An innovative and efficient method for interpolation of sea-level anomaly: the Data-Interpolating Variational Analysis (Diva)

Application to the Mediterranean Sea

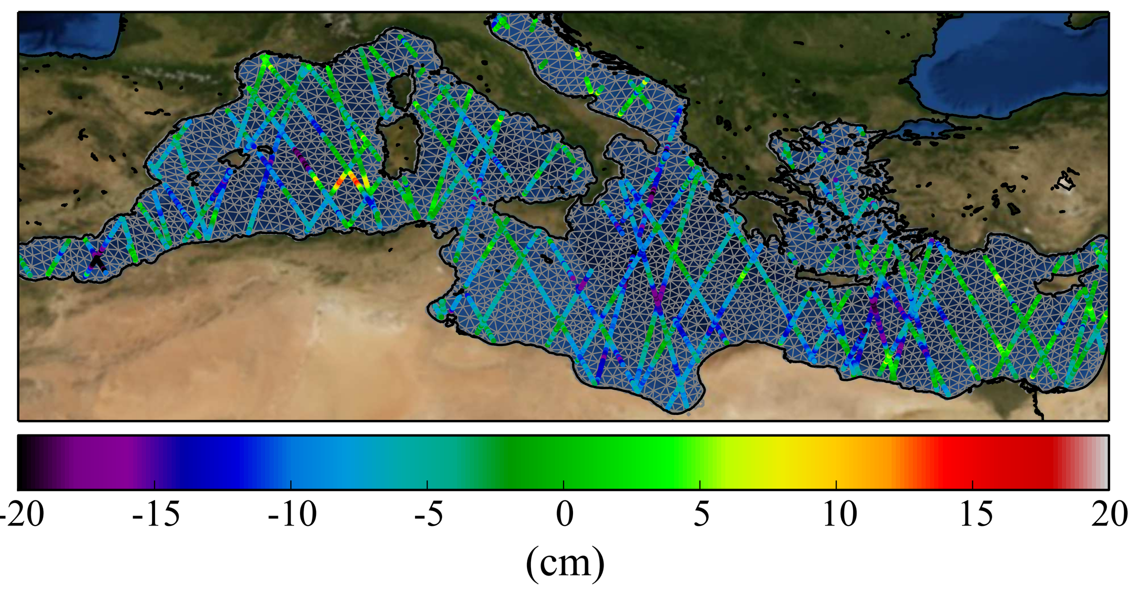
CharlesTroupin[1], Alexander Barth[2], Jean-Marie Beckers[2] & Ananda Pascual[1]

[1]IMEDEA (CSIC-UIB), Esporles, SPAIN
[2]GHER (ULg), Sart-Tilman, BELGIUM

Introduction - what are the problems?

Interpolating sparse in situ data onto a regular grid (*gridding*) is a common task in ocean sciences. There are numerous ways to do it, leading to a wide range of result qualities and numerical performances.

We present the application and the adaptations of the Data-Interpolating Variational Analysis (Diva) to perform gridding on along-track sea-level anomaly (SLA) measurements.



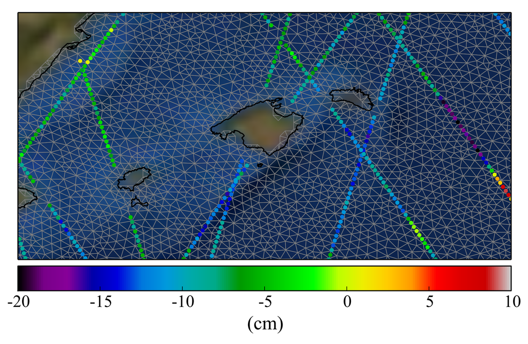


Figure 1: *The domain of interest and the data points (5189 measurements) for the period 6–13 May 2009, during which Envisat, Jason-1 and Jason-2 were available. The triangles constitute the finite-element mesh employed to solve the interpolation problem. For the global mesh on the Mediterranean Sea, the typical size of a triangle is 0.3°, while for the close-up view, this size is 0.1°.*

The method is particularly well adapted to this type of data, since:

- it is designed to easily deal with large numbers of measurements (millions if desired, see Figure 5): no sub-sampling, no clustering;
- the geometry of the domain directly influences the interpolation through a natural adaptation of the covariance function: no need to separate the sub-basins prior to the interpolation;
- the low number of parameters allows an automatic implementation to process a complete data set.

Data preparation

Download: SLA data (delayed-time, updated products, unfiltered and not sub-sampled) are obtained from the AVISO FTP server.

Conversion: the NetCDF files are converted into simple, column-formatted, text files with a format compatible with the Diva software.

Blending: measurements from different missions covering the same period (week or day) are gathered.

Time weighting: data points are assigned a weight depending on the time separation with respect to the selected date.

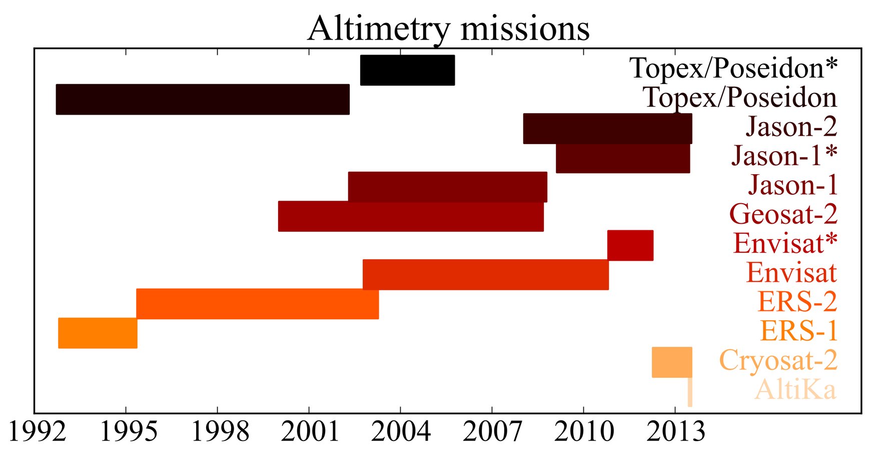
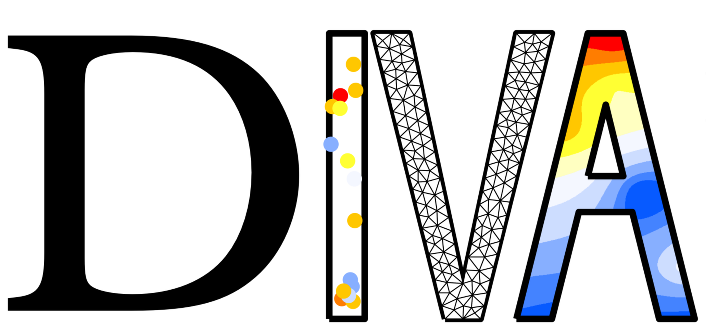


Figure 2: *Altimetry missions: the available measurements are unevenly distributed with respect to time. The* $*$ *denote change in mission orbits.*

Method - variational analysis

The gridded field (or analysis) is obtained as the solution of a cost function that penalizes:

i) The misfit between the measurements and the gridded field.
ii) The regularity of the gridded field.



The minimization of the cost function is done with the help of a finite-element technique: the solution is computed in a set of triangles covering the domain of interest, and the continuity of the solution across the triangles is assured.

Two main parameters determine an analysis with Diva:

The correlation length scale L, which measures the radius of influence of a data point. L is estimated by fitting the data correlation function to a theoretical function. The estimated value is $L \approx 0.5°$.

The signal-to-noise ratio λ, which translates the confidence in the data, not only considering the measurement noise, but also the representativity error. λ is estimated by cross-validation: several tracks are removed, then the analysis field is computed at the locations of the removed points, and finally the value of λ that minimize the RMS of the misfits (data minus analysis at data location) is selected. The estimated value is $\lambda \approx 3.0$ (no units), though the analysis is not sensitive to this parameter.

The error field, which reflects the data coverage and quality, can be calculated with different approaches using various approximations and hence leading to different computation times. More details concerning the methods are available in Beckers et al. (*in prep.*).

Results

The results obtained with Diva, using the same data, are very close to AVISO maps (Figure 3). The different processing of the data (sub-sampling, filtering, ...) as well as the different interpolation methods explain the slight discrepancies between the fields. The error field (Figure 4) evidences the need of a good coverage.

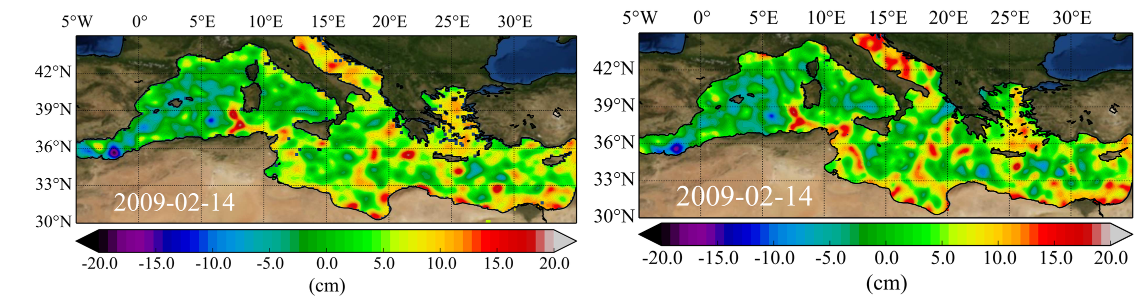


Figure 3: *Gridded SLA produced by AVISO (left) and Diva for the 14th of February 2009.*

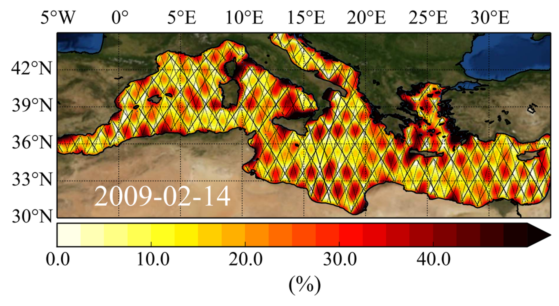


Figure 4: *Error field obtained with Diva for the 14th of February 2009. Black lines show the tracks used for the interpolation (40 days of data centred on the considered day).*

Numerical cost

In order to produce daily fields for a 20-year period, the interpolation process has to be repeated hundreds of times, thus the computation time for a single interpolation plus the generation of the corresponding error field has to be kept as low as possible.

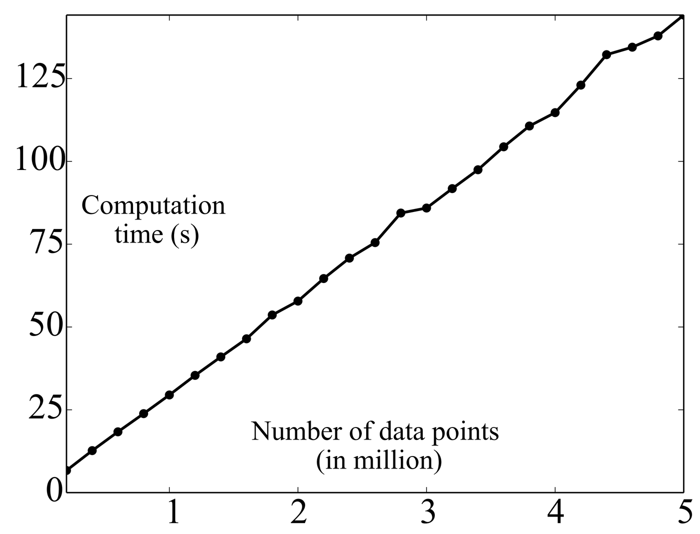


Figure 5: *Compiled with a standard compiler (gfortran), the code provides an analysis with 5 millions points in less than **2'30"** on a 426 × 161 grid. Computer: processor Intel(R) Core(TM) i7-3820 CPU, 3.60 GHz, cpu: 1800 MHz, cache size: 10240 KB.*

The time necessary to generate the finite-element mesh, only required once, is negligible. Moreover, different executions of the interpolation can be easily run simultaneously using multi-core processors.

Space and time-averaged analysed fields

The time series of mean SLA (Figure 6), computed from the data or from the gridded fields, display the same features, with a typical seasonal cycles with minimal (maximal) values in winter (summer). The close-up view evidences the smoothing effect of the gridding.

The time-averaged fields (Figure 7) obtained with Diva and AVISO onky have little differences, but the main features are localised at the same places and with the same SLA amplitude.

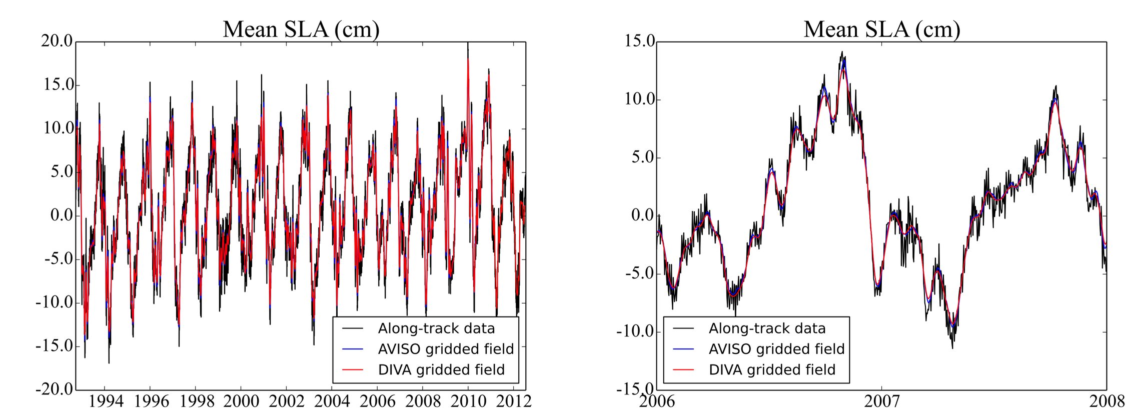


Figure 6: *Evolution of the space-averaged SLA (whole period and close-up view on 2006–2008) obtained from the along-track data and the gridded fields: the seasonal cycle and the local extrema are correctly reproduced with both methods.*

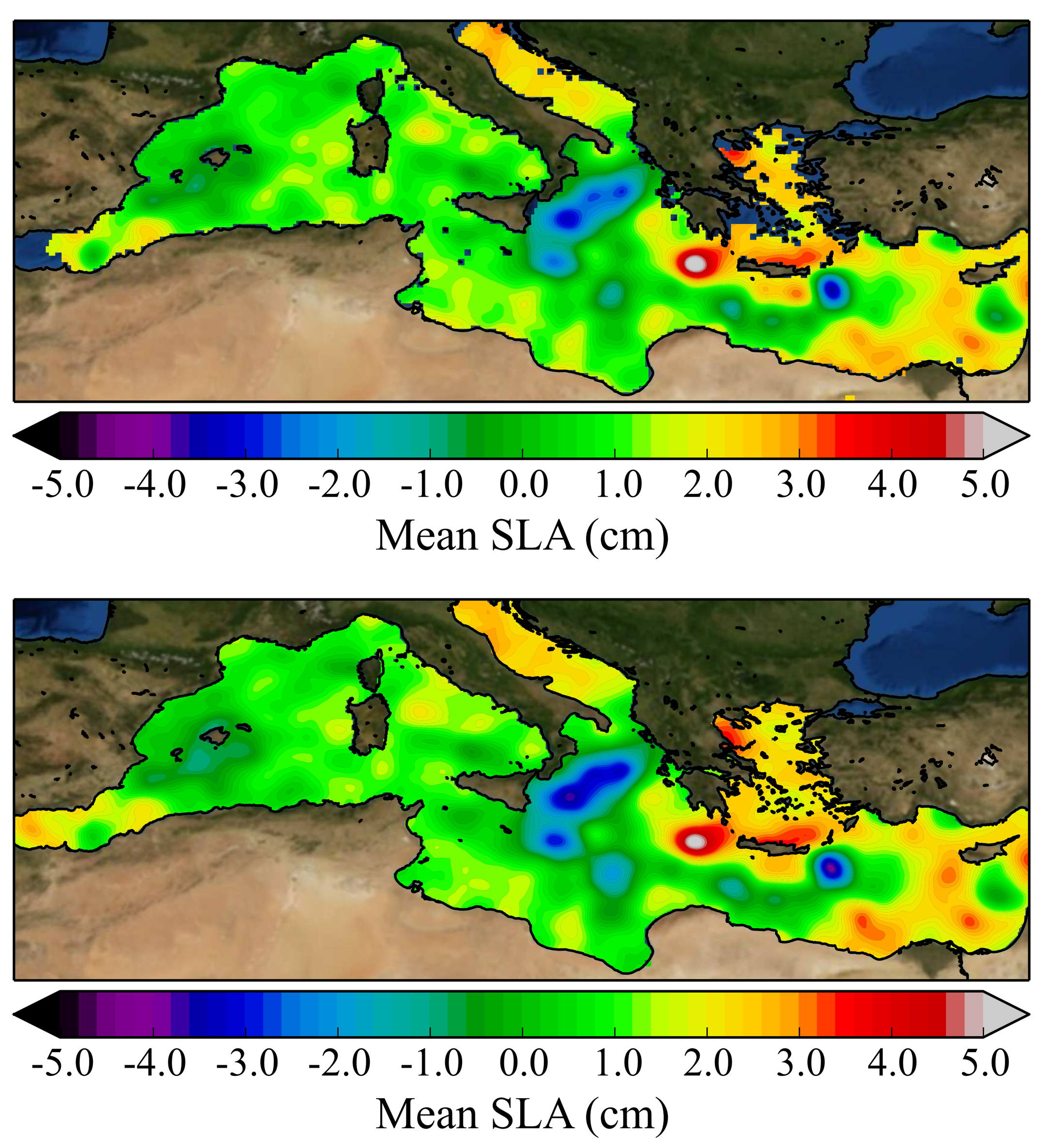


Figure 7: *Time-averaged SLA obtained for AVISO (14 October 1992 – 19 January 2011) and Diva (24 September 1992 – 9 July 2012).*

Space and time-averaged error fields

The evolution of the mean error field (Figure 8) strongly depends on the number of available observations. The largest error is due to a day (17 March 1994) without any data.

The time-averaged error field (Figure 9) reflects the data coverage. The largest errors are observed in coastal areas.

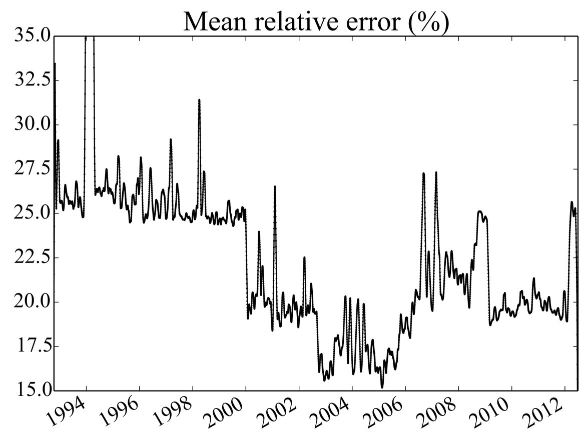


Figure 8: *Space-averaged error field obtained with Diva: the time evolution is mainly driven by the data availability (see Figure 2).*

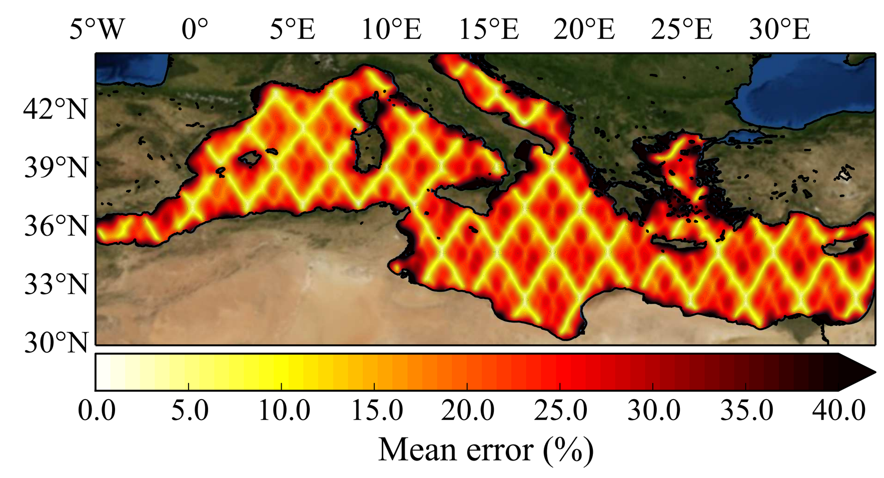


Figure 9: *Time-averaged error field obtained with Diva. The error is normalized by the variance of the background field. The geometry and repetitiveness of the altimeter tracks as well as the lower availability of coastal data are clearly visible.*

The signal variance of Diva and AVISO products are consistent (Figure 10) and the noise reduction due to the interpolation is evidenced in the right panel.

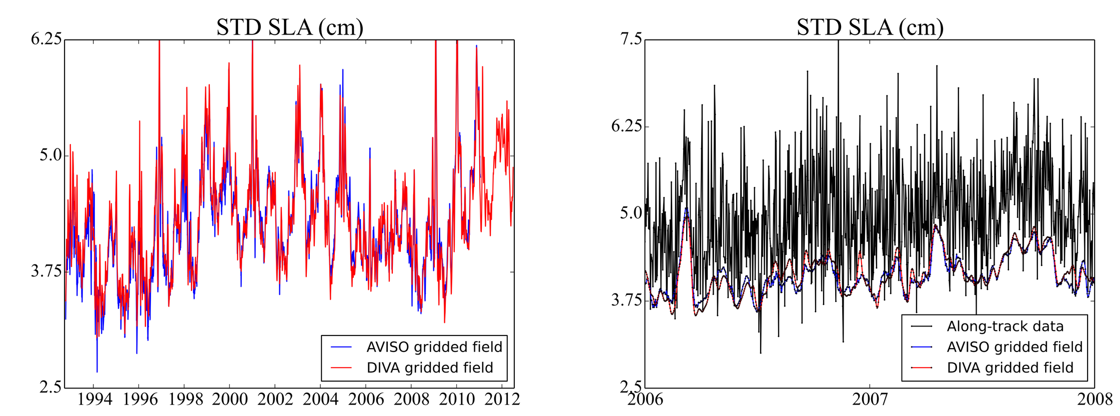


Figure 10: *Standard deviation of the SLA with respect to time (whole period and close-up view on 2006–2008), estimated from the along-track data and the gridded fields.*

Conclusions - assets of the method

i) Diva is consistent method with a limited number of parameters and a natural consideration of the domain geometry.
ii) The software can be easily tailored to allow for automatic process and deal with large quantity of observations, without the need of sub-sampling or data reduction.
iii) The results of the interpolation are close to those provided by AVISO: the daily fields display the same features in the Mediterranean Sea (Figure 3), the seasonal cycles of the SLA are consistent with the along-track data (Figure 6) and the variance of the gridded fields follow the same evolution (Figure 10).

Acknowledgements

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- Troupin, C.; Sirjacobs, D.; Rixen, M.; Brasseur, P.; Brankart, J.-M.; Barth, A.; Alvera-Azcárate, A.; Capet, A.; Ouberdous, M.; Lenartz, F.; Toussaint, M.-E. & Beckers, J.-M. Generation of analysis and consistent error fields using the Data Interpolating Variational Analysis (Diva), *Ocean Modelling*, 2012, 52-53, 90-101. doi:10.1016/j.ocemod.2012.05.002

How to get the code?

Diva source is available at
`http://modb.oce.ulg.ac.be/mediawiki/index.php/DIVA`

and distributed under the terms of the GNU General Public License (GPL):
`https://gnu.org/licenses/gpl.html` while an on-line version is available for simple analysis:
`http://gher-diva.phys.ulg.ac.be/web-vis/diva.html`