AN EFFICIENT FORMULATION FOR THE SIMULATION OF ELASTIC WAVE PROPAGATION IN 1-DIMENSIONAL COLLIDING BODIES

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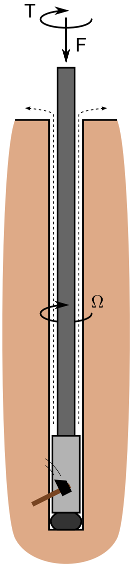
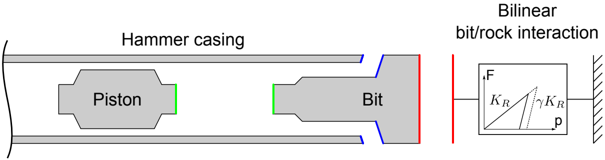
PERCUSSIVE DRILLING: DOWN-THE-HOLE HAMMER





Model specificities

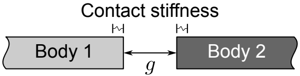
- Linear elasticity
- Multibody with contact
- Bilinear bit/rock interaction (BRI) law
  - Piecewise linear
  - Requires history variable

FE discretization → linear EOM for each body

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} + \mathbf{K}\mathbf{u} = \mathbf{f}^{\text{ext}} + \mathbf{f}^{\text{con}} + \mathbf{f}^{\text{bri}}$$

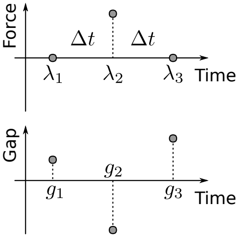
CONTACT HANDLING: PENALTY METHOD

- Regularizes unilateral constraints by introducing numerical stiffness at interface



$$\lambda^{\text{con}} = f([g]_{-}), \quad [g]_{-} = \min(0, g)$$

- Quadratic contact potential possibly leads to energetic instability



$$\begin{aligned} \Delta W^{\text{con}}_{1\to 3} &= \frac{\lambda_1 + \lambda_2}{2}(g_2 - g_1) \\ &+ \frac{\lambda_2 + \lambda_3}{2}(g_3 - g_2) \\ &= \lambda_2(g_3 - g_1) \\ &\neq 0 \end{aligned}$$

- Stores energy during persistent contact

CONTACT HANDLING: PENALTY METHOD

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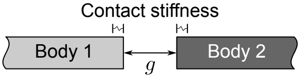

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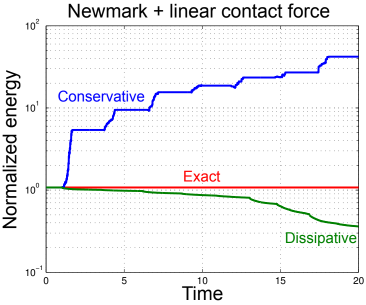

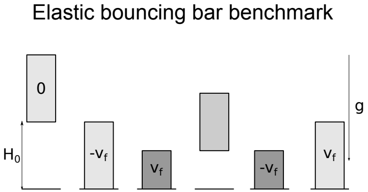

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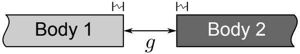

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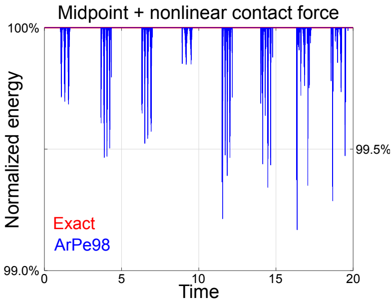
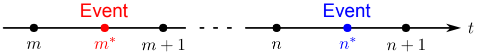
- Quadratic contact potential possibly leads to energetic instability
- Stores energy during persistent contact, Armero & Petőcz (1998)



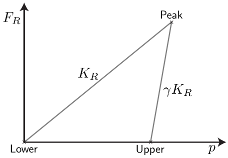
EVENT-DRIVEN INTEGRATION SCHEME

Concept



1. March EOM in time using any integration scheme
2. Detect occurrence of events related to BRI & contact status
3. Update EOM, go to 1

Pros & cons



$$\rightarrow [g]_- = g$$

- \+ Enables switch between BRI modes & correct handling of history variables
- \+ Maintains linearity of semi-discrete equations
- \+ Unilateral constraints become bilateral (easier)
- \+ Control of contact work
- − Nonlinear robust event detection/localization required
- − Possible troubles: chattering, zeno behavior

DISSIPATIVE MIDPOINT SCHEME

Armero & Romero (2001)

- One-step, four stage, 2nd order accurate
- Spectral annihilation property ($\rho_\infty = 0$)

$$\Gamma_0 \mathbf{z}_{n+1} = \Gamma_1 \mathbf{z}_n + \ell_n$$

with ($\chi > 0$)

$$\mathbf{z}_n = \begin{pmatrix} \mathbf{u}_n \\ \mathbf{v}_n \\ \tilde{\mathbf{u}}_{n-1} \\ \tilde{\mathbf{v}}_{n-1} \end{pmatrix} \qquad \Gamma_0 = \begin{pmatrix} \mathbf{K} & \frac{2\mathbf{M}}{\Delta t} + \mathbf{C} & \mathbf{K} & \mathbf{0} \\ 2\mathbf{I} & -\Delta t \mathbf{I} & \mathbf{0} & -\Delta t \mathbf{I} \\ \mathbf{0} & \chi \Delta t \mathbf{I} & \mathbf{I} & -\chi \Delta t \mathbf{I} \\ -\chi \Delta t \mathbf{K} & \mathbf{0} & \chi \Delta t \mathbf{K} & \mathbf{M} \end{pmatrix}$$

$$\ell_n = \begin{pmatrix} 2(\mathbf{f}_n + \mathbf{f}_{n+1}) \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \quad \Gamma_1 = \begin{pmatrix} \mathbf{0} & \frac{2\mathbf{M}}{\Delta t} - \mathbf{C} & \mathbf{0} & \mathbf{0} \\ 2\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

- Also possible to reduce system to displacement-based formulation

$$\mathbf{0} = \mathbf{Z}_1 \mathbf{u}_{n+1} + \mathbf{Z}_2 \mathbf{u}_n + \mathbf{Z}_3 \mathbf{v}_n - \mathbf{h}_n \quad \rightarrow \quad \mathbf{v}_{n+1} = \mathbf{H}_1 \mathbf{u}_{n+1} + \mathbf{H}_2 \mathbf{u}_n + \mathbf{H}_3 \mathbf{v}_n$$

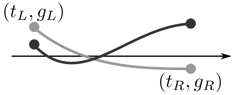
EVENT DETECTION ALGORITHM

Key points

- Parallel search
- Constraint reset

Index set of active constraints

$$\mathscr{I}_n = \{ i \in \mathscr{C} : (\mathbf{g}_L)_i \cdot (\mathbf{g}_R)_i < 0 \}$$



Algorithm

Given time bracket $[t_L, t_R]$ and $\mathbf{z}_L$, $\mathbf{z}_R$, $\mathbf{g}_L$, $\mathbf{g}_R$, $\mathscr{I}_n$

1. Locate earliest event $\forall i \in \mathscr{I}_n$ (bisection, inverse interpolation, etc.)
2. Compute state vectors using the leftmost state and time step $\Delta t^* = t^* - t_L$: $\mathbf{u}^*$, $\mathbf{v}^*$
3. Compute event functions using updated state $\forall i \in \mathscr{C}$: $\mathbf{g}^*$
4. Update $\mathscr{I}_n$
5. Assess convergence:

   `If` $\left\| \mathbf{g}^*_{\mathscr{I}_n} \right\| <$ `tol` : reset constraints, proceed with integration
   `Else` : update bracket $[t_L, t_R]$, proceed with event detection

EVENT LOCALIZATION

1. All procedures rely on the bracketing of an event after detection through a sign change of the event function.
2. Bisection is used if convergence is too slow.
3. Iterators:
   - Bisection

$$t^* = \frac{1}{2}(t_L + t_R)$$

   - Inverse linear interpolation

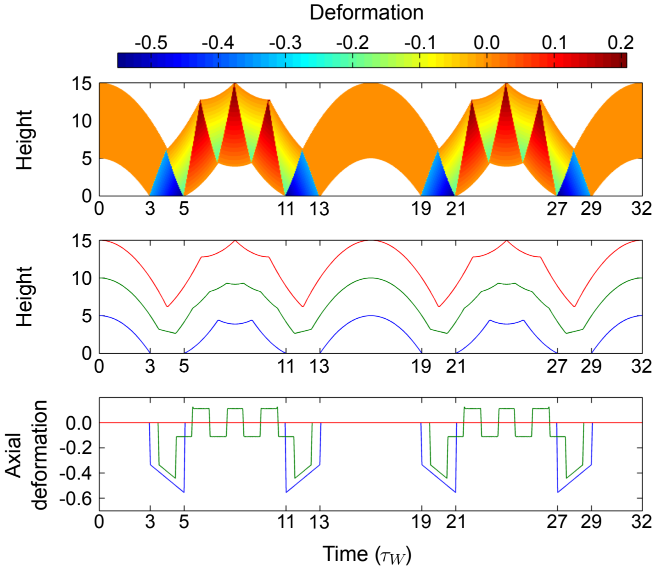
$$t^* = t_L - \frac{t_R - t_L}{g_R - g_L} g_L$$

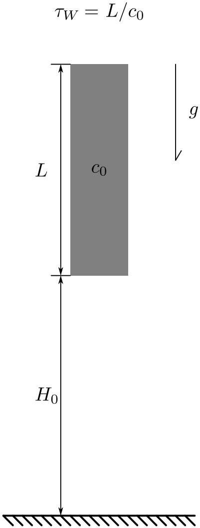
   - Barycentric hermite interpolation (Corless et al. (2007))

$$\mathbf{C}_0 = \begin{pmatrix} 0 & & & & g_L \\ 1 & 0 & & & Tg'_L \\ & & 1 & & g_R \\ & & 1 & 1 & Tg'_R \\ -2 & -1 & 2 & -1 & 0 \end{pmatrix}, \quad \mathbf{C}_1 = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 0 \end{pmatrix}$$

$$t^* = T \min_{\Lambda \in (0,1)} \Lambda(C_0, C_1), \quad T = t_R - t_L$$

ELASTIC BOUNCING BAR





$$(H_0, L, c_0, g) = (5, 10, 30, 10)$$

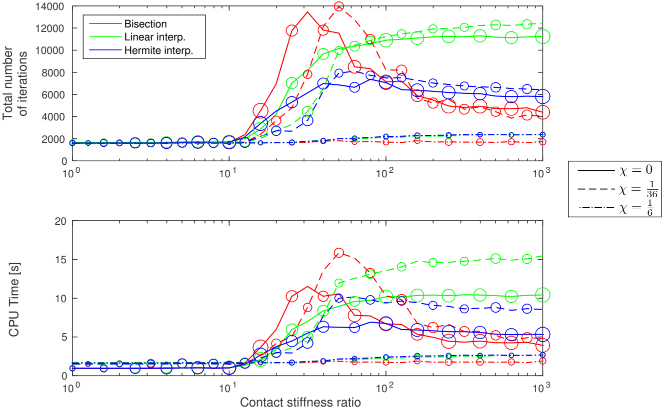
BOUNCING BAR – BENCHMARKING

Problem data

- Single event: gap at contact
- 100 linear elements, CFL $= 1$

Error measure: mechanical energy after 1 period

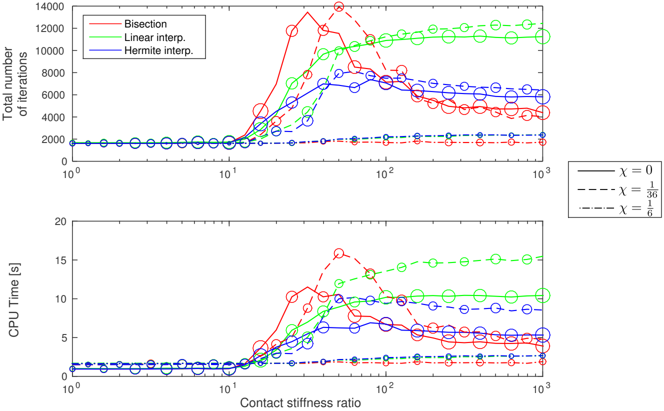
$$E = \frac{1}{2}(\mathbf{u}^T\mathbf{K}\mathbf{u} + \mathbf{v}^T\mathbf{M}\mathbf{v})_{t=16\tau_W}$$



BOUNCING BAR – BENCHMARKING

Learnings

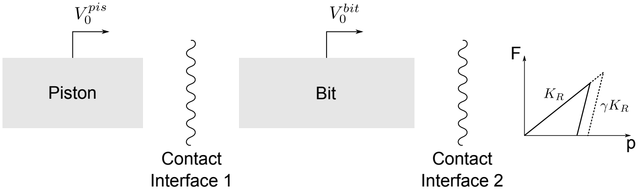
- Contact stiffness strongly influences convergence of ED
- $\chi = 1/6$: better convergence and lower error ($\sim 3^{\text{rd}}$-order TDG)
- Bisection method not necessarily more expensive



DTH SYSTEM

Problem data

- 5 events: contact + BRI
- Linear interpolation + bisection
- Uniform velocity prior to impact (no defo.)



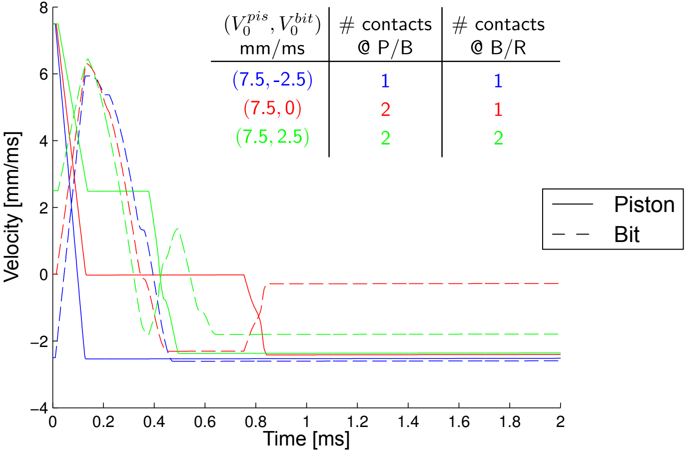
$A_{\text{bit}} = 6525$ mm^2 $\quad L_{\text{bit}} = 357$ mm $\quad E_{\text{steel}} = 210$ GPa $\quad \rho_{\text{steel}} = 7850 \cdot 10^{-6}$ g/mm^3

$A_{\text{pis}} = 6525$ mm^2 $\quad L_{\text{pis}} = 300$ mm $\quad K_R = 10^6$ N/mm $\quad \gamma = 10$

DTH SYSTEM – CG VELOCITIES

Analysis

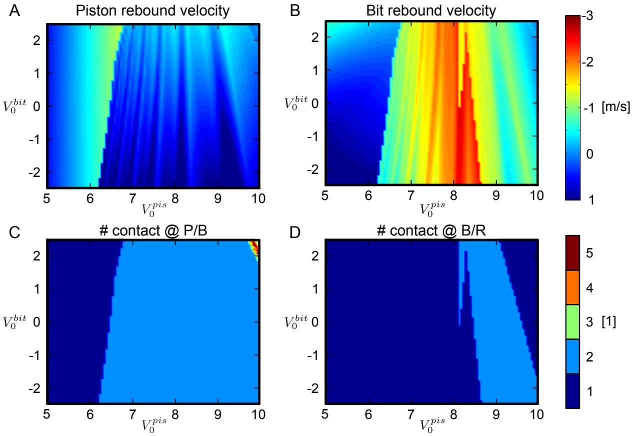
- Number of persistent contact phases varies with impact velocities
- Given nominal impact velocities design can ensure double impact



DTH SYSTEM – PARAMETRIC ANALYSIS

Analysis

- Rebound velocities depend discontinuously on impact velocities
- Persistent contact phases play a critical role in post-impact velocities



SUMMARY

Framework for the handling of contact & events

- Contact is handled via penalty method
- Scheme is energetically stable
- Applies to
  - Wave propagation in DTH systems (motivation) & chain systems
  - Linear structural dynamics

Benchmark 1 – Elastic bouncing bar

- Best performance/accuracy with dissipative midpoint for $\chi = 1/6$
- Influence of contact stiffness

Benchmark 2 – DTH system

- Discontinuous dependence & persistent contact phases

$\rightarrow$ Possible to model DTH systems with RBD? Tough ...

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