Sterile neutrinos and R_K

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Abstract. We consider an enhancement in the violation of lepton flavour universality in light meson decays arising from modified $W\ell\nu$ couplings in the standard model minimally extended by sterile neutrinos. Due to the presence of additional mixings between the active neutrinos and the new sterile states, the deviation from unitarity of the leptonic mixing matrix intervening in charged currents might lead to a tree-level enhancement of $R_P = \Gamma(P \to e\nu)/\Gamma(P \to \mu\nu)$, with $P = K, \pi$. These enhancements are illustrated in the case of the inverse seesaw, showing that one can saturate the current experimental bounds on Δr_K (and Δr_π), while in agreement with the different experimental and observational constraints.

1. Introduction

Lepton flavour universality (LFU) is a distinctive feature of the Standard Model (SM). The different lepton families couple with exactly the same strength to the gauge bosons. This leads to concrete predictions in electroweak precision tests, which can only distinguish among lepton families by the different charged lepton masses. Any deviation from the expected SM theoretical results would signal the presence of New Physics (NP). In this work we concentrate on light meson (K and π) leptonic decays which, in view of the expected experimental precision, have a unique potential to probe deviations from the SM regarding lepton universality.

In the SM, the dominant contribution to $\Gamma(P \to \ell\nu)$ ($P = K, \pi$) arises from W boson exchange. One may be afraid about potential hadronic uncertainties; however, by considering the ratios

$$R_K \equiv \frac{\Gamma(K^+ \to e^+\nu)}{\Gamma(K^+ \to \mu^+\nu)}, \qquad R_\pi \equiv \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)}, \tag{1}$$

the hadronic uncertainties are expected to cancel out to a good approximation. In order to compare the experimental bounds with the SM predictions, it is convenient to introduce a quantity, Δr_P, which parametrizes deviations from the SM expectations:

$$R_P^{\rm exp} = R_P^{\rm SM}\,(1 + \Delta r_P) \quad \text{or equivalently} \quad \Delta r_P \equiv \frac{R_P^{\rm exp}}{R_P^{\rm SM}} - 1\,. \tag{2}$$

The comparison of theoretical analyses [1, 2] with the recent measurements from the NA62 collaboration [3] and with the existing measurements on pion leptonic decays [4]

$$R_K^{\rm SM} = (2.477 \pm 0.001) \times 10^{-5}\,, \qquad R_K^{\rm exp} = (2.488 \pm 0.010) \times 10^{-5}\,, \tag{3}$$

$$R_\pi^{\rm SM} = (1.2354 \pm 0.0002) \times 10^{-4}\,, \qquad R_\pi^{\rm exp} = (1.230 \pm 0.004) \times 10^{-4} \tag{4}$$

suggests that observation agrees at 1σ level with the SM's predictions for

$$\Delta r_K = (4 \pm 4) \times 10^{-3}, \qquad \Delta r_\pi = (-4 \pm 3) \times 10^{-3}. \tag{5}$$

The current experimental uncertainty in Δr_K (of around 0.4%) will be further reduced in the near future, as one expects to have $\delta R_K / R_K \sim 0.1\%$ [5], which can translate into measuring deviations $\Delta r_K \sim \mathcal{O}(10^{-3})$. Similarly, there are also plans for a more precise determination of Δr_π [6, 7].

New contributions to Δr_P have been extensively discussed in the literature, especially in the framework of models with an enlarged Higgs sector. In the presence of charged scalar Higgs, new tree-level contributions are expected. However, as in the case of most Two Higgs Doublet Models (2HDM), or supersymmetric (SUSY) extensions of the SM, these new tree-level corrections are lepton universal [8]. In SUSY models, higher order non-holomorphic couplings can indeed provide new contributions to R_P [9, 10, 11, 12, 13], but in view of current experimental bounds (collider, B-physics and τ-lepton decays), one can have at most $\Delta r_K \leq 10^{-3}$ in the framework of unconstrained minimal SUSY models [13]. Corrections to the $W\ell\nu$ vertex can also induce violation of LFU in charged currents. However, if these appear at the loop level, as referred to in [10], the effect is expected to be of order $(\alpha/4\pi) \times (m_W^2/\Lambda_{\rm NP}^2)$ ($\Lambda_{\rm NP}$ being the new physics scale), generally well below experimental sensitivity.

Here we consider a different alternative: The tree-level corrections to charged current interactions once neutrino oscillations are incorporated into the SM [15, 16]. In this case, and working in the basis where the charged lepton mass matrix is diagonal, the flavour-conserving term $\propto g \bar{l}_j \gamma^\mu P_L \nu_j W_\mu^-$ now reads

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} U_\nu^{ji} \bar{l}_j \gamma^\mu P_L \nu_i W_\mu^- + \text{c.c.}, \tag{6}$$

where U_ν^{ji} is a generic leptonic mixing matrix, $i = 1, \ldots, n_\nu$ denoting the physical neutrino states (not necessarily corresponding to the three left-handed SM states $\equiv \nu_L$) and $j = 1, \ldots, 3$ the charged lepton flavour. In the case of three neutrino generations, U_ν^{ji} corresponds to the unitary PMNS matrix and flavour universality is preserved in meson decays: since one cannot tag the flavour of the final state neutrino (missing energy), the meson decay amplitude is proportional to $(U_\nu U_\nu^\dagger)_{jj} = 1$, and thus no new contribution to R_P is expected. However, in the presence of sterile states, the $W\ell\nu$ vertex is proportional to a rectangular $3 \times n_\nu$ matrix U_ν^{ji}, and the mixing between the left-handed leptons ν_L, ℓ_L corresponds to a 3×3 block of U_ν^{ji},

$$U_{\rm PMNS} \to \tilde{U}_{\rm PMNS} = (\mathbb{1} - \eta)\, U_{\rm PMNS}. \tag{7}$$

The larger the mixing between the active (left-handed) neutrinos and the new states, the more pronounced the deviations from unitarity of $\tilde{U}_{\rm PMNS}$, parametrized by the matrix η [14]. The active-sterile mixings and the departure from unitarity of $\tilde{U}_{\rm PMNS}$ can be at the source of the violation of LFU in different neutrino mass models which introduce sterile fermionic states to generate non-zero masses and mixings for the light neutrinos [15, 16].

Corrections to the $W\ell\nu$ vertex can arise in several scenarios with additional (light) singlet states, as is the case of νSM [17], low-scale type-I seesaw [18] and the Inverse Seesaw (ISS) [19], among other possibilities. This clearly shows the potentiality of the mechanism under discussion, which can be present in many different models.

In the next section we provide a model-independent computation of Δr_P in the presence of additional fermionic sterile states; we then briefly review in Section 3 the most important experimental and observational constraints on the mass of the additional singlet states. In Section 4, we consider the case of the inverse seesaw to give a numerical example of the impact of sterile neutrinos on Δr_P. Our concluding remarks are summarised in Section 5.

2. Δr_P in the presence of sterile neutrinos

Let us consider the SM extended by N_s additional sterile states. The matrix element for the meson decay $P \to l_j \nu_i$ can be generically written as

$$\mathcal{M}_{ij} = \bar{u}_{\nu_i} (\mathcal{A}^{ij} P_R + \mathcal{B}^{ij} P_L) v_{l_j} \,. \tag{8}$$

No sum is implied over the indices of the outgoing leptons i, j. Notice that $i = 1, \ldots, 3+N_s$. The expressions for $\mathcal{A}$ and $\mathcal{B}$ can be easily obtained from the usual 4-fermion effective hamiltonian obtained after integrating out the W boson in Eq. (6). These are

$$(\mathcal{A})^{ij} \;=\; (\mathcal{A}^W)^{ij} \;=\; -4\, G_F\, V_{\text{CKM}}^{us}\, f_P\, U_\nu^{ji\,*}\, m_{l_j} \,; \tag{9}$$

$$(\mathcal{B})^{ij} \;=\; (\mathcal{B}^W)^{ij} \;=\; 4\, G_F\, V_{\text{CKM}}^{us}\, f_P\, U_\nu^{ji\,*}\, m_{\nu_i} \,, \tag{10}$$

where f_P denotes the meson decay constant and m_{l_j,ν_i} the mass of the outgoing leptons.

The expression for R_P is finally given by

$$R_P \;=\; \frac{\sum_i F^{i1} G^{i1}}{\sum_k F^{k2} G^{k2}} \,, \quad \text{with} \tag{11}$$

$$F^{ij} \;=\; |U_\nu^{ji}|^2 \quad \text{and} \quad G^{ij} \;=\; \left[m_P^2 (m_{\nu_i}^2 + m_{l_j}^2) - (m_{\nu_i}^2 - m_{l_j}^2)^2\right] \left[(m_P^2 - m_{l_j}^2 - m_{\nu_i}^2)^2 - 4 m_{l_j}^2 m_{\nu_i}^2\right]^{1/2} . \tag{12}$$

The result of Eq. (11) has a straightforward interpretation: F^{ij} represents the impact of new interactions (absent in the SM), whereas G^{ij} encodes the mass-dependent factors. The SM result can be easily recovered from Eq. (11), in the limit $m_{\nu_i} = 0$ and $U_\nu^{ji} = \delta_{ji}$,

$$R_P^{SM} = \frac{m_e^2}{m_\mu^2} \frac{(m_P^2 - m_e^2)^2}{(m_P^2 - m_\mu^2)^2} \,, \tag{13}$$

to which small electromagnetic corrections should be added [1].

Using the results in Eqs. (11) and (13), we obtain a general expression for Δr_P

$$\Delta r_P \;=\; \frac{m_\mu^2 (m_P^2 - m_\mu^2)^2}{m_e^2 (m_P^2 - m_e^2)^2} \, \frac{\sum_{m=1}^{N_{\max}^{(e)}} F^{m1}\, G^{m1}}{\sum_{n=1}^{N_{\max}^{(\mu)}} F^{m2}\, G^{n2}} - 1 \,. \tag{14}$$

Thus, depending on the masses of the new states (and their hierarchy) and most importantly, on their mixings to the active neutrinos, Δr_P can considerably deviate from zero. In order to illustrate this, we consider two regimes:

- **Regime (A):** All sterile neutrinos are *lighter* than the decaying meson, but heavier than the active neutrino states, i.e. $m_\nu^{\text{active}} \ll m_{\nu_s} \lesssim m_P$
- **Regime (B):** All sterile neutrinos are *heavier* than m_P

Notice that in case (A), all the mass eigenstates can be kinematically available and one should sum over all $3+N_s$ states; furthermore there is an enhancement to Δr_P arising from phase space factors, see Eq. (12).

3. Constraints on sterile neutrinos

We review in this section the experimental and observational bounds on the mass regimes and on the size of the active-sterile mixings that must be satisfied.

First, it is clear that present data on neutrino masses and mixings [20] should be accounted for. Second, there are robust laboratory bounds from direct sterile neutrinos searches [21, 22], since the latter can be produced in meson decays such as $\pi^\pm \to \mu^\pm \nu$, with rates dependent on their mixing with the active neutrinos. Negative searches for monochromatic lines in the muon spectrum can be translated into bounds for $m_{\nu_s} - \theta_{i\alpha}$ combinations, where $\theta_{i\alpha}$ parametrizes the active-sterile mixing. The non-unitarity of the leptonic mixing matrix is also subject to constraints. Bounds on the non-unitarity parameter η (Eq. (7)), were derived using Non-Standard Interactions [23]; although not relevant in case (A), these bounds will be taken into account when evaluating scenario (B).

The modified $W\ell\nu$ vertex also contributes to lepton flavour violation (LFV) processes. The radiative decay $\mu \to e\gamma$, searched for by the MEG experiment [24], is typically the most constraining observable [1]. The rate induced by sterile neutrinos must satisfy [31, 32]

$$\mathrm{BR}(\mu \to e\gamma) = \frac{\alpha_W^3 s_W^2 m_\mu^5}{256\pi^2 m_W^4 \Gamma_\mu} |H_{\mu e}|^2 \leq 2.4 \times 10^{-12}\,, \tag{15}$$

where $H_{\mu e} = \sum_i U_\nu^{2i} U_\nu^{1i\,*} G_\gamma(\frac{m^2_{\nu,i+3}}{m_W^2})$, with G_γ the loop function and U_ν the mixing matrix defined in Eq. (6). Similarly, any change in the $W\ell\nu$ vertex will also affect other leptonic meson decays, in particular $B \to \ell\nu$; the following bounds were enforced in the analysis: $\mathrm{BR}(B \to e\nu) < 9.8 \times 10^{-7}$, $\mathrm{BR}(B \to \mu\nu) < 10^{-6}$ and $\mathrm{BR}(B \to \tau\nu) = (1.65 \pm 0.34) \times 10^{-4}$ [33].

Important constraints can also be derived from LHC Higgs searches [?] and electroweak precision data [35]. They will also be considered in our numerical analysis.

Under the assumption of a standard cosmology, the most constraining bounds on sterile neutrinos stem from a wide variety of cosmological observations [36, 22]. These include Large Scale Structure data, X-ray searches (which can be produced in $\nu_i \to \nu_j\gamma$), Lyman-α limits, the existence of additional degrees of freedom at the epoch of Big Bang Nucleosynthesis and Cosmic Microwave Background data. However, all the above cosmological bounds can be evaded if a non-standard cosmology is considered. In fact, the authors of Ref. [37] showed that the above cosmological constraints disappear in scenarios with low reheating temperature. Therefore, we will allow for the violation of the latter bounds, explicitly stating it.

4. A numerical example: Δr_K in the inverse seesaw

Although the generic idea explored in this work applies to any model where the active neutrinos have sizeable mixings with some additional singlet states, we consider the case of the Inverse Seesaw [19] to illustrate the potential of a model with sterile neutrinos regarding tree-level contributions to light meson decays. As mentioned before, there are other possibilities [17, 18].

4.1. The inverse seesaw

In the ISS, the SM particle content is extended by n_R generations of right-handed (RH) neutrinos ν_R and n_X generations of singlet fermions X with lepton number $L = -1$ and $L = +1$, respectively [19] (such that $n_R + n_X = N_s$). In our numerical application we will focus on

[1] Recently, it has been also noticed that in the framework of low-scale seesaw models, the expected future sensitivity of $\mu - e$ conversion experiments can also play a relevant rôle in detecting or constraining sterile neutrino scenarios [25, 26, 27, 28]. This is also the case in the supersymmetric version of these models, even when the sterile neutrinos are heavier [29, 30].

the case $n_R = n_X = 3$. The lagrangian is given by

$$\mathcal{L}_{\rm ISS} = \mathcal{L}_{SM} + Y^{ij}_\nu \bar\nu_{Ri} L_j \tilde H + M_{Rij}\, \bar\nu_{Ri} X_j + \frac{1}{2}\mu_{X ij} \bar X^c_i X_j + \text{h.c.} \tag{16}$$

where $i, j = 1, 2, 3$ are generation indices and $\tilde H = i\sigma_2 H^*$. Notice that the present lepton number assignment, together with $L = +1$ for the SM lepton doublet, implies that the "Dirac"-type right-handed neutrino mass term $M_{R_{ij}}$ conserves lepton number, while the "Majorana" mass term $\mu_{X_{ij}}$ violates it by two units.

The left-handed neutrinos mix with the right-handed ones after electroweak symmetry breaking. This leads to an effective Majorana mass for the active (light) neutrinos. Assuming $\mu_X \ll m_D \ll M_R$, where $m_D = \frac{1}{\sqrt{2}} Y_\nu v$, with v the vacuum expectation value of the SM Higgs boson, one obtains

$$m_\nu \simeq m_D^T {M_R^T}^{-1} \mu_X M_R^{-1} m_D\,. \tag{17}$$

The remaining 6 sterile states have masses approximately given by $M_\nu \simeq M_R$. Small corrections can be added to these results, but they are typically negligible [38].

In what follows, and without loss of generality, we work in a basis where M_R is a diagonal matrix (as are the charged lepton Yukawa couplings). Y_ν can be written using a modified Casas-Ibarra parametrisation [39] (thus automatically complying with light neutrino data),

$$Y_\nu = \frac{\sqrt{2}}{v} V^\dagger \sqrt{\hat M}\, R\, \sqrt{\hat m_\nu}\, U^\dagger_{\rm PMNS}\,, \tag{18}$$

where $\sqrt{\hat m_\nu}$ is a diagonal matrix containing the square roots of the three eigenvalues of m_ν (cf. Eq. (17)); likewise $\sqrt{\hat M}$ is a (diagonal) matrix with the square roots of the eigenvalues of $M = M_R \mu_X^{-1} M_R^T$. V diagonalizes M as $V M V^T = \hat M$, and R is a 3×3 complex orthogonal matrix, parametrized by 3 complex angles, encoding the remaining degrees of freedom.

The nine neutrino mass eigenstates enter the leptonic charged current through their left-handed component (see Eq. (6), with $i = 1, \ldots, 9$, $j = 1, \ldots, 3$). The unitary leptonic mixing matrix U_ν is now defined as $U_\nu^T \mathcal{M} U_\nu = \text{diag}(m_i)$. Notice however that only the rectangular 3×9 sub-matrix (first three columns of U_ν) appears in Eq. (6) due to the gauge-singlet nature of ν_R and X.

4.2. Numerical evaluation of Δr_K in the inverse seesaw

We numerically evaluate the contributions to R_K in the framework of the ISS and address the two scenarios discussed before, which can be translated in terms of ranges for the (random) entries of the M_R matrix: *regime (A)* ($m_{\nu_s} < m_P$) - $M_{Ri} \in [0.1, 200]$ MeV; *regime (B)* ($m_{\nu_s} > m_P$) - $M_{Ri} \in [1, 10^6]$ GeV. The entries of μ_X have also been randomly varied in the $[0.01\text{ eV}, 1\text{ MeV}]$ range for both cases.

The adapted Casas-Ibarra parametrisation for Y_ν, Eq. (18), ensures that neutrino oscillation data is satisfied (we use the best-fit values of the global analysis of Ref. [20] and set the CP violating phases of $U_{\rm PMNS}$ to zero). The R matrix angles are taken to be real (thus no contributions to lepton electric dipole moments are expected), and randomly varied in the range $\theta_i \in [0, 2\pi]$. We have verified that similar Δr_K contributions are found when considering the more general complex R matrix case.

In Figs. 1, we collect our results for Δr_K in scenarios (A) - left panel - and (B) - right panel, as a function of $\tilde\eta$, which parametrizes the departure from unitarity of the active neutrino mixing sub-matrix $\tilde U_{\rm PMNS}$, $\tilde\eta = 1 - |\text{Det}(\tilde U_{\rm PMNS})|$. Although the cosmological constraints are not always satisfied, we stress that all points displayed comply with the different experimental and laboratory bounds discussed before. For the case of scenario (A), one can have very large

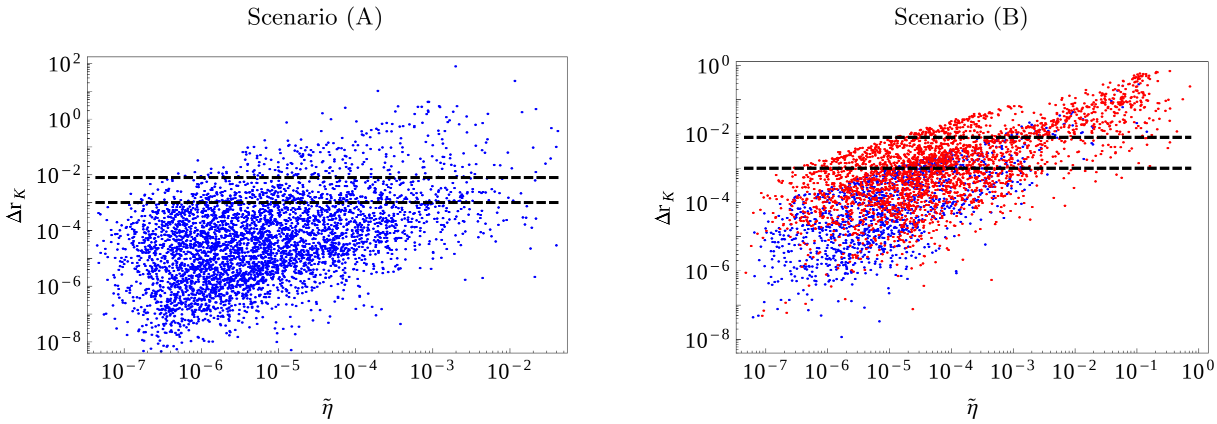

Figure 1. Contributions to Δr_K in the inverse seesaw as a function of $\tilde{\eta} = 1 - |\mathrm{Det}(\tilde{U}_{\mathrm{PMNS}})|$: regimes A (left) and B (right). The upper (lower) dashed line denotes the current experimental limit (expected sensitivity). On the right panel, red points denote cases where $Y_\nu \gtrsim 10^{-2}$. All points comply with experimental and laboratory constraints. Points in (B) are also in agreement with cosmological bounds, while those in (A) require considering a non-standard cosmology.

contributions to R_K, which can even reach values $\Delta r_K \sim \mathcal{O}(1)$ (in some extreme cases we find Δr_K as large as ~ 100). The hierarchy of the sterile neutrino spectrum in case (A) is such that one can indeed have a significant amount of LFU violation, while still avoiding non-unitarity bounds. Although this scenario would in principle allow to produce sterile neutrinos in light meson decays, the smallness of the associated Y_ν ($\lesssim \mathcal{O}(10^{-4})$), together with the loop function suppression (G_γ), precludes the observation of LFV processes, even those with very good associated experimental sensitivity, as is the case of $\mu \to e\gamma$. The strong constraints from CMB and X-rays would exclude scenario (A); in order to render it viable, one would require a non-standard cosmology.

Despite the fact that in case (B) the hierarchy of the sterile states is such that non-unitarity bounds become very stringent (since the sterile neutrinos are not kinematically viable meson decay final states), sizeable LFU violation is also possible, with deviations from the SM predictions again as large as $\Delta r_K \sim \mathcal{O}(1)$. Although one cannot produce sterile states in meson decays in this case, the large Y_ν open the possibility of having larger contributions to LFV observables so that, for example, $\mathrm{BR}(\mu \to e\gamma)$ can be within MEG reach.

Although we do not explicitly display it here, the prospects for Δr_π are similar: in the same framework, one could have $\Delta r_\pi \sim \mathcal{O}(\Delta r_K)$, and thus $\Delta r_\pi \sim \mathcal{O}(1)$ in both scenarios. Depending on the singlet spectrum, these observables can also be strongly correlated: if all the sterile states are either lighter than the pion (as it is the case of scenario (A)) or then heavier than the kaon, one finds $\Delta r_\pi \approx \Delta r_K$. This is a distinctive feature of our mechanism.

5. Concluding remarks

The existence of sterile neutrinos can potentially lead to a significant violation of lepton flavour universality at tree-level in light meson decays. As shown in this study, provided that the active-sterile mixings are sufficiently large, the modified $W\ell\nu$ interaction can lead to large contributions to lepton flavour universality observables, with measurable deviations from the standard model expectations, well within experimental sensitivity. This mechanism might take place in many different frameworks, the exact contributions for a given observable being model-dependent.

As an illustrative (numerical) example, we have evaluated the contributions to R_K in the inverse seesaw extension of the SM - a truly minimal extension of the SM - , for distinct hierarchies of the sterile states. Our analysis reveals that very large deviations from the SM predictions can be found ($\Delta r_K \sim \mathcal{O}(1)$) - or even larger, well within reach of the NA62 experiment at CERN. This is in clear contrast with other models of new physics (for example unconstrained SUSY models, where one typically has $\Delta r_K \lesssim \mathcal{O}(10^{-3})$).

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