Mode identification from spectroscopy

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Overview

* Why do we need empirical mode identification?
* Modelling of line-profile variations due to NRP
* Spectroscopic mode identification techniques
* Generalities
Why do we need empirical mode identification?

Asteroseismology

To constrain a stellar model, need of observational pulsational characteristics:

Pulsation frequencies + Pulsation modes

Mode identification from spectroscopy
WHY DO WE NEED EMPIRICAL MODE IDENTIFICATION?

ASTEROSEISMOLOGY

Observational constraints from spectroscopy:

- Pulsation frequencies
- Pulsation modes

Time series of high-resolution high S/N ratio spectroscopic measurements

Basic stellar parameters
- Chemical abundances

Mode identification from spectroscopy
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

OVERVIEW

* What causes LPVs?
* Basic line profile model
* Sophisticated line profile model
* Line profile model in FAMIAS
At the stellar surface:

Oscillatory displacements due to pulsation

Periodic temporal variations of

* velocity field
* local temperature

Doppler shift
local brightness
local line profile (width and EW changes)
Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- Intensity

Sum up all the contributions of all the visible surface elements

* Weighted by the on the line-of-sight projected area of the surface element

* Doppler shifted by the on the line-of-sight velocity fields caused by rotation and pulsation

Mode identification from spectroscopy
Distorted stellar surface divided into many surface elements

For each surface element, one computes:
- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Approximations

Spherical stellar surface (not distorted)
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL

Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Approximations

Gaussian absorption line profile

\[ 1 - \frac{EW}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(\lambda - \lambda_0)^2}{2\sigma^2} \right) \]

Constant in time and over the stellar surface

+ linear limb-darkening law for continuum intensity

\[ I_c = I_0 \left( 1 - u + u \cos \chi \right) \]

Mode identification from spectroscopy
Distorted stellar surface divided into many surface elements

For each surface element, one computes:
- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

**Approximations**

**Uniform and time-independent stellar rotation**

\[ v_{\text{rot}}(\theta, \varphi) = v_e \sin i \sin \theta \sin \varphi \]

**Rotational broadening of spectral line**

**Mode identification from spectroscopy**
Distorted stellar surface divided into many surface elements

For each surface element, one computes:
- Intensity
- Rotation velocity
- Pulsation velocity

In the linear approximation (i.e. small amplitude of pulsation)

For a star rotating sufficiently slowly (i.e. neglecting effects of rotation on pulsation)

\[
\vec{v}_{\text{puls}} = (v_r, v_\theta, v_\varphi) = N^m_\ell v_p \left( 1, K \frac{\partial}{\partial \theta}, \frac{K}{\sin \theta} \frac{\partial}{\partial \varphi} \right) Y^m_\ell (\theta, \varphi) \exp (i\omega t)
\]
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL

\[ Y_{\ell}^m(\theta, \varphi) \]

- **Zonal**
  \[ m = 0 \]
- **Tesseral**
  \[ 0 \neq |m| \neq \ell \]
- **Sectoral**
  \[ \ell = |m| \]

Mode identification from spectroscopy
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL

Intensity

\[
1 - \frac{\text{EW}}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(\lambda - \lambda_0)^2}{2\sigma^2} \right)
\]

\[I_c = I_0 \left(1 - u + u \cos \chi\right)\]

Rotation velocity

\[v_{\text{rot}}(\theta, \varphi) = v_e \sin i \sin \theta \sin \varphi\]

Pulsation velocity

\[N_{\ell m} v_p \left(1, \frac{\partial}{\partial \theta}, \frac{\partial}{\sin \theta \partial \varphi}\right) Y_{\ell m}(\theta, \varphi)\]

\[K = \frac{GM}{(\omega^2 R^3)}\]

* Adopted parameters: EW, u, K
* Free parameters: (l,m), \(v_p\), vsini, i, \(\sigma\)

Mode identification from spectroscopy
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL

\[ \nu_p = 5 \text{ km/s} \]
\[ \nu \sin i = 30 \text{ km/s} \]
\[ \sigma = 4 \text{ km/s} \]
\[ i = 55^\circ \]

Spectroscopy allows determination of both \( l \) and \( m \) while \( m \) is not accessible from photometry.

Mode identification from spectroscopy
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL

Moving bumps in certain high degree mode

Modes with high degree only visible in spectroscopy

Mode identification from spectroscopy
Different parameter sets can give the same time series of basic line profile
Distorted stellar surface divided into many surface elements

For each surface element, one computes:
- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Computation of orientation and area of each surface element

Mode identification from spectroscopy
Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- **Intensity**
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Pre-computed intensity spectra calculated for a given $T_{\text{eff}}$ and log g and $\cos \lambda$ with a stellar atmosphere code

Local $T_{\text{eff}}$ and log g vary in time

- Intensity varies in time
- Local line profile varies in time, i.e. both time varying width and EW

Mode identification from spectroscopy
Distorted stellar surface divided into many surface elements

For each surface element, one computes:
- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight
Sum up all the contributions of all the visible surface elements

Improved formalism that takes into account the influence of the rotation on pulsation
Inclusion of Coriolis correction terms

Mode identification from spectroscopy
### MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

### LINE PROFILE MODEL IN FAMIAS

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Rotation velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Gaussian intrinsic line profile</td>
<td>- Uniform and time-independent</td>
</tr>
<tr>
<td>- Quadratic limb-darkening law</td>
<td>- Slow</td>
</tr>
<tr>
<td>- Brightness variations</td>
<td></td>
</tr>
<tr>
<td>- Parameterized variable equivalent width due to temperature variations</td>
<td></td>
</tr>
</tbody>
</table>

| Pulsation velocity                                                        |
|---------------------------------------------------------------------------|                                                                                  |
| - Linear pulsation                                                        |
| - Effects of the Coriolis force to the first order taken into account     |

Zima (2006)

Mode identification from spectroscopy
* Observations required

* The methods
Ideally, use of isolated non-blended lines with
- High S/N ratio (> 200)
- High resolution (R > 40000)

Ideally, covering entire cycle of all modes
# > several hundred (say 100 per mode)

Ideally, accompanied by photometry
To increase S/N ratio:
Use of average of several lines formed in the same line-forming region

For too faint star with unavoidable low S/N ratio:
Use of cross-correlation profile

Assumption:
all used lines show the same temporal behavior
Silicon lines for pulsating B-type stars

- Sufficiently strong without being much affected by blending
- Dominated by thermal broadening → Gaussian profile
- LPVs little affected by temperature variations at the stellar surface

Si II lines for SPBs
Si III lines for β Cephei stars

LPVs for the Si III 4552 Å line of the β Cephei star 12 Lac
Pulsation frequencies unambiguously determined

Frequency analysis of both radial velocity and pixel-by-pixel variations + photometry
SPECTROSCOPIC MODE IDENTIFICATION TECHNIQUES

THE METHODS

* Line-profile fitting technique
* The moment method
* The IPS and pixel-by-pixel method
Mode identification from spectroscopy

**THE METHODS - Line-profile fitting technique**

- Theoretically computed LPVs for different values of $(l,m)$ and for the other parameters
- Observed line profile variations
- Goodness of fit measure e.g. based on least squares
- Set of “best fitting parameters”

Ledoux (1951), Osaki (1971), Smith (1977), etc.

Mode identification from spectroscopy
THE METHODS - **Line-profile fitting technique**

- Not only \((l,m)\) is determined but also the other parameters, such as the amplitude of the mode, the inclination angle and the rotational equatorial velocity

**BUT**

- Extremely CPU-time consuming
  * If no thorough investigation of parameter space, not sure to find the best fitting models
  * Simultaneous identification of multiple modes is unrealistic

- Depends much on the theoretical model
To decrease computational time:

Use of line profiles folded in only several bins for each detected frequency, such that the variations of other modes are assumed to cancel out

**BUT**

Phase binning is equivalent to extending the exposure times of the spectra

Phase smearing which can have an impact on the mode identification results
The entire absorption line profile can be replaced by the first few moments of a line profile, which are integrated quantities over the profile. Intensity information of each wavelength bin across the line profile.
THE METHODS

Theoretically computed
* moments
* bin intensities

Observed
* moments
* bin intensities

Goodness of fit measure

Set of “best fitting parameters”
In particular, identification of (l,m)
Integration over the whole line profile $p(v,t)$

$$M_n(t) \equiv \int_{-\infty}^{+\infty} dv \, v^n \, (1 - p(v,t))$$

$$\langle v^n \rangle \equiv \frac{M_n}{M_0}$$

$\equiv$ EW

Aerts et al. (1992)

The first three velocity moments

Radial velocity

Width

Skewness

Mode identification from spectroscopy
- Less CPU-time consuming
  * Thorough investigation of parameter space possible
  * Simultaneous identification of multiple modes feasible for a few modes (without using phase binning)

- Less model dependent
  * Not very sensitive to EW variations
  * Only assumption on the local line profile: it is symmetric, e.g. local line profile approximated with a constant Voigt function

BUT

Use of integrated quantities → only for low degree mode \((l \leq 4)\)
For every wavelength bin, for each detected pulsation frequency, computation of zero point, amplitude and phase using a multi-periodic least-squares fit with fitting formula as follows

\[ p(v, t) = C(v) + A_0(v) \sin(\sigma t + \Psi_0(v)) \]

\[ + A_1(v) \sin(2\sigma t + \Psi_1(v)) \]

\[ + A_2(v) \sin(3\sigma t + \Psi_2(v)) \]

Determining \((l,m)\) from phase distributions across the line profile
Extensive numerical simulations by Telting & Schrijvers (1997):

\[
\ell \approx 0.10 + 1.09 \left| \Delta \Psi_0 \right|/\pi,
\]

\[
|m| \approx -1.33 + 0.54 \left| \Delta \Psi_1 \right|/\pi
\]

Where maximum red-to-blue phase difference of detected frequency \( f \): \( \Delta \Psi_0 \)

of first harmonic of \( f \): \( \Delta \Psi_1 \)
Phase diagrams contain mostly information about $l$ and $|m|$ for direct identification without having to model the pulsation. However:

- Amplitude of the first harmonic of a frequency may be very low → need of very high S/N ratio (> 300)
- Method fails for stars with low $v_s\sin i$
- No information about the other parameters
- Uncertainty on $l$ and $m$ relatively large for low-degree modes
  - Error for $l$: ± 1
  - Error for $m$: ± 2
THE METHODS - The pixel-by-pixel method

For every wavelength bin, for each detected pulsation frequency, computation of $Z, A_i$ and $\phi_i$

$$Z + \sum_i A_i \sin(2\pi(f_i t + \phi_i))$$

For each detected pulsation frequency, use of the zero point, amplitude and phase to compute 10 profiles evenly distributed across one pulsation cycle

Direct line-profile fitting to this mono-mode profile
SPECTROSCOPIC MODE IDENTIFICATION TECHNIQUES

THE METHODS - The pixel-by-pixel method

Mantegazza (2000)

Allows identification of multiple modes without limits for (l,m)

BUT

- Very small value of vsini can prevent mode identification
- Method fails for stars whose dominant mode has high-amplitude relative to the projected rotational velocity
- No statistical significance limit of the derived identifications

Fourier Parameter Fit method by Zima (2006)
- Methods successfully applied to δ Scuti stars and β Cephei stars, applicable to all main-sequence pulsators hotter than the Sun

- The azimuthal order $m$ and its sign can be determined by both the moment method and the pixel-by-pixel method

  In FAMIAS, a positive value of $m$ denotes a prograde-mode, i.e. propagating in the direction of the stellar rotation

**BUT**

The degree $l$ is usually not determined unambiguously
GENERALITIES

- Apply both the pixel-by-pixel method (FPF method by Zima 2006) and the moment method (Briquet & Aerts 2003)

- The moment method is better suited than the FPF method
  * when \( \text{vsini} \) has a very small value (\( \text{vsini} < 10 \text{ km/s} \))
  * when the pulsation velocity is large relative to the projected rotational velocity

- The FPF method is better suited than the moment method for high-degree modes (\( l > 4 \))

- If both photometry and spectroscopy available:
  * search for frequencies in both of them
  * use photometric mode identification for \( l \) and fix this in spectroscopic mode identification
Mode identification from spectroscopy

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