Non-Local Incremental-Secant Mean-Field-Homogenization of Damage-Enhanced Elasto-Plastic Composites

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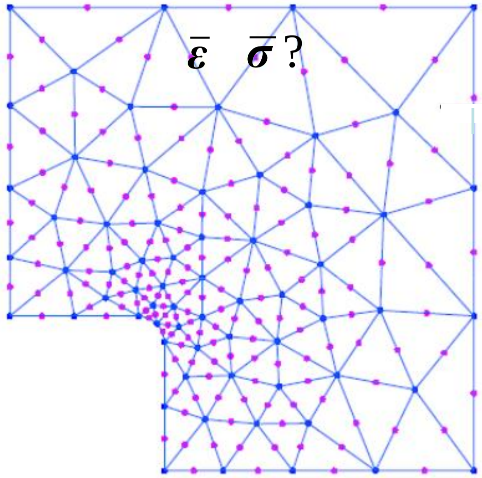
Content

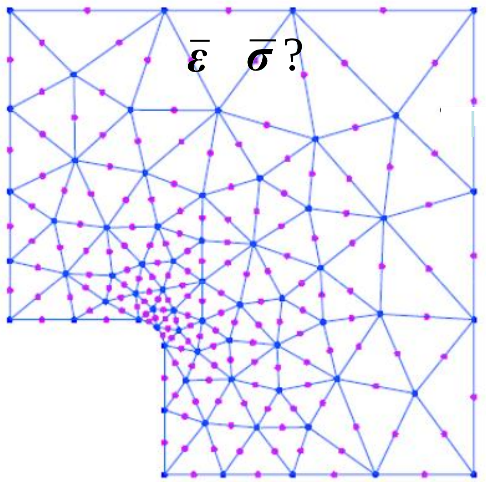
- Introduction
  - Mean-field homogenization Multi-scale modelling
- Mean-Field-Homogenization with non-local damage
  - Incremental secant approach idea
  - Non-local damage-enhanced incremental secant approach
- Finite-element implementation
  - Direct resolution
  - Staggered resolution
- Applications
  - Laminate with unloading
  - Laminate with a hole
- Conclusions

- Multiscale methods
  - Macro-scale
    - FE model
    - At one integration point $\bar{\varepsilon}$ is know, $\bar{\sigma}$ is sought

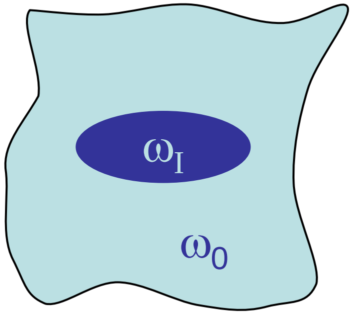


Multi-scale modelling: How?

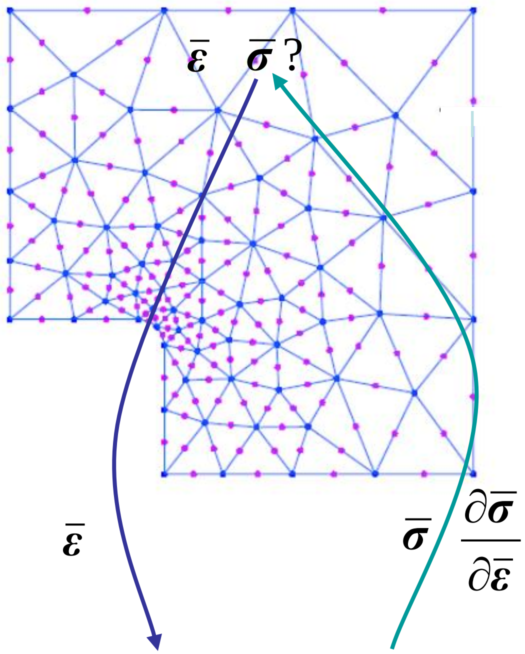
- Multiscale methods
  - Macro-scale
    - FE model
    - At one integration point $\bar{\boldsymbol{\varepsilon}}$ is know, $\bar{\boldsymbol{\sigma}}$ is sought

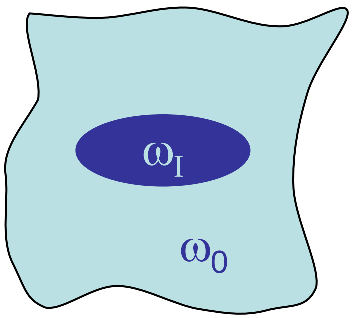


  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models



- Multiscale methods
  - Macro-scale
    - FE model
    - At one integration point $\bar{\varepsilon}$ is know, $\bar{\sigma}$ is sought
  - Transition
    - Downscaling: $\bar{\varepsilon}$ is used as input of the MFH model
    - Upscaling: $\bar{\sigma}$ is the output of the MFH model
  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models





Assumptions:

$L_{macro} >> L_{RVE} >> L_{micro}$

- Semi analytical Mean-Field Homogenization
  - Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V}\int_V a(\boldsymbol{X})\mathrm{d}V$$

  - Meso-response
    - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)

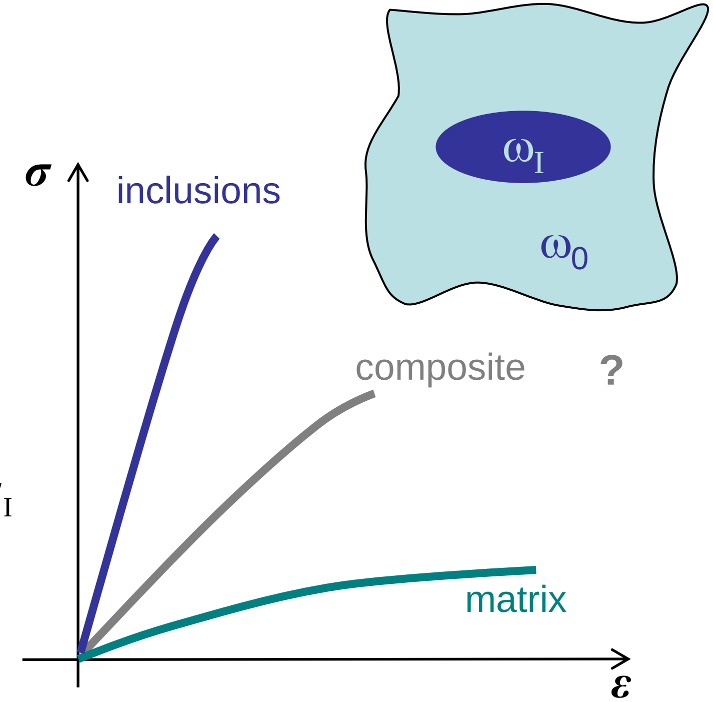
$$\begin{cases} \bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$

    - One more equation required

$$\boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0$$

  - Difficulty: find the adequate relations

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{I} = f(\boldsymbol{\varepsilon}_\mathrm{I}) \\ \boldsymbol{\sigma}_0 = f(\boldsymbol{\varepsilon}_0) \\ \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0 \end{cases} \qquad \boldsymbol{B}^{\varepsilon}\ ?$$



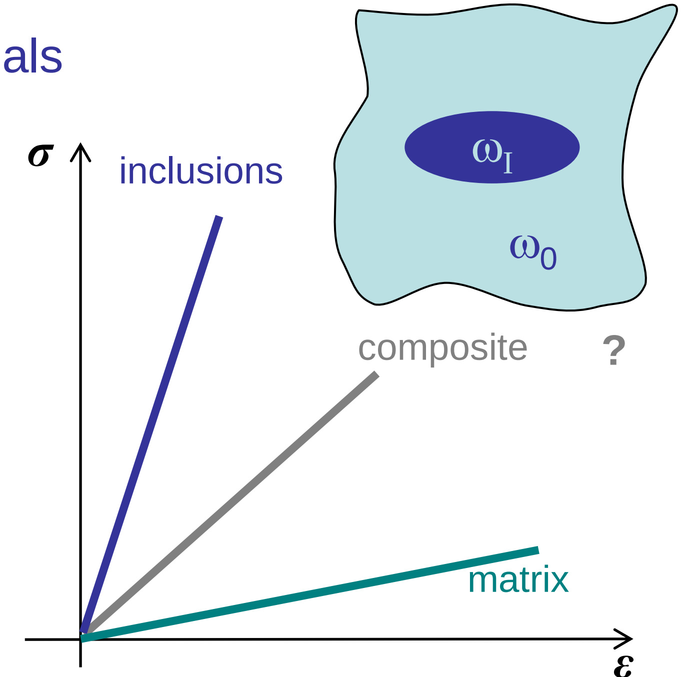
- Mean-Field Homogenization for different materials
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_0 = \overline{\boldsymbol{C}}_0 : \boldsymbol{\varepsilon}_0 \end{cases}$$

    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
    - Use Eshelby tensor

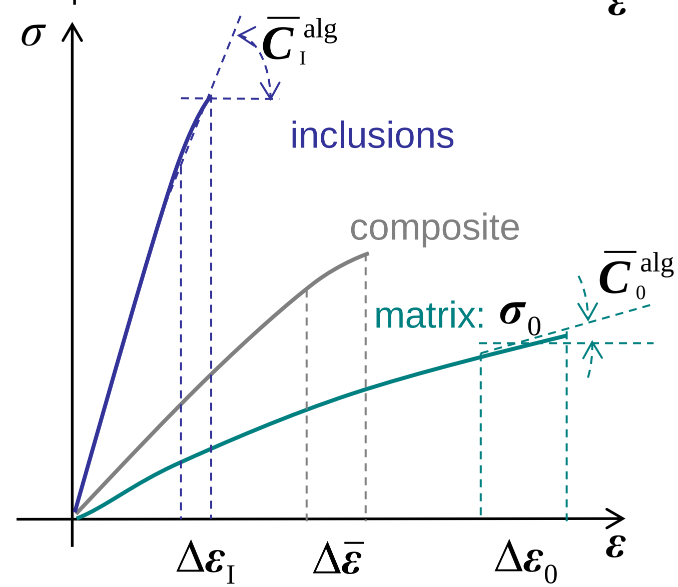
$$\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0, \overline{\boldsymbol{C}}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0$$

with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$

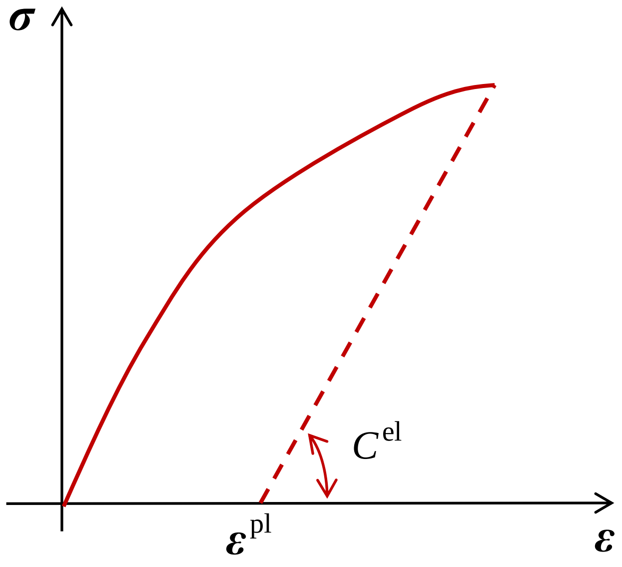


  - Non-linear materials
    - Define a Linear Comparison Composite
    - Common approach: incremental tangent

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_0$$

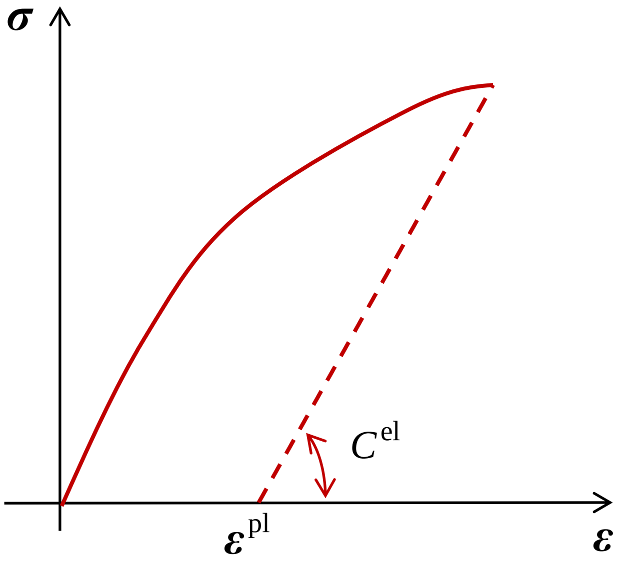


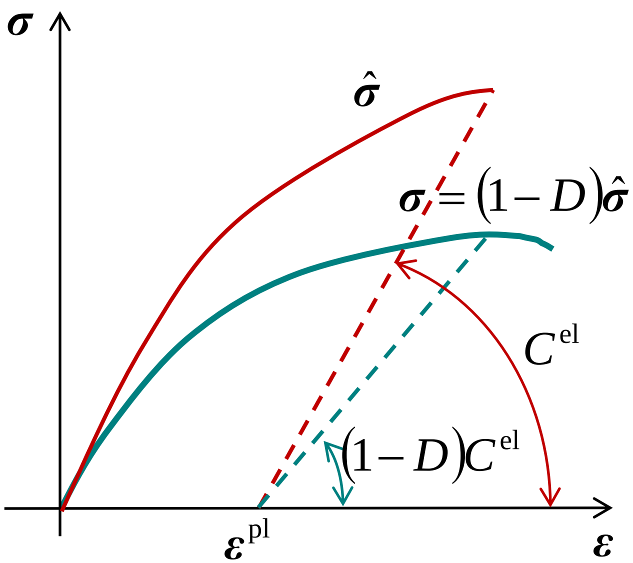
- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$



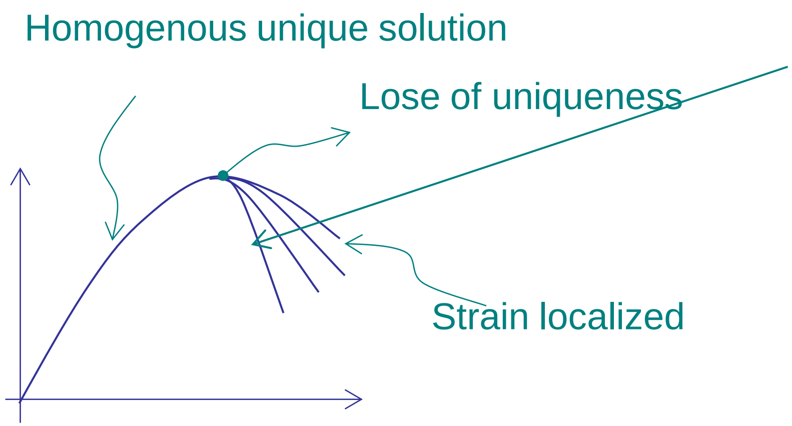
Mean-Field-Homogenization with non-local damage

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$

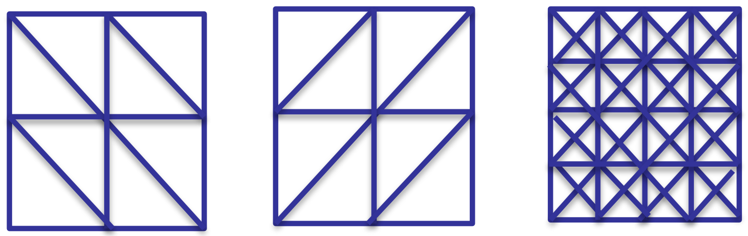




- Finite element solutions for strain softening problems suffer from:
  - The loss the uniqueness and strain localization
  - Mesh dependence



The numerical results change with the size of mesh and direction of mesh



The numerical results change without convergence

- Implicit non-local approach [Peerlings et al 96, Geers et al 97, …]
  - A state variable is replaced by a non-local value reflecting the interaction between neighboring material points

$$\tilde{a}(\boldsymbol{x}) = \frac{1}{V_C}\int_{V_C} a(\boldsymbol{y})w(\boldsymbol{y};\boldsymbol{x})\mathrm{d}V$$

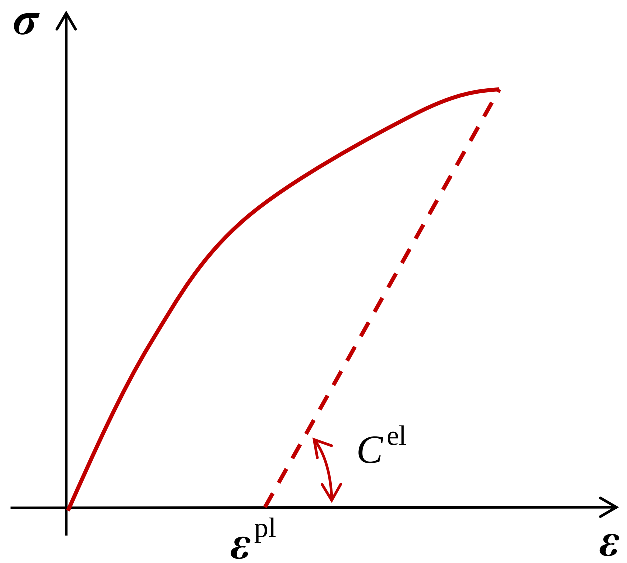
  - Use Green functions as weight $w(y; x)$

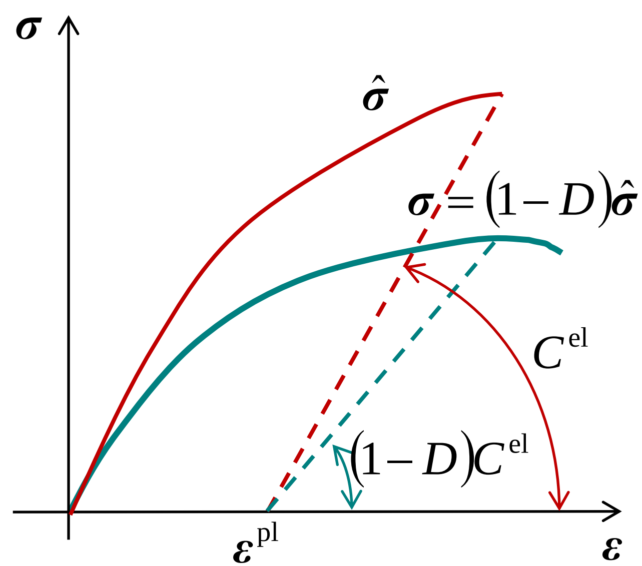
⇒ $\tilde{a} - c\nabla^2\tilde{a} = a$ ⇒ New degrees of freedom

Mean-Field-Homogenization with non-local damage

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$
  - Non-Local damage model
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta \tilde{p})$
    - Anisotropic governing equation $\tilde{p} - \nabla \cdot (\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p}) = p$
    - Linearization

$$\delta\boldsymbol{\sigma} = \left[(1 - D)\boldsymbol{C}^{\mathrm{alg}} - \hat{\boldsymbol{\sigma}} \otimes \frac{\partial F_D}{\partial \boldsymbol{\varepsilon}} \right] : \delta\boldsymbol{\varepsilon} - \hat{\boldsymbol{\sigma}} \frac{\partial F_D}{\partial \tilde{p}} \delta\tilde{p}$$

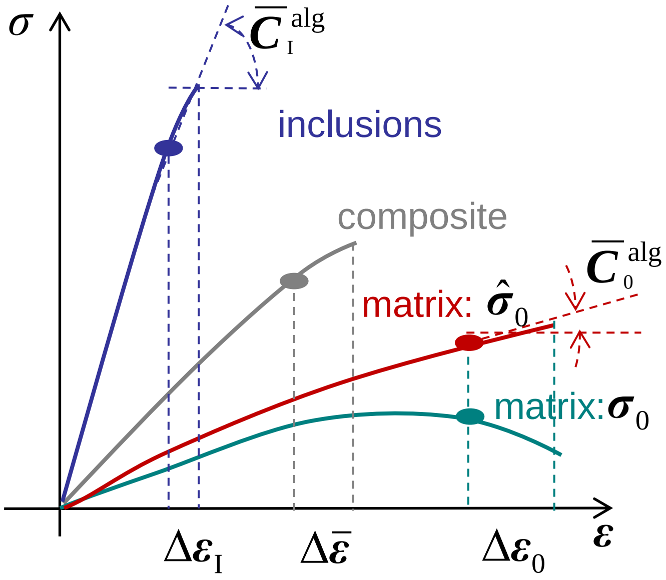




- Problem
  - We want the fibres to get unloaded during the matrix damaging process
    - For the incremental-tangent approach

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\overline{\boldsymbol{C}}_{0}^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_{0}$$

    - To unload the fibres ($\boldsymbol{\varepsilon}_{\mathrm{I}} < 0$) with such approach would require $\overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}} < 0$
    - We cannot use the incremental tangent MFH
  - We need to define the LCC from another stress state



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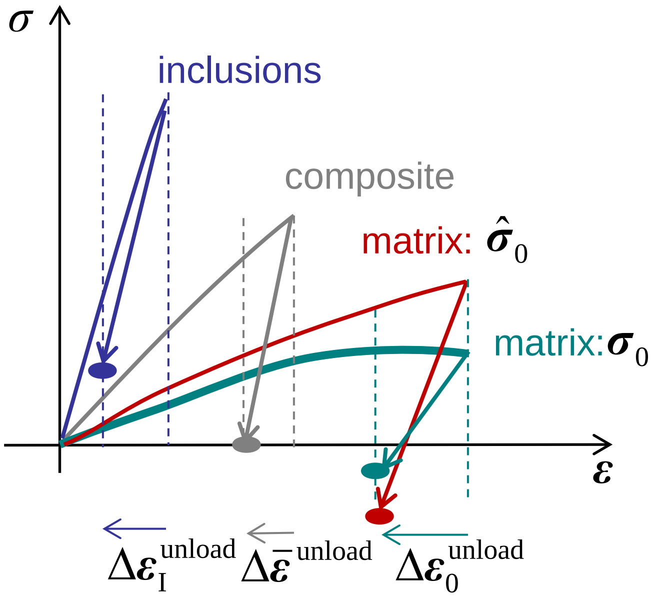
Mean-Field-Homogenization with non-local damage

- Idea
  - New incremental-secant approach
    - Perform a virtual elastic unloading from previous solution
      - Composite material unloaded to reach the stress-free state
      - Residual stress in components



Mean-Field-Homogenization with non-local damage

- Idea
  - New incremental-secant approach
    - Perform a virtual elastic unloading from previous solution
      - Composite material unloaded to reach the stress-free state
      - Residual stress in components
    - Apply MFH from unloaded state
      - New strain increments (>0)

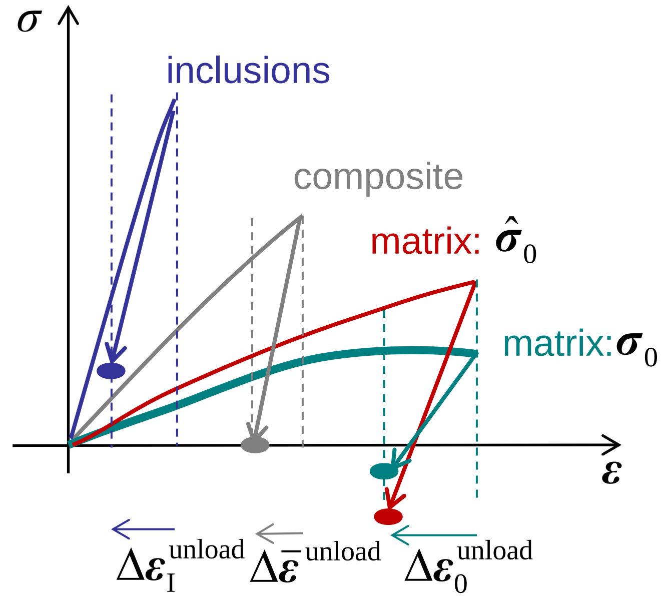
$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I/0}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I/0}} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I/0}}$$

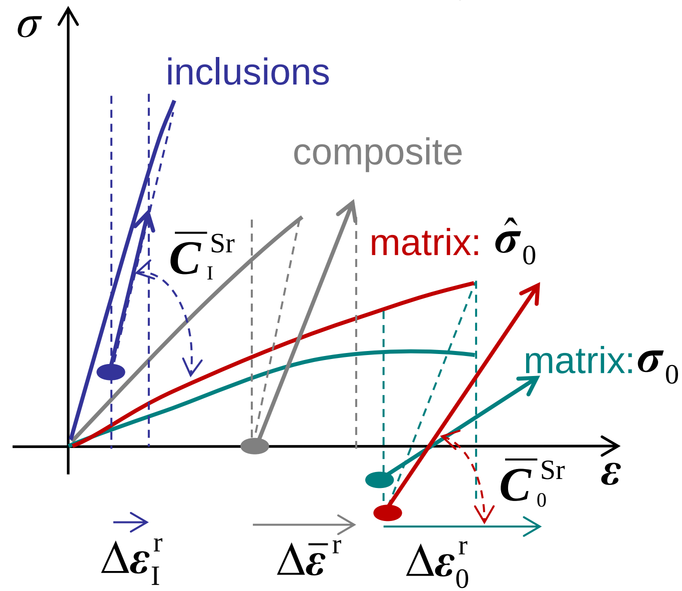
      - Use of secant operators

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \bar{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$

      - Possibility of have unloading

$$\begin{cases} \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} > 0 \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} < 0 \end{cases}$$





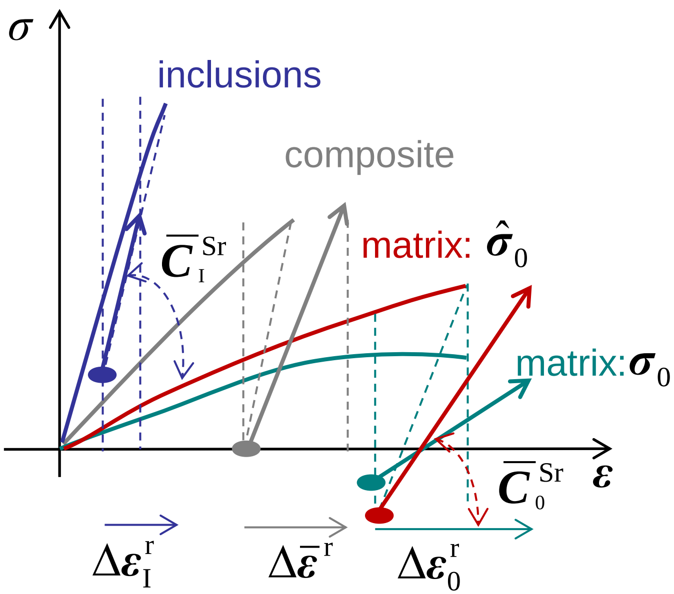
Mean-Field-Homogenization with non-local damage

- New incremental-secant approach
  - Equations summary
    - Inputs
      - Internal variable at last increment
      - Residual tensor after virtual unloading
      - $\Delta\bar{\boldsymbol{\varepsilon}}, \Delta\tilde{p}$ from FE resolution
    - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_\mathrm{I} \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{r} = \Delta\boldsymbol{\varepsilon}_\mathrm{I} + \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{unload} \\ \Delta\boldsymbol{\varepsilon}_0^\mathrm{r} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^\mathrm{unload} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{r} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}_0^\mathrm{Sr}, \bar{\boldsymbol{C}}_\mathrm{I}^\mathrm{Sr}\right) : \Delta\boldsymbol{\varepsilon}_0^\mathrm{r} \end{cases}$$

    - With the stress tensors

$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \boldsymbol{\sigma}_\mathrm{I} = \boldsymbol{\sigma}_\mathrm{I}^\mathrm{res} + \bar{\boldsymbol{C}}_\mathrm{I}^\mathrm{Sr} : \Delta\boldsymbol{\varepsilon}_\mathrm{I}^\mathrm{r} \\ \boldsymbol{\sigma}_0 = (1-D)\hat{\boldsymbol{\sigma}}_0^\mathrm{res} + (1-D)\bar{\boldsymbol{C}}_0^\mathrm{Sr} : \Delta\boldsymbol{\varepsilon}_0^\mathrm{r} \end{cases}$$

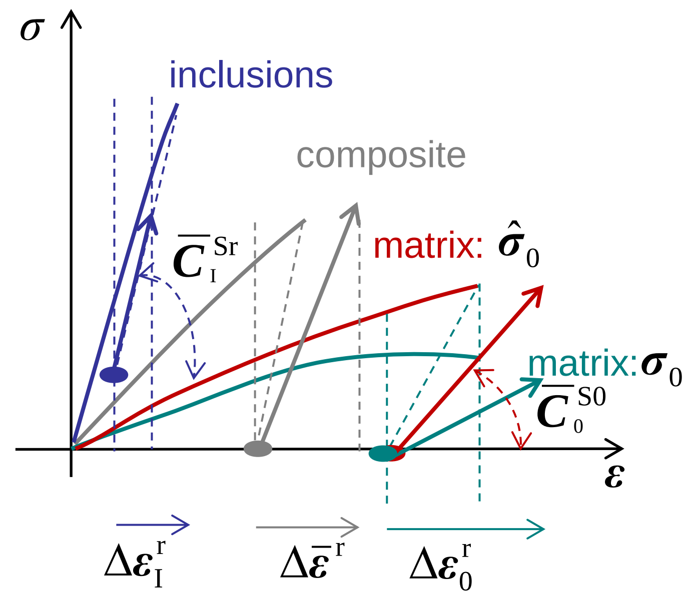


- New incremental-secant approach (2)
  - Alternative
    - For soft matrix response
      - Remove residual stress in matrix
      - Avoid adding spurious internal energy
    - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_\mathrm{I} \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_\mathrm{I} + \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{S0}}, \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

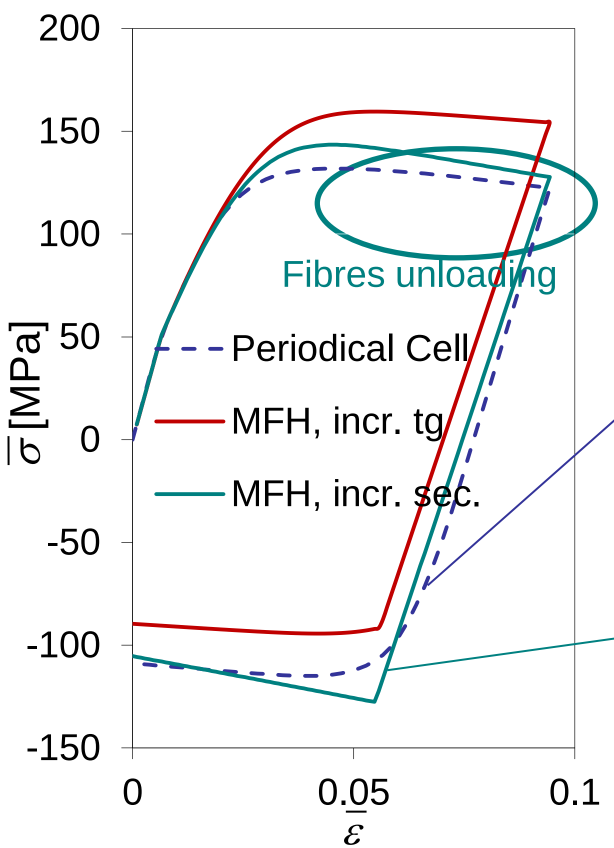
    - With the stress tensors

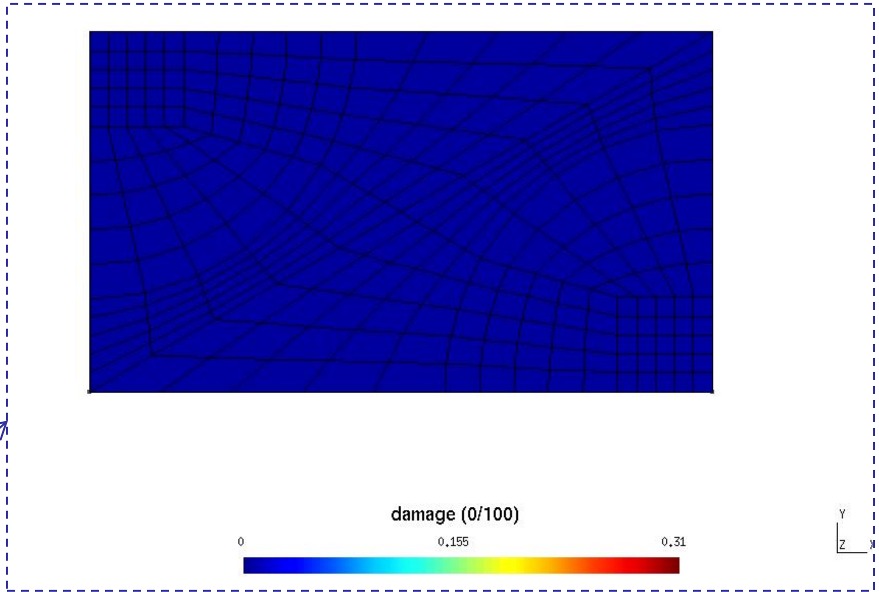
$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \boldsymbol{\sigma}_\mathrm{I} = \boldsymbol{\sigma}_\mathrm{I}^{\mathrm{res}} + \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{S0}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

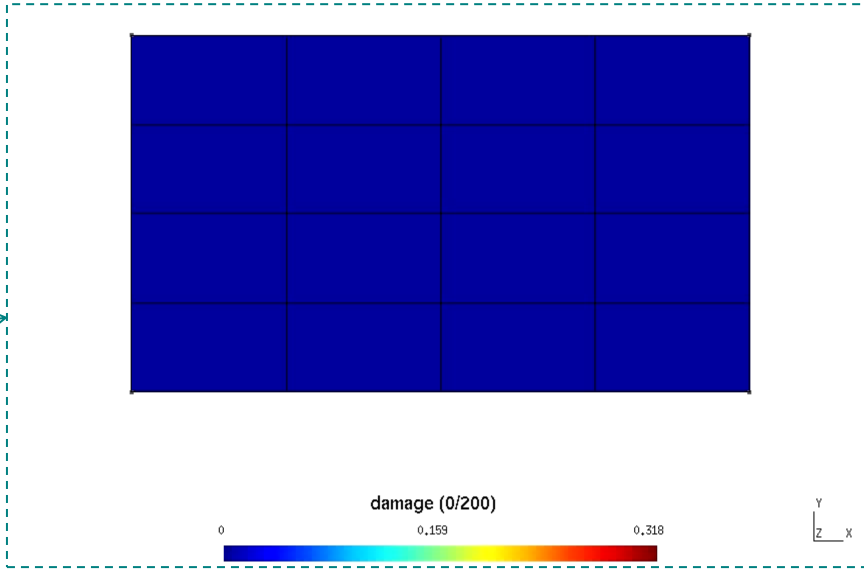


Mean-Field-Homogenization with non-local damage

- New results for damage
  - Fictitious composite
    - 50%-UD fibres
    - Analyse phases behaviours







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- Weak formulation
  - Strong form

$$\begin{cases} \nabla \cdot \bar{\boldsymbol{\sigma}}^T + \boldsymbol{f} = \boldsymbol{0} & \text{for the homogenized composite material} \\ \tilde{p} - \nabla \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p} \right) = p & \text{for the matrix phase} \end{cases}$$

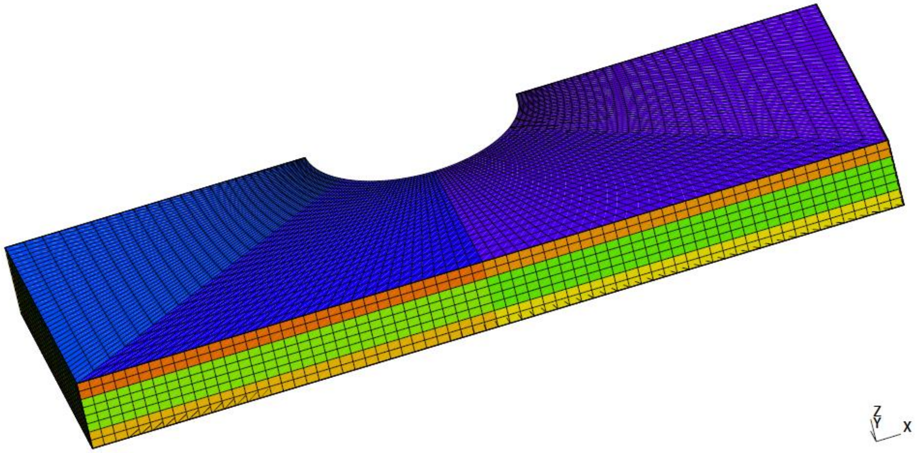
  - Boundary conditions

$$\begin{cases} \boldsymbol{\sigma} \cdot \boldsymbol{n} = \boldsymbol{T} \\ \boldsymbol{n} \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p} \right) = 0 \end{cases}$$

  - Finite-element discretization

$$\begin{cases} \tilde{p} = N_{\tilde{p}}^{a} \tilde{\boldsymbol{p}}^{a} \\ \boldsymbol{u} = N_{u}^{a} \boldsymbol{u}^{a} \end{cases}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\tilde{p}} \\ \boldsymbol{K}_{\tilde{p}u} & \boldsymbol{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\boldsymbol{u} \\ d\tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{ext}} - \boldsymbol{F}_{\mathrm{int}} \\ \boldsymbol{F}_{p} - \boldsymbol{F}_{\tilde{p}} \end{bmatrix}$$



- Resolution strategies
  - Fully coupled resolution

$$\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\tilde{p}} \\ \boldsymbol{K}_{\tilde{p}u} & \boldsymbol{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\boldsymbol{u} \\ d\tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\text{ext}} - \boldsymbol{F}_{\text{int}} \\ \boldsymbol{F}_{p} - \boldsymbol{F}_{\tilde{p}} \end{bmatrix}$$

  - Staggered dynamic resolution
    - Explicit resolution of the displacement dofs

$$\ddot{\boldsymbol{u}}^{n+1} = \frac{1}{1-\alpha_M} \boldsymbol{M}\left[\boldsymbol{F}_{\text{ext}}^{\,n} - \boldsymbol{F}_{\text{int}}^{\,n}\right] - \frac{\alpha_M}{1-\alpha_M} \boldsymbol{u}^{n}$$

$$\dot{\boldsymbol{u}}^{n+1} = \dot{\boldsymbol{u}}^{n} + \Delta t\left[1-\gamma_M\right]\ddot{\boldsymbol{u}}^{n} + \Delta t \gamma_M \ddot{\boldsymbol{u}}^{n+1}$$

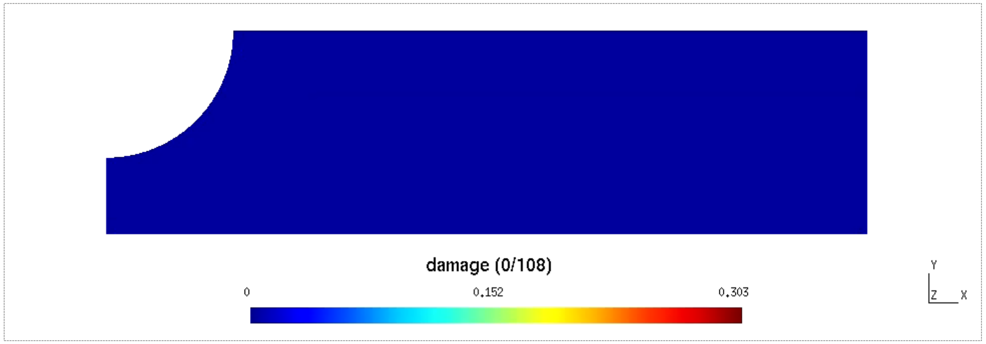
$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^{n} + \Delta t \dot{\boldsymbol{u}}^{n} + \Delta t^{2}\left[\frac{1}{2} - \beta_M\right]\ddot{\boldsymbol{u}}^{n+1} + \Delta t^{2} \beta_M \ddot{\boldsymbol{u}}^{n+1}$$

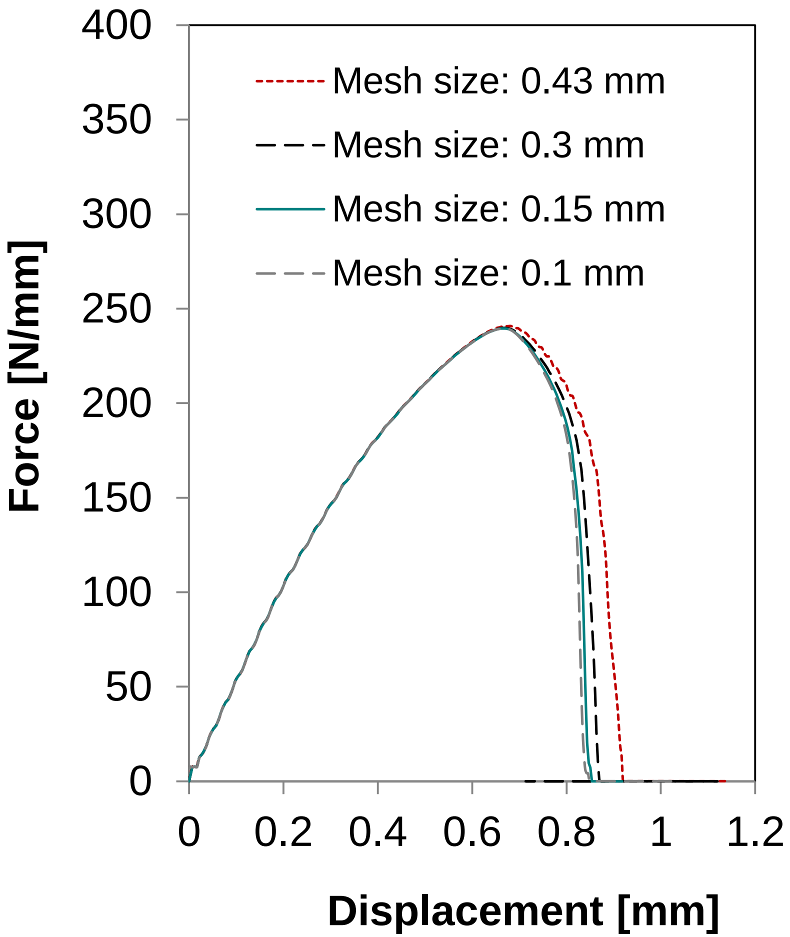
    - Resolution of the non-local equation once every N steps

$$\boldsymbol{K}_{\tilde{p}\tilde{p}} d\tilde{\boldsymbol{p}} = \boldsymbol{F}_{p} - \boldsymbol{F}_{\tilde{p}}$$

Finite-element implementation

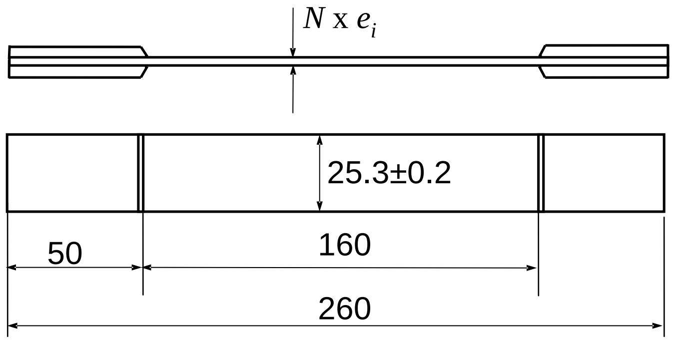
- Mesh-size effect
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply

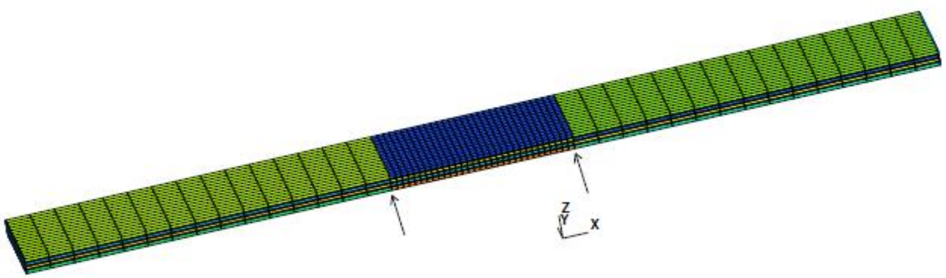


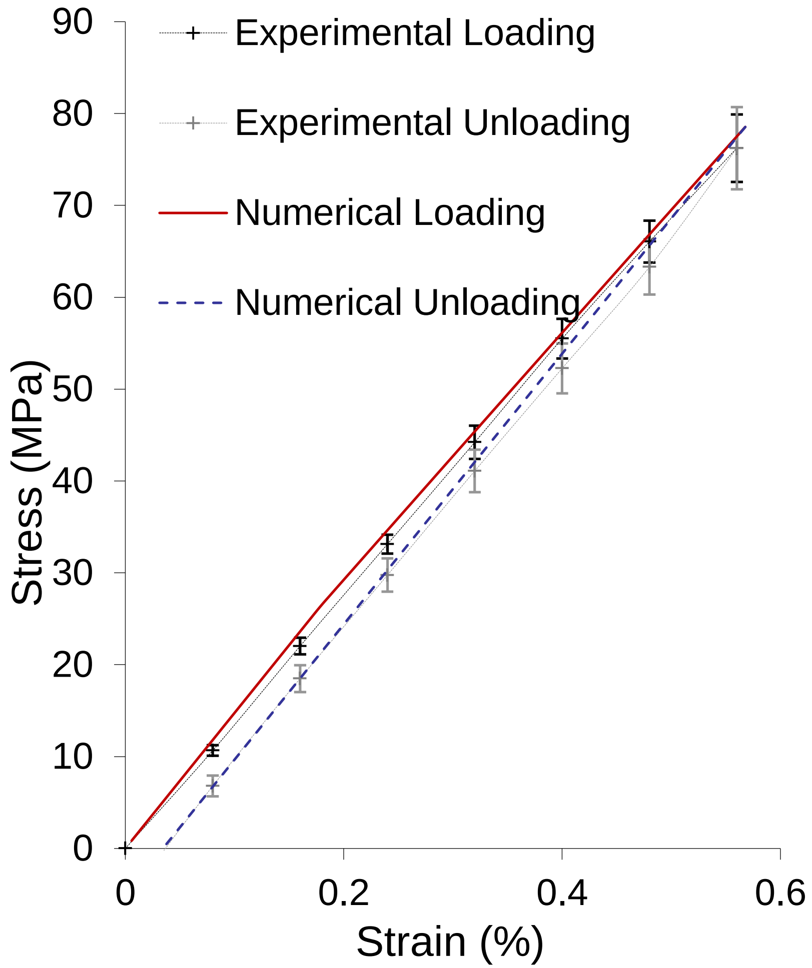


Applications

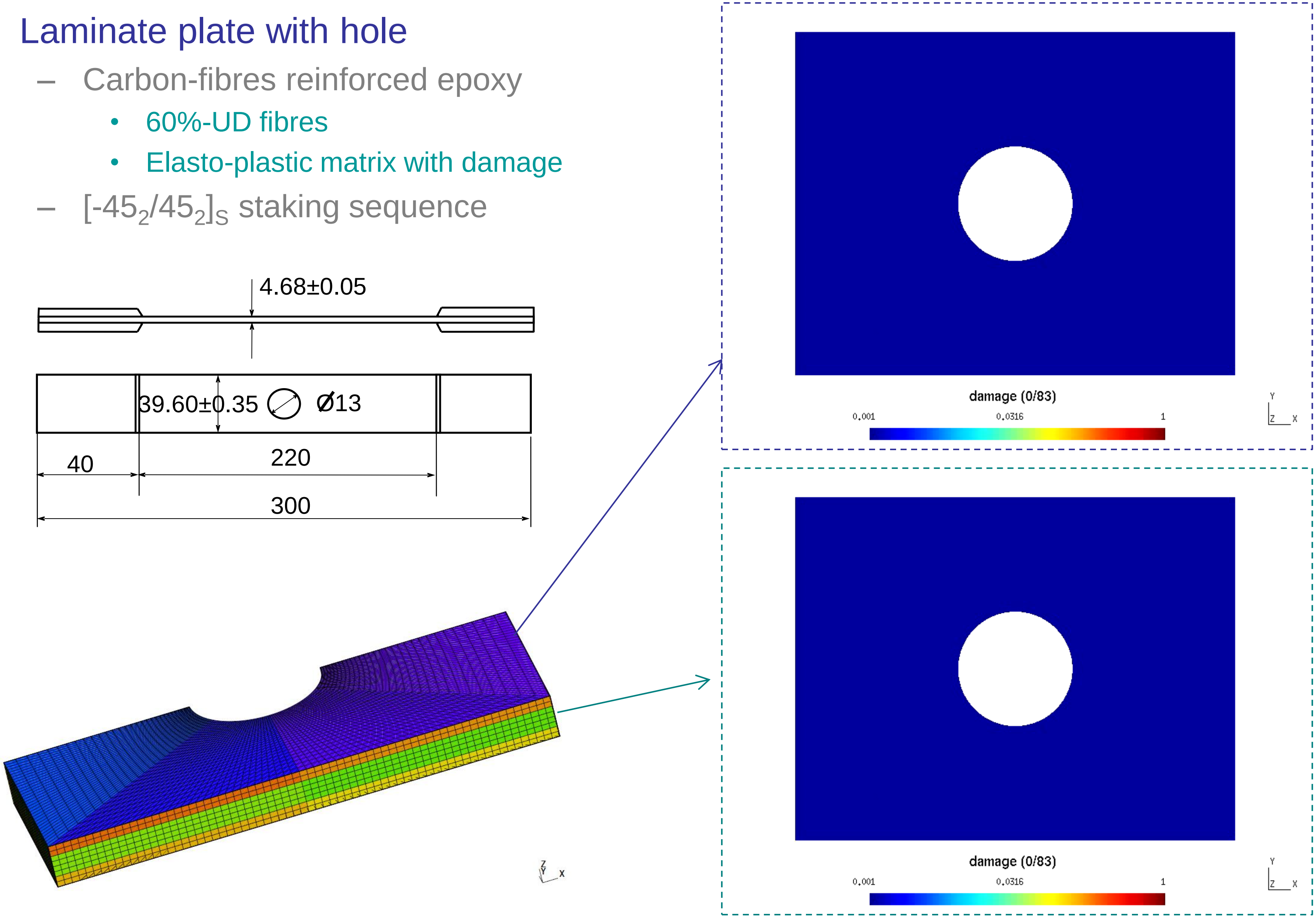
- Laminate: calibration
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$ staking sequence



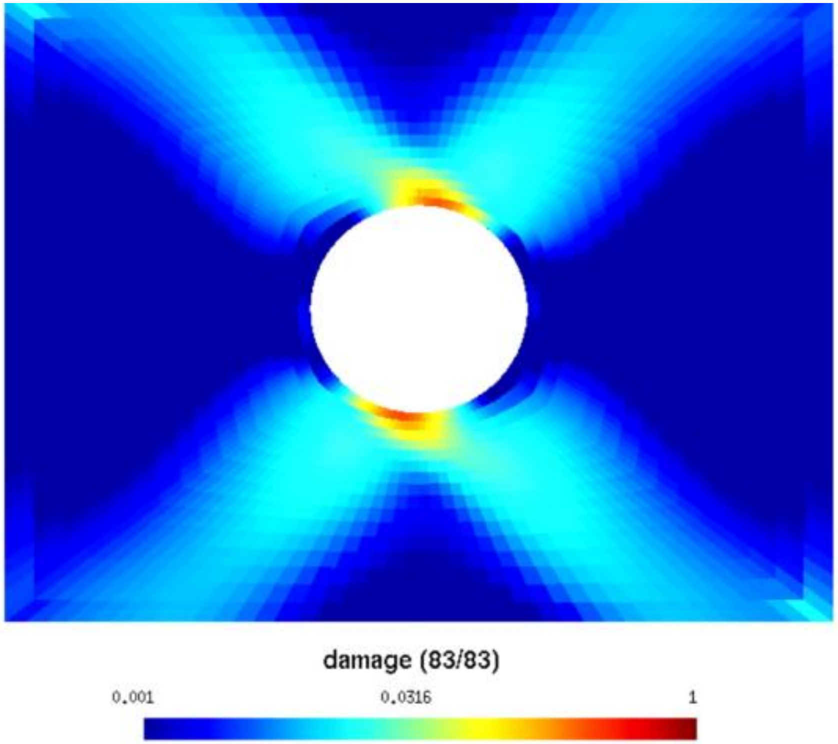


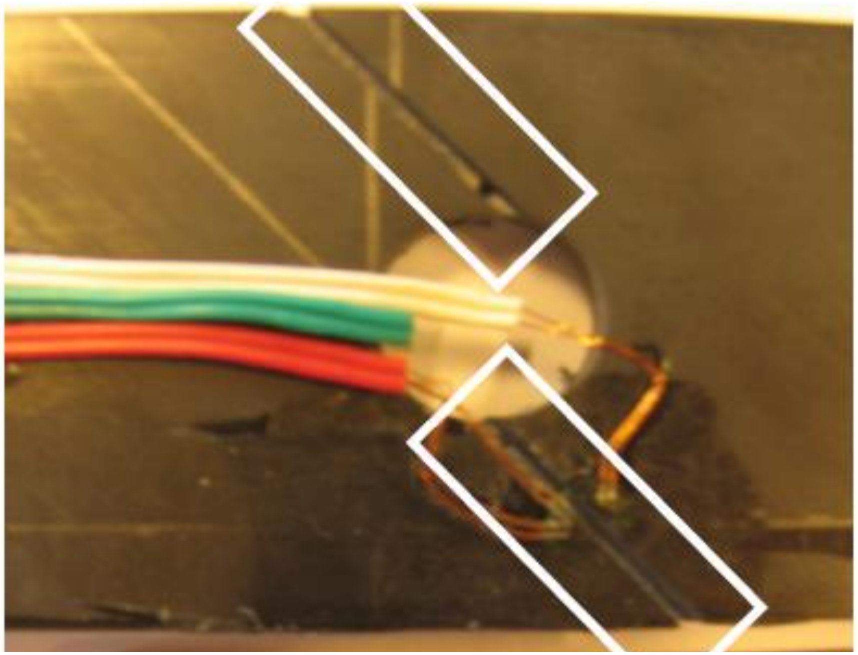


- Laminate plate with hole
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$ staking sequence

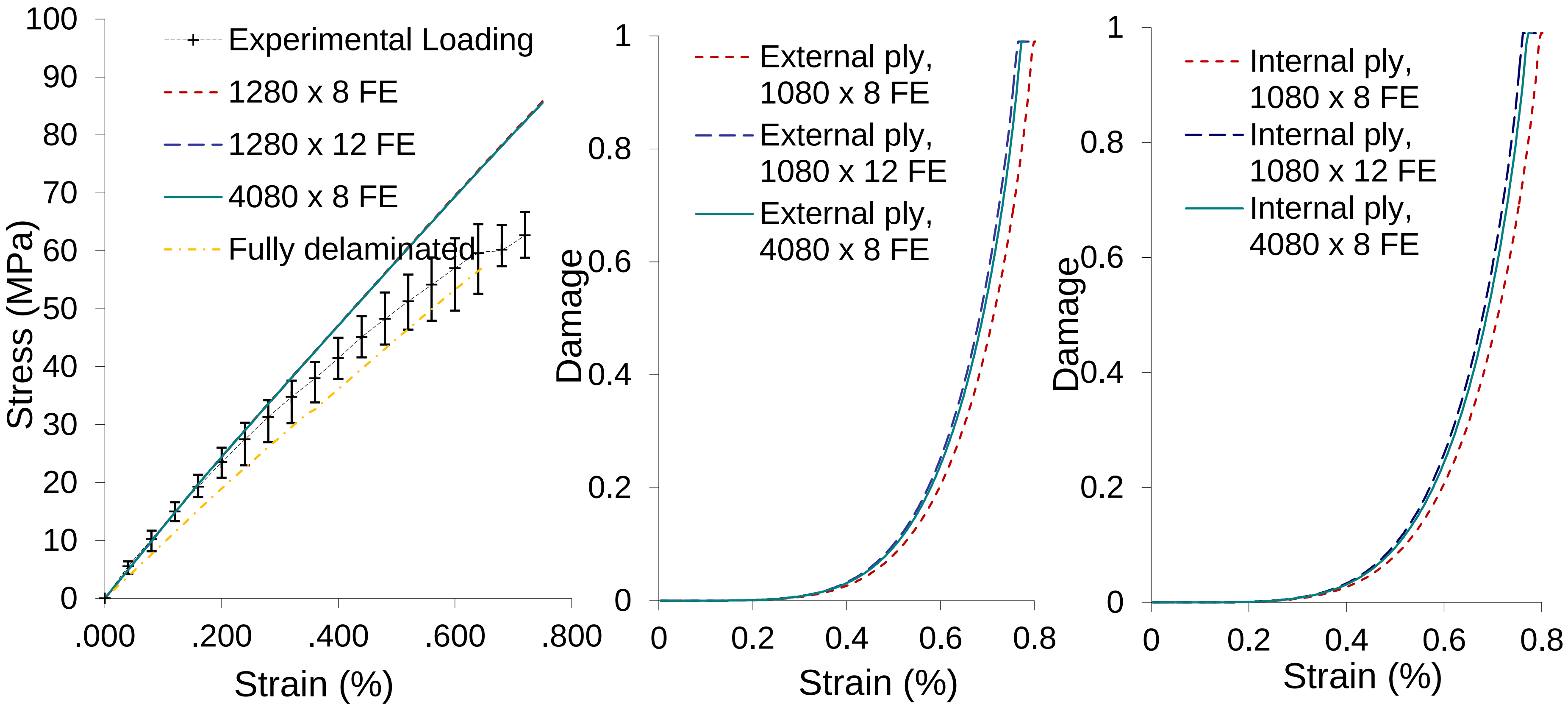


- Laminate plate with hole (2)
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$ staking sequence





- Laminate plate with hole (3)
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$ staking sequence



Conclusions

- New damage-enhanced incremental secant MFH approach
  - Efficient computationally
  - Allows fibres unloading during matrix softening

- Non-local damage-enhanced MFH
  - Good description of the meso-scale response
  - Can be used to study coupons problems

- Perspective

  - From damage to crack