Enhancing $l_i \to 3l_j$ with the Z^0-penguin

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Lepton flavor violation (LFV) has been observed in neutrino oscillations. For charged lepton FV decays only upper limits are known, but sizable branching ratios are expected in many neutrino mass models. High scale models, such as the classical supersymmetric seesaw, usually predict that decays $l_i \to 3l_j$ are roughly a factor α smaller than the corresponding decays $l_i \to l_j\gamma$. Here we demonstrate that the Z^0-penguin diagram can give an enhancement for decays $l_i \to 3l_j$ in many extensions of the MSSM. We first discuss why the Z^0-penguin is not dominant in the MSSM with seesaw and show that much larger contributions from the Z^0-penguin are expected in general. We then demonstrate the effect numerically in two example models, namely, the supersymmetric inverse seesaw and R-parity violating supersymmetry.

Introduction: Neutrino oscillation experiments [1, 2] have firmly established that lepton flavor is violated in the neutrino sector, with two of the three measurable mixing angles being surprisingly large. Observation of the characteristic "neutrino dip" leaves no doubt that neutrinos have mass [3, 4] and quite accurate values for the mass squared differences are known now [5]. In the charged lepton sector, however, only upper limits on LFV branching ratios, such as $\mu \to e\gamma$ [6] or $\mu \to 3e$ [7], exist.

Extending the standard model (SM) only by neutrino masses does not automatically lead to measurable charged LFV (CLFV), but sizable branching ratios are expected in many models. In fact, on quite general grounds one expects large CLFV, if physics beyond the SM exists at the TeV scale. A prime example for this observation is supersymmetry (SUSY). Here, the mass matrices of the new scalar particles need not (and in general will not) be aligned with those of the SM fermions. CLFV decays will occur and one can estimate roughly the branching ratios for radiative decays as [8]

$$\text{Br}(l_i \to l_j\gamma) \simeq \frac{48\pi^3\alpha}{G_F^2} \frac{|(m_{\tilde{f}}^2)_{ij}|^2}{m_{SUSY}^8}\text{Br}(l_i \to l_j\nu_i\bar{\nu}_j) \quad (1)$$

where $(m_{\tilde{f}}^2)_{ij}$ parameterizes the dominant off-diagonal elements of the soft SUSY breaking slepton mass matrices and m_{SUSY} is the typical mass of the SUSY particles, expected to be in the ballpark of $\mathcal{O}(0.1-1)$ TeV. Rather small off-diagonal elements are required to satisfy experimental bounds [6, 7].

In the Constrained Minimal Supersymmetric extension of the SM (CMSSM), on the other hand, CLFV is zero, just as in the SM, simply because neutrinos are assumed to be massless. Extending the CMSSM to include neutrino masses (and mixings), for example by a seesaw mechanism, then leads to CLFV decays, because the flavor violation necessarily present in the Yukawa couplings is transmitted to the slepton mass matrices in the RGE (renormalization group equation) running [9]. In such high-scale neutrino mass models, with only MSSM particle content at the electroweak scale, it has been shown that the photonic penguin diagram gives the dominant contribution to $l_i \to 3l_j$ decays in large regions of parameter space.[1] In this case a simple relation can be derived [11]

$$\text{Br}(l_i \to 3l_j) \simeq \frac{\alpha}{3\pi} \left(\log\left(\frac{m_{l_i}^2}{m_{l_j}^2}\right) - \frac{11}{4} \right) \text{Br}(l \to l'\gamma) \quad (2)$$

Thus, usually it is concluded that the decays $l_i \to l_j\gamma$ are more constraining than the decays $l_i \to 3l_j$.

Apart from the photonic penguin, there are also box diagrams, Higgs- and Z^0-penguin contributing to the decays $l_i \to 3l_j$. The latter diagram is not *per se* smaller than the photonic penguin. Rather, as we will show below, in models with only the MSSM particle content and couplings, the Z^0-penguin is suppressed by a subtle cancellation among different terms in the amplitude. Such a cancellation, however, can be easily spoiled if there are (a) new couplings and/or (b) a larger particle content than in the MSSM. Then, as we will discuss, the Z^0-penguin can easily give the dominant contribution to $l_i \to 3l_j$. We will demonstrate this fact numerically with two typical example models: (i) a supersymmetric inverse seesaw and (ii) R-parity violating SUSY. The former is an example of a model with extended particle content, while the latter is an example of a model with the MSSM particle content but new interactions. As we will show, in such models $l_i \to 3l_j$ can be more constraining than $l_i \to l_j\gamma$. This is the main result of the present paper.

Analytical discussion: The total width of the $l_i \to 3l_j$ decay contains contributions from the photon penguin,

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[1] An exception from this rule is the decay $\tau \to 3\mu$ in the limit of large $\tan\beta$ [10].

the Higgs penguin, the Z^0-penguin and boxes. Considering only contributions from photon and Z^0-penguin, which are the ones of interest to us and numerically the most important ones, the total width $\Gamma \equiv \Gamma(l_i^- \to l_j^- l_j^- l_j^+)$ can be written as [12]:

$$
\begin{aligned}
\Gamma &= \frac{e^4}{512\pi^3} m_{l_i}^5 \Big[\left|A_1^L\right|^2 + \left|A_1^R\right|^2 - 2\left(A_1^L A_2^{R*} + A_2^L A_1^{R*} + h.c.\right) + \left(\left|A_2^L\right|^2 + \left|A_2^R\right|^2\right) \left(\frac{16}{3}\log\frac{m_{l_j}}{m_{l_i}} - \frac{22}{3}\right) \\
&\quad + \frac{1}{3}\left\{2\left(|F_{LL}|^2 + |F_{RR}|^2\right) + |F_{LR}|^2 + |F_{RL}|^2\right\} + I_{AF}\Big]
\end{aligned}
\tag{3}
$$

Here, terms denoted A (F) are due to photon (Z^0) exchange and I_{AF} denotes their interference terms, irrelevant for the following discussion. Both, photon and Z^0 penguins have chargino-sneutrino and neutralino-slepton contributions. Exact definitions can be found in [11]. We will focus on the chargino loops for brevity here, since the effects we are interested in are most pronounced in these loops. The photon contributions are

$$
A_a^{(c)L,R} = \frac{1}{m_{\tilde{\nu}}^2} \mathcal{O}_{A_a}^{L,R} s(x^2)
\tag{4}
$$

whereas the Z-contributions read

$$
F_X = \frac{1}{g^2 \sin^2\theta_W m_Z^2} \mathcal{O}_{F_X}^{L,R} t(x^2)
\tag{5}
$$

with $X = \{LL, LR, RL, RR\}$. In these expressions $\mathcal{O}_y^{L,R}$ denote combinations of rotation matrices and coupling constants and $s(x^2)$ and $t(x^2)$ are short-hands for the Passarino-Veltman loop functions which depend on $x^2 = m_{\tilde{\chi}^-}^2 / m_{\tilde{\nu}}^2$. For precise definitions see [11].

The scaling $A \sim m_{SUSY}^{-2}$ and $F \sim m_Z^{-2}$ can be understood, in principle, from simple dimensional analysis. The width of the decay is proportional to $m_{l_i}^5$, so both A and F must be $A, F \propto m^{-2}$. In penguin diagrams it is the smallest mass term in the loop which sets the scale, which in F is m_Z. Due to the masslessness of the photon in case of A the smallest mass scale in the loop is m_{SUSY}. With $m_Z^2 \ll m_{SUSY}^2$ the Z^0 penguin can, in principle, be even more important than the photonic one.

Numerically, however, it has been found in case of the MSSM that the photonic penguin is dominant [11]. This can be understood as follows. To simplify the discussion, we neglect first F_R, since it is always proportional to the charged lepton Yukawa couplings. Consider then n generations of sneutrinos and neglect the chargino mixing. In this simplified scenario only the wino contributes to $F_L^{(c)}$ and it can be written as

$$
F_L^{(c)} = M_{\text{wave}} + M_{\text{p1}} + M_{\text{p2}}
\tag{6}
$$

with

$$
\begin{aligned}
M_{\text{wave}} &= \frac{1}{2}g_2^2(g_2 c_W - g_1 s_W) Z_V^{ik} Z_V^{ij*} f_{\text{wave}}^i \tag{7}\\
M_{\text{p1}} &= -g_2^3 c_W Z_V^{ik} Z_V^{ij*} f_{\text{p1}}^i \tag{8}\\
M_{\text{p2}} &= \frac{1}{2}g_2^2(g_2 c_W + g_1 s_W) Z_V^{ik} Z_V^{ij*} f_{\text{p2}}^i \tag{9}
\end{aligned}
$$

Summing over the index i is implied. The terms in the sum come from different types of diagrams: wave function diagrams (M_{wave}), penguins with the Z^0-boson attached to the chargino line (M_{p1}) or the sneutrino line (M_{p2}). Moreover, $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, Z_V is a $n \times n$ unitary matrix that diagonalizes the mass matrix of the sneutrinos and we used the abbreviations $f_{\text{wave}}^i = -B_1(m_{\tilde{\chi}^\pm}^2, m_{\tilde{\nu}_i}^2)$, $f_{\text{p1}}^i = \frac{1}{2}\tilde{C}_0(m_{\tilde{\nu}_i}^2, m_{\tilde{\chi}^\pm}^2, m_{\tilde{\chi}^\pm}^2) - m_{\tilde{\chi}^\pm} C_0(m_{\tilde{\nu}_i}^2, m_{\tilde{\chi}^\pm}^2, m_{\tilde{\chi}^\pm}^2)$, $f_{\text{p2}}^i = \frac{1}{2}\tilde{C}_0(m_{\tilde{\chi}^\pm}^2, m_{\tilde{\nu}_i}^2, m_{\tilde{\nu}_i}^2)$. The sum in eq. (6) vanishes exactly as can be seen by grouping the different terms

$$
F_L^{(c)} = \frac{1}{2}g_2^3 c_W Z_V^{ik} Z_V^{ij*} X_1^i + \frac{1}{2}g_2^2 g_1 s_W Z_V^{ik} Z_V^{ij*} X_2^i
\tag{10}
$$

with $X_1^i = f_{\text{wave}}^i - 2f_{\text{p1}}^i + f_{\text{p2}}^i$, $X_2^i = f_{\text{p2}}^i - f_{\text{wave}}^i$. Using the exact expressions for the loop functions [11] one finds that the masses cancel out and these combinations become just numerical constants: $X_1^i = -\frac{3}{4}$ and $X_2^i = -\frac{1}{4}$. Therefore, one is left with $F_L^{(c)} \propto \sum_i Z_V^{ik} Z_V^{ij*}$, which vanishes due to unitarity of the Z_V matrix.

This cancellation can be spoiled by two effects, either (1) the sneutrinos mix with other particles which are not $SU(2)_L$ doublets so that the factorization no longer holds, or (2) the charginos are not pure wino and higgsino states. The last effect is of course present in the MSSM and therefore this cancellation is not exact. Nevertheless, the Z^0-contributions are suppressed due to their proportionality to the square of the chargino mixing angle (two wino-higgsino insertions are necessary since there is no $\tilde{H}^\pm - \tilde{\nu}_L - l_L$ coupling). We neglected so far Higgsino interactions because in many models these couplings are very small in comparison to the gauge interactions (for example, a SUSY scale type-I seesaw model would have $Y_\nu \sim 10^{-6}$). However, in models where the Higgsino can

have much larger Yukawa interactions, a large enhancement of the Z^0-contributions can be expected. This will be addressed numerically in the next section.

Before turning to a numerical discussion, consider for simplicity a toy model consisting of two generations of left-handed sneutrinos which can mix with one generation of right-handed sneutrinos. A 3×3 rotation matrix is in general parametrized by 3 angles, but we will assume here for simplification that two of them vanish and call the third one Ψ. In addition, we introduce a new interaction for the Higgsinos $\kappa \nu^c \tilde{H}_u \tilde{l}_L$. We give in Fig. 1 the computed $F_L^{(c)}$ for arbitrarily chosen sneutrino and chargino masses as a function of Ψ for different values of κ. The dashed line shows the case for $\Psi = \kappa = 0$. As clearly seen, $F_L^{(c)}$ depends on the left-right mixing already for small values of κ. However, increasing κ, $F_L^{(c)}$ becomes totally dominated by the new κ interactions and enhances $\mathrm{Br}(l_i \to 3l_j)$.
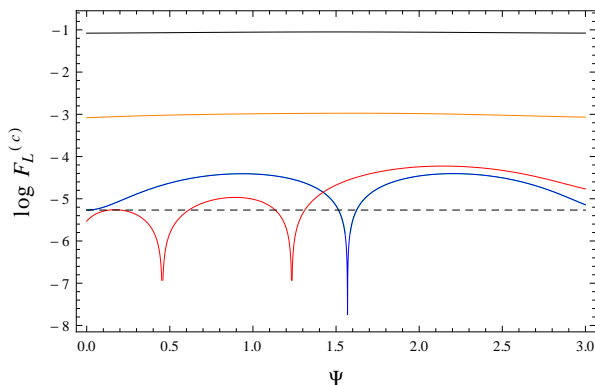

FIG. 1: $F_L^{(c)}$ for our toy model as a function of the sneutrino left-right mixing angle Ψ and for different values of κ: 10^{-4} (blue), 10^{-2} (red), 0.1 (orange) and 1.0 (black).

Numerical examples: We turn to the full fledged numerical study of two examples: an inverse seesaw model and the MSSM with R-parity violation. For this purpose, we have created for both models `SPheno` modules [13, 14] using the Mathematica package `SARAH` [15–17]. These modules calculate the low-energy observables exactly including all possible diagrams [17].

Inverse Seesaw: In inverse seesaw the MSSM particle content is extended by three generations of right handed sneutrino superfields $\hat{\nu}^c$ and of gauge singlets $\hat{N}_S$ which carry lepton number [18, 19]. The superpotential reads

$$W_{IS} = W_{\mathrm{MSSM}} + Y_\nu \hat{\nu}^c \hat{L} \hat{H}_u + M_R \hat{\nu}^c \hat{N}_S + \frac{\mu_N}{2} \hat{N}_S \hat{N}_S \quad (11)$$

After electroweak symmetry breaking (EWSB) the effective mass matrix for the light neutrinos is approximately $m_\nu \simeq \frac{v_u^2}{2} Y_\nu (M_R^T)^{-1} \mu_N M^{-1} Y_\nu^T$. Since μ_N can be of $\mathcal{O}(10^{-1})$ keV or even smaller while M_R is of $\mathcal{O}(m_{SUSY})$, the neutrino Yukawa couplings have to be much larger
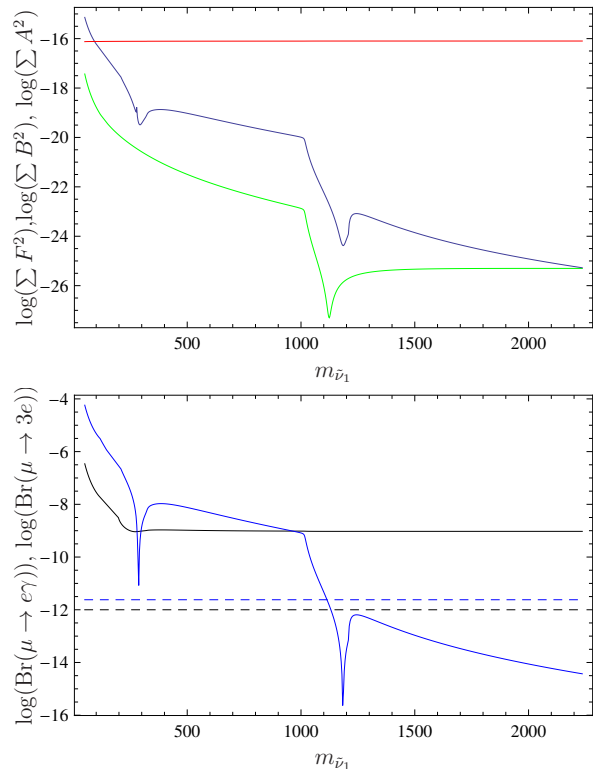

FIG. 2: Top: Different contributions to $\mathrm{Br}(\mu \to 3e)$ as function of the lightest sneutrino mass: Z^0-penguins (red), photonic penguins (blue), combined Higgs-penguins/box diagrams (green). Bottom: $\mathrm{Br}(\mu \to 3e)$ (black) and $\mathrm{Br}(\mu \to e\gamma)$ (blue) and the current experimental bounds (dashed lines). The dips are an effect of a mass crossing between charginos and sneutrinos.

than for a standard weak-scale seesaw to explain neutrino data.

Due to the extended particle content, new contributions for $\mathrm{Br}(l_i \to 3l_j)$ are expected in the inverse seesaw. For example, the Higgs mediated contributions were recently studied in [20]. In Fig. 2 (top) we show the different contributions to $\mathrm{Br}(\mu \to 3e)$ for a variation of the SUSY masses. To disentangle RGE effects we have calculated once the spectrum for a GUT input ($m_0 = 500$ GeV, $M_{1/2} = 1$ TeV, $\tan(\beta) = 10$, $A_0 = -300$ GeV) and rescaled all dimensionful parameters at the SUSY scale. This changes the sfermion masses but not the mixing matrices. Y_ν has been chosen to explain neutrino data for $\mathrm{diag}(\mu_N) = 10^{-1}$ keV and $M_R = 1$ TeV. Clearly, the Z^0-penguins dominate and are nearly independent of the SUSY scale. Only in the limit $m_{SUSY} \to m_Z$ the other contributions can compete. In Fig. 2 (bottom) the branching ratios for $\mu \to e\gamma$ and $\mu \to 3e$ and the current experimental bounds of $2.4 \cdot 10^{-12}$ and $1.0 \cdot 10^{-12}$ are depicted [6, 7]. While $\mathrm{Br}(\mu \to e\gamma)$ would be in conflict with experiment only for $m_{\tilde{\nu}_1} < 1.2$ TeV, $\mathrm{Br}(\mu \to 3e)$ rules out the entire range. In this example, we have assumed μ_N and M_R to be diagonal and all flavor violation comes from
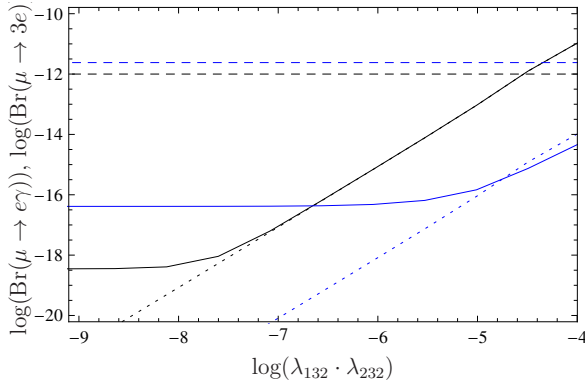
FIG. 3: $\mathrm{Br}(\mu \to 3e)$ (black) and $\mathrm{Br}(\mu \to e\gamma)$ (blue) varying $\lambda_{132} \cdot \lambda_{232}$ in a mixed bi- and trilinear (solid lines) and a pure tri-linear (dotted lines) RpV scenario. The dashed lines show the experimental limits.

Y_ν, as is usually done in literature. However, neutrino oscillation data could equally well be fitted with the flavor violation coming from μ_N and M_R. In that case CLFV observables would be much smaller and consistent with experimental data. However, the relative order between 2- and 3-body decays won't change, i.e. $\mathrm{Br}(l_i \to 3l_j)$ will be most likely observed before $\mathrm{Br}(l_i \to l_j\gamma)$ if inverse seesaw is realized in nature.

R-parity violation: As second example, we take the MSSM particle content but extend the superpotential by the lepton number violating terms [22]

$$W_{\not{R}} = W_{MSSM} + \frac{1}{2}\lambda_{ijk}\hat{L}_i\hat{L}_k\hat{E}_k^c + \frac{1}{2}\lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}_k^c + \epsilon_i\hat{L}_i\hat{H}_u \tag{12}$$

While the ϵ-parameters are highly constrained by neutrino data [21], the bounds for the tri-linear couplings are much weaker and some entries can be of $\mathcal{O}(1)$ [23]. In the following, all entries of λ and λ' are set to zero but λ_{132} and λ_{232}. We give in Fig. 3 the results for $\mathrm{Br}(\mu \to 3e)$ and $\mathrm{Br}(\mu \to e\gamma)$ for a mixed bi- and tri-linear as well as for the pure tri-linear scenario varying $\lambda_{132} \cdot \lambda_{232}$. In the mixed case ϵ_i and v_L^i have been chosen to be consistent with neutrino data and a moderate flavor violation in the sneutrino sector has been induced by $m_{\tilde{l}_iH_d}^2 = (45~\mathrm{GeV})^2$. It can be seen that in the mixed case $\mathrm{Br}(\mu \to 3e) > \mathrm{Br}(\mu \to e\gamma)$ holds when $\lambda_{132} \cdot \lambda_{232}$ crosses $2.5\cdot10^{-7}$, while for the pure tri-linear case without any flavor violation at tree level in the sneutrino sector the three body decays dominate even for much smaller values.

In both cases we get an upper limit for $\lambda_{132} \cdot \lambda_{232}$ of $2.5 \cdot 10^{-4}$ from the bounds on $\mathrm{Br}(\mu \to 3e)$ for sneutrino masses of 730 GeV. So far, in the literature just the limits for $m_{\tilde{\nu}} = 100$ GeV from the photonic penguins [24] have been published. These are much weaker, after rescaling the bound $\sim 7.1 \cdot 10^{-5} \left(\frac{730~\mathrm{GeV}}{100~\mathrm{GeV}}\right)^4 \simeq 0.2$.

Summary: We have shown in this letter that the Z^0-penguin can give the dominant contribution in lepton flavor violating three body decays in many models. The importance of the Z^0-penguin increases with increasing SUSY particles masses. As numerical examples, we have briefly discussed the supersymmetric inverse seesaw and the MSSM with R-parity violation.

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