

Local Head-Loss Coefficient at the Rectangular Transition from a Free-Surface Channel to a Conduit

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Abstract: Experimental tests have been performed to observe and determine, in stationary flow conditions, the local head loss at the transition from a free-surface channel to a conduit. These investigations considered a wide range of discharge and varied dimensions and positions of a rectangular cross-sectional conduit connected to the downstream extremity of a rectangular free-surface channel. From the head-loss evaluation results, simple analytical expressions to predict the local head-loss coefficient value at the rectangular transition from a channel to a conduit are proposed and validated. DOI: 10.1061/(ASCE)HY.1943-7900.0000790. © 2013 American Society of Civil Engineers.

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Introduction

In hydraulics, two kinds of energy losses may be distinguished (Hager 2008). The first kind is due to shear stresses along the boundary walls, which is distributed along the flow boundaries. It is designated as friction loss due to surface resistance and may be evaluated using the so-called friction formulas. The second one is intimately linked with the variations in the flow-path geometry, resulting in local-flow contraction, expansion, or deviation. The energy loss provoked by such local modifications in the flow conditions is called local loss due to form resistance. The local losses are known to be proportional to the dynamic pressure or kinetic energy of the flow. The proportion factor, depending on the type of geometry variation and its dimensions, is the head-loss coefficient (k). It is a nondimensional number.

Usual formulations to compute a local head-loss coefficient in case of flow contraction or expansion mainly consider the flow cross-sectional values upstream and downstream of the transition (Gardel 1962; Idel'cik 1986). Extensive study of the local head-loss coefficient value has been performed in conduits (Idel'cik 1986), considering a wide spectra of geometry variations. Idel'cik considered

varied circular conduit inlet configurations from a reservoir with negligible flow velocity. Local head-loss coefficient can be approximated in this case as $k = 0.5 \exp(-15r_v/D)$, where r_v is the radius of rounding of the conduit inlet and D is the conduit diameter (Hager 2008). Idel'cik also proposed local head-loss coefficients for square conduit inlet protruding in a reservoir, depending on the conduit location and sidewall thickness. Head-loss coefficient is equal to 0.63 when the conduit bottom is aligned with the reservoir bottom. When a sidewall and the bottom of the conduit are those of the reservoir, the head-loss coefficient is equal to 0.77 (Idel'cik 1986). Each conduit had a wall thickness that equals 0.03–0.04 times the width of the square conduit (Idel'cik 1986). Hager summarized several local head-loss coefficient expressions for a conduit expansion/contraction. Based on a paper by Gardel (1962), Hager (2008) presented an expression as $k = 0.5(1 - A_2/A_1)$ for a conduit contraction when the angle of the contraction is equal to 90° , where A_1 , A_2 are the wetted areas at the upstream and downstream cross sections of the contraction, respectively. This head-loss coefficient is related to the flow velocity downstream of the conduit expansion/contraction. Norman et al. (2001) provided detailed information about the hydraulic design of highway culverts, considering varied geometries of the inlet. Tullis et al. (2008) experimentally determined the entrance-loss coefficients for circular/elliptical buried-invert culverts in both unsubmerged and submerged culvert inlet conditions. These experiments have been carried out for circular culverts with invert burial depths of 20%, 40%, and 50% and an elliptical culvert with 50% invert burial depth (Tullis et al. 2008). The obtained coefficients varied in the range $k = 0.32$ – 1.10 , depending on the geometry and the culvert inlet end treatments (Tullis et al. 2008).

To the best knowledge of the authors, no work has been done to date to determine the local head-loss coefficient expression at the rectangular transition from a free-surface flow to a pressurized flow, neither experimentally nor numerically. Such situation may, however, be regularly encountered in hydraulic engineering, for instance in culverts, water intakes, or sewer systems. It is therefore of practical interest.

In this context, this paper presents the results of a study aiming at defining the local head loss taking place at the transition from a rectangular free-surface channel to a rectangular conduit whose height is smaller than the upstream water depth.

This study has been carried out at the laboratory of the Research Group of Hydraulics in Environmental and Civil Engineering,

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University of Liege, using an experimental approach. The tests considered a wide range of steady discharges and varied dimensions and positions of a rectangular cross-sectional conduit connected at the bottom of a rectangular free-surface channel. The channel axis and the conduit longitudinal axis are parallel.

Based on the whole set of results, some analytical expressions to predict the local head-loss coefficient at the rectangular transition from the channel to the conduit have been proposed and validated.

Test Characteristics

Geometry

Investigations have been performed on the basis of a 10-m long, 0.98-m wide, and 0.5-m deep horizontal glass flume. All the tested configurations consider a 4.5-m long rectangular conduit between two free-surface-flow channels, respectively, 4.5 m long (upstream) and 1.6 m long (downstream). The bottom elevation is constant along the system (flume bottom) and the channels and conduit axis are parallel. The upstream free-surface-flow channel uses the whole flume width ($B = 0.98$ m). The conduit cross section is rectangular with a height d of 0.1 m and a width b equal to B , $0.75B$, $0.5B$ or $0.25B$, depending on the configuration (Table 1). The downstream free-surface channel has a rectangular cross section with a width equal to the conduit width.

Two positions of the conduit have been considered: along the right bank of the flume (Configuration I, asymmetric) and aligned with the flume axis (Configuration II, symmetric). They are illustrated in Figs. 1 and 2, respectively.

The conduit and downstream channels have been built using exterior-type wood on the walls and faces that are not those of the glass flume.

Boundary Conditions

Discharges up to 90 L/s have been injected upstream of the flume through a permeable screen ensuring uniform velocity distribution over the cross section. The upstream boundary condition was a steady discharge. Downstream, the opening height of a flat rising gate is regulated to control the water level in the flume, depending on the discharge. The regulation is done to create a pressurized flow along the whole conduit while limiting the upstream water levels to the height of the flume sidewalls. Table 1 summarizes the discharges and the ranges of upstream energy values considered in this study (the minimum energy value corresponds to the smallest discharge value and vice versa for each geometric configuration).

Parameters and Measures

In this study, the width b of the conduit has been varied considering a constant conduit height d . The upstream water depth h_{up} varies depending on the discharge Q and gate opening. The upstream channel width B is constant. This results in varied ratios of the free-surface area to the conduit cross-sectional area. The influence of the conduit location along the channel width has also been investigated (Configurations I and II).

Specific cross sections have been selected to compute the flow energy upstream and downstream of the transition. The location of these sections is shown in Fig. 3. Sections 1 and 6 are far enough

Table 1. Geometric and Hydraulic Configurations

Configuration	Test geometry	b (m)	Discharge (L/s)	Upstream energy (E_3) (m)
I	A	$B = 0.980$	40, 50, 60, 70, 80, 90	(0.164, 0.212) ^a
	B	$0.75B = 0.735$	20, 30, 40, 50, 60, 70, 80	(0.159, 0.244) ^a
	C	$0.5B = 0.490$	10, 20, 30, 40, 50, 60, 70	(0.157, 0.334) ^a
	D	$0.25B = 0.245$	10, 15, 20, 25, 30, 35	(0.166, 0.354) ^a
II	B	$0.75B = 0.735$	20, 30, 40, 50, 60, 70, 80	(0.153, 0.239) ^a
	C	$0.5B = 0.490$	10, 20, 30, 40, 50, 60, 70	(0.156, 0.324) ^a
	D	$0.25B = 0.245$	10, 15, 20, 25, 30, 35	(0.163, 0.350) ^a

^aRange of the upstream energy values, the minimum value corresponding to the smallest value of discharge and vice versa for each geometric configuration.

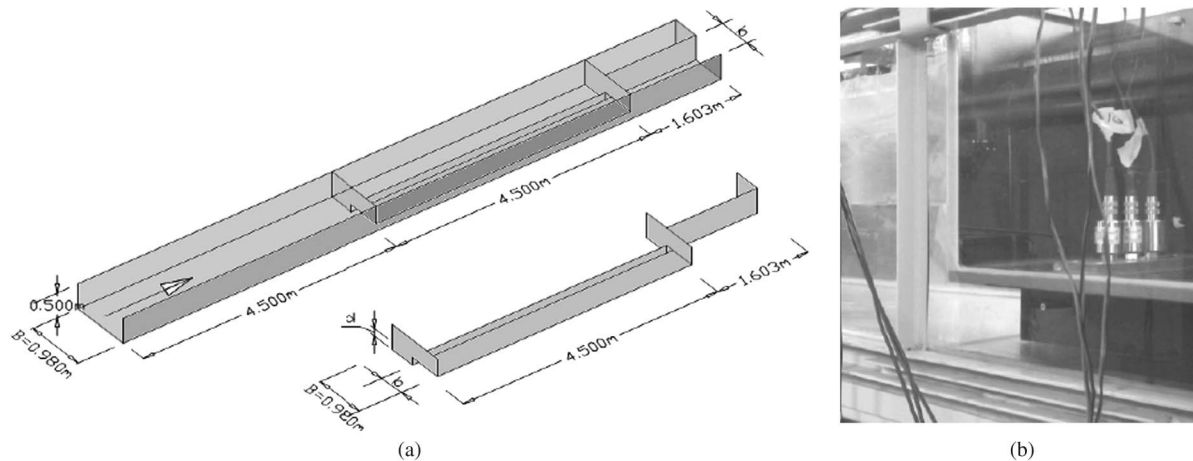


Fig. 1. Configuration I: conduit along the right bank of the channel: (a) three-dimensional sketch; (b) photo of the conduit inlet in the physical model, looking downstream (image by Nguyen Van Nam)

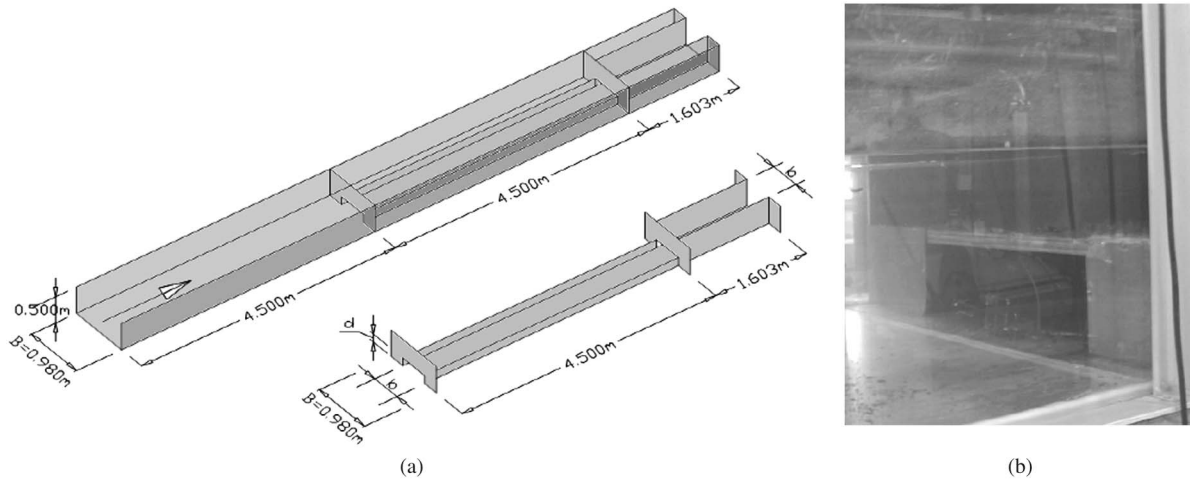


Fig. 2. Configurations II: conduit centered on the channel cross section: (a) three-dimensional sketch; (b) photo of the conduit inlet in the physical model, looking upstream (image by Nguyen Van Nam)

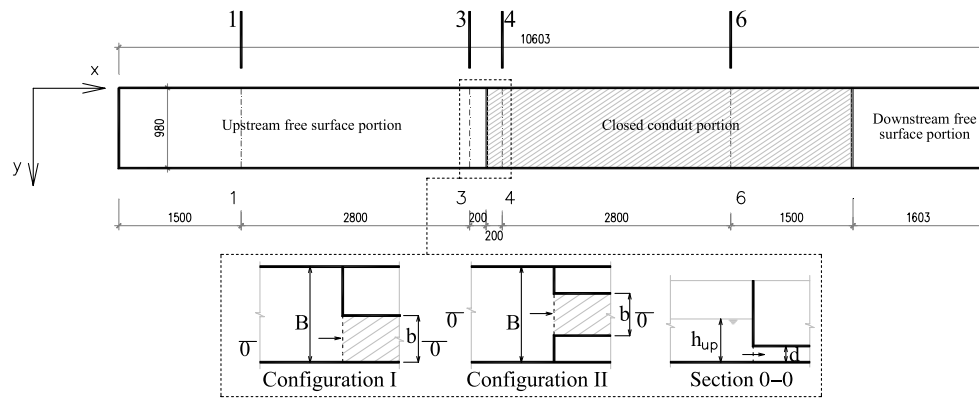


Fig. 3. Measurement cross sections (on the sketch of geometric Configuration I-A) and details of the transition location

from the transition to ensure uniform flow-velocity conditions and thus to be used to compute the flow energy from water depth and pressure measurements, respectively, in the free-surface channel and in the conduit. The difference between energy values in Sections 3 and 4 provides the local head loss. Energy in Sections 3 and 4 is computed from the measurements in Sections 1 and 6, considering friction losses between the measurement in Sections 1 and 6. Each test has been repeated three to five times to ensure the consistency of the results.

Measurement Devices

The experimental model has been equipped with the following measurement devices:

- The upstream discharge (inflow) is measured with an electromagnetic flowmeter (accuracy of $\pm 1\%$) and adjusted with a frequency regulator on the pumping system;
- The water-level measurement system (free-surface channels) includes five ultrasound sensors (placed in Sections 1 and 3 in Fig. 3) from Microsonic, ranging from 350 to 65 mm in height from the water surface (accuracy of ± 0.5 mm);
- The pressure-measurement system (closed conduit) is made of five Keller piezoresistive pressure transducers (placed in

Sections 4 and 6 in Fig. 3), connected to a LabVIEW device and software for signal treatment (accuracy of $\pm 0.2\%$);

- Two Pitot tubes were also used to measure the pressure head and velocity inside the conduit (accuracy of ± 1 mm). They are placed at Section 6 in Fig. 3;
- In addition, two limnimeters are used. The first one is placed at the upstream channel (accuracy of ± 0.5 mm) to determine the water depths during the sensors calibration process and the second one is fixed at the sluice gate to determine the gate opening and;
- Several tubes are also located at the top of the conduit in Sections 4 and 6 to measure the pressure head directly (accuracy of ± 1 mm).

Results and Discussion

Mathematical Processing of the Investigation Results

Experimental tests provide the mean water depth h_1 in Section 1, where the energy transverse slope is very small (difference between two measurement points on this section is equal to ± 0.1 mm whatever the discharge and configurations are). Pressure p_6 was also

measured experimentally in Section 6, where the transverse variations remain very small. From these values, the mean flow velocity has been computed using Eqs. (1a) and (1b) whatever the discharge and configurations are.

At the free-surface reach

$$V_1 = \frac{Q}{Bh_1} \quad (1a)$$

At the closed conduit portion

$$V_6 = \frac{Q}{bd} \quad (1b)$$

The mean flow energy E has been computed in Cross Sections 1 and 6 (with an elevation reference at the channel bottom) as follows:

$$E_1 = h_1 + \frac{V_1^2}{2g} \quad (2a)$$

$$E_6 = p_6 + \frac{V_6^2}{2g} \quad (2b)$$

From E values at Sections 1 and 6, the energy loss at the transition ΔE_T is as follows:

$$\Delta E_T = \Delta E_{1-6} - \Delta E_{1-3} - \Delta E_{4-6} \quad (3)$$

where ΔE_{1-6} = energy difference from Section 1 to 6, and ΔE_{1-3} and ΔE_{4-6} = friction losses between Section 1 and the conduit inlet section, and between the conduit inlet section and Section 6, respectively. The friction losses may be estimated using Darcy–Weisbach formula [Eq. (5)] and Colebrook–White equation [Eq. (6)] as follows:

$$\Delta E_j = S_j x_j \quad (4)$$

$$S_j = \frac{f_j V_j^2}{D_{h,j} 2g} \quad (5)$$

$$\frac{1}{\sqrt{f_j}} = -2 \log \left(\frac{\delta}{3.7D_{h,j}} + \frac{2.51}{R_j \sqrt{f_j}} \right) \quad (6)$$

where the subscript j represents free-surface channel Reaches 1–3 or the conduit Reaches 4–6; S = energy slope; x = reach length; V and D_h = flow velocity and hydraulic diameter, respectively, computed from the wetted areas at Sections 1 and 6; f = friction factor; δ = equivalent sand roughness, and is equal to 0.0014 mm for the free-surface channel, 0.05 mm for the conduit reach of Configuration II and 0.0014 mm for conduit in Configuration I. These values have been determined from previous backwater-curve measurements in the flume and energy-slope measurement in the closed conduit. For the considered wall materials, these values are in good agreement with the results presented by McGovern (2011). As the friction coefficient in the free-surface channel is very small, friction loss is of weak amplitude ($\Delta E_{1-3} = 2\text{--}3\% \Delta E_{1-6}$).

From ΔE_T values, the local head-loss coefficient k at the transition may be computed as a function of the flow kinetic energy

$$k = \frac{2g\Delta E_T}{V^2} \quad (7)$$

It is important to correctly define the reference velocity V . In particular, it should be selected so that no problem arises for its determination in further applications. In this investigation, V values are referred to the Cross Section 6, computed from Eq. (1b) whatever the discharge and geometrical configurations are. As explained in the “Boundary Conditions” section, the downstream boundary condition has been regulated to always create pressurized flow along the whole conduit length.

Local Head-Loss Coefficient

The local head-loss amplitudes ΔE_T at the transition are presented graphically in Fig. 4 as a function of the tested discharges for each geometric configuration. Fig. 5 shows the local head-loss coefficient k , referring to the downstream cross-sectional velocity as a function of the ratio of the downstream cross section A_4 to the upstream one A_3 (Sections 4 and 3, respectively, in Fig. 3). The following formulations fit the experimental results with a coefficient of determination (r^2), as defined by John et al. (1998), equal to 0.94:

$$k = 0.72 \times (1 - A_4/A_3) \quad (8a)$$

$$k = 0.63 \times (1 - A_4/A_3) \quad (8b)$$

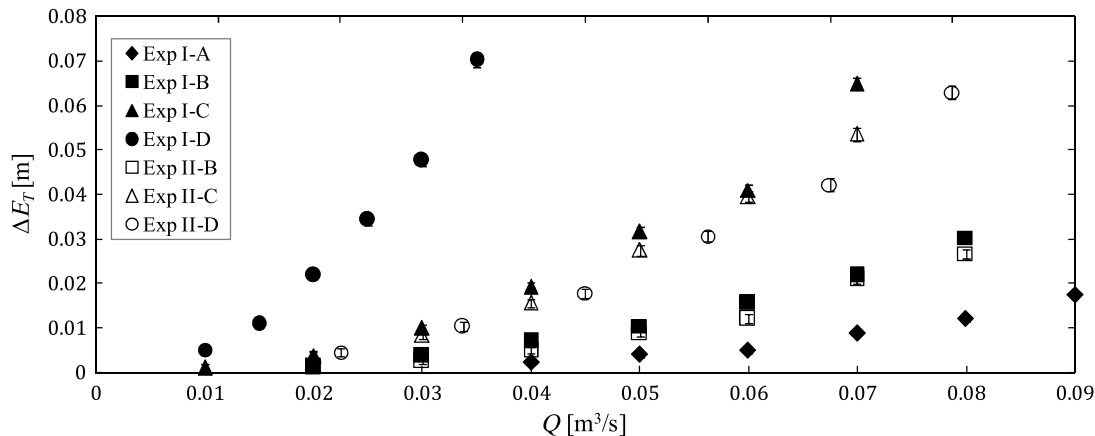


Fig. 4. Amplitude of the local head loss as a function of the tested discharges; error bars represent the accuracy of the measurements on the physical model

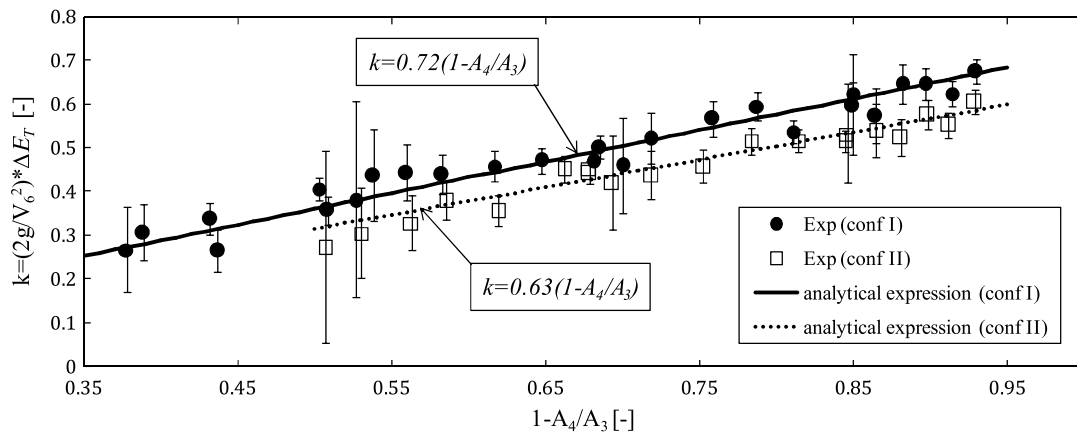


Fig. 5. Local head-loss coefficient variation depending on the transition geometry; k is defined by the experiments; error bars represent the accuracy of the measurements on the physical model

Eq. (8a) is used to compute the local head-loss coefficient for Configuration I, whereas Eq. (8b) is used for Configuration II. These formulas are also in good agreements with the data given by Idel'cik (1986) for upstream reservoir configuration ($A_4/A_3 \rightarrow 0$ and $V_3 \rightarrow 0$) as well as no flow contraction ($A_4 = A_3 \rightarrow k = 0$).

Both Figs. 4 and 5 show that there is a significant influence of the conduit location on the head loss. At constant cross-sectional area ratio, k values for Configuration I are always higher than those of Configuration II. This observation is consistent with the results of Idel'cik (1986) for square conduit inlet protruding in a reservoir. Despite not being investigated in detail in this study, this observation may be explained by more important transverse flow-velocity components induced by the nonsymmetric configuration in comparison with the corresponding symmetric one, creating a larger recirculation area at the conduit inlet. In addition, it has been observed that the maximum flow velocity at the transition is always higher in the nonsymmetric configuration compared with the corresponding symmetric one.

Conclusions

Experimental investigations have been carried out to determine the local head loss in the rectangular transition from a free-surface channel to a closed conduit. A wide range of discharges and geometric configurations as well as two different locations of the conduit have been carefully considered, providing a large data set.

Simple formulas are proposed to compute the local head-loss coefficient value from the geometries of the transition and the discharges. They are in good agreement with the experimental data and are of practical interest to design culverts for instance. In addition, an influence of the conduit position on the local head-loss coefficient is observed.

In the next step, a similar work will be done for varied conduit heights to enlarge the scope of application of the proposed formulation.

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Notation

The following symbols are used in this paper:

- A = wetted area of cross section (m^2);
- B = upstream channel width (m);
- b = conduit width (m);
- D_h = hydraulic diameter (m);
- d = conduit height (m);
- f_j = friction factor;
- g = gravity acceleration (m/s^2);
- h_i = water depth at cross section i of the free-surface channel (m);
- h_{up} = water depth upstream of the transition location (m);
- i = number of the cross section;
- j = designated free-surface channel Reaches 1–3 or the conduit Reaches 4–6;
- k = local head-loss coefficient;
- p_i = pressure at cross section i of the conduit (m);
- Q = discharge ($L/s, m^3/s$);
- R = Reynolds number;
- r = coefficient of determination;
- S_j = friction slope in the free-surface channel/conduit reach (m/m);
- V_i = mean flow velocity at cross section i (m/s);
- x_j = free-surface channel/conduit reach length (m);
- ΔE_{1-6} = energy difference from Section 1 to 6 (m);
- ΔE_j = friction loss in the free-surface channel/conduit reach (m);
- ΔE_T = local head loss at the transition position (m); and
- δ_j = equivalent sand roughness at the free-surface channel/conduit reach (mm).

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