R&D Investment, Capital Structure, and Agency Conflict: A Contingent Claim Analysis

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Abstract

The paper explores agency conflict between bondholders and shareholders over sequential R&D investment in a real option framework, where the risk associated with the R&D activity emanates from (1) asset intangibility, (2) the success intensity of successfully completing the R&D project, and (3) asset specificity. We show that if the assets employed to conduct R&D are highly intangible and specific and if the success intensity is low, then shareholders overinvest in R&D to shifts the burden of risk on to the bondholders, whereby the value of outstanding debt is reduced. Moreover, specificity of the firm's assets after the commercialization of the new discovery compounds the conflict over R&D investment.

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Irreversible Investment, Regime Switching, Asset Intangibility, Success Intensity, Asset Specificity, Asset Substitution
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1. Introduction

Hall (1993, 1994), Opler and Titman (1993, 1994), and Blass and Yosha (2001) are some of the papers that provide empirical evidence that R&D intensive firms are less leveraged than those that are not. Brown et al. (2009) and Tiwari et al. (2014) find similar evidence for a panel of R&D performing firms. Brown et al. show that for R&D intensive firms equity might be preferred to debt as a means of financing R&D, especially for young firms. While Tiwari et al. find that financially unconstrained firms are more likely to finance innovative activity with internal equity and less likely to finance with debt. However, under constraint, larger and mature firms are better placed to borrow for financing innovative activity.

Despite the large literature that document R&D intensive firms exhibiting lower leverage, we find that little or no attempt has been made to study agency conflict over R&D investment. In this paper we explore how asset intangibility and specificity and technical uncertainty associated with R&D activity give rise to agency conflict between bondholders and shareholders of the firm.

Williamson (1988) and Shleifer and Vishny (1992) argue that asset specificity, or redeployability of an asset to alternative uses, is a key determinant of the liquidation value of the asset, where firms with more specific assets have lower liquidation value and lower debt level. Hall and Lerner (2010) state that "... although leverage may be a useful tool for reducing agency costs within a firm, it is of limited value for R&D intensive firms. Because the knowledge asset created by R&D investment is intangible, partly embedded in human capital, and ordinarily very specialized to the particular firm in which it resides, the capital structure of R&D intensive firms customarily exhibits considerably less leverage than that of other firms." Also, Aboody and Lev (2000) argue that because of the uniqueness of assets of R&D performing firms, which makes it difficult for outsiders to learn about the productivity and value firm's R&D activity from the performance and products of other firms in the industry, the extent of information asymmetry associated with R&D is larger than that associated with investment in tangible and financial assets.

While there are many empirical papers that have confirmed asset specificity as a significant determinant of capital structure, some of the papers that look at asset specificity of R&D intensive firm are Titman and Wessels (1988), Alderson and Betker (1996) and Bah and Dumontier (2001). Titman and Wessels find that because of lack of a secondary market for R&D and the non-collateralizability of R&D activity, having "unique" assets is associated with lower debt levels. Alderson and Betker find evidence that liquidation costs and R&D are positively related across firms. While Bah and Dumontier compare a sample of R&D intensive firms to non-R&D ones in Europe, Japan, the UK and the US to find that because of asset specificity, R&D intensive firms exhibit significantly lower debt levels.

Secondly, it is well known that R&D has a number of characteristics that make it different from ordinary investment: it is long-term in nature, high risk in terms of the probability of failure, unpredictable in outcome, labor intensive, and idiosyncratic. The risk of failure and unpredictability of outcomes are potential sources of asymmetric information that give rise to agency issues in which the inventor has better information about the likelihood of success and the nature of the contemplated innovation project than the investors. Hence, bond holders, ceteris paribus, may be unwilling to hold the risks associated with greater R&D activity.

Berk et al. (2004) study the implication of various uncertainties, such as, the technical uncertainty in successfully completing the R&D project, the uncertainty about the future cash flows, and risk of obsolescence, for the value of the R&D venture and risk premium. They show that in a multistage innovation process the risk premium is higher during the initial stages of process. However, as technical uncertainties get resolved and the project advances towards completion, the risk premium decreases. In an empirical paper Shi (2003) finds evidence that R&D activity, which increases the market value of equity, also increases bond default risk and debt risk premia.

Pindyck (1993), Childs and Triantis (1999), Schwartz and Moon (2000), Berk et al. (2004) and Miltersen and Schwartz (2004) are some of the papers that study R&D process in a contingent claim framework. However, these papers either treat the R&D project in isolation or as "pure growth" firms and not as a project which an existing firm, facing a

threat of liquidation, might undertake. Besides, these papers do not study the implications of the risk associated with R&D for agency conflict between the bondholders and the shareholders over R&D investment and its implication for debt valuation.

The wealth of papers that use real options models to study financing and investment decisions of firms is large. A partial list of such papers includes Mello and Parsons (1992), Leland (1994), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Leland (1998), Fan and Sundaresan (2000), Mauer and Sarkar (2004), Childs et al. (2005), and Egami (2009). While the paper differ in their scope and the issues they address, central to all issues is the agency conflict between the bondholders and the stockholders over investment decisions. Given bankruptcy costs and limited liability the agency problem these papers study is either the problem of risk-shifting or the problem of underinvestment. The investment decision considered in these papers is either that of asset substitution or scale expansion, but do not consider R&D investment, the associated risks, and the resultant agency conflict.

The aim of the paper is to study agency conflict between the bondholders and shareholders that could arise over sequential R&D investment. This paper differs from the other papers in the literature in that the risk associated with the new venture and the associated conflict does not emanate from higher volatility of the cash flows from the new venture, but from (1) intangibility and specificity of asset involved in the conduct of R&D activity, (2) the success intensity in successfully completing the R&D project, and (3) the specificity of the assets after the implementation/commercialization of the innovative output of the completed R&D project.

We find that if the assets involved in the conduct of R&D are highly intangible and unique to the firm and if the success intensity of successfully completing the project is low, then shareholders overinvest to shifts the burden of risk on to the bondholders, whereby the value of outstanding debt is reduced. Moreover, the agency conflict over R&D investment is exacerbated if the assets of the firm become specific to the firm after the implementation/commercialization of the new discovery.

The problem of risk substitution occurs because the maximum amount of default-risk free debt – which is function of asset intangibility and the success intensity – sustainable before and after starting the R&D project differ. If assets employed to conduct R&D are highly intangible and unique to the firm and if the intensity of successfully completing the R&D project is low, then the maximum amount of default-risk free debt sustainable after having started the R&D project is lower than some debt levels that are deemed safe before starting the R&D project. This implies that if the shareholders undertakes R&D investment, which may be the only growth option available to the firm, then certain levels of debt, which are risk-free before starting the R&D project, become risk prone upon embarking on the R&D project. In such a situation, even if no additional amount is borrowed for R&D investment, shareholders, maximizing their claim and unconcerned about the welfare of bondholders, overinvest and shift the burden or risk on to the bondholders to reduce the value of the outstanding debt.

Once the firm implements the innovative output from a successfully completed R&D project, it very much likely that the assets of the firm become less redeployable or unde-

ployable in alternative uses. Hence, specificity of assets after the implementation of the innovative output that reduces the liquidation value of the firm constitutes yet another risk factor. As a result, debt levels that could have been default risk-free before the start of the R&D project or during its conduct become risky after the commercialization of the innovative output. This compounds the agency conflict over investment in the R&D project that delivers the innovative output.

Because of the above elucidated risk inherent to R&D activity, which when undertaken reduces the value of outstanding debt, it is possible that rational investors may be unwilling to finance R&D intensive firms or firms that are likely to take up R&D and innovative activities. The results obtained here could thus explain the empirical finding that R&D intensive firms are less leveraged than those that are not.

The rest of the paper is organized as follows: In Section 2 we describe the model, where there is a description of the firm and the R&D project, debt characteristics, and the liquidation value of the firm at the various stages in the life of the firm. In Section 3 we discuss the investment and liquidation strategy of the firm for the various stages of the R&D process and also obtain equity and firm valuation when the level of outstanding debts at the various stages of the R&D process are default-risk free. In Section 4 we discuss and equity valuation and investment and liquidation strategy for the various stages of the R&D process when R&D is financed with risky debt. In section 5, given equityholders investment and liquidation strategy, we provide closed form solutions for the value of the outstanding debts at the various stages of the life of the firm. Finally, in Section 6 we conclude.

2. The Model

2.1. Description of the Firm and the R&D Project

Before starting the R&D project, shareholders of the firm operate a set of assets to derive an income/cash flow of ψx_t, where ψ, the constant revenue parameter, which also reflects the current state of technology that the firm employs for its production, and x_t represents exogenous demand/price, which follows a geometric Brownian motion,

$$dx_t = \alpha x_t dt + \sigma x_t dw_t.$$

The growth of the firm comes through a R&D project, and all discretionary investments of the firm is related to the R&D project. Investing in R&D entails making sequential decisions, where the problem faced by the firm is first to start the R&D project and then to continue investing in the project until completion at an uncertain date. To start the R&D project, the firm requires an initial investment of I, which is the amount spent in building a capacity to do research and development. This capacity or the asset comprises of assets that are *specific* to the firm as well as intellectual capital that are *intangible* in nature. Building a capacity to conduct R&D, among other, requires setting up an administrative organization that can ensure smooth functioning of R&D activity and creating mechanisms to monitor the progress of R&D activity. Moreover, there are costs involved in hiring of

scientific personnel and staff. By spending resources on such activities the firm is able to earn intellectual capital required for R&D activity.

To keep the analysis simple and tractable, we make some simplifying assumptions.

Assumption 1. *Investment in R&D is for developing a more efficient technology for production.*

Having started the R&D project, to successfully complete the project the firm incurs a fixed investment cost of a. If the firm manages to innovate a new technology by successfully completing the project, shareholders earn the option to invest K and implement/commercialize the new technology. The amount K captures the capital required and the labor and capital adjustment costs to be incurred in shifting to the more cost effective means of production. By switching to a new technology the firm receives a stochastic cash flow of πx_t per instant of time, where π is strictly greater than ψ, $(\pi > \psi)$; this, in a simple way, captures a move to a more efficient means of production. Also, after having successfully completed the R&D project, to maintain and operate the new technology the firm pays a fixed price of c per instant of time.

Here we would like to state that, though ASSUMPTION 1 encompasses aspects of process innovation, the newly developed technology can also be thought of as a new product. One could also think of it as investing in a venture for improving the quality of the product that the firm is engaged in producing[1]. The improved quality could then bring higher revenues to the firm for nearly the same cost as the existing operation. Through out the text, however, we will maintain that the new venture is for developing an efficient means of production.

Since the R&D project is for developing a new, more efficient technology, the demand, x_t, for the produce from the implemented new technology, the innovation output of the successfully completed R&D project, is the same as the demand for the output from its existing operation. Consequently, even when the project is not complete, shareholders can observe the cash flows the successfully completed and implemented R&D project would bring.

Assumption 2. *The success intensity, λ, of completing the R&D project is known to the firm.*

The firm successfully completes the R&D project at some random date τ, where τ is exponentially distributed with intensity, λ[2]. Hence, the expected time to completion, $E(\tau)$

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[1] In a more general model of R&D investment, where the new venture is for developing a new product, the value of the R&D project will depend on the y_t, the price process of the new product, which will be correlated with x_t. Investment, financing and liquidation strategy will then depend on both the stochastic behavior of the value of R&D project and the price process x_t, both of which will be correlated with one another. While this remains a more involved problem to solve, it seems unlikely that the qualitative results regarding investment, financing and liquidation strategy for new product development will be different from that obtained in this paper.

[2] When success intensity is known to the firm is a special case treated in Berk et al., distinct from the case which involves learning overtime.

is $1/\lambda$, which is independent of the process, $\{x_t\}_{t\in[0,\infty)}$.

After having embarked on the R&D project, if the demand/price for the firm's produce, x, falls low, shareholders have the option to mothball the project. When the project is mothballed the firm incurs a fixed cost m, where $m < a$. If the x falls further then shareholders can abandon the R&D project[3].

Assumption 3. *When the firm abandons the R&D project, the initial investment I retains only its scrap value.*

The assumption that the initial investment I, but for the scrap value, becomes valueless once the R&D project is abandoned could imply that the firm does not have any other R&D project for which, with some additional cost, the initial investment I could be made useful. This is more likely to be true for smaller and younger firms than large established firms, who may have a choice between alternative R&D projects[4]. Alternatively, it is possible that the assets employed and generated in the conduct of the abandoned R&D project are so specific that they couldn't be employed in alternative ones. Now, since the initial investment I becomes valueless to the firm when the shareholders abandon the R&D project, shareholders wait too long to abandon the R&D project. Whereas, it might be optimal to liquidate the firm without waiting to first abandon the R&D project. In the event of liquidation the owners are able to retain a fraction f of the initial investment.

It is well known that the second hand market for intangibles and assets specific to the firm is fraught with friction and generally do not exist. Hence, after having invested the amount I, if the shareholders liquidate the firm, only a fraction, f, of I can be recouped. Since most of I is spent in acquiring intangible and firm specific assets, f is likely to be small. Investing I to start the project is thus an irreversible decision. R&D activity is also risky because of the uncertainty involved in successfully completing the R&D program. Since the interarrival time of success in R&D activity is modeled as an Exponential Process with parameter λ, this implies that both the expected time, $\mathrm{E}(\tau)$, at which success can be realized as well as the variance of that time increases as λ becomes low. Thus, if expected time to complete the R&D project is large, it might happen that before successfully completing the project the firm may experience a downfall of demand/price, forcing shareholders to liquidate, during which they stand to lose a large fraction of I. The third reason why R&D ventures are risky is because once the firm commercializes the innovative output from the R&D project, the assets of the firm become more specialized to the firm, which reduces the firm's liquidation value. This is discussed in detail in subsection 2.3.

Finally, we would like to note that most papers model risk associated with new investment through the volatility of cash flows to be earned from the new venture. While this source of risk could be incorporated in our model, in this paper we focus on risks more specific to R&D ventures.

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[3]In Berk et al. the decision to abandon or mothball a project is endogenously determined. To keep the analysis tractable and short, we do not employ their model.

[4]Modeling and incorporating the choice between alternative R&D projects, which could a lend greater generality to the issue we study, is beyond the scope of the paper.

2.2. Valuation of Cash Flows

Let $\underline{V}(x_t)$ be the present value at time t of the cash flows from the operation that yields a cash flow of ψx_t per instant of time, and $\overline{V}(x_\tau)$ the present value of the cash flows from the implemented/commercialized innovation output that is successfully obtained at the random date τ. By standard results in asset pricing, we know that

$$\underline{V}(x_t) = \int_t^\infty \mathrm{E}_t(\psi x_s)e^{-\alpha_p s}ds = \int_t^\infty \psi_1 x_t e^{\alpha s}e^{-\alpha_p s}ds = \frac{\psi_1 x_t}{\alpha_p - \alpha} = \frac{\psi_1 x_t}{r - \mu},$$

and similarly

$$\overline{V}(x_\tau) = \frac{\pi x_\tau}{r - \mu},$$

where r is the risk-free rate of return, μ is the risk adjusted rate of return on the cash flows, x and $r - \mu = \alpha_p - \alpha = \delta$, where α_p is the drift of the price, p, of the asset or dynamic portfolio of assets that span x and δ is the some kind of dividend that accrues for holding x.

2.3. Debt Characteristic, Liquidation, and Default

Assumption 4. *The firm's debt policy is static so that the debt principal remains fixed throughout the life of the loan.*

That is, we do not consider the possibility of dynamic restructuring of debt or renegotiation of debt contract. The reason why we keep capital structure static and abstain from issues such as maturity and dynamic restructuring is because we want to focus on the agency conflict over R&D investment, which is risky given its nature, when shareholders finance R&D activity with debt and/or when debt is in place. The firm finances part of its initial investment I with perpetual debt of the amount $D - \underline{D}$, where $\underline{D}$ is the level of outstanding debt before the start of the R&D project. The amount $D - \underline{D}$ is negotiated with the bondholders prior to the start of the R&D project. Such a contract, as Mauer and Sarkar explain, with the creditors for future financing is analogous to loan commitment or revolving line of credit.

We also make the standard, but necessary, assumption that the firm cannot extract concessions from the banks or bondholders, so that lenders get the full debt service of $rD = r(\underline{D} + (D - \underline{D}))$ as long as the firm continues as a going concern. Failure to service the debt allows lenders to take over. Lenders are willing to advance financing only if they have property rights to the firm's assets when it defaults. Their property rights can be protected by laws against fraudulent conveyance and by covenants. We assume these laws and covenants are effective and that lenders are protected against purely dilutive debt issues.

Apart from the problem of investing, equitholders also have the option to irreversibly liquidate the firm both before and after starting the R&D project, which makes the problem we study analogous to the two sided optimal stopping time problem examined in Egami (2009).

The value of the firm at closure after it has embarked on the R&D project but before implementing the innovation output of the successfully completed the R&D project is given by:

$$\Phi + \frac{\phi_1 x}{r - \mu}, \tag{1}$$

where $\phi_1 < \psi$. In the above, $\phi_1 x_t$ is the flow of income per instant of time in the alternative use of the firm's asset and $\phi_1 x (r - \mu)^{-1}$ is the present value of this income flow. When $\phi_1 > 0$, closure value and going-concern value move together. This is plausible if, as explained in Lambrecht and Myers (2008) (henceforth (LM)), the assets are used to produce a different product in the same industry, or if the values of all assets depend on some common macroeconomic factor. That ϕ_1 is strictly less than ψ reflects that the assets of the firm in their alternative use generate less revenue. This is because in the event of liquidation, in the spirit of Hart and Moore (1994), the incumbents' inalienable human capital cannot be used in the future. The difference between ψ and ϕ_1, thus, captures the degree to which assets are specific to the firm (see Mella-Barral, 1999, for more discussion on the topic).

In (1), $\Phi = \phi_0 + fI$, where ϕ_0 is the constant liquidation value at closure that results from that operation of the firm, which yields ψx_t every instant of time, and fI is the constant liquidation value at closure of the R&D investment project. If shareholders liquidate the firm prior to the starting of the R&D project, then its value at liquidation is given by:

$$\phi_0 + \frac{\phi_1 x}{r - \mu}.$$

When shareholders liquidate the firm after it has embarked upon the R&D project but before successfully implementing the innovation output, they irreversibly exchange their current claim for their residual one:

$$\max\{0, (\Phi + \frac{\phi_1 x_l}{r - \mu} - D)\},$$

where x_l is optimal liquidation threshold of exogenous demand, such that if x falls below x_l, then the residual value of the firm in liquidation states becomes more valuable to the shareholders . The maximum between 0 and $\Phi + \frac{\phi_1 x_l}{r-\mu} - D$ ensures that there are no violations of limited liability of the shareholders.

We know that when debt is default-risk free, the optimal liquidation threshold x_l is also the first-best (see LM). For debt D to be risk-free, it is required that

$$\max\{0, (\Phi + \frac{\phi_1 x_l}{r - \mu} - D)\} = \Phi + \frac{\phi_1 x_l}{r - \mu} - D.$$

Thus, the maximum level of debt, D, *after* it has started the R&D project such that D is risk-free is

$$D^* = \Phi + \frac{\phi_1 x_l}{r - \mu}. \tag{2}$$

Similarly, the maximum level of debt, $\underline{D}$, *before* starting the R&D project such that $\underline{D}$ is default-risk free is

$$\underline{D}^* = \phi_0 + \frac{\phi_1 x_L}{r - \mu}, \tag{3}$$

where x_L is the optimal liquidation threshold at which the shareholders liquidate the firm before embarking on the R&D project.

As stated earlier, after the firm successfully completes the R&D project the owners of firm have the option to invest K amount and implement the innovation output so that production commences with the new technology. Equityholders finance part of this investment with a perpetuity of the amount $\overline{D} - D$, the terms of which can be arranged during the conduct of the R&D project, or prior to the starting of the R&D project. Thus, the total amount of outstanding debt after the implementation of the project is $\overline{D}$. The maximum level of risk-free debt that the firm can sustain after the implementation is

$$\overline{D}^* = \varphi_0 + \frac{\varphi_1 x_{LI}}{r - \mu}, \tag{4}$$

where x_{LI} the threshold level of price at which the shareholders liquidate the firm post implementation. In (4), φ_0 is the constant liquidation value at closure and $\varphi_1 > 0$, but $\varphi_1 < \pi$. These two conditions reflect that the closure value of the firm and going-concern value move together, and that assets of the firm, post implementation, in their alternative use generate less revenue.

Now, since the assets employed for the new technology to be operational will be *specific* to the firm, in the event of closure the net revenue in the best alternative use of assets, φ_1, is lower than ϕ_1, the net revenue in the best alternative use of assets before the adoption of the new technology. Also, φ_1 as a fraction of π will be much lower than what would have been the case before the new technology was adopted; that is, $\varphi_1/\pi < \phi_1/\psi$.

The shareholders default if at closure, debt principals, $\underline{D}$, D, and $\overline{D}$, exceeds the value of the firm at closure. In the event of default, bondholders take control of the firm's assets and a fraction δ $(0 < \delta < 1)$ of the firm's value is destroyed. Default triggers bankruptcy costs, which depend on δ.

3. Firm Valuation and Investment and Liquidation Strategy with Risk-Free Debt

Let x_I be the optimal threshold of exogenous demand at which the shareholders invest I to start the R&D project. To ascertain x_I and the optimal liquidation threshold, x_L, we have to first obtain the value of the firm's equity, $E(x)$, after embarking on the R&D project. To obtain this equity value we first solve the second stage switching problem, where the shareholders decide if it is optimal to spend a every instant on the R&D project or switch to the passive state by mothballing the project.

3.1. The Switching Problem

After starting the R&D project, shareholders maximize the value of their equity, where the value of the firm's equity, $E(x_t)$, at time t is the solution to the following stochastic control problem:

$$
E(x_t) = \sup_{\mathcal{I}_s,\mathcal{L}_s,\mathbb{I}_s\in\{0,1\},s\in(t,T)} \mathbb{E}_t^Q\Bigg\{ \int_t^T e^{-r(s-t)}\bigg[(1-\mathcal{L}_s)\bigg((1-\zeta_s)\bigg(\psi x_s - rD - \mathcal{I}_s a - (1-\mathcal{I}_s)m\bigg)
$$
$$
+ \zeta_s\bigg(\mathbb{I}_s(\pi x_s - c - r\overline{D}) + (1-\mathbb{I}_s)(\psi x_s - c - rD)\bigg)\bigg)
$$
$$
+ \mathcal{L}_s[\max\{0,(r\Phi + \phi_1 x_s - rD)\}]\bigg]ds + e^{-r(T-t)}E(x_T)\Bigg\}, \tag{5}
$$

where T is an arbitrary point in the future, $\mathbb{E}^Q$ is the expectation under the risk-neutral measure Q, and $\zeta(s)$ is an indicator variable that takes value 1 if the firm successfully completes the R&D project at time s and 0 otherwise. The other indicator variables $\mathcal{I}_s$, $\mathcal{L}_s$, and $\mathbb{I}_s$ respectively are defined as follows:

$$
\mathcal{I}_s = \begin{cases} 1 & \text{is the decision to spend } a \text{ amount to complete the R\&D project} \\ 0 & \text{is the decision to mothball the project,} \end{cases}
$$

$$
\mathcal{L}_s = \begin{cases} 1 & \text{is the decision to liquidate the firm} \\ 0 & \text{is the decision to continue,} \end{cases}
$$

and

$$
\mathbb{I}_s = \begin{cases} 1 & \text{if the shareholders decide to pay the fixed investment cost, } K, \\ & \text{required to implement/commercialize the innovation output} \\ 0 & \text{if they decide to wait.} \end{cases}
$$

In (5), when $\mathcal{I}_s = 1$, the net revenue acquiring to shareholders is $\psi x_s - rD - a$; when the firm mothballs the project, ($\mathcal{I}_s = 0$), they obtain a net revenue of $\psi x_s - rD - m$. After the successful completion of the R&D project, if the shareholders decide to implement/commercialize the new discovery by investing K, ($\mathbb{I}_s = 1$), the net flow of revenue every instant is $\pi x_s - c - r\overline{D}$. When $\mathbb{I}_s = 0$, they receive a net revenue of $\psi x_s - rD - c$.

Now, since to maintain the R&D project in the mothballed state requires expending a fixed cost of m every instant, therefore, if the demand for the firm's produce falls further, shareholders would like to abandon the project in exchange of an option to liquidate the firm. When the firm abandons the R&D project the initial investment I is of no use to the firm, but is redeemed for a scrap value of fI. Let x_a be the level of exogenous demand such that if x falls below it, shareholders abandon the R&D venture. And let $\bar{x}_l$ be the threshold level of x such that, after having abandoned the project, if $x \leq \bar{x}_l$, the firm is liquidated. Since abandoning the venture is an irreversible decision, what we find is that the firm waits too long to abandon the R&D project, with the result that the abandonment threshold x_a

is lower than the liquidation threshold $\bar{x}_l$. This implies that the firm is liquidated as soon as the R&D venture is abandoned. In Appendix C we show how x_a and $\bar{x}_l$ are obtained.

Alternately, it might be more profitable for the firm to liquidate the firm along with R&D venture without waiting to first abandon the R&D venture. This is indeed what we find; for a broad range of parameter values we find that $x_l > x_a$, where x_l is the threshold level of price at which the firm is liquidated without first waiting to abandon the R&D project, so that

$$\Phi + \frac{\phi_1 x_l}{r - \mu} > \Phi + \frac{\phi_1 x_a}{r - \mu}.$$

That is, the value of the firm at the liquidation threshold at x_l is greater than the value of the firm at the abandonment threshold x_a at which the shareholders first abandon the R&D and then, since $x_a < \bar{x}_l$, immediately proceed to liquidate the firm. Given that it is optimal to liquidate the firm rather than wait to abandon the R&D project, the term $\max\{0, (r\Phi + \phi_1 x_s - rD)\}$ in (5) is the flow equivalent of the amount the shareholders receive when the firm is liquidated before the successful completion and implementation of the R&D project.

Assumption 5. *At the random date τ, when the firm successfully completes the R&D project, it immediately implements the innovative output.*

According to the above assumption, at the date τ when the firm comes up with an innovative output, the shareholders immediately implement it by investing K and obtain a value of $\mathcal{E}^a(x)$, where $\mathcal{E}^a(x)$ is the value of the firm's equity after the implementation of the new discovery. The above assumption holds true if at τ the exogenous demand/price is high enough; say, above a certain threshold x^{o}[5]. The assumption also holds true if x^o is less than x^*, where x^* is the threshold level of price below which the firm mothballs the project. If, however, x^o is greater than x^* and at the date τ the price x is between x^o and x^*, then the firm might want to wait until x hits x^o before investing K to implement the innovative output. This scenario, however, does not present any new insight that is not obtained in the paper, and is not considered here. ASSUMPTION 5 effectively implies that x^* is the lower bound of price x, such that if at the date τ, $x \geq x^*$, then the firm always implements the project. In Lemma 1 below we show that shareholders are better off by implementing the innovative output when at date τ, $x \geq x^*$.

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[5]At x^o,

$$\mathcal{E}^b(x^o) - (\overline{D} - D) = \mathcal{E}^a(x^o) - K,$$

where $\mathcal{E}^b(x)$ the value of the equity in the interim between the successful completion of the R&D project and the implementation of its innovative output. During this period the firm can patent its innovation and retain the right to commercialize at a date when the demand/price x is such that it is above a certain threshold x^o. The term, $\overline{D} - D$, is due to the fact that at x^o, when the shareholders shift to the new, cost-effective means of production, they increase the debt level to $\overline{D}$ and consequently lose a claim of the amount $\overline{D} - D$ on the value of their equity, $\mathcal{E}^b(x)$.

11

Given ASSUMPTION 5 and the fact that shareholders find it optimal to liquidate the firm without waiting to first abandon the R&D project, the control problem can be written as:

$$E(x_t) = \sup_{\mathcal{I}_s, \mathcal{L}_s \in \{0,1\}, s \in (t,T)} \mathbb{E}_t^Q \left\{ \int_t^T e^{-r(s-t)} \left[(1 - \mathcal{L}_s) \left((1 - \zeta_s)(\psi x_s - rD - \mathcal{I}_s a - (1 - \mathcal{I}_s)m) \right. \right. \right.$$
$$\left. \left. \left. + \zeta_s(\pi x_s - c - r\overline{D}) \right) + \mathcal{L}_s[\max\{0, (r\Phi + \phi_1 x_s - rD)\}] \right] ds + e^{-r(T-t)} E(x_T) \right\}.$$
(6)

Thus, the only two decision that the shareholders have to take are, (1) whether to keep the project active at time s or to mothball it and (2) whether to liquidate the firm or not.

The Hamilton-Jacobi-Bellman equation, derived in Appendix A, corresponding to the above problem when $\zeta_s = 0$ and $\mathcal{L}(s) = 0$ is given by

$$\frac{1}{2} x^2 \sigma^2 E_{xx}(x_s) + \mu x E_x(x_s) - rE(x_s) + \psi x_s - rD$$
$$+ \sup_{\mathcal{I}(s) \in \{0,1\}} \mathcal{I}(s) \left\{ \lambda \left((\mathcal{E}^a(x_s) - K) - (E(x_s) - (\overline{D} - D)) \right) - a \right\} + (\mathcal{I}_s - 1)m = 0.$$
(7)

Conditional on $\mathcal{L}(s) = 0$, the value of the control that maximizes the LHS in (6) is either $\mathcal{I}(s) = 1$ or $\mathcal{I}(s) = 0$. Now, when the R&D project is mothballed, it cannot be completed. Therefore, there will be a threshold level, x^*, such that, if x_t is above this threshold at a certain date, then it would be optimal for the firm to activate the R&D project, during which it incurs a fixed cost of a every instant. If x_t is below x^*, then it is optimal for them to mothball the project, which involves spending a fixed amount m every instant.

According to the above, when $\mathcal{I}(s) = 1$, with intensity λ the value of the firm's equity jumps from $E(x_s) - (\overline{D} - D)$ to its completion value $\mathcal{E}^a(x_s) - K$, where $E(x_s)$ is the value of the firm's equity when the R&D project is active. The term $(\overline{D} - D)$ is because at the date $s = \tau$ when the firm successfully completes the R&D project and shareholders implement the new technology, they borrow an additional amount $\overline{D} - D$ to finance K. Consequently, shareholders lose an amount of $\overline{D} - D$ on their claim, $E(x_s)$.

In the region $(x_l < x_t \leq x^*)$ where the owners mothballs the R&D project, the evolution of the equity value is driven entirely by the dynamics of x_t. Define $E^m(x)$ as the value of the firm's equity in the "mothball region" and $E^c(x)$ is the value of the firm's equity in the "continuation region" $(x_t \geq x^*)$, the region where the firm owners choose to spend the extra $a - m$ every instant to keep the project active. Thus, the value of the firm's equity in the various regions implied by the thresholds x_l and x^* can be written as

$$E(x) = \begin{cases} E^c(x) & \text{if } x \geq x^*: \zeta(s) = 0, \mathcal{L}(s) = 0, \mathcal{I}(s) = 1 \\ \\ E^m(x) & \text{if } x_l < x \leq x^*: \zeta(s) = 0, \mathcal{L}(s) = 0, \mathcal{I}(s) = 0 \\ \\ \Phi + \frac{\phi_1 x}{r - \mu} - D & \text{if } x \leq x_l: \zeta(s) = 0, \mathcal{L}(s) = 1, \mathcal{I}(s) = 0, \end{cases}$$

where the last equality follows from the fact that D is default-risk free and shareholders receive the residual value of the firm at closure. The above implies that the value of the firm's equity in the continuation region and the mothball region, respectively, satisfy the following differential equations:

$$\frac{1}{2}x^2\sigma^2 E_{xx}^c(x) + \mu x E_x^c(x) - rE^c(x) + \lambda\left(\mathcal{E}^a(x) - K - (E^c(x) - (\overline{D} - D))\right) +$$
$$\psi x - a - rD = 0 \text{ if } x \geq x^*, \qquad (8)$$

and

$$\frac{1}{2}x^2\sigma^2 E_{xx}^m(x) + \mu x E_x^m(x) - rE^m(x) + \psi x - m - rD = 0 \text{ if } x_l < x \leq x^*. \qquad (9)$$

From Proposition 1 in LM we know that when the level of outstanding debt, $\overline{D}$, after the commercialization of the new discovery is default-risk free,

$$\mathcal{E}^a(x) = \mathcal{E}_s^a(x) = \frac{\pi x}{r - \mu} - \frac{c}{r} - \overline{D} + \left[\varphi_0 + \frac{c}{r} - \frac{(\pi - \varphi_1)x_{LI}}{r - \mu}\right]\left(\frac{x}{x_{LI}}\right)^\theta$$
$$= \frac{\pi x}{r - \mu} - \frac{c}{r} - \overline{D} + Cx^\theta \qquad , \qquad (10)$$

where

$$x_{LI} = \frac{-\theta(\varphi_0 + \frac{c}{r})(r - \mu)}{(1 - \theta)(\pi - \varphi_1)}$$

is the threshold level of exogenous demand at which the shareholders liquidate the firm. When $\overline{D}$ is risky, LM in Proposition 2 show that

$$\mathcal{E}^a = \mathcal{E}_r^a(x) = \frac{\pi x}{r - \mu} - \frac{c}{r} - \overline{D} + \left[\overline{D} + \frac{c}{r} - \frac{\pi x_D}{r - \mu}\right]\left(\frac{x}{x_D}\right)^\theta$$
$$= \frac{\pi x}{r - \mu} - \frac{c}{r} - \overline{D} + C_r x^\theta \qquad , \qquad (11)$$

where

$$x_D = \frac{-\theta(\overline{D} + \frac{c}{r})(r - \mu)}{(1 - \theta)\pi}$$

is the threshold level of exogenous demand at which the owners of the firm default on $\overline{D}$. Assuming that $\overline{D}$ is risk-free, the solution of the differential equation in (8) is given by:

$$E^c(x) = C_{1c}x^\kappa + C_{2c}x^\gamma + Cx^\theta + \frac{\lambda\pi x}{(r - \mu)(\lambda + r - \mu)} - \frac{\lambda c}{r(\lambda + r)} - \frac{\lambda K}{\lambda + r}$$
$$+ \frac{\psi x}{\lambda + r - \mu} - \frac{a}{\lambda + r} - D \qquad , \qquad (12)$$

13

where $\kappa > 1$ and $\gamma < 0$. The solution to the differential equation in (9) is:

$$E^m(x) = C_{1m}x^\beta + C_{2m}x^\theta + \frac{\psi x}{r - \mu} - \frac{m}{r} - D, \tag{13}$$

where $\beta > 1$ and $\theta < 0$[6]. Thus, we have the constants, C_{1c}, C_{2c}, C_{1m}, and C_{2m}, and the two free boundaries, x^* and x_l, which we determine using boundary conditions.

The first boundary condition that we consider is the following:

$$\lim_{x \to \infty} E(x) \propto x. \tag{14}$$

Equation (14) rules out speculative bubbles as $x \to \infty$. The boundary condition in (14) is pertinent to value of equity in the continuation region, $(x_t \geq x^*)$. Since, $\kappa > 1$ for the boundary condition (14) to hold, $C_{1c} = 0$. Since the boundaries x^* and x_l of the mothball region are positive and finite, C_{1m} and C_{2m} are both non zero.

Now we are left five unknowns C_{2c}, C_{1m}, C_{2m}, x^*, and x_l, which can be determine by two value matching, two smooth pasting, and a super contact condition. The value matching conditions at the switching point, x^*, and at the liquidation point, x_l, respectively are:

$$E^m(x^*) = E^c(x^*) \tag{15}$$

$$\Phi + \frac{\phi_1 x_l}{r - \mu} - D = E^m(x_l) \tag{16}$$

The value matching conditions imply continuity of the value function, $E(x)$, at x^* and x_l. The smooth pasting conditions

$$\frac{\partial E^m(x^*)}{\partial x} = \frac{\partial E^c(x^*)}{\partial x} \tag{17}$$

$$\frac{\phi_1}{r - \mu} = \frac{\partial E^m(x_l)}{\partial x}, \tag{18}$$

respectively ensures differentiability of the value function at x^* and x_l respectively. The super contact condition

$$\frac{\partial^2 E^m(x^*)}{\partial x^2} = \frac{\partial^2 E^c(x^*)}{\partial x^2} \tag{19}$$

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[6] In (12) κ and γ are the roots of the characteristic polynomial,

$$\frac{1}{2}\sigma^2\eta^2 + [\mu - \frac{1}{2}\sigma^2]\eta - (r + \lambda) = 0.$$

As it turns out, $\kappa = \frac{-[\mu - \frac{1}{2}\sigma^2] + \{[\mu - \frac{1}{2}\sigma^2]^2 + 2\sigma^2(r+\lambda)\}^{1/2}}{\sigma^2} > 1$, and $\gamma = \frac{-[\mu - \frac{1}{2}\sigma^2] - \{[\mu - \frac{1}{2}\sigma^2]^2 + 2\sigma^2(r+\lambda)\}^{1/2}}{\sigma^2} < 0$. In (13) $\beta > 1$ and $\theta < 0$ are the roots of the characteristic polynomial,

$$\frac{1}{2}\sigma^2\eta^2 + [\mu - \frac{1}{2}\sigma^2]\eta - r = 0,$$

where $\beta = \frac{-[\mu - \frac{1}{2}\sigma^2] + \{[\mu - \frac{1}{2}\sigma^2]^2 + 2\sigma^2 r\}^{1/2}}{\sigma^2} > 1$, and $\theta = \frac{-[\mu - \frac{1}{2}\sigma^2] - \{[\mu - \frac{1}{2}\sigma^2]^2 + 2\sigma^2 r\}^{1/2}}{\sigma^2} < 0$.

is based on the following instantaneous trade-off argument:

$$\lambda(\mathcal{E}^a(x^*) - K) = \lambda(E^c(x^*) - (\overline{D} - D)) + a - m. \tag{20}$$

The right hand side is the increased instantaneous costs of switching to the active state, which comprises of: (i) the increased intensity of losing the R&D investment project and the firm that yields the revenue of ψx every instant (an inevitable consequence of completion), which has a flow value of $\lambda(E^c(x^*) - (\overline{D} - D))$ per unit of time and (ii) the additional cost of $a - m$ per unit of time. The left hand side of (20) is the increased instantaneous benefits from switching from a passive state to an active state. The instantaneous benefit is the increased intensity of completion which has a flow value of $\lambda(\mathcal{E}^a(x^*) - K)$ per unit of time. Substituting the value of $\lambda(\mathcal{E}^a(x^*) - K)$ as implied by (20) in equation (8), we obtain

$$\frac{1}{2}(x^*)^2\sigma^2 E_{xx}^c(x^*) = -\mu x^* E_x^c(x^*) + rE^c(x^*) - m - \psi x^* + rD. \tag{8a}$$

Since, $E^c(x^*) = E^m(x^*)$ (value matching condition) and $E_x^c(x^*) = E_x^m(x^*)$, (smooth pasting condition), the right hand side of equations (8a) and (9) are equal, which gives the super contact condition in (19).

Before proceeding further, we state a lemma that shows that it is profitable for the shareholders to implement the innovative output at the date τ when the firm successfully completes the R&D project. This ensures that ASSUMPTION 5 is not violated.

Lemma 1. $\mathcal{E}^a(x) - K > E^c(x) - (\overline{D} - D)$ *for all* $x \geq x^*$

Proof of Lemma 1. *Given in Appendix B*

The five equations, (15) to (19), determine the five unknowns C_{2c}, C_{1m}, C_{2m}, x^*, and x_l. Since no analytical solution exits for the above set of equations, we solve the system numerically. Table 1 lists the parameter values chosen to solve the above system of equations.

Table 1: Parameter Values

Parameters for		
Price Process	$r = 0.06$	$\mu = 0.03$
	$\sigma = 0.3$	
The firm undertaking	$\psi = 1$	$\phi_1 = 0.70$
the R&D project	$\phi_0 = 200$	
The R&D project	$I = 100$	$m = 0.5$
	$a = 1$	$K = 100$
The firm *after* implementing	$\pi = 1.5$	$c = 0.50$
the innovative output	$\varphi_1 = 0.50$	$\varphi_0 = 250$

The revenue parameter, ψ, of the firm before it implements the new technology is assumed to be $\psi = 1$. We set $\phi_1 = 0.70$, which implies that, given $\psi = 1$, net revenue in

the best alternative use of assets, post liquidation, is 70% of its pre-closure value[7]. For the R&D project, the assumptions regarding initial investment, fixed costs, success intensity, and the capital required to implement the new technology are arbitrary.

The revenue parameter, π, after the implementation of the new technology is assumed to be 1.5; that π is 50% higher than ψ is to suggest that the new technology, operated with an additional cost of $c = 0.5$, is 50% more efficient than the existing technology that yields ψx_t every instant. As stated earlier, the shareholders also have the option to liquidate the firm after the shifting to the new technology. Because of asset specificity of the new technology, the net revenue from the best alternative use of assets that operate the new technology, φ_1, as a fraction of π will be lower than what would have been the case before the new technology was adopted; that is, $\varphi_1/\pi < \phi_1/\psi$. If we assume $\varphi_1 = 0.5$, then φ_1 is about 33.3% of the π.

For the above chosen parameter values and holding λ at 0.5, we solve the nonlinear system of equations, (15) - (19), for different values of f. Likewise, we fix f at 0.05 and solve the system of equations for different values of λ.

In Figure 1(a) we plot x^*, the trigger point at which the firm resumes its R&D investment from the inactive state and x_l, the trigger point at which the firm liquidates from the inactive state for different values of f. We find that both x^* and x_l decrease as f decreases. That is, when R&D investment involves assets that are highly intangible in nature, then shareholders switch early from the mothballed state to the active state, but liquidate late when prices start to fall down.
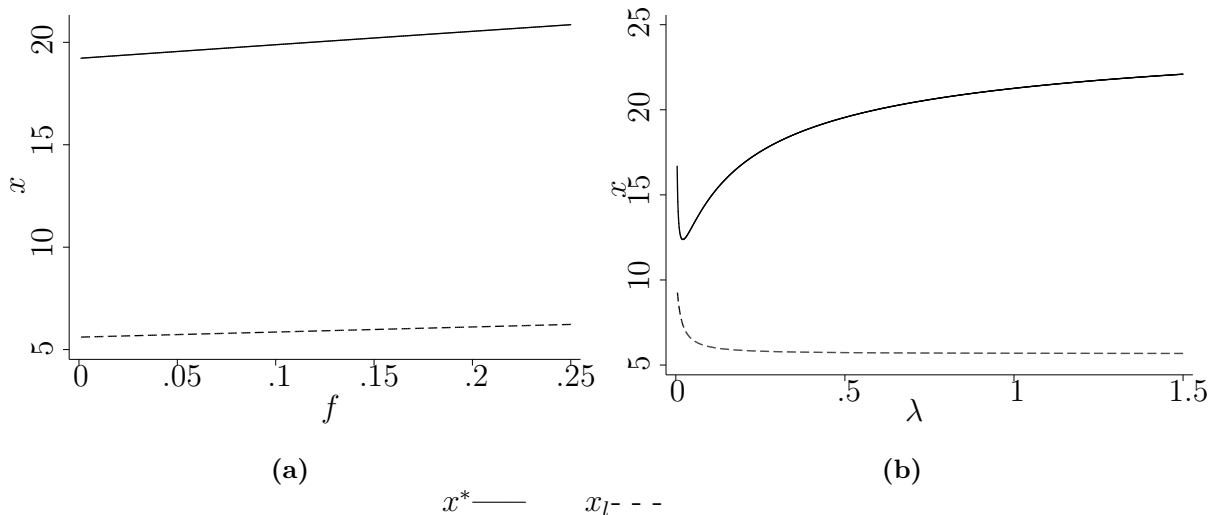

x^*—— x_l- - -

Figure 1. Mothballing/Resuming threshold, x^*, and Liquidation threshold, x_l.

In notational terms, the result suggests that $x^*(f') > x^*(\hat{f})$ when $f' > \hat{f}$. Now, suppose

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[7]LM assume net revenue in the best alternative use of assets, post liquidation, to be 70% of the pre-closure value. The values of r, σ, and μ too have been borrowed from LM.

the current level of exogenous demand, x, is such that $x^*(\hat{f}) < x < x^*(f')$. At x, it is clear that the firm with an R&D project involving higher f has mothballed the R&D project. This implies that the shareholders of firm with an R&D project with higher f wait longer to resume investment from the state of inaction. This is because, if starting from x, the demand/price does start falling to the point that the owners eventually have to liquidate the firm, then in the event of liquidation, the firm with higher f is able to retain a higher fraction of the initial investment, I. This gives an advantage to the firm with higher f, thereby making it possible for its owners to wait longer before resuming with the R&D project. By waiting longer shareholders ensure a greater chance for $E^c(x_\tau)$ to be high at the completion date τ. Also, by waiting longer the firm avoids paying the additional fixed cost, $a - m$.

The result also suggests that for $f' > \hat{f}$, $x_l(f') > x_l(\hat{f})$. This is because, having already invested I to start the R&D project, if in the event of liquidation the fraction, f, of the initial investment that can be recouped is small, then the shareholders of the firm in question would like to wait longer before they liquidate the firm, hoping that x might eventually rise. Thus, as a result of both lower f and a lower x_l, the maximum amount of default-risk free debt, D^*, the firm can borrow is lower.

In Figure 1(b) we plot x^* and x_l against the success intensity λ. We find that when the expected time for successfully completing the R&D project, $\mathrm{E}(\tau) = \lambda^{-1}$, becomes infinitely large, $x^*(\lambda)$ too tends towards infinity. In other words, when success intensity is very low, shareholders mothball the project and do not resume until the demand x for the firm's produce becomes extremely high. The relationship, however, is not monotone: $x^*(\lambda)$ first decreases rapidly and then increases with increasing λ. This is because as the expected time to completion becomes shorter, given that τ is exponentially distributed, the uncertainty, $\mathrm{Var}(\tau) = \lambda^{-2}$, regarding the completion of the project too decreases. This, again, makes it possible for the shareholders to wait longer before resuming the mothballed project, which ensures a greater chance for the value of equity after implementation to be high.

As far as $x_l(\lambda)$ is concerned, we find that when the expected time to completion becomes large, the shareholders find themselves too eager to liquidate the firm. As λ increases from a small value, $x_l(\lambda)$ first decreases rapidly then at a decreasing rate, which suggests that as the expected time to completion becomes small and the firm more certain of the outcome, shareholders wait longer before liquidating, thereby giving the firm a greater chance to resume innovative activity.

3.2. Starting the R&D Project: The Irreversible Investment Problem

Having obtained the value of firm's equity and the optimal liquidation and the switching threshold after having embarked on the R&D project, we can now solve backward to obtain the value of equity before staring the R&D project and along with it the optimal investment and liquidation thresholds, x_I and x_L respectively. Now, shareholders are not going to start the R&D project if after investing I they have to mothball the project. The gain, therefore, from investing I is $E^c(x) - I$, where $E^c(x)$ is the value of the firm's equity in the continuation region of the switching problem under the assumption that debt D is

default-risk free. Hence, at the time of the starting of the R&D project, the value of the equity is given by the following optimization problem:

$$E(x_t) = \max\left\{ E^c(x_t) - I, \max_{\mathcal{L}_s \in \{0,1\}, s \in (t, t+dt)} \mathbb{E}_t^Q \left\{ \int_t^{t+dt} e^{-r(s-t)} \Big[(1 - \mathcal{L}_s)(\psi x_s - r\underline{D}) \right. \right.$$
$$\left. \left. + \mathcal{L}_s \max\{0, (r\phi_0 + \phi_1 x_s - r\underline{D})\} \Big] ds + e^{-rdt} E(x_{t+dt}) \right\} \right\}.$$
(21)

According to the above, shareholders value the decision to continue without investing for a small period of time, dt, against $E^c(x) - I$. Continuing without investing, they receives a revenue of ψx_t every instant from the firm's existing operation and also maintains the option to liquidate firm. After the elapse of that time interval dt, the shareholders again review the situation for another small period of time.

Define $E^d(x)$ as the value of the firm's equity when shareholders are deciding whether to embark upon the R&D project or to liquidate the firm. Now, when it is neither optimal to liquidate the firm nor start the R&D project, then equation (21) becomes

$$E^d(x_t) = \mathbb{E}_t^Q \left\{ \int_t^{t+dt} e^{-r(s-t)} \Big[(\psi x_s - r\underline{D}) \Big] ds + e^{-rdt} E^d(x_{t+dt}) \right\}. \tag{21a}$$

Also, $E^d(x)$ is the value of firm's equity when x is such that it lies between x_L and x_I. Thus, we have

$$E(x) = \begin{cases} E^c(x) - I & \text{if } x \geq x_I;\ \mathcal{L}_s = 0 \\[2mm] E^d(x) & \text{if } x_L < x < x_I;\ \mathcal{L}_s = 0 \\[2mm] \phi_0 + \frac{\phi_1 x}{r-\mu} - \underline{D} & \text{if } x \leq x_L;\ \mathcal{L}_s = 1, \end{cases}$$

where $\mathcal{L}(s)$ is the indicator variable that takes value 1 if the owners decide to liquidate the firm.

Since we solved for $E^c(x)$ in the last subsection, we are left to solve for the value of the firm's equity in the decision region, $E^d(x)$. By employing Itô's lemma we get

$$e^{-rdt} E^d(x_T) = E^d(x_t) + \int_t^{t+dt} e^{-r(s-t)} \Big[\mathcal{D}E^d(x_s) - rE^d(x_s) \Big] ds + \int_t^{t+dt} e^{-r(s-t)} \Big[E_x^d(x_s)\sigma dw_s \Big],$$

where

$$\mathcal{D}E^d(x) = \frac{1}{2}x^2\sigma^2 E_{xx}^d(x) + \mu x E_x(x).$$

Taking expectations with respect to the risk neutral measure we obtain

$$E^d(x_t) = \mathbb{E}_t^Q \left\{ e^{-rdt} E^d(x_T) - \int_t^{t+dt} e^{-r(s-t)} \Big[\mathcal{D}E^d(x_s) - rE^d(x_s) \Big] ds \right\}. \tag{22}$$

Comparing equation (21a) and equation (22) we obtain the differential equation governing the value of equity in the decision region:

$$\frac{1}{2}x^2\sigma^2 E_{xx}^d(x) + \mu x E_x^d(x) - rE^d(x) + \psi x - r\underline{D} = 0, \ x_L \le x \le x_I.$$ (23)

We know that the solution to the above differential equation is given by

$$E^d(x) = C_{1d}x^\beta + C_{2d}x^\theta + \frac{\psi x}{r - \mu} - \underline{D}$$

where $\beta > 1$ and $\theta < 0$ are the roots of the characteristic polynomial, $\frac{1}{2}\sigma^2\eta^2 + [\mu - \frac{1}{2}\sigma^2]\eta - r = 0$.

Now we have four unknowns, C_{1d}, C_{2d}, x_I, and x_L, which we can determine by the boundary conditions: the value matching and smooth pasting conditions. Assuming that level of debt, D, is risk-free after investing I, the value matching conditions,

$$E^d(x_I) - (D - \underline{D}) = E^c(x_I) - I \text{ and}$$ (24)

$$E^d(x_L) = \phi_0 + \frac{\phi_1 x_L}{r - \mu} - \underline{D},$$ (25)

reflect the fact that $E(x)$ is continuous at the investment trigger point, x_I, and at the liquidation point, x_L respectively. The additional term, $D - \underline{D}$, on the LHS of (24) is due to the fact that at x_I when the shareholders embark on the project, they increase the debt level to D, and consequently they lose a claim of the amount $D - \underline{D}$ on the value of the firm. The smooth pasting conditions,

$$\frac{\partial E^d(x_I)}{\partial x} = \frac{\partial E^c(x_I)}{\partial x}$$ (26)

$$\frac{\partial E^d(x_L)}{\partial x} = \frac{\phi_1}{r - \mu},$$ (27)

reflect the fact that the value function is differentiable at x_I and x_L.

Since no analytical solution exits for the above set of nonlinear equations, (24) to (27), we solve them numerically for the parameter values in Table 1 and for different values of f and λ. In Figure 2(a) we plot x_I and x_L for different values of f and $\lambda = 0.5$, while in Figure 2(b) x_I and x_L have been plotted against λ, where f has been fixed at 0.05.

We find that both x_I and x_L decrease with f. This is because if, after investing I, x eventually does fall so that the firm has to be liquidated, then the loss to the shareholders of firm with higher f is less than the loss to the owners of the firm with lower f; that is, $(1-f')I < (1-\hat{f})I$, where $f' > \hat{f}$. Knowing this, the firm with a higher f would rather start the R&D project early than late. As far as x_L is concerned, we find that $x_L(f') < x_L(\hat{f})$ for $f' > \hat{f}$. This is because, ceteris paribus, stakes are low after investment for firms with R&D project that involve tangible assets. Therefore, the firm with R&D project with higher f would be willing to wait more before eventually liquidating the firm hoping that
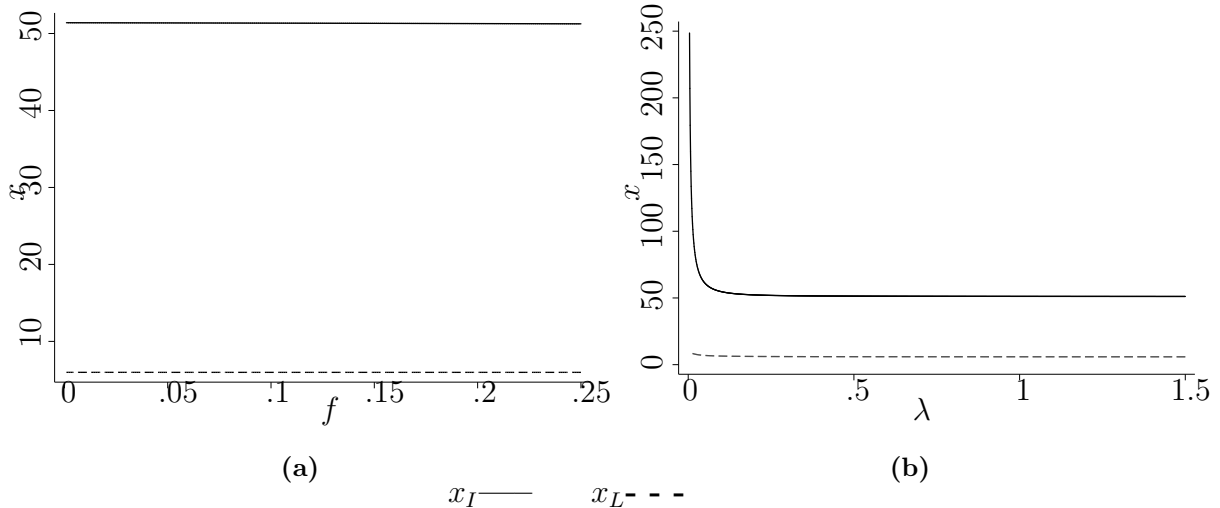
Figure 2. Starting the R&D project threshold, x_I, and liquidation threshold, x_{LI}, before starting the project.

x might rise in the near future to the point where it can start the R&D project. However, the decrease in x_L with increasing f is negligible, which suggests that the incentive to wait longer before embarking on the R&D project when f is high is minimal.

Comparing Figure 1(a) and Figure 2(a), we find that the investment threshold x_I is higher than the mothballing/resuming threshold x^* for all of f. This is because starting the R&D project involves sinking an initial amount I. Hence, shareholders would begin the R&D program only when they are sure that the demand for their firm's produce is sufficiently high.

As far as $x_I(\lambda)$ is concerned, Figure 2(b) suggests that when λ is small, the threshold demand $x_I(\lambda)$ at which shareholders think fit to invest I to start the R&D project is almost impossibly high. This is because when λ is small, the expected time to completion and the uncertainty regarding successful innovation are high. Consequently, only when the threshold level of demand for the firm's produce is high will the owners be willing to take the high risk of sinking I. As the expected time to completion and the associated uncertainty reduce, we find that the owners embark early on the R&D project.

The liquidation threshold $x_L(\lambda)$ prior to starting the R&D project is found to be similar to that of $x_l(\lambda)$; that is, as the expected time to completion becomes small and the firm more certain of the outcome, shareholders wait longer before liquidating, which gives the firm a greater chance to start the R&D. However, we find that $x_l(\lambda) < x_L(\lambda)$ for lower values of f and that there is lesser variation in $x_L(\lambda)$ as compared to $x_l(\lambda)$. The reason why the owners wait longer in deciding to liquidate the firm when the R&D project is operational, as compared to liquidation decision prior to starting of the project, is because, when f is small, in the event of liquidation after the firm has started the project, a large fraction of I too is lost.

Now, since the liquidation threshold, $x_l(f)$, after sinking the initial amount I varies

more with f as compared to $x_L(\mathrm{f})$, the implication is that the maximum level of default-risk free debt that a firm can afford, before and after starting the R&D project differ. This is illustrated in Figure 3(a).
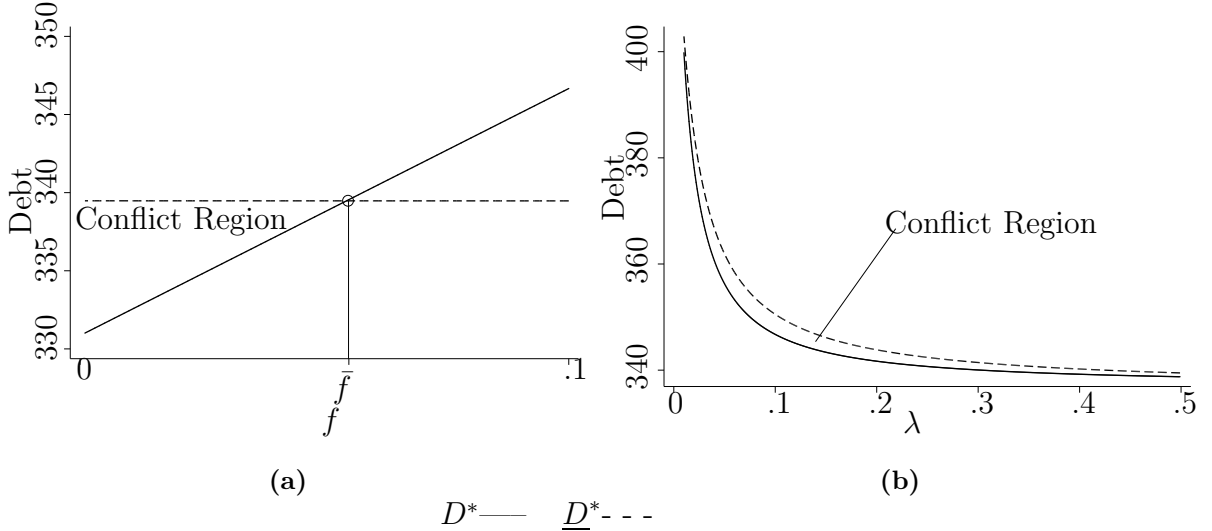

D^*—— $\underline{D}^*$- - -

Figure 3. Maximum level of safe debt after starting the R&D project, D^*, and before starting the project, $\underline{D}^*$, as a function of f and λ

The result suggests that there exits a "Conflict Region" defined by $\bar{f}$, such that $D^*(f) \leq \underline{D}^*(f)$ when $f \leq \bar{f}$. In other words, even when no additional debt is incurred for starting the project, when the investment needed to start the project involves highly intangible assets, then the debt levels that are deemed safe before starting the project, becomes risky upon embarking on the R&D project. What drives this result is the fact that when assets involved in the conduct of R&D are highly intangible, shareholders *after* the starting the R&D project, if they have to liquidate the firm, liquidate late; while *before* embarking on the R&D project, when they liquidate, they do it early.

As we will see in the next section, if the outstanding debt level, D, when the R&D project is operational is higher than what is deemed safe, then the shareholders start the R&D project early; that is, they overinvest[8]. This also implies that even if no additional debt is incurred and if $D = \underline{D}$ lies in the "Conflict Region", where $f < \bar{f}$, then shareholders by overinvesting in R&D turn existing level of safe debt into a risky one. Similar to the risk-shifting problem of Jensen and Meckling (1976), this occurs because, given limited liability, shareholders can transfer the risk of prematurely starting the R&D project to the bondholders while preserving the upside potential.

For given $f = 0.05$, we plot $D^*(\lambda)$ and $\underline{D}^*(\lambda)$ in Figure 3(b). The plot shows that $D^*(\lambda) < \underline{D}^*(\lambda)$ for all values of λ and the difference between the two becomes larger as λ

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[8]See Leland (1998) and Mauer and Sarkar (2004) on why early exercise decision can be characterized as overinvestment.

approaches 0. This is seen more clearly in Figure 4, where we plot $D^*(f)$ and $\underline{D}^*(f)$ for λ = 1 and $\lambda = 0.1$.
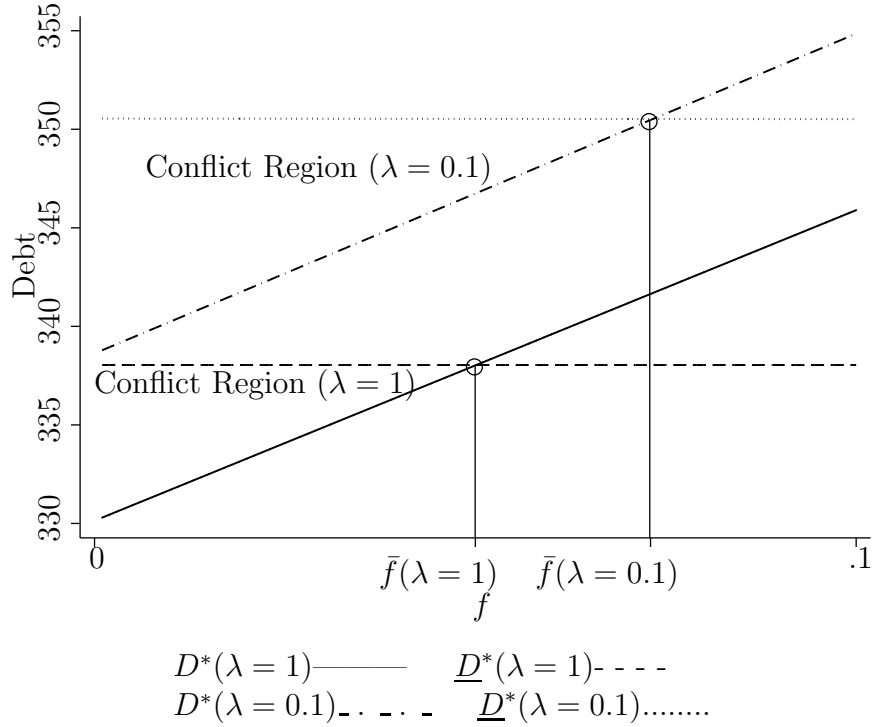

Figure 4. D^* and $\underline{D}^*$ for $\lambda = 1$ and $\lambda = 0.1$

We find that as the expected time to completion, $E(\tau) = 1/\lambda$, increases, the "Conflict Region" becomes larger and $\bar{f}$, the critical level of f, below which $D^* < \underline{D}^*$, also increases. Moreover, the maximum amount of safe debt prior to starting the R&D project, $\underline{D}^*$, increases with decreasing λ for all f. This is because if the firm knows that the expected time to completion is large, then shareholders, prior to starting the R&D program, will be all too eager to liquidate the firm early when x drops. With declining λ, the same is true of x_l. However, the overall effect of declining λ is a larger difference between $\underline{D}^*$ and D^* for each value of f below $\bar{f}$. This suggests that as the expected time for successful completion of the R&D program increases, the scope for conflict between bondholders and shareholders increases.

4. Firm Valuation and Investment and Default Strategy when Innovative Activity is Financed with Risky Debt

Until now we have looked at the shareholders optimal investment and liquidation strategy under the assumption that the level of outstanding debts D and $\overline{D}$ are such that the firm does not default on its debt. In this section we study the firm's investment and default strategy when additional debt of the amount $\overline{D} - D$ committed by the bondholders

to finance implementation and the debt of the amount $D - \underline{D}$ to finance R&D investment are large, so that the level of outstanding debt $\overline{D}$ after the implementation and the debt level D during the conduct of R&D activity exceeds the value of the firm in liquidation states.

4.1. Outcomes when Implementation is Financed with Risky Debt

When the outstanding debt levels $\underline{D}$ and D are risk-free and when additional debt of the amount $\overline{D} - D$ agreed upon to finance K is such that the outstanding amount $\overline{D}$ after the implementation becomes default prone, then shareholders find a strong incentive to invest early or aggressively. This can be seen in Figure 5(a), where the mothballing/resuming threshold, $x^*(\mathcal{E}_r^a)$, when $\overline{D}$ *is risky* is lower than mothballing/resuming threshold, $x^*(\mathcal{E}_s^a)$, when $\overline{D}$ *is safe* for all values of f[9]. In other words, when implementation of the new technology is financed with risky debt, shareholders resume with the R&D project too early from the mothballed state as compared to when debt level is risk-free. As can be evinced from Figure 5(c), the same is true of the investment threshold x_I, where we find $x_I(\mathcal{E}_r^a)$ to be lower than $x_I(\mathcal{E}_s^a)$.

This happens because shareholders, maximizing their claim, are unconcerned about the welfare of bondholders and, therefore, are indifferent to the increased risk of default and bankruptcy cost resulting from their decisions to invest early. The early exercise decisions makes it possible for the shareholders to successfully complete the R&D project at an earlier date, which when commercialized, generates higher revenue at an earlier date. While shareholders potentially gain from early investment decision, they also transfer the risk of early exercise decision on to the bondholder.

Besides, as can be seen from Figure 5(b) and 5(d), shareholders also liquidate the firm late, both, before starting the R&D venture and when the project is operational. Choosing to liquidate at a later time is also symptomatic of overinvestment. By waiting longer to liquidate, shareholders allow the firm a greater chance to start the R&D project as well as a greater chance to resume with the project from the mothballed state. However, in doing so shareholders also increase the probability of default on outstanding debt levels, D and $\underline{D}$. This happens because at the lower liquidation threshold, the value of the firm in liquidation state become lower than certain levels of debt that were held to be default-risk free.

As a result of late liquidations the "Conflict Region", as shown in Figure 6, enlarges. This results from the fact that when implementation is financed with risky debt, shareholders wait much longer to liquidate the firm when the R&D project is operational than when it is yet to embark on the R&D project. In other words, they give the firm a greater opportunity to complete the R&D project once the project has been started than what they give for starting the project. The enlargement of the "Conflict Region" suggests that when implementation is financed with risky debt, the scope for conflict between shareholders and the bondholders over R&D investment increases.

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[9]We set the risky debt level $\overline{D}$ at 365. Given the parameter values in Table 1, we find that the maximum amount of risk-free debt is $\overline{D}^* = 314.58$.
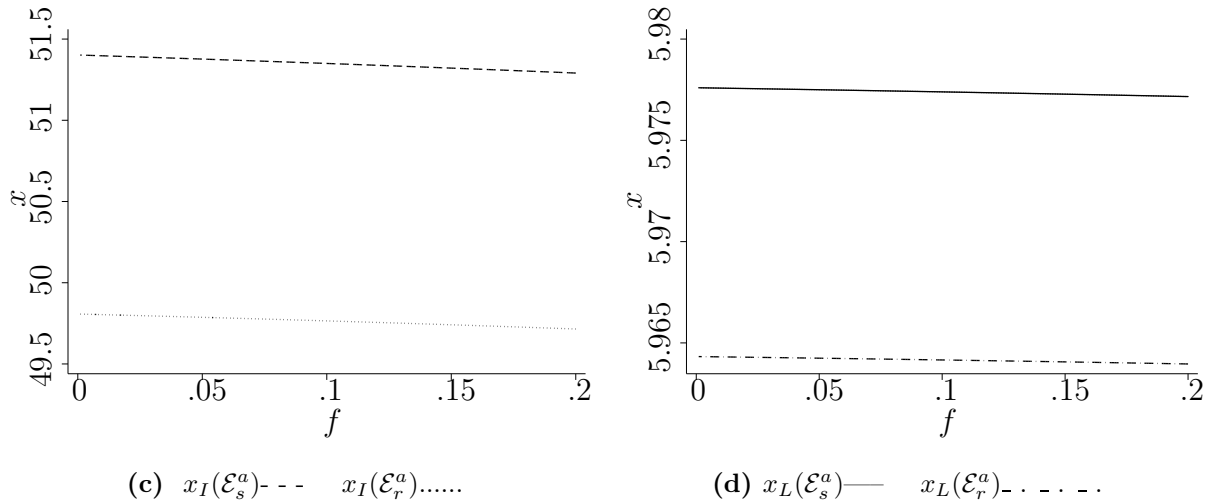
Outcomes for the Switching Problem

(a) $x^*(\mathcal{E}_s^a)$- - - $x^*(\mathcal{E}_r^a)$......

(b) $x_l(\mathcal{E}_s^a)$—— $x_l(\mathcal{E}_r^a)$- . - - - .
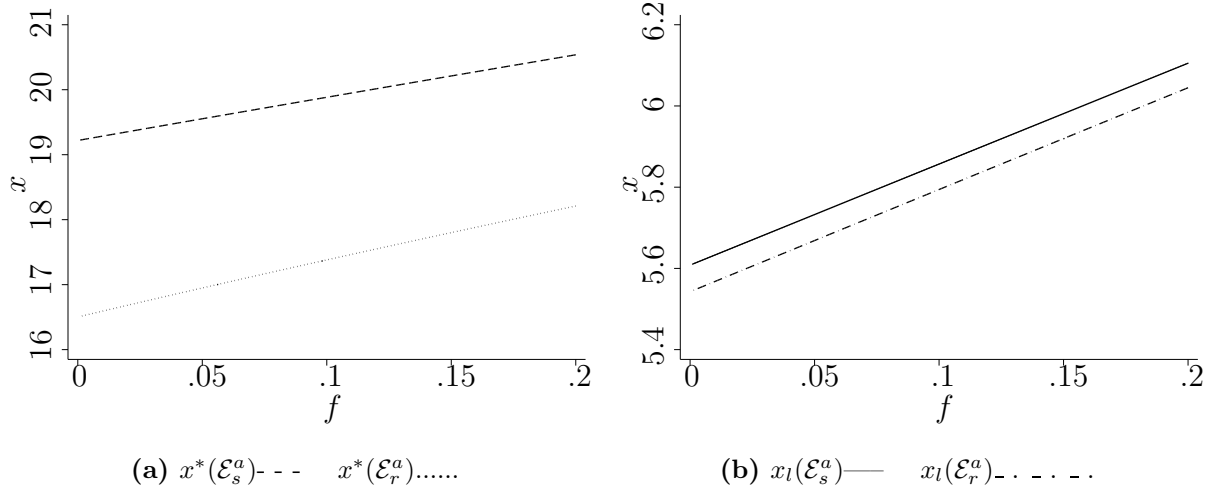
Outcomes for the Starting Problem

(c) $x_I(\mathcal{E}_s^a)$- - - $x_I(\mathcal{E}_r^a)$......

(d) $x_L(\mathcal{E}_s^a)$—— $x_L(\mathcal{E}_r^a)$- . - - - .

Figure 5. Investment and Liquidation Thresholds when Implementation is Financed with Risky Debt

Here we would like to note that for the parameter value in Table 1, we find that $\overline{D}^* = 314.58$, which shows that $D^*(f, \lambda) > \overline{D}^*$ and that $\underline{D}^*(f, \lambda) > \overline{D}^*$ for almost all values of f and λ. That is, the maximum level of risk-free debt that the firm can sustain when it is engaged with the R&D project and before the start of the project are both higher than the maximum level of risk-free debt that it can sustain after implementing of the innovative output from the successfully completed R&D project.

We get this result because after the implementation, when the firm shifts to a new, more efficient mode of production, assets of the firm become more specific to the firm. Hence, in the event of closure the net revenue in the best alternative use of assets, $\varphi_1 x$, is much lower than what would have been the case before the new technology was adopted;
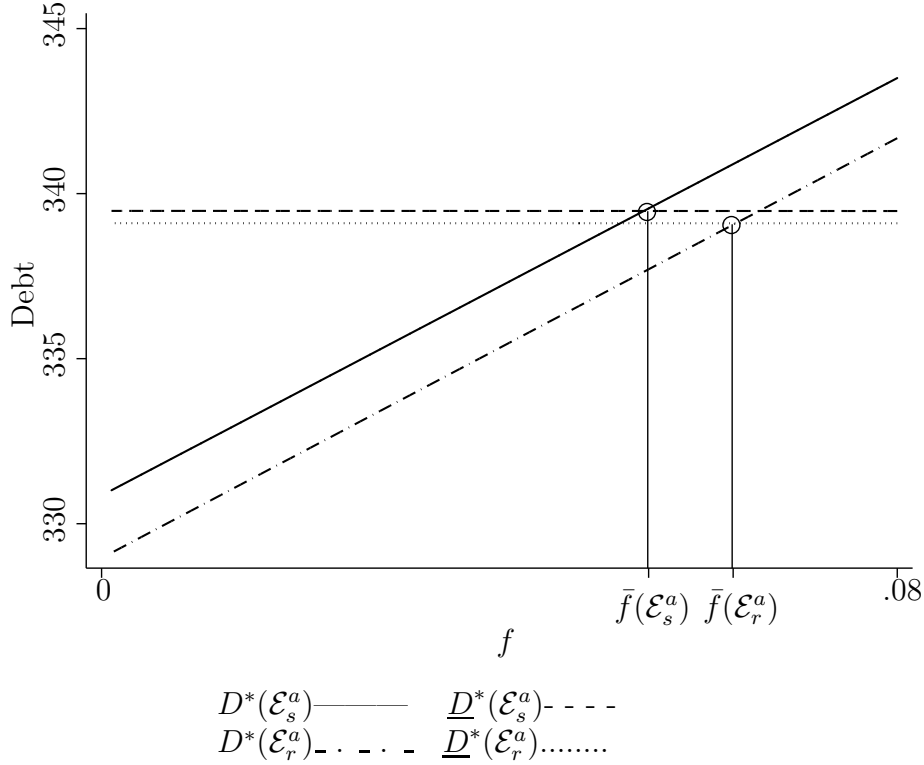
Figure 6. D^* and $\underline{D}^*$ for $\mathcal{E}_s^a$ and $\mathcal{E}_r^a$

that is, $\varphi_1 < \phi_1$. The implication of this is that after the implementation, if x starts to fall, then shareholders wait longer before eventually liquidating the firm. This, in turn, implies that the maximum amount of risk-free debt, $\overline{D}^*$, that the firm can sustain after the implementation is lower than amount of risk-free debt it could sustain prior to the start of the R&D venture.

Thus, it is possible that even if no extra debt of the amount $\overline{D} - D$ for implementation were committed and if debt level D were such that $D^*(f, \lambda) \geq D > \overline{D}^*$, shareholders by investing early turn existing level of risk-free debt level, D, to a default prone one after the implementation[10]. Hence, we see that specificity of assets, which lowers the liquidation value of the firm post implementation, constitutes a risk factor that makes an erstwhile risk-free debt level, D, default prone after the implementation. Thus, asset specificity, which lowers the value of existing debt, with the concomitant result that the "Conflict Region" enlarges, also increases the scope for agency conflict over R&D investment.

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[10]While it could be possible for the firm to raise equity and retire some of its debt at the time of implementation, this paper in its scope does not consider any such renegotiation of contractual arrangements; see ASSUMPTION 3.

4.2. Outcomes when R&D is Financed with Risky Debt

We first solve for investment and default threshold for the switching problem when debt in place, D, is risky. Given the outcomes for the switching problem and equity value with risky debt in place, we can work backward to obtain the investment threshold $\hat{x}_I$ and the liquidation threshold $\hat{x}_L$ when investment I is financed with debt of the amount $D - \underline{D}$.

4.2.1. Outcomes for the Switching Problem with Risky Debt in Place

When D is risky, the value of the firm at closure is less than the D; that is, $\Phi + \frac{\phi_1 x_d}{r-\mu} < D$, where x_d is the trigger point, such that if x_t falls below x_d, the firm defaults and bankruptcy is declared. Since during bankruptcy the seniority of claim of the creditors is respected, the creditors in the event of default receive

$$(1 - \delta)\left[\Phi + \frac{\phi_1 x_d}{r - \mu}\right], \tag{28}$$

where, δ, $(0 \leq \delta \leq 1)$, is the fraction of the firm's value that is destroyed when the firm defaults on its debt at closure. Being protected by limited liability, the shareholder's claim in the event of default at closure is given by $E(x) = 0$.

Analogous to the optimization problem in (6), the shareholders optimization problem when R&D is financed with risky debt can be written as:

$$\widetilde{E}(x_t) = \sup_{\mathcal{I}_s, \mathcal{D}_s \in \{0,1\}, s \in (t,T)} \mathbb{E}_t^Q \Bigg\{ \int_t^T e^{-r(s-t)} \Big[(1 - \mathfrak{D}_s)\Big((1 - \zeta_s)(\psi x_s - rD - \mathcal{I}_s a - (1 - \mathcal{I}_s)m)$$

$$+ \zeta_s(\pi x_s - c - r\overline{D}) \Big) + \mathfrak{D}_s(0) \Big] ds + e^{-r(T-t)} \widetilde{E}(x_T) \Bigg\}, \tag{29}$$

where, for notational convenience, $\widetilde{E}(x_t)$ denotes the value of equity when the firm has risky debt in place. In the above optimization problem, the two decision that the owners of the firm have to take are (a) whether to keep the R&D project active by investing a ($\mathcal{I}_s = 1$) or to suspend operation ($\mathcal{I}_s = 0$), and (b) whether to default on the firm's obligation ($\mathfrak{D}_s = 1$) or to continue ($\mathfrak{D}_s = 0$).

Conditional on $\mathfrak{D}_s = 0$, the Hamilton-Jacobi-Bellman equation corresponding to the above problem is given by:

$$\frac{1}{2}x^2\sigma^2\widetilde{E}_{xx}(x_s) + \mu x\widetilde{E}_x(x_s) - r\widetilde{E}(x_s) + \psi x_s - rD +$$

$$\sup_{\mathcal{I}(s)\in\{0,1\}} \left\{ \mathcal{I}(s)\left[\lambda\Big((\mathcal{E}^a(x_s) - K) - (\widetilde{E}(x_s) - (\overline{D} - D)) \Big) - a \right] + (\mathcal{I}_s - 1)m \right\} = 0, \tag{30}$$

where $\mathcal{E}^a(x_s) - K$ is the value of the firm's equity after the implementation of the innovative output minus the fixed cost of K required for implementation. As in the case when D was default-risk free, the above reflects that with intensity λ the value of the firm's equity jumps from $\widetilde{E}(x_s) - (\overline{D} - D)$ to its completion value $\mathcal{E}^a(x_s) - K$.

Now, given our assumption that debt policy of the firm is static, which does not allow reduction of debt level though the life of the firm, it is necessary that we ensure that in accordance to our assumption, debt levels are consistent across the stages in the life of the firm. To ensure consistency and for the sake of exposition, we make some assumptions that are without any loss of generality.

- To begin with, we assume that the level of outstanding debt, $\underline{D}$, before the start of the R&D activity is risk-free, and that f and λ are such that $\underline{D}$ lies in the "Conflict Region".

- Secondly, of the investment I needed to start the R&D project, a fraction F_I is financed with additional debt. So that the level of outstanding debt when the firm is engaged in R&D is $D = \underline{D} + F_I I$; when $F_I = 0$, $D = \underline{D} = D_0$, and when $F_I = 1$, $D = \underline{D} + I = D_1$.

- Thirdly, at the time of the implementation of the new discovery, the additional debt of the amount $\overline{D} - D$ is used to finance that a fraction F_K of the investment K needed for implementation. Hence, the level of outstanding debt after the implementation is $\overline{D} = \underline{D} + F_I I + F_K K$.

We have seen that when $\underline{D}$, the level of outstanding debt before the commencement of the R&D activity, is in the "Conflict Region", then it is greater then the maximum amount of risk-free debt, D^*, the firm could sustain when it is engaged in conducting R&D. Therefore, any level of D, where $D = \underline{D} + F_I I$, is risk prone.

Also, in the last subsection we saw that $D^*(f, \lambda) > \overline{D}^*$ for almost all values of f and λ. Hence, when $\underline{D}$ is in the "Conflict Region", then

$$\underline{D} > D^*(f, \lambda) > \overline{D}^*, \text{ which implies that}$$
$$\overline{D} = \underline{D} + F_I I + F_K K > \overline{D}^*.$$

In other words, when $\underline{D}$ is in the "Conflict Region", then every level of debt, $\overline{D}$, after the implementation will be risky. Hence, the value of equity, $\mathcal{E}^a(x_s)$, post implementation, in (30) is given by $\mathcal{E}^a(x_s) = \mathcal{E}^a_r(x_s)$, where $\overline{D}$ in the expression of $\mathcal{E}^a_r(x_s)$ in equation (11) is equal to $\underline{D} + F_I I + F_K K$.

Let $\tilde{x}^*$ be the switching threshold level of price such that if x_t rises above this level, then the firm becomes actively involved in the R&D project and optimally mothballs the project if x_t falls below it. Thus, as in the case of safe debt, we have the continuation region defined by $[\tilde{x}^*, \infty)$ and the mothball region defined by $(x_d, \tilde{x}^*)$, where x_d is threshold level of price at which the firm defaults.

The solution to the HJB equation when $\mathfrak{D}_s = 0$ and $\mathcal{I}_s = 1$ is given by

$$\widetilde{E}^c(x) = \widetilde{C}_{1c} x^\kappa + \widetilde{C}_{2c} x^\gamma + C_r x^\theta + \frac{\lambda \pi x}{(r-\mu)(\lambda + r - \mu)} - \frac{\lambda c}{r(\lambda + r)} - \frac{\lambda K}{\lambda + r}$$
$$+ \frac{\psi x}{\lambda + r - \mu} - \frac{a}{\lambda + r} - D \qquad , \quad (31)$$

27

where $\widetilde{E}^c(x)$ the value of the firm's equity in the continuation region. The boundary condition in equation (3.10) implies that $\tilde{C}_{1c} = 0$. When x_t lies in the mothball region, that is when $\mathcal{I}(s) = 0$ optimally, the solution to the HJB equation is given by

$$\widetilde{E}^m(x) = \tilde{C}_{1m}x^\beta + \tilde{C}_{2m}x^\theta + \frac{\psi x}{r - \mu} - D - \frac{m}{r}, \tag{32}$$

where the constants $\tilde{C}_{1m}$, and $\tilde{C}_{2m}$ are to be determined.

Thus, we are left with five unknowns, $\tilde{C}_{2c}$, $\tilde{C}_{1m}$, and $\tilde{C}_{2m}$ and the two threshold levels of exogenous demand or the free boundaries x_d and $\tilde{x}^*$. These are determined by the boundary conditions at the free boundaries x_d and $\tilde{x}^*$.

The boundary conditions – the value matching, smooth pasting, and the super contact conditions, which we have discussed earlier – at $\tilde{x}^*$ respectively are:

$$\widetilde{E}^c(\tilde{x}^*) = \widetilde{E}^m(\tilde{x}^*), \tag{33}$$

$$\frac{\partial \widetilde{E}^c(\tilde{x}^*)}{\partial x} = \frac{\partial \widetilde{E}^m(\tilde{x}^*)}{\partial x}, \tag{34}$$

and

$$\frac{\partial^2 \widetilde{E}^c(\tilde{x}^*)}{\partial x^2} = \frac{\partial^2 \widetilde{E}^m(\tilde{x}^*)}{\partial x^2}. \tag{35}$$

The standard boundary conditions, the value matching and smooth pasting conditions, respectively, at x_d are:

$$\widetilde{E}^m(x_d) = 0 \tag{36}$$

and

$$\frac{\partial \widetilde{E}^m(x_d)}{\partial x} = 0. \tag{37}$$

We solve the set of equation, (33)–(37), numerically to obtain the mothballing/resuming threshold $\tilde{x}^*$ and the default threshold x_d as a function of risky debt, $D = \underline{D} + F_I I$. For $\underline{D}$ to be in the "Conflict Region", we fix $\underline{D}$ at D^*, where D^* is computed at $\tilde{f} = 0.05$ and $\tilde{\lambda} = 0.5$, while the other parameters are the same as in Table 1. Also, we set $F_K = 0.25$; that is, the extra loan of the amount $\overline{D} - D$ finances a quarter of investment K need for implementation. Given that $K = 100$, $\overline{D} - D = 25$, so that the level of outstanding debt after the implementation is $\overline{D} = D + 25$.

In Figure 7, the dash-dotted and the dotted lines respectively are the motballing/resuming threshold and the default threshold when debt is risky; that is, when $D > D^*$. The solid and the dashed lines respectively are the motballing/resuming threshold and the liquidation threshold when the debt level D is risk-free.
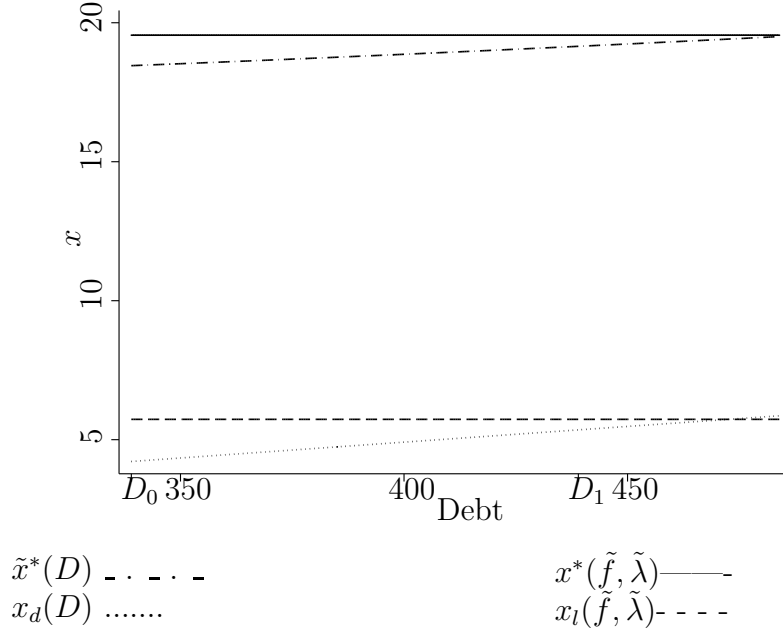
Figure 7. Mothballing/resuming and default threshold with risky debt in place as compared to mothballing/resuming and liquidation threshold when outstanding debt is risk-free.

For parameter values in Table 1 we find that $D^*(\tilde{f} = 0.05, \tilde{\lambda} = 0.5) = 338.76$. As we increase D above this level, we find that, first, $\tilde{x}^*(D)$ shifts below x^* from where it increases as we increase D. In other words, what we find is that when the burden of risky debt is small, shareholders resume innovative activity from the mothballed state too early. However, as the burden of servicing debt increases, shareholders resume the innovative activity late. We get this outcome because when the level of risky debt is small and if the next stage implementation is financed with risky debt, shareholders, unconcerned about the welfare of bondholders, invest early. The early exercise decisions with some uncertainty makes it possible for the shareholders to complete the R&D project and implement the output at an earlier date. While shareholders potentially gain from early investment decision, they also transfer the risk of early exercise decision on to the bondholder.

This, however, does not contradict Myers (1977)'s debt overhang result, which predicts that with risky debt in place shareholders underivest. This can be evinced from the fact that as we increase D, shareholders resume the R&D activity late as compared to when the level of risky debt was small.

We find a similar result for the default threshold x_d, where x_d, while lower than the liquidation threshold x_l, increases with the level of outstanding debt D. Also, there exists a risky debt level $\tilde{D}$, such that for $D < \tilde{D}$, $x_d < x_l$ and for $D \geq \tilde{D}$, $x_d \geq x_l$[11]. This

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[11]When $\phi_1 = 0$, $x_d > x_l$ for all values of $D > D^* = \phi_0 + \tilde{f}I$.

result essentially follows from the fact that when the outstanding debt is risky, in the event of default shareholders receive nothing and do not internalize the deadweight cost of bankruptcy; consequently, the firm ignores the net revenue of the firm as a going concern in liquidation states. That is, with risky debt in place the firm's optimal default threshold is determined by the burden of debt service. Thus, when the burden of debt services are low, the firm defaults late, and when they are high, the firm defaults early. Due to the presence of high level of debt when firm defaults early, they also risk forgoing valuable investment opportunity; that is, they underinvest à la Myers (1977).

To conclude, what we find is that when outstanding debt, D, is default prone, but small, and if the next stage implementation were to be financed with risky debt, then shareholders overinvest. As the level of risky debt in place increases, the firm invests at a later date and liquidates at an earlier date compared to when the level of risky debt is small. If, however, the outstanding debt level is very large, then shareholders might even invest later than and liquidate earlier than when D is risk-free, which is symptomatic of underinvestment.

4.3. Outcomes for the Starting of the R&D Project when it is Financed with Risky Debt

Once we have obtained the value of the firm's equity, $\widetilde{E}^c(x)$, we can now solve for the investment, $\tilde{x}_I$, and liquidation threshold, $\tilde{x}_L$, when the loan of the amount $D - \underline{D}$ for financing the initial investment I is such that the outstanding debt level D becomes default prone. To solve for the outcomes $\tilde{x}_I$ and $\tilde{x}_L$ we solve the system of equations:

$$E^d(\tilde{x}_I) - (D - \underline{D}) = \widetilde{E}^c(\tilde{x}_I) - I, \tag{38}$$

$$\frac{\partial E^d(\tilde{x}_I)}{\partial x} = \frac{\partial \widetilde{E}^c(\tilde{x}_I)}{\partial x}, \tag{39}$$

$$E^d(\tilde{x}_L) = \phi_0 + \frac{\phi_1 \tilde{x}_L}{r - \mu}, \tag{40}$$

$$\frac{\partial E^d(\tilde{x}_L)}{\partial x} = \frac{\phi_1}{r - \mu}, \tag{41}$$

where equations (38) and (39) are the value matching and smooth pasting conditions at $\tilde{x}_I$ and equations (40) and (41) are the value matching and smooth pasting conditions at $\tilde{x}_L$. Along with $\tilde{x}_I$ and $\tilde{x}_L$ we also obtain the values of C_{1d} and C_{2d}, which are the coefficients in

$$E^d(x) = C_{1d} x^\beta + C_{2d} x^\theta + \frac{\psi x}{r - \mu} - \underline{D}$$

that need to be determined to obtain $E^d(x)$ for different values of D. The results of the numerical solution are illustrated in Figure 8.

Figure 8 (a) we plot $\tilde{x}_I(D)$, where $D > \underline{D} = D^*$ and compare it with x_I. We find that the investment threshold, $\tilde{x}_I(D)$, is lower than x_I and that $\tilde{x}_I(D)$ decreases as D decreases. Which is to say that the shareholders exercise the investment option prematurely relative
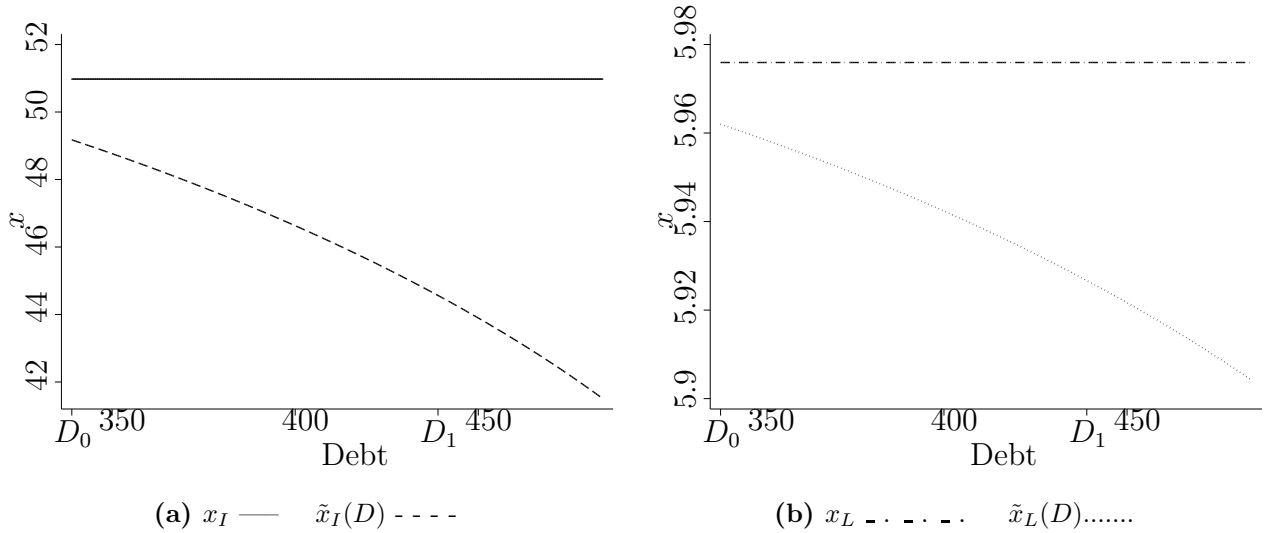
Figure 8. Investment and closure thresholds when investment I is financed with risky debt as compared to when debt levels are risk-free.

to the case when D is default-risk free; in other words, they overinvest. Again, this is because shareholders have a strong incentive to exercise the investment option quickly and reap the benefits of a successfully completed R&D project. This incentive is higher for higher levels of D because shareholders are not concerned about the welfare of bondholders, and are indifferent to the increased risk of bankruptcy resulting from their early exercise decision. Having limited liability, shareholders shift the burden of default risk of an early exercise decision on to the bondholders, while simultaneously enjoying a real possibility of reaping higher cash flows form the implementation of the innovative output of a successfully completed R&D project.

We also find that $\tilde{x}_L(D)$ is lower as compared to x_L, and decreases as the level of debt increases. Choosing to liquidate at a later time is also symptomatic of overinvestment. By waiting longer to liquidate, the owners give a greater chance for the firm to make the investment I, but at the same time they also risk defaulting on $\underline{D}$.

Thus, what is important about the "Conflict Region" is that even when no additional debt is incurred to finance I, existing debt level, $\underline{D}$, which is risk-free prior to starting of the R&D project turns risky upon embarking on the R&D project. Thus, when f and λ are low, unless the existing debt level is also low, it is possible that the firm may not be able to finance its R&D project with additional debt.

5. Valuation of Risky Debt

We now turn to the valuation of risky debt in place. Let the value of debt, D, in the mothball region and the continuation region be denoted respectively by $B^m(x_t)$ and $B^c(x_t)$. Using Itó's formula and standard hedging argument we know that the value of debt in the

mothball region satisfies the following differential equation:

$$\frac{1}{2}x^2\sigma^2 B^m_{xx}(x) + \mu x B^m_x(x) - rB^m(x) + rD = 0, \ x_d \le x \le \tilde{x}^*, \tag{42}$$

and the equation governing the value of debt in the continuation region is given by

$$\frac{1}{2}x^2\sigma^2 B^c_{xx}(x) + \mu x B^c_x(x) - rB^c(x) + rD + \lambda(\mathcal{B}(x) - (B^c(x) + \overline{D} - D)) = 0 \text{ for } x \ge \tilde{x}^*. \tag{43}$$

The term $\lambda(\mathcal{B}(x) - (B^c(x) + \overline{D} - D))$ in the above equation is due to the fact that with intensity λ, the value of debt in the continuation region jumps from $B^c(x) + \overline{D} - D$ to $\mathcal{B}(x)$. The term $(\overline{D} - D)$ is because at the random date τ when the firm successfully completes the R&D project and shareholders implement the new technology, they borrow an additional amount $\overline{D} - D$ to finance K. Consequently, bondholders gain an additional amount of $\overline{D} - D$ to their claim, $B(x)$.

From Proposition 2 in LM we know that the value of the debt level $\overline{D}$ is given by

$$\begin{aligned} \mathcal{B}(x) &= \overline{D} - \left[\overline{D} - (1-\delta)\left(\varphi_0 + \frac{\varphi_1 x_D}{r-\mu}\right)\right]\left(\frac{x}{x_D}\right)^\theta \\ &= \overline{D} - Bx^\theta, \end{aligned} \tag{44}$$

where x_D, the default threshold, is given with equation (11).

The solution to the differential equation in (42) is given by

$$B^m(x) = B_{1m}x^\beta + B_{2m}x^\theta + D,$$

and, given (44), the solution to the differential equation in (43) is given by

$$B^c(x) = B_{1c}x^\kappa + B_{2c}x^\gamma - Bx^\theta + D,$$

where β and θ are the roots of the characteristic polynomial, $\frac{1}{2}\sigma^2\eta^2 + [\mu - \frac{1}{2}\sigma^2]\eta - r = 0$, and κ and γ are the roots of the characteristic polynomial, $\frac{1}{2}\sigma^2\eta^2 + [\mu - \frac{1}{2}\sigma^2]\eta - (r+\lambda) = 0$, and the constants B_{1m}, B_{2m}, B_{1c}, and B_{2c} are to be determined by the boundary conditions.

The first boundary condition that we consider is absence of speculative bubbles:

$$\lim_{x\to\infty} B(x) = D. \tag{45}$$

The condition states that as the value of the underling exogenous demand for the firm's produce tends to infinity the value of debt converges to D, the debt principal. This is due to the fact that for high values of x, the firm is easily able to service its debt and hence quite unlikely to default. This boundary condition is only pertinent to the value of debt in the continuation region. This is because, given

$$B(x) = \begin{cases} B^c(x) & \text{if } x \ge \tilde{x}^* \\ B^m(x) & \text{if } x_d \le x \le \tilde{x}^*, \end{cases}$$

the continuation region is bounded below by $\tilde{x}^*$ but is not bounded above, but the mothball region is bounded below and above by x_d and $\tilde{x}^*$ respectively. The boundary condition (45) implies that $B_{1c} = 0$. Thus, the value of debt in the continuation region is given by

$$B^c(x) = B_{2c}x^\gamma - Bx^\theta + D. \tag{46}$$

Now, we are left with three constants, B_{1m}, B_{2m}, and B_{2c}, which we can determine from the value matching and smooth pasting conditions at the default threshold x_d and the mothballing/resuming threshold $\tilde{x}^*$, both of were determined earlier as a solution to the system of equations (33)-(37). The value matching and smooth pasting conditions respectively at x_d are the following:

$$B^m(x_d) = B_{1m}x_d^\beta + B_{2m}x_d^\theta + D = (1-\delta)\left[\Phi + \frac{\phi_1 x_d}{r-\mu}\right] \tag{47}$$

$$\frac{\partial B^m(x_d)}{\partial x} = \beta B_{1m}x_d^{(\beta-1)} + \theta B_{2m}x_d^{(\theta-1)} = \frac{(1-\delta)\phi_1}{r-\mu}. \tag{48}$$

The value matching condition (47) states that when the firm defaults at x_d, the creditors receive $(1-\delta)\left[\Phi + \frac{\phi_1 x_d}{r-\mu}\right]$, where, δ, as explained earlier, is the fraction of the value of the firm that gets destroyed during bankruptcy. Since, the above two set of equations, (47) and (48), are sufficient to determine B_{1m} and B_{1m}, the only remaining constant B_{2c} can be determined by

$$B^m(\tilde{x}^*) = B^c(\tilde{x}^*), \tag{49}$$

the value matching condition at $\tilde{x}^*$.

Given x_d and $\tilde{x}^*$, using equations (47) and (48) to solve for B_{1m} and B_{2m}, we get

$$B_{1m} = -\frac{1}{\beta-\theta}\left\{-\theta\left[D - (1-\delta)\left(\Phi + \frac{\phi_1 x_d}{(r-\mu)}\right)\right] - \frac{(1-\delta)\phi_1 x_d}{(r-\mu)}\right\}\left(\frac{1}{x_d}\right)^\beta$$

and

$$B_{2m} = -\frac{1}{\beta-\theta}\left\{\beta\left[D - (1-\delta)\left(\Phi + \frac{\phi_1 x_d}{(r-\mu)}\right)\right] + \frac{(1-\delta)\phi_1 x_d}{(r-\mu)}\right\}\left(\frac{1}{x_d}\right)^\theta.$$

Thus, we arrive at the value of debt in the mothball region:

$$B^m(x) = D - \left\{D - (1-\delta)\left[\Phi + \frac{\phi_1 x_d}{(r-\mu)}\right]\right\}\left\{\frac{\beta}{\beta-\theta}\left(\frac{x}{x_d}\right)^\theta + \frac{\theta}{\theta-\beta}\left(\frac{x}{x_d}\right)^\beta\right\}$$
$$- \left[\frac{(1-\delta)\phi_1 x_d}{(r-\mu)(\beta-\theta)}\right]\left\{\left(\frac{x}{x_d}\right)^\theta - \left(\frac{x}{x_d}\right)^\beta\right\}. \tag{50}$$

It can be easily verified that

$$\left\{D - (1-\delta)\left[\Phi + \frac{\phi_1 x_d}{(r-\mu)}\right]\right\}\left\{\frac{\beta}{\beta-\theta}\left(\frac{x}{x_d}\right)^{\theta} + \frac{\theta}{\theta-\beta}\left(\frac{x}{x_d}\right)^{\beta}\right\} > 0$$

and that

$$\left[\frac{(1-\delta)\phi_1 x_d}{(r-\mu)(\beta-\theta)}\right]\left\{\left(\frac{x}{x_d}\right)^{\theta} - \left(\frac{x}{x_d}\right)^{\beta}\right\} > 0,$$

which implies that when D is risky, the value of debt in the mothball region, $B^m(x)$, is less than the debt principal, D. Given $B^m(x)$, equation (49) yields the value of debt in the continuation region as

$$B^c(x) = (B^m(\tilde{x}^*) - D + B\tilde{x}^{*\theta})\left(\frac{x}{\tilde{x}^*}\right)^{\gamma} - Bx^{\theta} + D. \tag{51}$$

Since $B^m(\tilde{x}^*) - D < 0$ and since in the continuation region $x \geq \tilde{x}^*$, which implies that $B\tilde{x}^{*\theta}(x/\tilde{x}^*)^{\gamma} - Bx^{\theta} \leq 0$, we have $B^c(x) < D$.
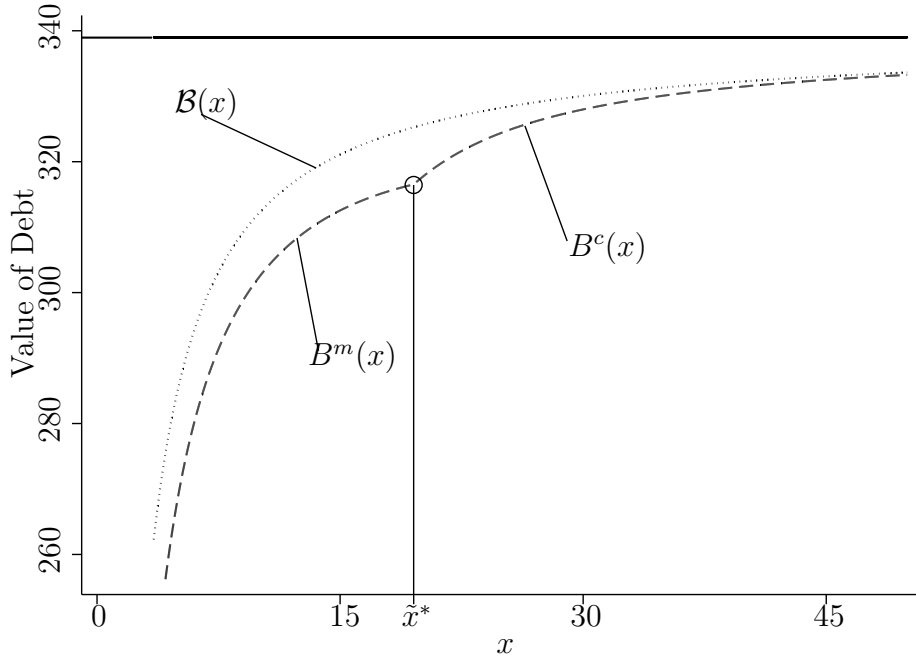

Figure 9. Value of debt D in the mothball region, $B^m(x)$, the continuation region, $B^c(x)$, and the value post implementation, $\mathcal{B}(x)$.

In Figure 9 we plot the value of debt in the mothball region, continuation region, and the value after the implementation/commercialization of the new discovery. To compute the value of debt we assume $\delta = 0.15$; that is, bankruptcy costs are assumed to be 15% of the

liquidation value of firm's assets in bankruptcy states[12]. We set $\underline{D} = 339$, which is slightly higher that $D^*(\tilde{f} = 0.05, \tilde{\lambda} = 0.5) = 338.76$, but less than $\underline{D}^*(\tilde{f} = 0.05, \tilde{\lambda} = 0.5) = 339.5$; that is, $\underline{D} = 339$ belongs to the "Conflict Region". Moreover, we assume that no additional debt of the amount $D - \underline{D}$ have been borrowed to finance I and no additional debt of the amount $\overline{D} - D$ have been borrowed to finance K, so that $\underline{D} = D = \overline{D} = 339$.

Alternatively, one can assume that $\underline{D}=300$ and that $D - \underline{D} = 39$, which would give us exactly the same result as in Figure 9. The purpose, however, of the above assumption is to show that if the level of outstanding debt $\underline{D}$ is in the "Conflict Region", so that $\underline{D}$ is risk-free before the start of the R&D project and even if no additional debt to finance the investments I or K is incurred, then, too, shareholders by overinvesting in R&D shift the burden of risk on to the bondholders, where the risk associated with R&D stems from (1) asset intangibility, (2) time uncertainty in successfully coming up with an innovation, and (3) asset specificity.

In the figure the solid line, constant at 339, depicts the level of debt principal, D. It is clear from the above figure that shareholders by engaging prematurely in risky venture as R&D lower the value of risk-free debt, $\underline{D}$, that existed prior to the start of the R&D project. The results presented here, thus, show how risks inherent to R&D and innovation activity lead to lowering of the value of debt. In such a situation it is possible that bondholders, rationally anticipating the behavior of shareholders of firms that are R&D intensive or those that are likely to engage in R&D activity, may not provide the requisite finance to such firms or their R&D activity. Our analysis, thus, explains the observed empirical regularity that R&D intensive firms are less leveraged than those that are not.

6. Conclusion

In this paper we study the implication of risk associated with R&D investment for agency conflict between bondholders and shareholders, where the associated risk derives from (a) intangibility and specificity of assets employed in conducting R&D, most of which is lost in the event of liquidation, (b) the time-uncertainty or the success intensity of successfully obtaining an innovation output, and (c) the resulting specificity of assets after the implementation/commercialization of the innovation output.

We found that when the R&D project involves assets that are highly intangible and specific and when the success intensity of obtaining an innovation output is low, then certain outstanding debt levels that are default-risk free before the start of the R&D venture become risk prone after the firm embarks on the R&D project. In such a situation, shareholders overinvest in the R&D venture to shift the burden of risk on to the bondholders while at the same time potentially gaining from the implementation/commercialization of the innovation output. Moreover, the agency conflict over R&D investment is compounded

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[12] Given that empirical estimates of bankruptcy costs range from 10–20% of firm value (see Mauer and Sarkar, 2004), our assumed value of δ is reasonable. Also, the assumed value of δ is greater than the critical value of δ, δ^*. When $\delta \geq \delta^*$, LM show that the firm, post implementation, always defaults early. For parameter values in Table 1, we get $\delta^* = 0.1204$.

when the assets of the firm become specific to the firm after the commercialization of the new discovery.

As a result of shareholders overinvesting in R&D, that sifts the burden of risks associated with R&D on to the bondholders, bondholders lose value on their claim. Consequently, investors may not be willing to invest in firms that are likely to engage in R&D and innovative activity. These results help us to understand why, as documented in many empirical studies, firms engaging in R&D and innovative activity are less leveraged than those that do not engage.

However, the results presented in this paper only establishes the potential for the agency conflict between the bondholders and the owners of the firm over R&D investment. For future work, it would be worthwhile to quantify the agency cost due to the risk involved in R&D activity and its implication for optimal debt financing.

Acknowledgment

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Appendix A. Derivation of the Hamilton-Bellman-Jacobi Equation for the Switching Problem

After having started the R&D project when the firm has still not liquidated itself and when innovation output is yet to be obtained, that is when $\mathcal{L}_s = 0$ and $\zeta_s = 0$, Itô's Lemma imply that

$$e^{-r(T-t)}E(x_T) = E(x_t) + \int_t^T e^{-r(s-t)}\left[\mathcal{D}E(x_s) - rE(x_s)\right]ds$$

$$\int_t^T e^{-r(s-t)}\left[E_x(x_s)\sigma dw_s\right] + \int_t^T e^{-r(s-t)}\left[\mathcal{E}(x_s) - K - (E(x_s) - (\overline{D} - D))\right]dq_s,$$

$$(A.1)$$

where $E(x_s)$ is the value of the firm's equity and $\mathcal{E}(x_s)$ is the value of the firm after the implementation of the R&D project, which has been defined in the main text. In (A.1), q_t denotes the jump process, whose intensity is $\lambda \mathcal{I}_s$, and

$$\mathcal{D}E(x) = \frac{1}{2}x^2\sigma^2 E_{xx}(x) + \mu x E_x(x).$$

Taking expectations under the risk neutral measure on both sides of the equation (A.1) and after some rearrangement and considering that

$$\mathbb{E}_t^Q \left\{ \int_t^T e^{-r(s-t)} E_x(x_s)\sigma dw_s \right\} = 0$$

we obtain

$$E(x_t) = \mathbb{E}_t^Q \left\{ e^{-r(T-t)} E(x_T) \right. $$
$$\left. - \int_t^T e^{-r(s-t)} \left[\mathcal{D}(x_s) - rE(x_s) + \lambda \mathcal{I}(s)\Big(\mathcal{E}(x_s) - K - (E(x_s) - (\overline{D} - D)) \Big) \right] ds \right\}.$$
$$(A.2)$$

Comparing equations (A.2) and (3.2) when $\mathcal{I}_s = 1$ we obtain

$$\psi x_s - a - rD = -\left[\mathcal{D}E(x_s) - rE(x_s) + \lambda\Big(\mathcal{E}(x_s) - K - (E(x_s) - (\overline{D} - D))\Big)\right],$$
$$(A.3)$$

the H-B-J equation in (3.3). When $\mathcal{I}_s = 0$ we obtain

$$\psi x_s - m - rD = -(\mathcal{D}E(x_s) - rE(x_s)), \tag{A.4}$$

the H-B-J equation in (3.5).

Appendix B. Proofs

Lemma 1. $\mathcal{E}^a(x) - K > E^c(x) - (\overline{D} - D)$ *for all* $x \geq x^*$

Proof 1.

Since $a - m > 0$, it is clear from the optimality condition in equation (3.16) that at x^* $\mathcal{E}^a(x^*) - K > E^c(x^*) - (\overline{D} - D)$. That is,

$$\frac{\pi x^*}{(r-\mu)} - \frac{c}{r} - K > C_{2c}x^{*\gamma} + \frac{\lambda\pi x^*}{(r-\mu)(\lambda+r-\mu)} - \frac{\lambda c}{r(\lambda+r)} - \frac{\lambda K}{\lambda+r} + \frac{\psi x^*}{\lambda+r-\mu} - \frac{a}{\lambda+r}.$$
$$(B.1)$$

Adding and subtracting $\pi x^*(\lambda + r - \mu)^{-1}$ in the RHS of the above and after a few algebraic manipulation we find that equation (B.1) implies

$$\frac{(\pi - \psi)x^*}{(\lambda + r - \mu)} + \frac{a - c}{r + \lambda} > C_{2c}x^{*\gamma} + \frac{rK}{\lambda + r}.$$

To show that $\mathcal{E}^a(x) - K > E^c(x) - (\overline{D} - D)$ for $x > x^*$, we then have to show that

$$\frac{(\pi - \psi)x}{(\lambda + r - \mu)} + \frac{a - c}{r + \lambda} > C_{2c}x^{\gamma} + \frac{rK}{\lambda + r} \tag{B.2}$$

when $x > x^*$. Since $\pi > \psi$, we have

$$\frac{(\pi - \psi)x}{(\lambda + r - \mu)} > \frac{(\pi - \psi)x^*}{(\lambda + r - \mu)}$$

for all $x > x^*$. Now, since $C_{2c}x^{\gamma}$ is the value of the option to mothball the R&D project, $C_{2c} > 0$, and because $\gamma < 0$ implies $C_{2c}x^{*\gamma} > C_{2c}x^{\gamma}$ when $x > x^*$, the inequality in (B.2) holds true when $x > x^*$.

Lemma 2. $\mathcal{E}^a(x) - K > E^c(x) - (\overline{D} - D)$ for all $x \geq x^*$

Proof 2.

Since in the event of default $\overline{D} > (1 - \delta)\left(\varphi_0 + \frac{\varphi_1 x_D}{r - \mu}\right)$, $B > 0$. Therefore, after implementation the value of risk-prone debt level, $\overline{D}$

Appendix C. Outcomes and Firm Valuation with Option to Abnadon the R&D Project

Now, since the firm incurs a fixed cost of m every instant when it is has mothballed the R&D project, if the demand for the firm's produce falls further then the firm might want to abandon the R&D project. In this Appendix we solve for the mothballing/resuming threshold, x^*, and x_a and $\bar{x}_l$, where x_a is the threshold level of price/demand such that if x falls below it then the firm abandons the R&D project and $\bar{x}_l$ is the threshold level of price below which the owners of the firm liquidate the firm. Here we would also like to mention that the level of outstanding debt at the various stages of the firms life is risk free.

According to ASSUMPTION 3, when the firm abandons the R&D project, but for the scrap value of fI, the initial investment I becomes valueless to the firm. Since we have maintained that the firm does not have any other growth option, therefore the only option left once the R&D project is abandoned is the option to liquidate the firm. This implies that the value of the firm's equity after the firm abandons the project is given by

$$E^a(x) = C_a x^{\theta} + \frac{\psi x}{r - \mu} + fI - D,$$

where $C_a x^\theta$ is the value of the option for liquidating the firm and $\frac{\psi x}{r - \mu}$ is the present value of the cash flow, ψx, which the firm receives every instant if the firm continues for ever. The value of the equity when the firm is actively engaged in the R&D process – that is, when x is in the continuation region – $E^c(x)$, is given by

$$E^c(x) = C_{2c} x^\gamma + C x^\theta + \frac{\lambda \pi x}{(r - \mu)(\lambda + r - \mu)} - \frac{\lambda c}{r(\lambda + r)} - \frac{\lambda K}{\lambda + r} + \frac{\psi x}{\lambda + r - \mu} - \frac{a}{\lambda + r} - D,$$

and the value of equity in the mothball region given in equation (3.9) is

$$E^m(x) = C_{1m} x^\beta + C_{2m} x^\theta + \frac{\psi x}{r - \mu} - \frac{m}{r} - D.$$

After abandoning the project if x falls further, then it would optimal to liquidate the firm. At the liquidation threshold, $\bar{x}_l$, the value of the firm's equity is

$$f I + \phi_0 + \frac{\phi_1 \bar{x}_l}{r - \mu} - D = \Phi_0 + \frac{\phi_1 \bar{x}_l}{r - \mu} - D,$$

where the scrap value of $f I$ is what remains with the shareholders when the firm is liquidated.

Thus, we seven unknowns – C_{2c}, C_{1m}, C_{2m}, C_a, x^*, x_a, and $\bar{x}_l$ – to be determined. By determining C_{2c} we will obtain $E^c(x)$, determining C_{1m} and C_{2m} will give us the value of the firm's equity in the mothball region, and C_a will help us obtain the value of the firm after abandoning the R&D project. The seven unknowns can be determined by the following system of seven nonlinear equations:

$$E^c(x^*) = E^m(x^*), \tag{C.1}$$

$$\frac{\partial E^c(x^*)}{\partial x} = \frac{\partial E^m(x^*)}{\partial x}, \tag{C.2}$$

$$\frac{\partial^2 E^c(x^*)}{\partial x^2} = \frac{\partial^2 E^m(x^*)}{\partial x^2}, \tag{C.3}$$

$$E^m(x_a) = E^a(x_a), \tag{C.4}$$

$$\frac{\partial E^m(x_a)}{\partial x} = \frac{\partial E^a(x_a)}{\partial x}, \tag{C.5}$$

$$E^a(\bar{x}_l) = \Phi_0 + \frac{\phi_1 \bar{x}_l}{r - \mu}, \tag{C.6}$$

$$\frac{\partial E^a(\bar{x}_l)}{\partial x} = \frac{\phi_1}{r - \mu}, \tag{C.7}$$

Equations (C.1), (C.2), and (C.3) respectively are value matching, smooth pasting, and super contact conditions at x^*. The value matching and smooth pasting conditions at x_a are given by equations (C.4) and (C.5), while (C.6) and (C.7) are value matching and smooth pasting conditions respectively at the liquidation threshold $\bar{x}_l$.

Since no closed form solution exists for the above set of equations, we solve the system numerically. For the $f = 0.05$, $\lambda = 0.5$ and for the parameter value given in Table 1, we find that $x^* = 23.02$, $x_a = 1.31$, and $\bar{x}_l = 10$. The reason why we find the abandonment threshold so low is because once the firm abandons the R&D project, a decision that cannot be reversed, it only retains scrap value. Hence, the firm waits much longer before finally abandoning it. However, since there are growth option left with the firm, the liquidation threshold, $\bar{x}_l$, is higher than x_a, which suggests that once the firms abandons the R&D project, shareholders immediately proceed to liquidate the firm.

Waiting to first abandon the R&D project and then liquidate the firm is therefore a suboptimal decision. Instead, as argued in the main text, the firm would be better off by liquidating the firm without waiting to first abandon the project.