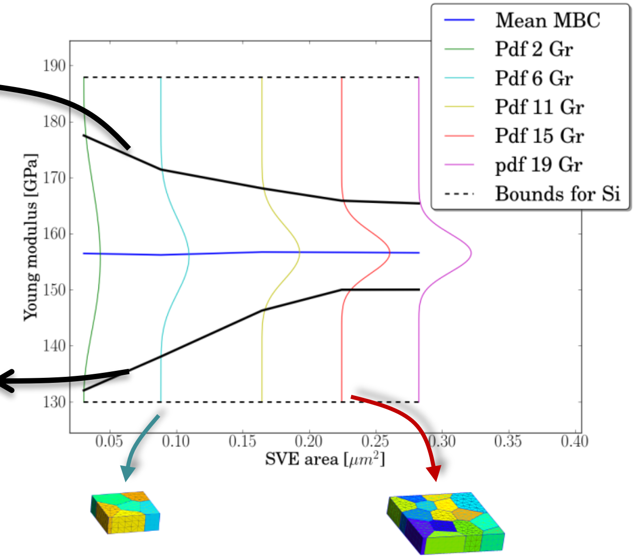
- The bigger the SVE, the lower the COV

From micro to meso: results

1D case

$$E_x^u = Max(E_x^{KUBC})$$

$$E_x^l = Min(E_x^{SUBC})$$



Matrix case [3,4]

$$\boldsymbol{C}_l = \arg\min_{\boldsymbol{C}\in \boldsymbol{C}_{ad}^l} \sum_{k=1}^{N_s} \|\boldsymbol{C}^{SUBC}(\theta_k) - \boldsymbol{C}\|_F$$

$$\boldsymbol{C}_{ad}^l = \{\boldsymbol{C} \in \mathbb{M}_n^+(\mathbb{R}) | \boldsymbol{C} < \boldsymbol{C}^{SUBC}(\theta_k), k = 1, \dots, N_s\}$$

$$\boldsymbol{C}_u = \arg\min_{\boldsymbol{C}\in \boldsymbol{C}_{ad}^u} \sum_{k=1}^{N_s} \|\boldsymbol{C} - \boldsymbol{C}^{KUBC}(\theta_k)\|_F$$

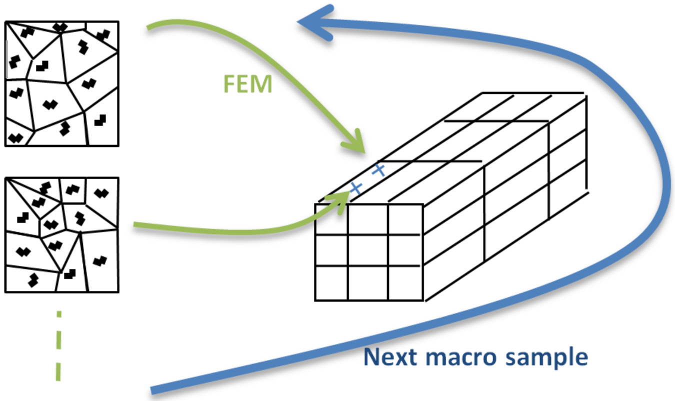
$$\boldsymbol{C}_{ad}^u = \{\boldsymbol{C} \in \mathbb{M}_n^+(\mathbb{R}) | \boldsymbol{C}^{SUBC}(\theta_k) < \boldsymbol{C}, k = 1, \dots, N_s\}$$

[3] "*A probabilistic model for bounded elasticity tensor random fields with application to polycrystalline microstructures* ", Guilleminot J., Noshadravan A., Soize C., Ghanem R.G., Elsevier, 2011

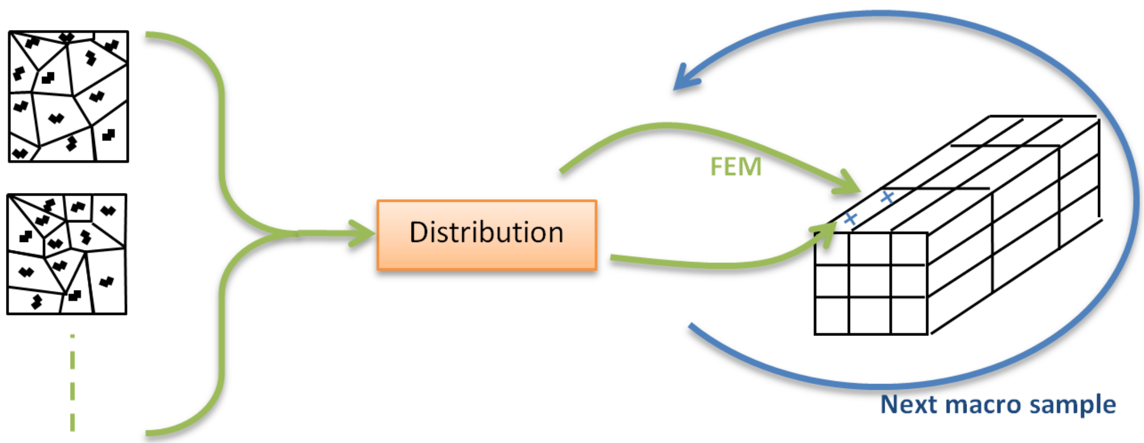
[4] "*Validation of a probabilistic model for mesoscale elasticity tensor of random polycrystals*", Noshadravan A., Ghanem R., Guilleminot J., Atodaria I, International journal for uncertainty quantification, 2011

The problem

- If a Monte-Carlo procedure is applied with FEM over a microbeam:
  - No distribution and no generator:
    - For each macro realization
      - At each Gauss points:
        - » a SVE should be computed

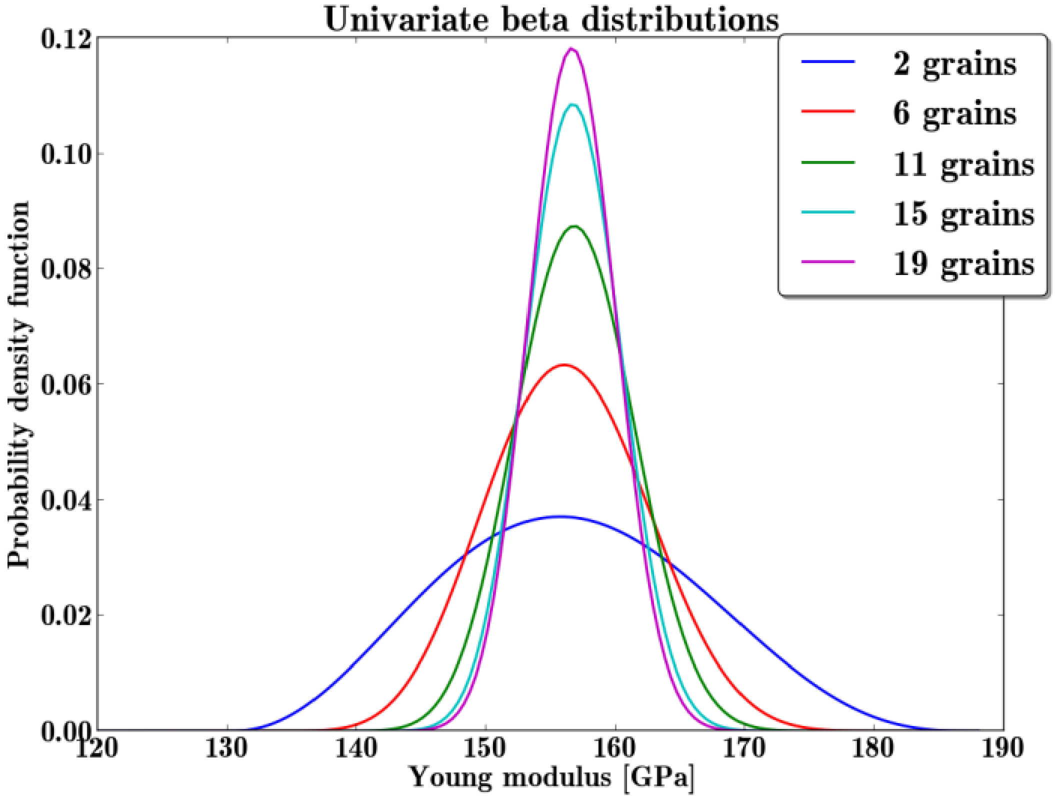


  - With distribution and generator:
    - Define a distribution encompassing the microstructure statistical behaviour
    - At each Gauss point:
      - Use the generator



- 1D case of E_x: Beta-based distribution of 4 parameters

$$P(E_x) = \frac{1}{\beta(\alpha,\beta)}\left[\frac{E_x - E_x^l}{E_x^u - E_x^l}\right]^{\alpha-1}\left[\frac{E_x^u - E_x}{E_x^u - E_x^l}\right]^{\beta-1}$$



Extension to matrix case

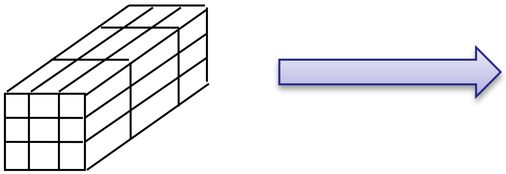
- Maximize the information entropy → $\max_{\mathbb{C}} - \int_{M_n^+(\mathbb{R})} p([\mathbb{C}]) \ln\big(p([\mathbb{C}])\big)\, d\mathbb{C}$
  - Under constraints encompassing the available information
    - Samples of $\mathbb{C}$
    - Bounds $\mathbb{C}_u$ and $\mathbb{C}_l$
  - No change of variable
    - Matrix-variate Kummer-beta distribution [6]
  - Using the N space [3,4] → $N = \frac{1}{C - C_l} - \frac{1}{C_u - C_l}$
    - Non-linear change of variable
    - Matrix-variate Gamma distribution [7]
  - Using the N' space → $N' = C - C_l$
    - Linear change of variable
    - Matrix-variate Gamma distribution
    - Loose the bound information

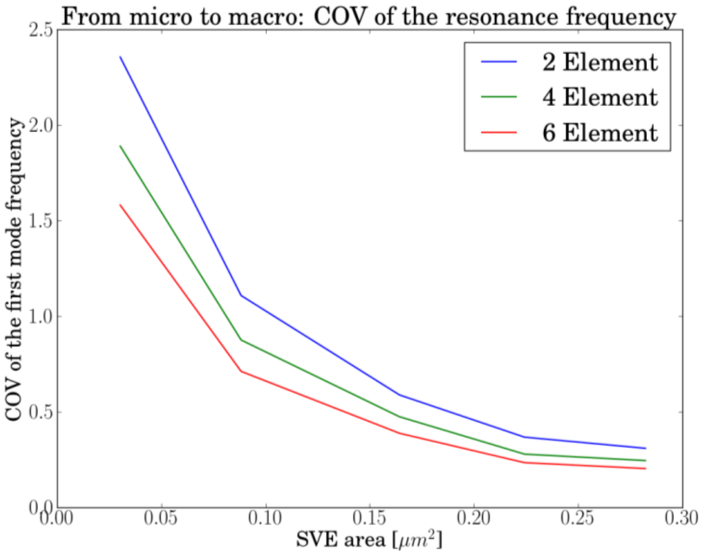
[6] *"A bounded random matrix approach for stochastic upscaling",* S. Das and R. Ghanem, Society for industrial and appl. Math., 2009,
[7] "*Random matrix theory for modeling uncertainties in computational mechanics*", Soize C.,Comp. meth. In Appl. Mech. And Engrg., 2004

Conclusions & Perspectives

- From samples of grain orientations, elasticity tensors can be generated at meso-scale.

- How can we propagate the uncertainties up to the macroscopic scale?
  - Relevant SVE size
  - Correlation
  - …





- Upgrades can take various forms:
  - 3D meso to macro part
  - Perturbation or Spectral Finite Element
  - Periodic boundary condition for the microstructure
  - …

Thank you for your attention !