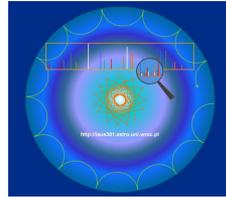




# IAU Symposium 301 Precision Asteroseismology



Reaching the 1% accuracy level on stellar mass and radius determinations from asteroseismology

The case of hot subdwarf B stars

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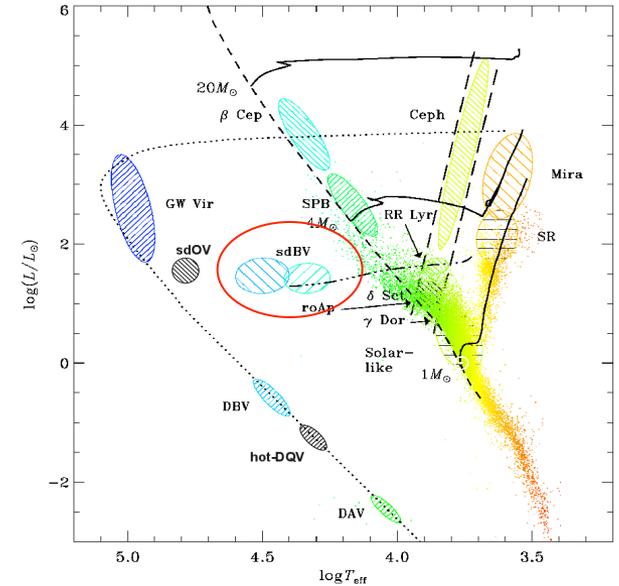
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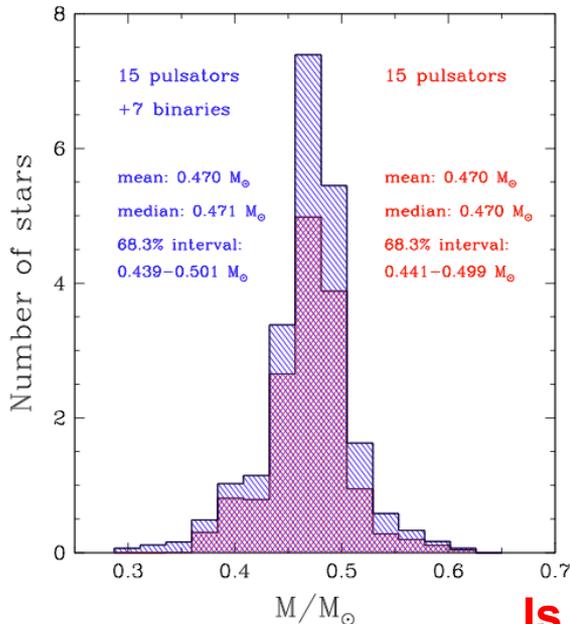
# 1. Introduction to hot subdwarf B (sdB) stars

Hot ( $T_{\text{eff}} = 20\,000 - 40\,000\text{ K}$ ) and compact ( $\log g = 5.2 - 6.2$ ) stars

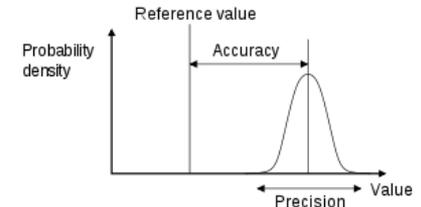
- Core He-burning, extremely thin H-rich envelope
- sdBs are thought to be post-RGB stars having lost most of their H-envelope through binary interaction (stellar, sub-stellar and planet)
- p-mode and g-mode pulsators



Mass distribution of sdB stars



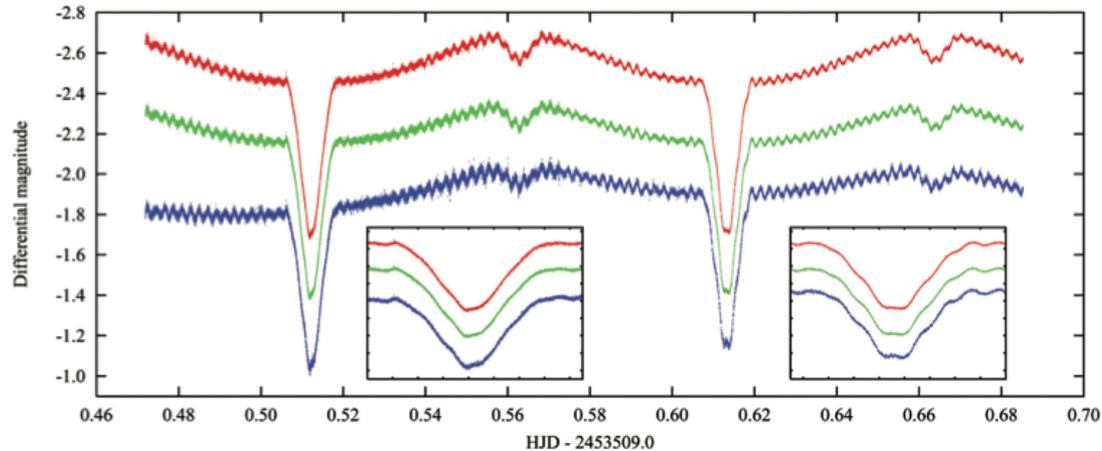
- To date: 15 sdB pulsators modeled by seismology
  - Mass:  $\sim 1\%$  precision
  - $\log g$ :  $\sim 0.1\%$  precision
  - Radius:  $\sim 0.6\%$  precision



**Is this reliable? Is this accurate? Is this really so precise?**

## 2. PG 1336-018, the Rosetta stone of sdB seismology

pulsating sdB+dM eclipsing binary ( $P_{\text{orb}}=2.4244\text{h}$ )

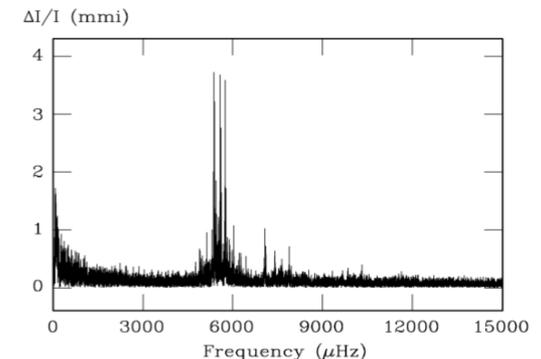


Eclipses and light curve modeling (multicolor photometry ULTRACAM@VLT)  
+accurate spectroscopy (RV curve UVES@VLT):

**orbital solutions for mass, radius (and log g) of the sdB component**

(Vuckovic et al. 2007, A&A, 471, 605)

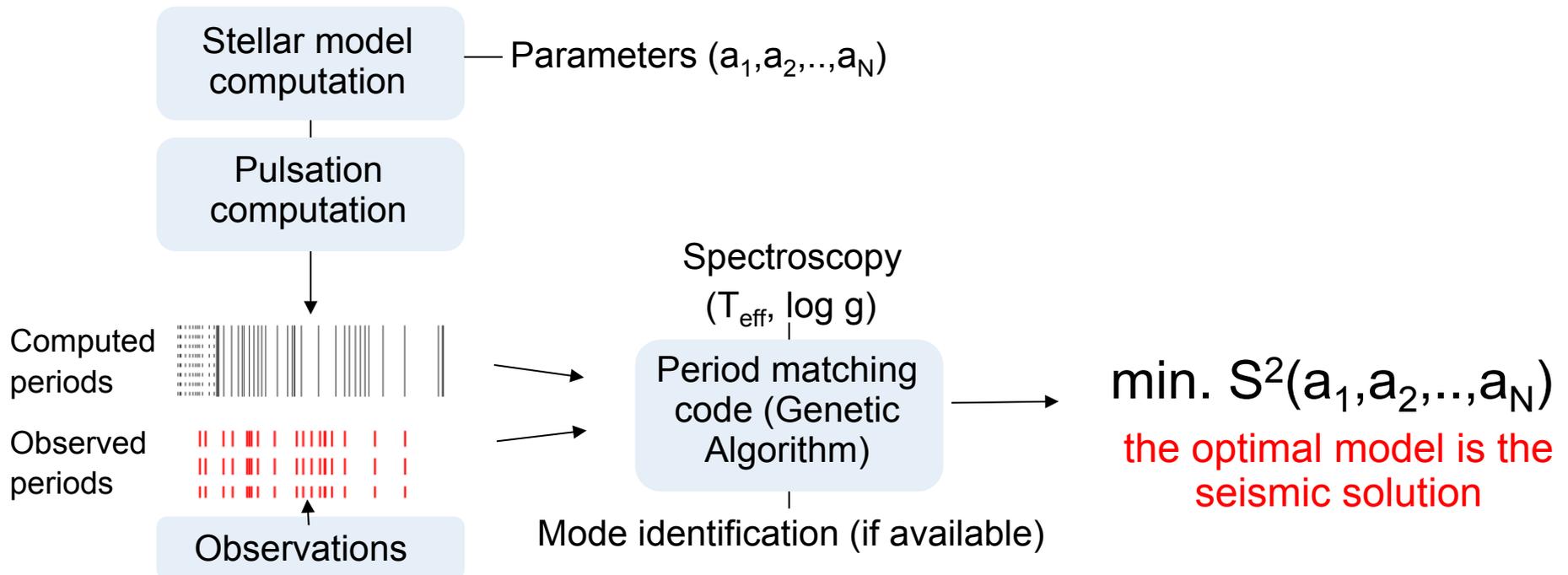
- Whole Earth Telescope campaign: 25 pulsation periods for the sdB component in the range 96-205s (p-modes), Kilkenney et al. 2003 (MNRAS, 345, 843)



### 3. The forward modeling approach for asteroseismology

The method consists of finding the best possible match between the observed frequencies and those computed from models → optimization procedure

$$S^2 = \sum_{i=1}^{N_{\text{obs}}} \left( \frac{P_{\text{obs}}^i - P_{\text{th}}^i}{\sigma_i} \right)^2$$



### 3. The forward modeling approach for asteroseismology

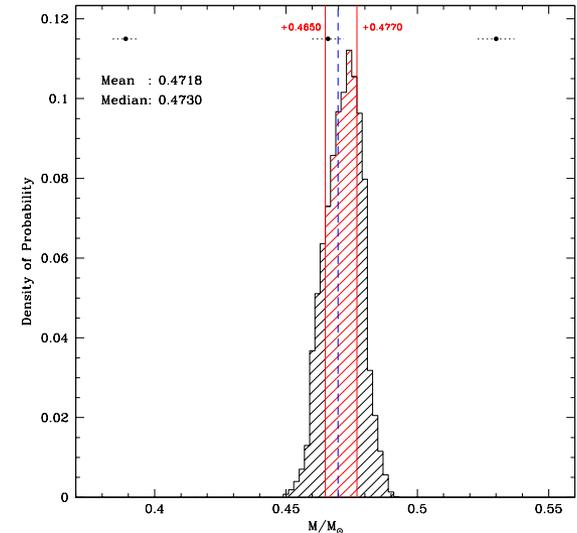
Error estimates: using probability distributions (Van Grootel et al. 2013, A&A, 553, 97)

Likelihood function  $\mathcal{L}(a_1, a_2, a_3, a_4) \propto e^{-\frac{1}{2}S^2}$

Probability density function for parameter  $a_1$  (ex. mass):

$$\mathcal{P}(a_1)da_1 \propto da_1 \iiint \mathcal{L}(a_1, a_2, a_3, a_4) da_2 da_3 da_4$$

(with  $\int \mathcal{P}(a_1)da_1 = 1$ )



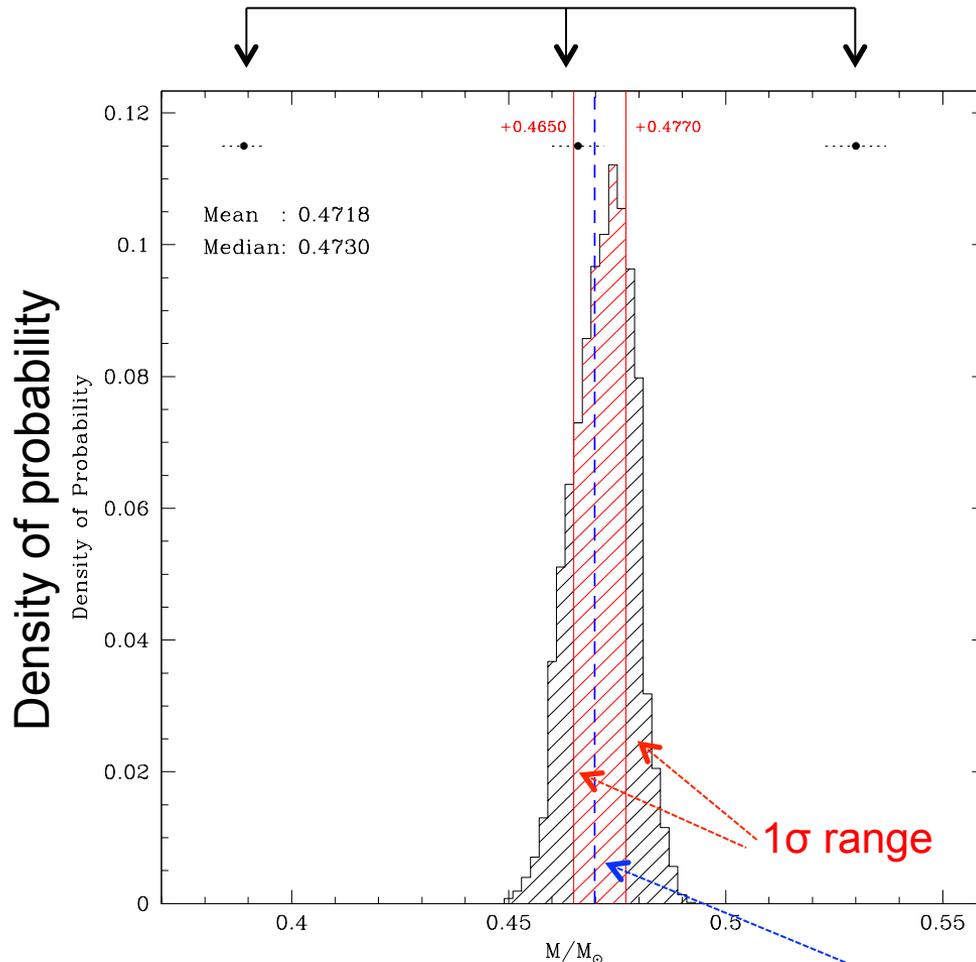
#### Questions:

- Is the best-fit model the **most representative model** of the star?
- Are seismic estimates **accurate** (do model uncertainties introduce systematics on parameters determined from seismology)?
- Are seismic estimates **precise** (error estimates reliable)?

## 4. Seismic analysis of the pulsating sdB PG 1336-018

- Optimization procedure is launched in a vast parameter space where sdB stars are found (details in Van Grootel et al. 2013, A&A, 553, 97)
- Best-fit solution to the 25 observed periods:

3 orbital solutions (Vuckovic et al. 2007)  
for the sdB mass



sdB mass from seismology:

$$0.471 \pm 0.006 M_{\odot}$$

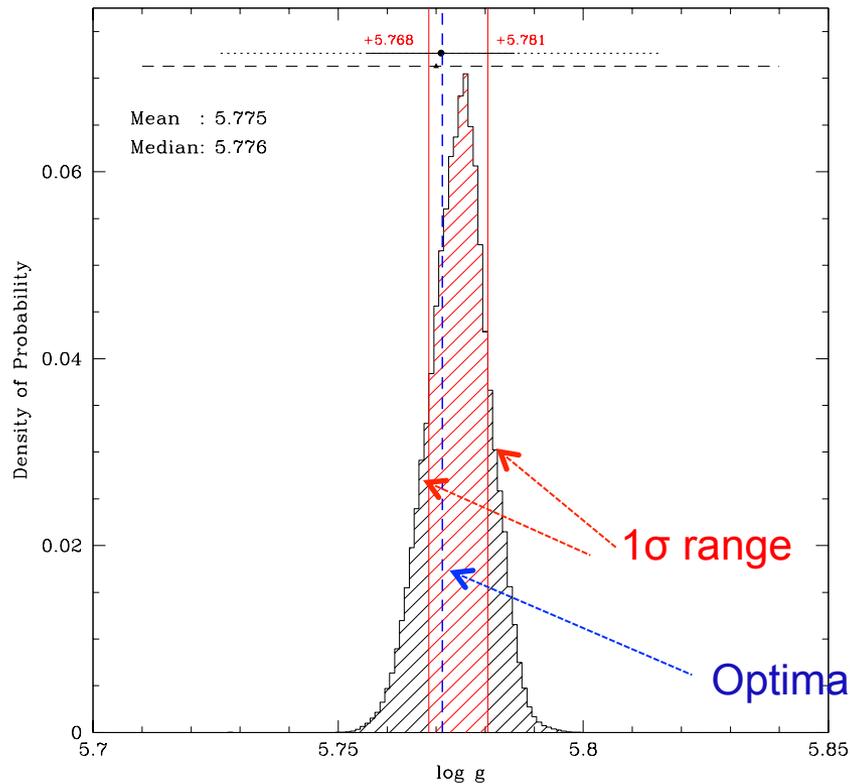
(1.3% precision)

- Consistent within  $1\sigma$  the orbital mass  $0.466 \pm 0.006 M_{\odot}$
- No significant difference, the mass measurement is accurate
- The best-fit mass (min.  $S^2$ ) is in the  $1\sigma$  range (not an outlier)

## 4. Seismic analysis of the pulsating sdB PG 1336-018

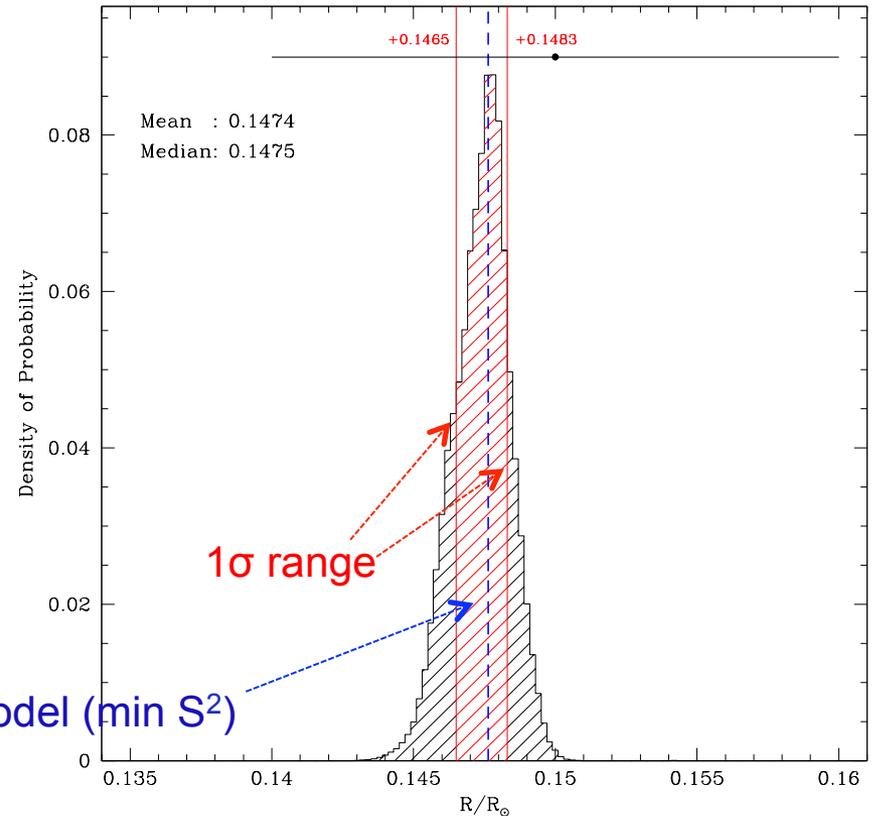
### Surface gravity $\log g$

- **Seismic solution:**  $5.775 \pm 0.007$   
(0.1% precision)
- **Orbital solution:**  $5.77 \pm 0.06$
- **Spectroscopy:**  $5.771 \pm 0.015$



### Stellar radius

- **Seismic solution:**  $0.147 \pm 0.001 R_s$   
(0.6% precision)
- **Orbital solution:**  $0.15 \pm 0.01 R_s$



- No significant difference,  $R$  and  $\log g$  are accurately derived
- Best-fit (min  $S^2$ )  $R$  and  $\log g$  in the  $1\sigma$  range

## 5. Precision and accuracy of seismology

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In summary:

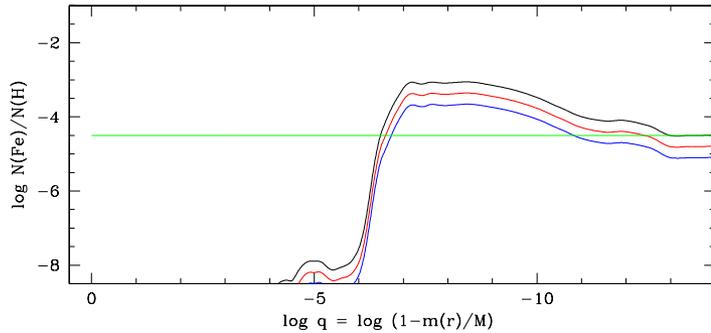
Stellar models for asteroseismology of sdB stars allow for both precise and accurate determinations of the stellar parameters, in particular mass and radius.

But how do the model uncertainties impact this result ?

3 main sources of uncertainties in sdB models:

- Envelope Iron profile (standard: radiative levitation=gravitational settling)
- Core/envelope transition profile (standard: not smoothed by diffusion)
- He-burning nuclear reaction rates (standard: Caughlan & Fowler 1988)

# 5. Precision and accuracy of seismology: envelope iron profile



Green: uniform solar profile

Black: standard profile (radiative levitation = gravitational settling)

Red: standard/2

Blue: standard/4

We redo 3 seismic analyses of PG 1336-018 with the modified models. Results:

**Table 4.** Parameters derived for PG 1336–018 using various iron abundance profiles.

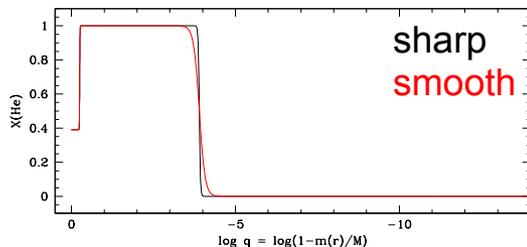
Parameter	Uniform/Solar	$\frac{1}{4} \cdot \frac{N(\text{Fe})}{N(\text{H})}$	$\frac{1}{2} \cdot \frac{N(\text{Fe})}{N(\text{H})}$	$1 \cdot \frac{N(\text{Fe})}{N(\text{H})}$	Largest drift	Likely drift
$M/M_{\odot}$	$0.471 \pm 0.009$	$0.474 \pm 0.009$	$0.474 \pm 0.009$	$0.471 \pm 0.006$	$+0.003 \pm 0.011$	$+0.003 \pm 0.011$
$\log g$	$5.814 \pm 0.007$	$5.795 \pm 0.008$	$5.786 \pm 0.009$	$5.775 \pm 0.007$	$+0.039 \pm 0.010$	$+0.020 \pm 0.011$
$R/R_{\odot}$	$0.1406 \pm 0.0011$	$0.1443 \pm 0.0012$	$0.1459 \pm 0.0012$	$0.1474 \pm 0.0009$	$-0.0068 \pm 0.0014$	$-0.0031 \pm 0.0015$
$\log q(H)$	$-4.11 \pm 0.08$	$-3.70 \pm 0.09$	$-3.74 \pm 0.09$	$-3.83 \pm 0.06$	$-0.28 \pm 0.10$	$+0.07 \pm 0.11$
$X(C + O)$	$0.46 \pm 0.10$	$0.48 \pm 0.10$	$0.51 \pm 0.09$	$0.58 \pm 0.06$	$-0.12 \pm 0.12$	$-0.10 \pm 0.12$
$L/L_{\odot}$	$20.6 \pm 0.7$	$21.8 \pm 0.7$	$22.3 \pm 0.8$	$22.9 \pm 0.6$	$-2.3 \pm 0.9$	$-1.1 \pm 0.9$
$S^2$ (opt.)	5.95	5.93	5.46	4.81		
$\overline{\Delta X/X}$ (%)	0.24	0.21	0.19	0.18		
$\Delta \nu$ ( $\mu\text{Hz}$ )	14.07	12.75	11.43	11.37		

No drift on stellar mass  
 $2\sigma$  drift on  $\log g$  and radius

Despite significant changes in the iron abundance profiles, the derived parameters are mostly unaffected (e.g. the mass) or only subject to very small systematic drifts

# 5. Precision and accuracy of seismology

## Core/Envelope transition profile



**Table 5.** Parameters derived for PG 1336–018 using a sharp or a smooth He/H transition profile.

Parameter	Sharp transition	Smooth transition	Drift
$M/M_{\odot}$	$0.471 \pm 0.006$	$0.476 \pm 0.009$	$+0.005 \pm 0.011$
$\log g$	$5.775 \pm 0.007$	$5.777 \pm 0.007$	$+0.002 \pm 0.010$
$R/R_{\odot}$	$0.1474 \pm 0.0009$	$0.1476 \pm 0.0011$	$+0.0002 \pm 0.0014$
$\log q(\text{H})$	$-3.83 \pm 0.06$	$-3.79 \pm 0.08$	$+0.04 \pm 0.10$
$X(\text{C} + \text{O})$	$0.58 \pm 0.06$	$0.53 \pm 0.09$	$-0.05 \pm 0.11$
$L/L_{\odot}$	$22.9 \pm 0.6$	$22.8 \pm 0.7$	$-0.1 \pm 0.9$
$S^2$ (opt.)	4.81	4.91	
$\overline{\Delta X/X}$ (%)	0.18	0.18	
$\overline{\Delta \nu}$ ( $\mu\text{Hz}$ )	11.37	11.48	

## He-burning rate: $^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \times 2$

(Angulo et al. 1999)

**Table 6.** Parameters derived for PG 1336–018 using models with modified reaction rates for the triple- $\alpha$  and  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  reaction rates.

Parameter	CF88	$2 \times \text{CF88}$	Drift
$M/M_{\odot}$	$0.471 \pm 0.006$	$0.474 \pm 0.010$	$+0.003 \pm 0.012$
$\log g$	$5.775 \pm 0.007$	$5.775 \pm 0.008$	$0.00 \pm 0.011$
$R/R_{\odot}$	$0.1474 \pm 0.0009$	$0.1479 \pm 0.0010$	$+0.0005 \pm 0.0013$
$\log q(\text{H})$	$-3.83 \pm 0.06$	$-3.84 \pm 0.09$	$-0.01 \pm 0.11$
$X(\text{C} + \text{O})$	$0.58 \pm 0.06$	$0.54 \pm 0.10$	$-0.04 \pm 0.12$
$L/L_{\odot}$	$22.9 \pm 0.6$	$23.0 \pm 0.7$	$+0.1 \pm 0.9$
$S^2$ (opt.)	4.81	5.12	
$\overline{\Delta X/X}$ (%)	0.18	0.19	
$\overline{\Delta \nu}$ ( $\mu\text{Hz}$ )	11.37	11.52	

No significant drift on stellar mass, radius and  $\log g$

**The seismic solution is robust against uncertainties in the constitutive physics of the models**

## 6. Conclusion

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### Conclusion:

- Seismic parameters determined from asteroseismology for sdB stars are both **precise**, **accurate**, and **robust** against model uncertainties
- The best-fit (optimal) model is the most representative model of the star

**We can indeed achieve ~1% accuracy for mass and radius determinations from asteroseismology**

### Remarks:

- We are still (very) far from reproducing the precision of the observations (1 nHz for Kepler, 0.1  $\mu$ Hz for ground-based; vs 10  $\mu$ Hz for seismic model)

**Asteroseismology has still not delivered its full potential**

- To provide extremely accurate global parameters (M, R, log g...)
- To probe internal structure, composition and physics (effects of diffusion...). Sensitivity of g-modes to the stellar interior.