The moment method

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Overview

* Monoperiodic pulsation
* Multiperiodic pulsation
* Computation of the observed moments
* Newness in FAMIAS

The moment method

MONOPERIODIC PULSATION

Aerts et al. (1992)
Aerts (1996)

$$<v> = v_p A(\ell, m, i) \sin[(\omega - m\Omega)t + \psi],$$

$$<v^2> = v_p^2 C(\ell, m, i) \sin\left[2(\omega - m\Omega)t + 2\psi + \frac{3\pi}{2}\right]$$
$$+ v_p v_\Omega D(\ell, m, i) \sin\left[(\omega - m\Omega)t + \psi + \frac{3\pi}{2}\right]$$
$$+ v_p^2 E(\ell, m, i) + \sigma^2 + b_2 v_\Omega^2,$$

$$<v^3> = v_p^3 F(\ell, m, i) \sin[3(\omega - m\Omega)t + 3\psi]$$
$$+ v_p^2 v_\Omega G(\ell, m, i) \sin\left[2(\omega - m\Omega)t + 2\psi + \frac{3\pi}{2}\right]$$
$$+ \left[v_p^3 R(\ell, m, i) + v_p v_\Omega^2 S(\ell, m, i) + v_p \sigma^2 T(\ell, m, i)\right]$$
$$\times \sin[(\omega - m\Omega)t + \psi].$$

Second moment varies with 2 ω_{obs}
≡
Zonal mode (m=0)

Odd moments have constant = 0

Slow rotation
Constant symmetric local line profile

MONOPERIODIC PULSATION

$$< v > = v_{\mathrm{p}} A(\ell, m, i) \sin\left[(\omega - m\Omega)t + \psi\right],$$

$$< v^2 > = v_{\mathrm{p}}^2 C(\ell, m, i) \sin\left[2(\omega - m\Omega)t + 2\psi + \tfrac{3\pi}{2}\right]$$

$$+ v_{\mathrm{p}} v_{\Omega} D(\ell, m, i) \sin\left[(\omega - m\Omega)t + \psi + \tfrac{3\pi}{2}\right]$$

$$+ v_{\mathrm{p}}^2 E(\ell, m, i) + \sigma^2 + b_2 v_{\Omega}^2,$$

$$< v^3 > = v_{\mathrm{p}}^3 F(\ell, m, i) \sin\left[3(\omega - m\Omega)t + 3\psi\right]$$

$$+ v_{\mathrm{p}}^2 v_{\Omega} G(\ell, m, i) \sin\left[2(\omega - m\Omega)t + 2\psi + \tfrac{3\pi}{2}\right]$$

$$+ \left[v_{\mathrm{p}}^3 R(\ell, m, i) + v_{\mathrm{p}} v_{\Omega}^2 S(\ell, m, i) + v_{\mathrm{p}} \sigma^2 T(\ell, m, i)\right]$$

$$\times \sin\left[(\omega - m\Omega)t + \psi\right].$$

$<v>$ does not depend on σ and *vsini*

Constant of $<v^2>$:
- **width of the intrinsic profile σ**
- **stellar rotation *vsini***
- **stellar pulsation**

Free parameters:
(l,m), v_p, *vsini*, *i*, σ

IACC

Amplitude of <v>:

$$v_p \underbrace{A(\ell, K, u)} \underbrace{d^{\ell}_{0m}(i)}$$

In general, becomes smaller for higher degree l

≡

Partial cancellation effect

For each (l,m), at least one inclination angle so that

$$d^{\ell}_{0m}(i) = 0$$

≡

Inclination angle of complete cancellation (IACC)

≡

<v> shows no time variation

IACC and IALC

(ℓ, m)	IACC						IALC
$(1,0)$						$90°$	$0°$
$(2,0)$			$54.7°$				$0°$
$(3,0)$		$39.2°$				$90°$	$0°$
$(4,0)$		$30.6°$			$70.1°$		$0°$
$(5,0)$	$25.0°$		$57.4°$			$90°$	$0°$
$(1,1)$	$0°$						$90°$
$(2,2)$	$0°$						$90°$
$(3,3)$	$0°$						$90°$
$(4,4)$	$0°$						$90°$
$(5,5)$	$0°$						$90°$
$(2,1)$	$0°$					$90°$	$45.0°$
$(3,1)$	$0°$			$63.4°$			$31.1°$
$(3,2)$	$0°$					$90°$	$54.7°$
$(4,1)$	$0°$	$49.1°$				$90°$	$23.9°$
$(4,2)$	$0°$			$67.8°$			$40.9°$
$(4,3)$	$0°$					$90°$	$60.0°$
$(5,1)$	$0°$	$40.1°$			$73.4°$		$19.4°$
$(5,2)$	$0°$		$54.7°$			$90°$	$32.9°$
$(5,3)$	$0°$				$70.5°$		$46.9°$
$(5,4)$	$0°$					$90°$	$63.4°$

* IACCs and IALCs same for (l,m) and (l,-m)
* i = 0° ≡ IACC for non-zonal modes
* i = 90° ≡ IACC if l - m odd
* i = 0° ≡ IALC for zonal modes
* i = 90° ≡ IALC for sectoral modes

May explain missing peaks in a multiplet

DISCRIMINANT FOR A MONOPERIODIC PULSATION

Comparison of the amplitude of theoretically calculated moments and observed moments, through

$$
\begin{aligned}
\Gamma_\ell^m(v_\mathrm{p}, i, v_\Omega, \sigma) \equiv \Bigg[& \Big| AA - v_\mathrm{p} |A(\ell,m,i)| f_{AA} \Big|^2 \\
& + \left(\Big| CC - v_\mathrm{p}^2 |C(\ell,m,i)| \Big|^{1/2} f_{CC} \right)^2 \\
& + \left(\Big| DD - v_\mathrm{p} v_\Omega |D(\ell,m,i)| \Big|^{1/2} f_{DD} \right)^2 \\
& + \left(\Big| EE - v_\mathrm{p}^2 |E(\ell,m,i)| - \sigma^2 - b_2 v_\Omega^2 \Big|^{1/2} f_{EE} \right)^2 \\
& + \left(\Big| FF - v_\mathrm{p}^3 |F(\ell,m,i)| \Big|^{1/3} f_{FF} \right)^2 \\
& + \left(\Big| GG - v_\mathrm{p}^2 v_\Omega |G(\ell,m,i)| \Big|^{1/3} f_{GG} \right)^2 \\
& + \Big(\Big| RST - v_\mathrm{p}^3 |R(\ell,m,i)| - v_\mathrm{p} v_\Omega^2 |S(\ell,m,i)| \\
& \quad - v_\mathrm{p} \sigma^2 |T(\ell,m,i)| \Big|^{1/3} f_{RST} \Big)^2 \Bigg]^{1/2}.
\end{aligned}
$$

$$
f_{AA} \equiv \frac{AA}{s_{AA} W}, f_{CC} \equiv \frac{CC}{s_{CC} W}, f_{DD} \equiv \frac{DD}{s_{DD} W},
$$
$$
f_{EE} \equiv \frac{EE}{s_{EE} W}, f_{FF} \equiv \frac{FF}{s_{FF} W}, f_{GG} \equiv \frac{GG}{s_{GG} W},
$$
$$
f_{RST} \equiv \frac{RST}{s_{RST} W},
$$

$$
W \equiv \frac{AA}{s_{AA}} + \frac{CC}{s_{CC}} + \frac{DD}{s_{DD}} + \frac{EE}{s_{EE}} + \frac{FF}{s_{FF}} + \frac{GG}{s_{GG}} + \frac{RST}{s_{RST}}
$$

Aerts (1996)

- * Discriminant chosen so that contributions of $<v>$, $<v^2>$ and $<v^3>$ are similar
- * Less weight to amplitudes with a larger standard error

DISCRIMINANT FOR A MONOPERIODIC PULSATION

$$\Gamma_\ell^m(v_\mathrm{p}, i, v_\Omega, \sigma)$$

$$\gamma_\ell^m \equiv \min_{v_\mathrm{p}, i, v_\Omega, \sigma} \Gamma_\ell^m$$

ℓ	$\|m\|$	γ_ℓ^m	v_p	i	$v \sin i$	σ
0	0	0.08	5.6	-	15.3	6.5
1	1	0.13	10.0	38°	14.8	5.9
2	1	0.17	12.1	64°	16.4	2.2
1	0	0.18	5.0	7°	19.6	1.7
2	2	0.23	15.0	53°	10.3	4.8
⋮	⋮	⋮	⋮	⋮	⋮	⋮

δ Scuti star ρ Puppis

MULTIPERIODIC PULSATION

$$
\begin{aligned}
&< v^2 >_{f*g} = \\
&\sum_{j=1}^{N} (v_{\mathrm{p}}^{j})^2 C(\ell_j, m_j, i) \sin\left(2(\omega_j - m_j\Omega)t + 2\psi_j + \frac{3\pi}{2}\right) \\
&+ \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i \neq j}}^{N} v_{\mathrm{p}}^{j} v_{\mathrm{p}}^{i} C_b(\ell_j, \ell_i, m_j, m_i, i) \\
&\times \sin\left((\omega_i - \omega_j - (m_i - m_j)\Omega)t + \phi_{ij}^{b}\right) \\
&+ \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i \neq j}}^{N} v_{\mathrm{p}}^{j} v_{\mathrm{p}}^{i} C_s(\ell_j, \ell_i, m_j, m_i, i) \\
&\times \sin\left((\omega_i + \omega_j - (m_i + m_j)\Omega)t + \phi_{ij}^{s}\right) \\
&+ v_{\Omega} \sum_{j=1}^{N} v_{\mathrm{p}}^{j} D(\ell_j, m_j, i) \sin\left((\omega_j - m_j\Omega)t + \psi_j + \frac{3\pi}{2}\right) \\
&+ \sum_{j=1}^{N} (v_{\mathrm{p}}^{j})^2 E(\ell_j, m_j, i) + \sigma^2 + b_2 v_{\Omega}^2
\end{aligned}
$$

Multiple modes are identified separately

- **using the amplitudes appearing in the moment expressions expect those coupling frequencies**
- **with the same discriminant as in the monoperiodic case but without including the constant of $<v^2>$**

MULTIPERIODIC PULSATION

$$
\begin{aligned}
< v^2 >_{f*g} = & \sum_{j=1}^{N} (v_{\rm p}^j)^2 C(\ell_j, m_j, i) \sin\left(2(\omega_j - m_j\Omega)t + 2\psi_j + \frac{3\pi}{2}\right) \\
& + \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i\neq j}}^{N} v_{\rm p}^j v_{\rm p}^i C_b(\ell_j, \ell_i, m_j, m_i, i) \\
& \times \sin\left((\omega_i - \omega_j - (m_i - m_j)\Omega)t + \phi^b_{ij}\right) \\
& + \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i\neq j}}^{N} v_{\rm p}^j v_{\rm p}^i C_s(\ell_j, \ell_i, m_j, m_i, i) \\
& \times \sin\left((\omega_i + \omega_j - (m_i + m_j)\Omega)t + \phi^s_{ij}\right) \\
& + v_\Omega \sum_{j=1}^{N} v_{\rm p}^j D(\ell_j, m_j, i) \sin\left((\omega_j - m_j\Omega)t + \psi_j + \frac{3\pi}{2}\right) \\
& + \sum_{j=1}^{N} (v_{\rm p}^j)^2 E(\ell_j, m_j, i) + \sigma^2 + b_2 v_\Omega^2
\end{aligned}
$$

BUT

- **Separate identifications lead to different values for *i* and *vsini***
- **One does not use the information from the coupling terms and from the constant of $<v^2>$**
- **No simultaneous identification mainly because of still very CPU–time consuming**

MULTIPERIODIC PULSATION

Slowly Pulsating B stars:

– **Multiperiodic**
– **The rotation period is of the same order as the pulsation periods (of the order of days)**

$\longrightarrow$ **Need of improved version of the moment method:**

– **Improved efficiency of computers + clever implementation:**
Use of a discriminant which identifies all the modes simultaneously, leading to one value for *vsini* and *i*

– **Use of Lee & Saio (1992)'s formalism for the pulsation velocity field, taking into account the Coriolis force in a way appropriate to g-modes**
(use of routines by Richard Townsend 1997)

MULTIPERIODIC PULSATION

$$< v > = \sum_{j=1}^{N} A_1^j \cos(\omega_j t + \psi_j),$$

$$< v^2 > = \sum_{j=1}^{N} C_1^j \cos(2\omega_j t + 2\psi_j)$$

$$+ \sum_{j=1}^{N} D_2^j \sin(\omega_j t + \psi_j)$$

$$+ \sum_{j=1}^{N} \sum_{k \neq j}^{N} C_{b1}^{jk} \cos((\underline{\omega_j - \omega_k})t + \psi_j - \psi_k)$$

$$+ \sum_{j=1}^{N} \sum_{k \neq j}^{N} C_{s1}^{jk} \cos((\underline{\omega_j + \omega_k})t + \psi_j + \psi_k)$$

$$+ \underline{\sum_{j=1}^{N} E_{12}^j + E_{\rm rot} + \sigma^2},$$

$$A_1^j = A_{\rm p}^j \left\{ I[C_{\ell_j}^{m_j}] + K_j I[D_{\ell_j}^{m_j}] \right\}$$

***I*[x] ≡ integration over the visible stellar surface**

To gain computational time, a grid of such integrals is pre-computed for all (l,m,i) combinations

Briquet & Aerts (2003)

DISCRIMINANT FOR A MULTIPERIODIC PULSATION

Comparison of theoretically calculated moments and observed moments at each time of observations, through

$$\Sigma = \left\{ \frac{1}{N_{\rm obs}} \sum_{k=1}^{N_{\rm obs}} \left[\left(< v > (t_k) - < v >_{\rm obs} (t_k) \right)^2 + \left| < v^2 > (t_k) - < v^2 >_{\rm obs} (t_k) \right| + \left(< v^3 > (t_k) - < v^3 >_{\rm obs} (t_k) \right)^{2/3} \right] \right\}^{1/2}$$

Main drawback:
no statistical significance for the derived solutions

APPLICATION TO A β CEPHEI STAR

The β Cephei star θ Ophiuchi

ID	Frequency (d^{-1})	(ℓ, m)
ν_1	7.1160	**(2,?)**
ν_2	7.2881	**(2,?)**
ν_3	7.3697	**(2,?)**
ν_4	7.4677	$(0,0)$
ν_5	7.7659	$(1,-1)$
ν_6	7.8742	$(1,0)$
ν_7	7.9734	$(1,1)$

In photometry, three components of a quintuplet

↓

Two possible m-values

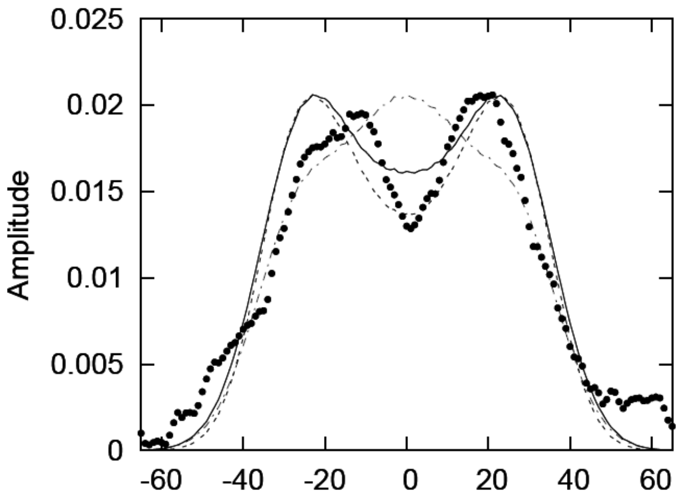
Ambiguity lifted by spectroscopy

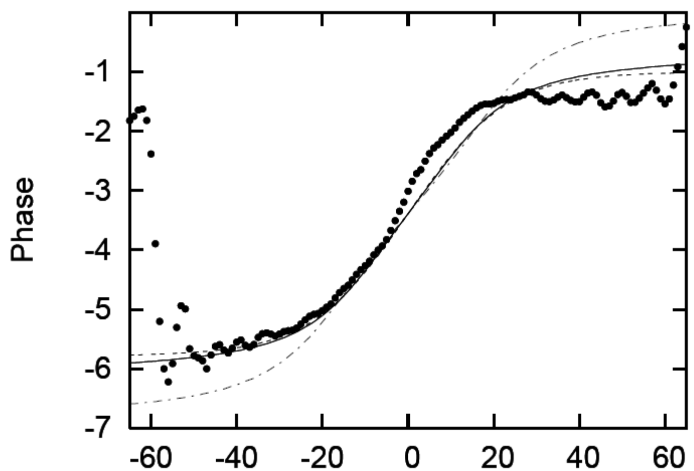
(ℓ_1,m_1)	*(2, −1)*	*(1, −1)*	(3, −1)	(2, 0)	(0, 0)	(1, 0)	*(2, −2)*	(3, 0)	(1, 1)	(2, 1)	...
(ℓ_2,m_2)	*(0, 0)*	*(0, 0)*	(0, 0)	(0, 0)	(0, 0)	(0, 0)	*(0, 0)*	(0, 0)	(0, 0)	(0, 0)	...
A_p^1	*33.96*	*12.65*	46.28	20.80	13.11	8.98	*16.98*	47.43	12.65	38.73	...
A_p^2	*10.74*	*10.74*	10.74	10.74	10.74	10.74	*10.74*	10.74	10.74	10.74	...
$v_{r,max}$	3.23	7.23	17.46	9.34	6.58	3.05	9.36	3.39	7.24	3.21	...
$v_{t,max}$	3.28	0.55	1.86	0.04	0.00	0.53	1.70	6.39	0.51	3.47	...
i_{rot}	*75*	*89*	90	90	-	5	*90*	69	90	77	...
$v \sin i$	*31*	*33.5*	30	38	38.5	38.5	*31.5*	30	32	30.5	...
σ	*10*	*8.5*	10	2.5	1	1	*10*	9.5	10	10	...
Σ	*14.40*	*14.43*	14.54	14.55	14.56	14.57	*14.77*	14.77	15.01	15.13	...

APPLICATION TO A β CEPHEI STAR

The β Cephei star θ Ophiuchi

(ℓ_1,m_1)	(2, −1)	(1, −1)	(3, −1)	(2, 0)	(0, 0)	(1, 0)	(2, −2)	(3, 0)	(1, 1)	(2, 1)	...
(ℓ_2,m_2)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	...
A_p^1	33.96	12.65	46.28	20.80	13.11	8.98	16.98	47.43	12.65	38.73	...
A_p^2	10.74	10.74	10.74	10.74	10.74	10.74	10.74	10.74	10.74	10.74	...
$v_{\mathrm{r,max}}$	3.23	7.23	17.46	9.34	6.58	3.05	9.36	3.39	7.24	3.21	...
$v_{\mathrm{t,max}}$	3.28	0.55	1.86	0.04	0.00	0.53	1.70	6.39	0.51	3.47	...
i_{rot}	75	89	90	90	-	5	90	69	90	77	...
$v \sin i$	31	33.5	30	38	38.5	38.5	31.5	30	32	30.5	...
σ	10	8.5	10	2.5	1	1	10	9.5	10	10	...
Σ	14.40	14.43	14.54	14.55	14.56	14.57	14.77	14.77	15.01	15.13	...





(l,m) = (2,-1)

COMPUTATION OF THE OBSERVED MOMENTS

– **Use the zero point of the least-squares fit to the first moment as velocity of the star v_γ with respect to the Sun**

The computed observed moments are then corrected for v_γ so that odd moments have a constant equal to zero

– **Chose the integration boundaries of the moments dynamically in order to avoid to noisy continuum**

NEWNESS IN FAMIAS

Changes in the moment method implemented in FAMIAS compared to version by Briquet & Aerts (2003):

*** Computation of statistical uncertainties of observed moments from S/N ratio of the spectrum:** $\sigma_{<v^1>_o}$ $\sigma_{<v^2>_o}$

Use of these uncertainties to derive a chi-square goodness-of-fit value using only the first and second moment

$$\chi^2_\nu = \frac{1}{2N} \sum_{i=1}^{N} \left[\left(\frac{<v^1>_o - <v^1>_t}{\sigma_{<v^1>_o}} \right)^2 + \left(\frac{<v^2>_o - <v^2>_t}{\sigma_{<v^2>_o}} \right)^2 \right]$$

*** Limb darkening coefficient of 4th order**

*** Genetic optimization**

The moment method

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