Back Analysis and optimization methods with LAGAMINE

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First International Workshop on the Finite Element Code LAGAMINE

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Outline

- **What is back analysis?**
- **Deterministic approach with *Optim***
- **Stochastic approach with *AI_Lagamine***

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- What is back analysis?
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- Stochastic approach with *AI_Lagamine*

What is back analysis?

- Main performances of numerical modelling depend on:
  - Choice of constitutive models
  - Identification of parameters
    - Tests are often expensive and can be difficult to interpret:

complex laws, heterogeneity and/or noise in measurements …

→ manual calibration is often difficult

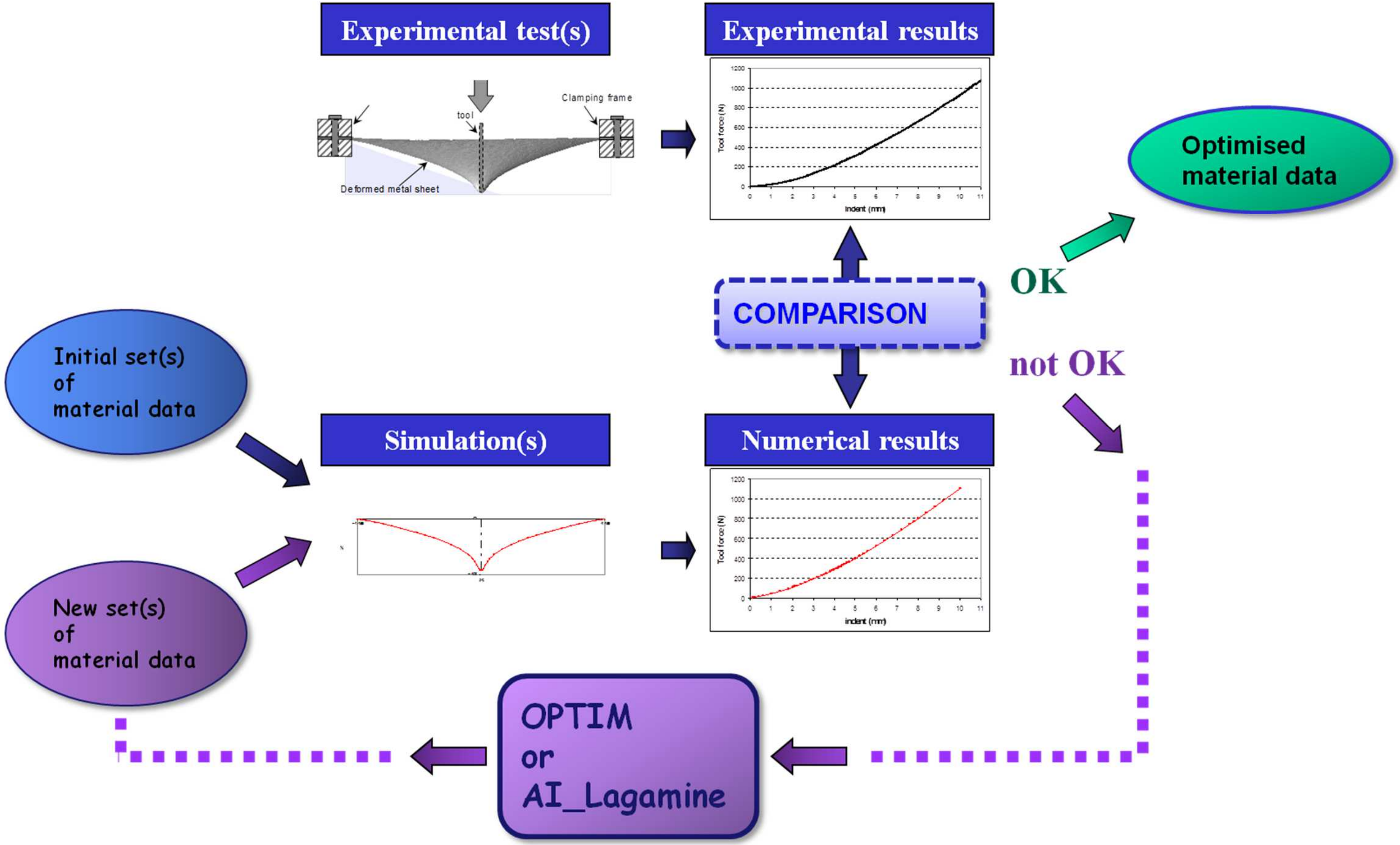
⇨ **Back analysis as a tool to help identification**

Automatic strategy to fit material parameters

until numerical results ≈ experimental results

option included in LAGAMINE FE code

What is back analysis?



What is back analysis?

In LAGAMINE:

- Two optimization approaches are available:
  - Levenberg-Marquardt method through ***Optim***
  - Genetic algorithm method through ***AI_Lagamine***

Advantages and drawbacks for both approaches

- Applicable on all parameters of all constitutive laws
- Efficiency depends on
  - well-adapted model
  - accuracy of experimental results that have to be fited by the model
  - application that should be simulated with limited CPU time

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Optim – Levenberg-Marquardt optimization

- Iterative method
- **Levenberg-Marquardt algorithm** (multivariate optimization)
- Minimization of the difference between the experimental and numerical results (for each test)

Optim – Levenberg-Marquardt optimization

For each set of data and for each test:

Several simulations are performed in parallel:

- 1 with the initial parameters: $p_1 , \dots, p_j , \dots p_k$
- and for each parameter to fit p_i, 2 simulations:

$$p_1 , \dots, p_i + dp_i , \dots p_k$$
$$p_1 , \dots, p_i - dp_i , \dots p_k$$

The perturbation dp_i is small

$dp_i = \delta * p_i$ with perturbation factor $\delta = 0.001$ (for example)

→ convergence quickly obtained

Optim – Sensitivity analysis

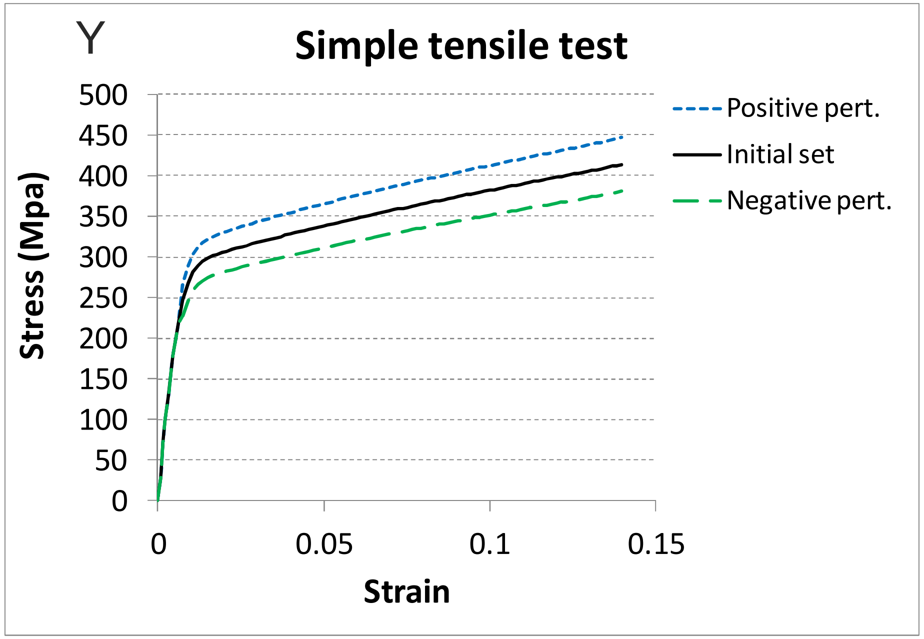
Sensitivity $S(p_i)$

→ computed for each test and each parameter p_i to fit

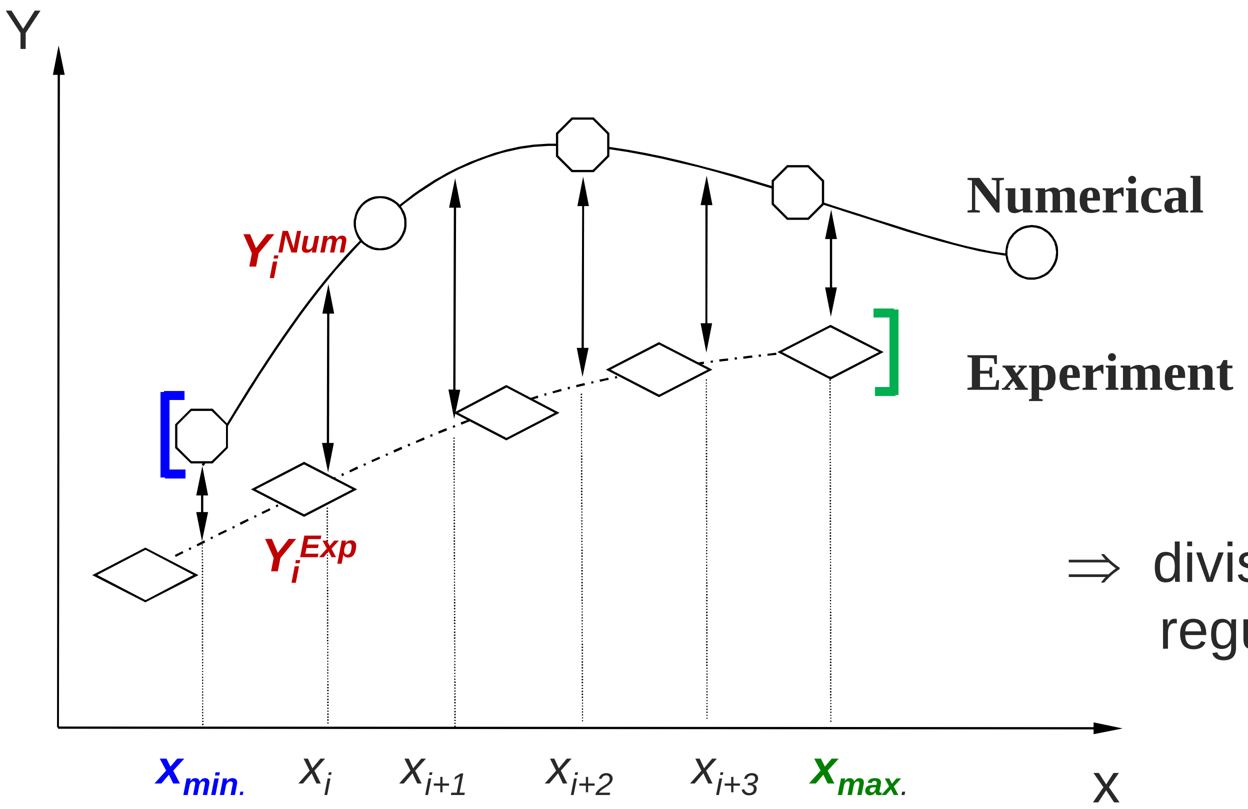
→ at each Lagamine step

$$S(p_i) = \frac{Y_{p_i+dp_i} - Y_{p_i-dp_i}}{2 * dp_i}$$

Example



Optim – Error function (to minimize)



⇒ division of $[x_{min.}, x_{max.}]$ into regular sub-intervals

$$\text{Error} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_i^{Num} - Y_i^{Exp}\right)^2}$$

Optim – Example

Characterization (Aluminium AlMgSc)

Elastic part: Hooke's law (E, ν)

Plastic part: Hill's law (Hill48):

$$F_{HILL}(\underline{\sigma}) = \frac{1}{2}\left[H(\sigma_{xx} - \sigma_{yy})^2 + G(\sigma_{xx} - \sigma_{zz})^2 + F(\sigma_{yy} - \sigma_{zz})^2 + 2N(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2)\right] - \sigma_F^2 = 0$$

Isotropic hardening: Voce's formulation:

$$\sigma_F = \sigma_0 + K(1 - \exp(-n.\varepsilon^{pl}))$$

Back-stress (kinematic hardening): Ziegler's equation:

$$\dot{\underline{X}} = C_A \frac{1}{\sigma_F}(\underline{\sigma} - \underline{X}).\dot{\varepsilon}^{pl} - G_A.\underline{X}.\dot{\varepsilon}^{pl}$$

Optim – Example

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Young modulus: **E** and Poisson ratio: **ν** defined by tensile tests

F, **G** & **H** defined by tensile tests in 3 directions (RD, TD, 45°)

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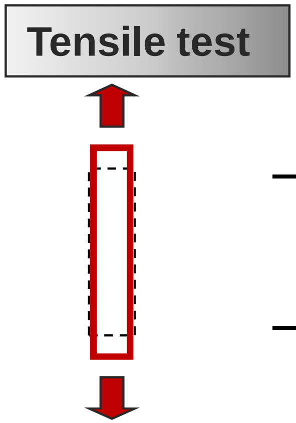
F, **G** & **H** defined by tensile tests in 3 directions (RD, TD, 45°)

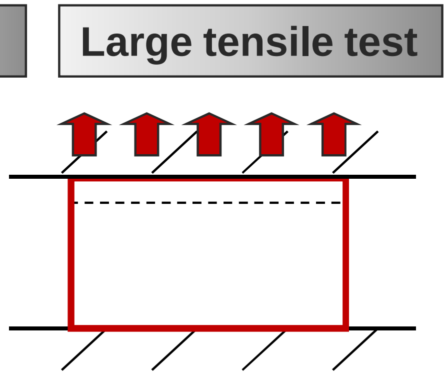
N, **σ_0**, **K**, **n**, **C_A**, **G_A** defined by Optim

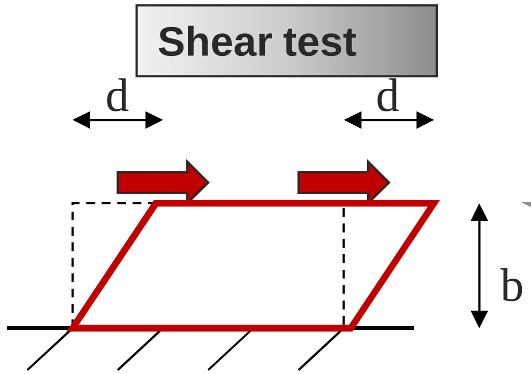
Optim – Example

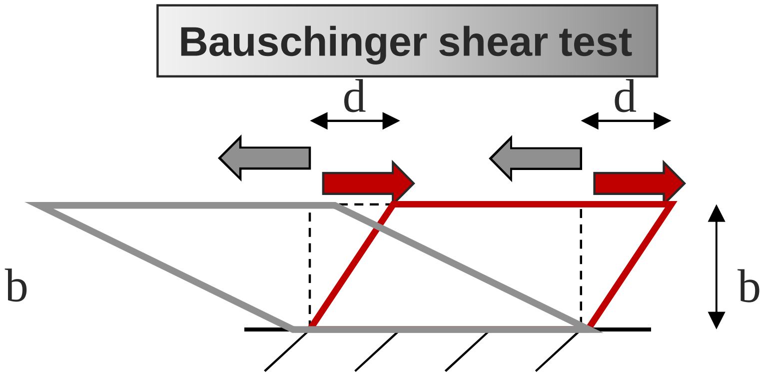
Example of tests chosen for the characterization (Aluminium AlMgSc):

- Tensile test, large tensile test
- Monotonic simple shear test, Bauschinger simple shear tests (2 levels)
- Orthogonal tests (2 levels)
- Indent test

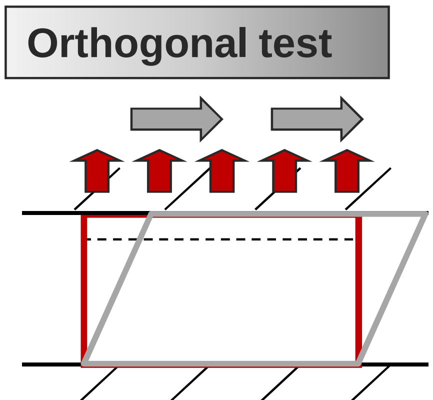


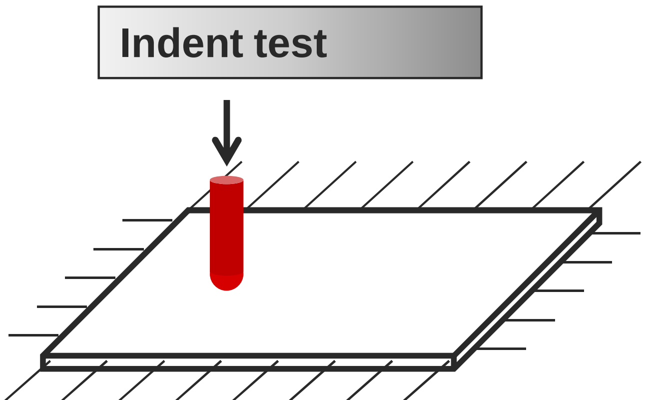






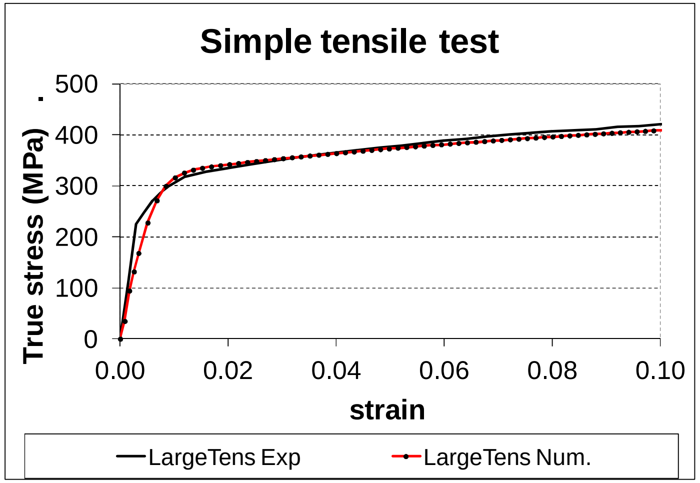
2 levels of pre-strain:
gamma= d/b= 10 & 30%

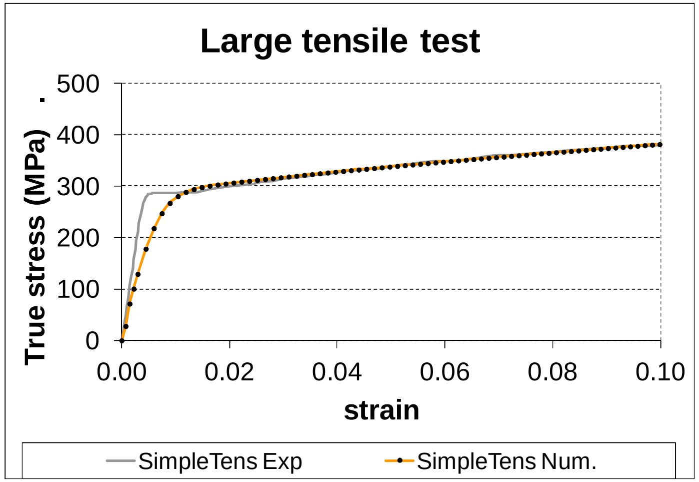


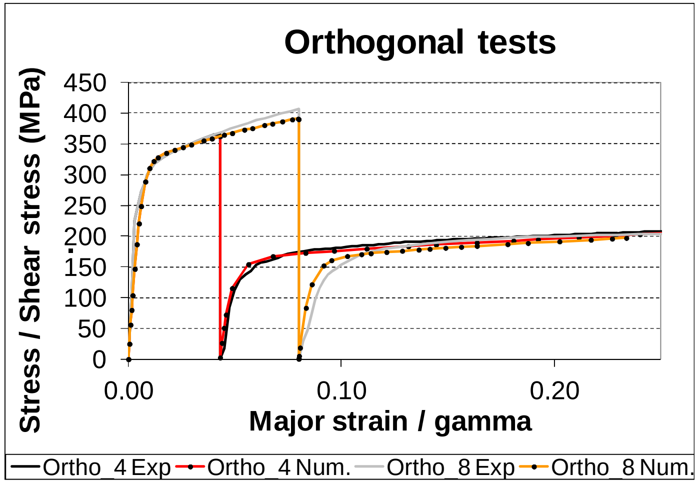


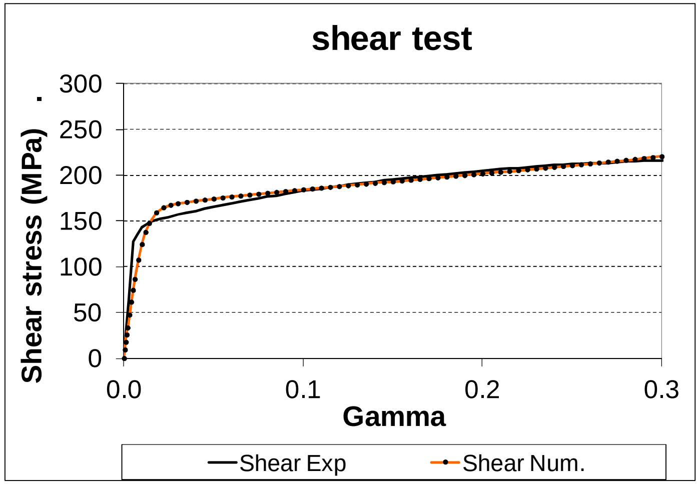
Optim – Example

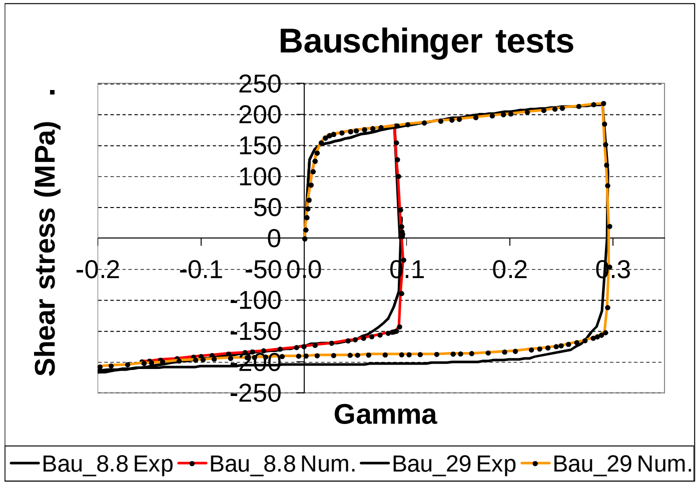
Comparison: experiments and numerical results

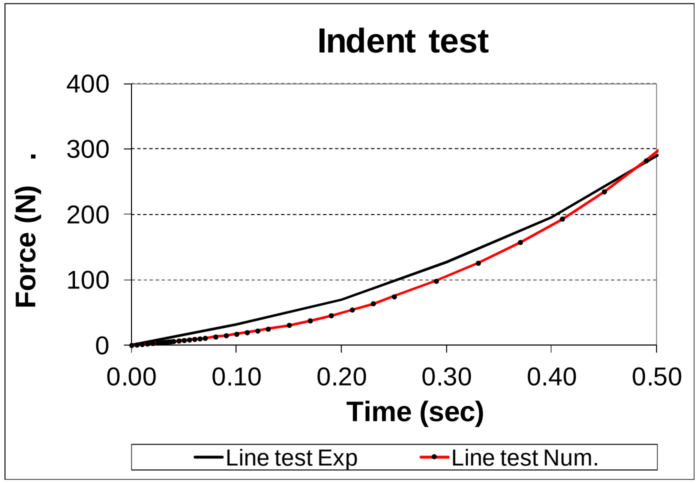












Optim – Comments

- The method is efficient for complex laws
- Possibility of fitting several data simultaneously
- The tests chosen must be sensitive to the parameters to fit
- The range of each parameter must be defined
- Several initial sets of data are to be tested to avoid local minimum
- The efficiency of the method is linked to the initial set of data
- Advantage: possibility of choosing complex tests inducing heterogeneous stress and strain fields close to the ones reached during the real process (but CPU !!!)

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- Deterministic approach with *Optim*
- **Stochastic approach with *AI_Lagamine***

AI_Lagamine – Genetic algorithm optimization

- Numerical assumptions of complex problems
- Uncertainties on experimental measurements
- Spatial variability of parameters

⇨ **Uniqueness of the parameter set is not always guarantee, Parameters can be interdependent** (mainly in geomaterials)

⇨ ***GENETIC ALGORITHM* approach to quickly converge to several approximated parameter sets**

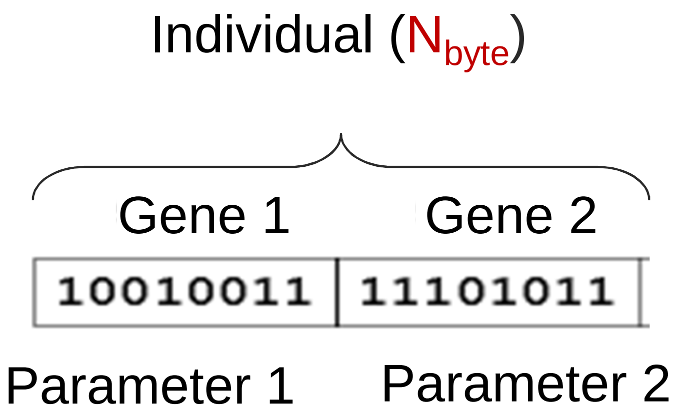
S. Levasseur. 2007. *Analyse inverse en géotechnique : Développement d'une méthode à base d'algorithmes génétiques.* PhD thesis, Université Joseph Fourier, Grenoble.

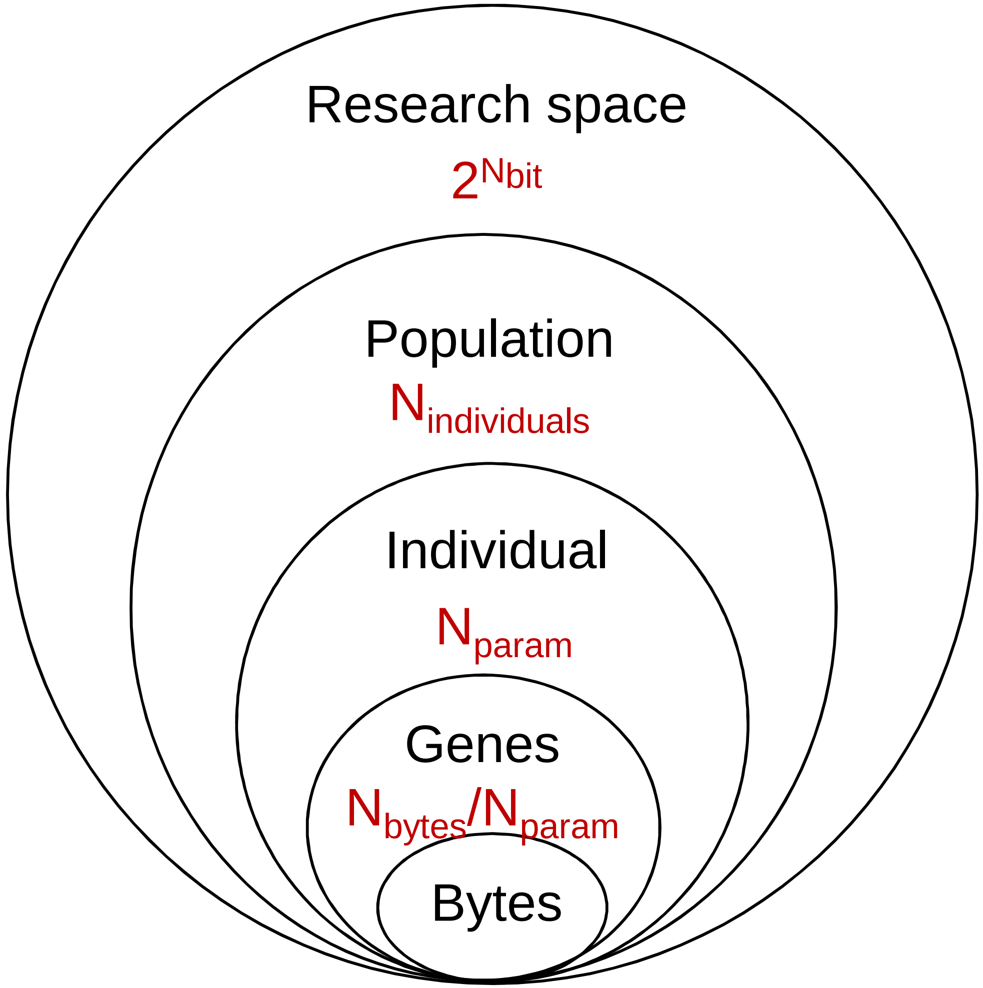
G. Sanna. 2011. *Geoenvironmental study on Boom Clay by inverse analysis.* Master thesis, Université Joseph Fourier, Grenoble.

AI_Lagamine – Genetic algorithm optimization

- Inspired by Dawin theroy of evolution

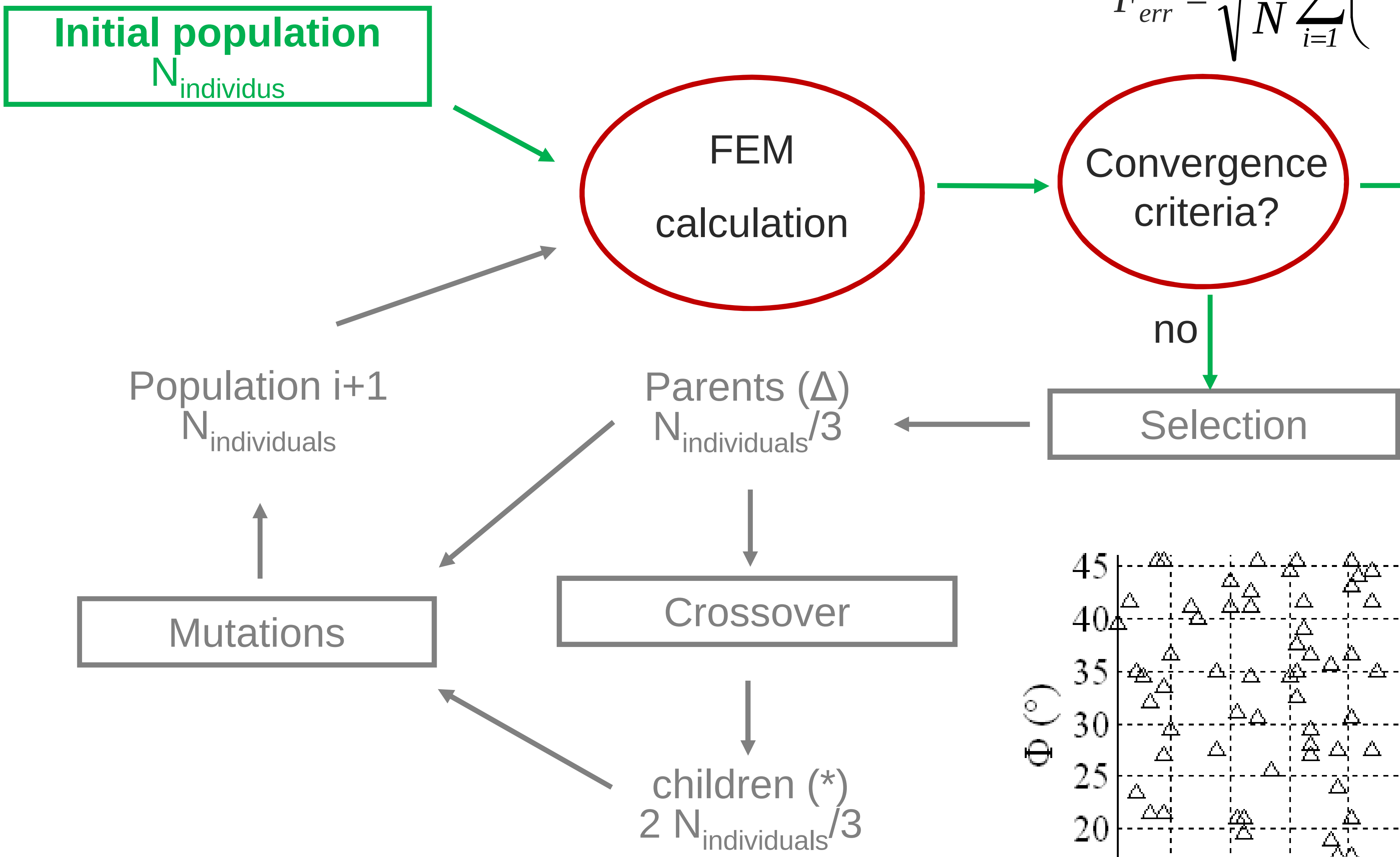
Example of two parameters identification

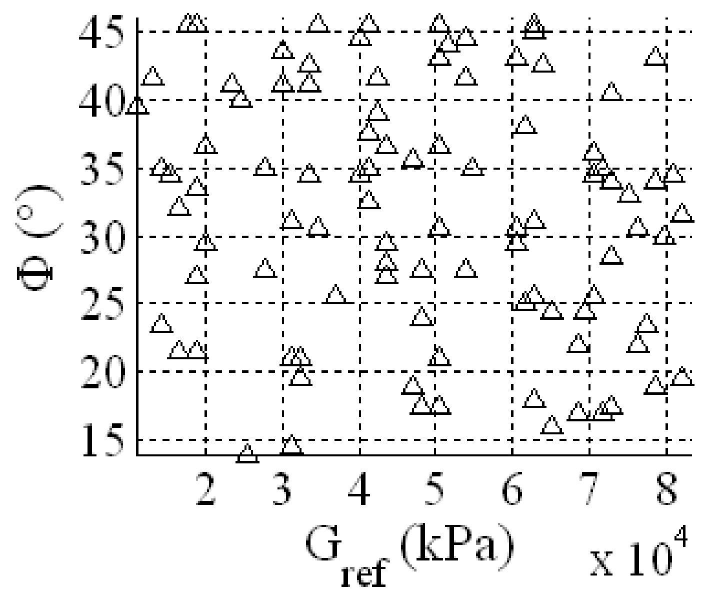




Al_Lagamine – Genetic algorithm optimization

$$F_{err} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{Ue_i - Un_i}{\Delta U_i}\right)^2}$$





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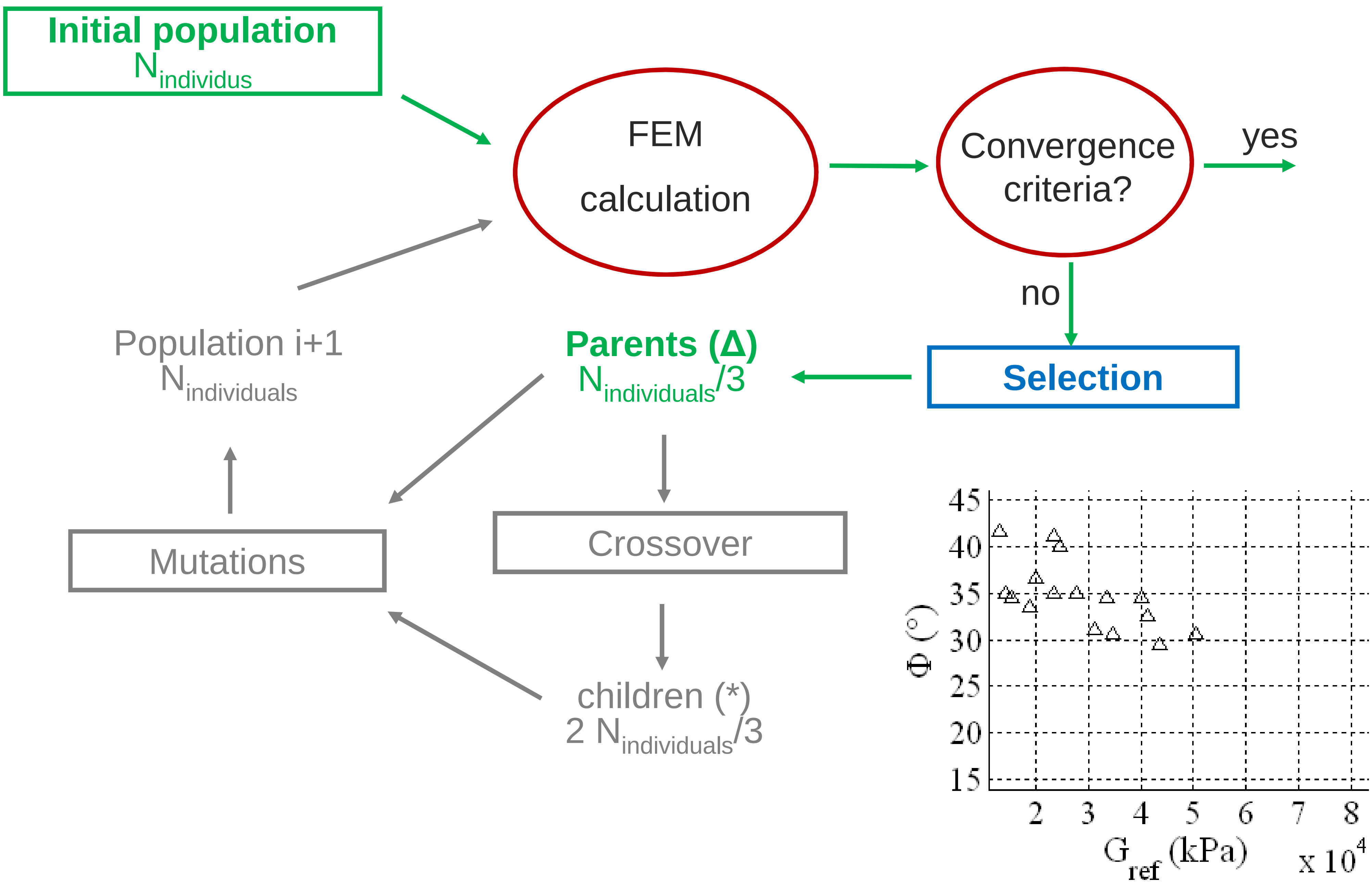

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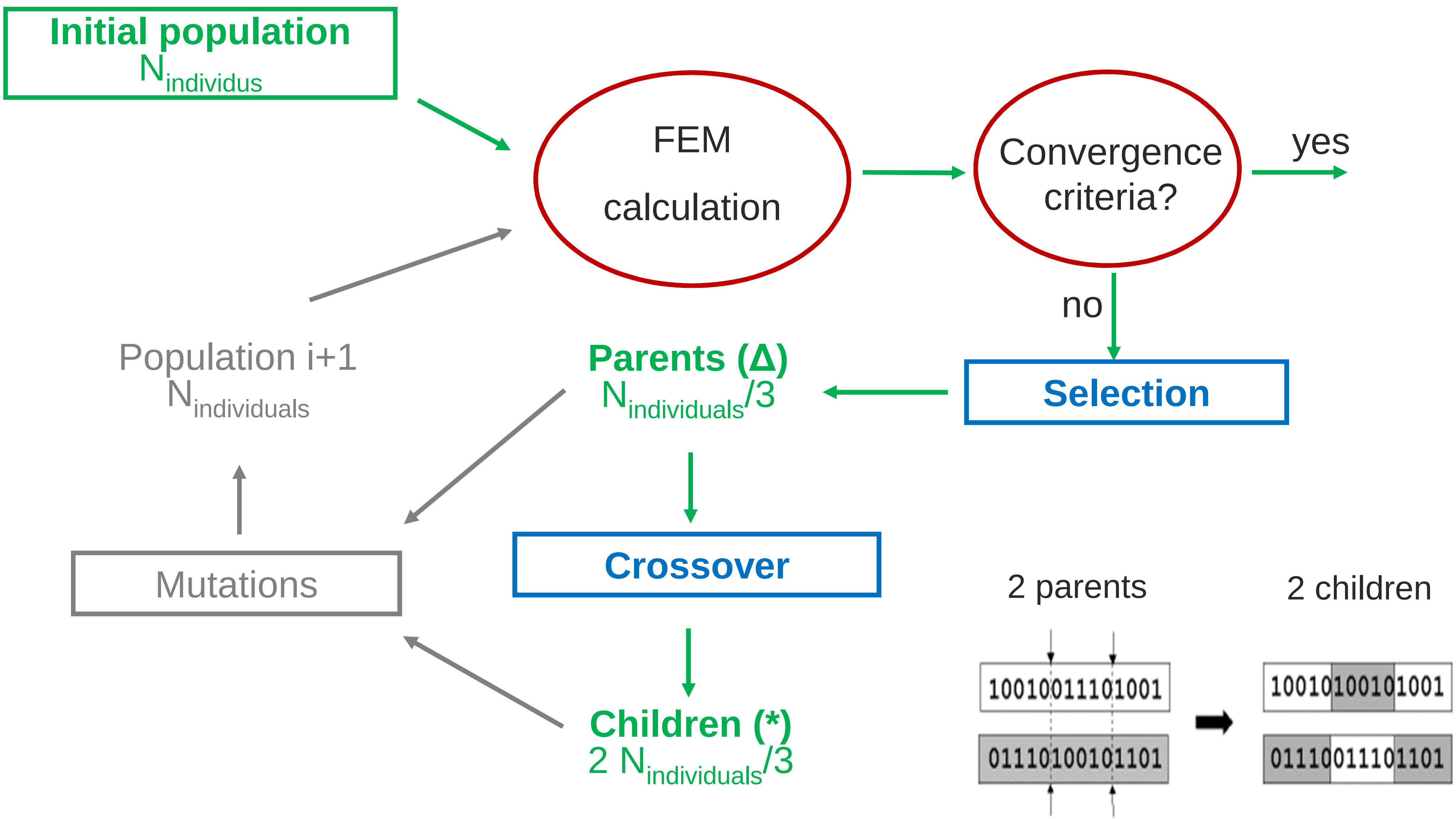

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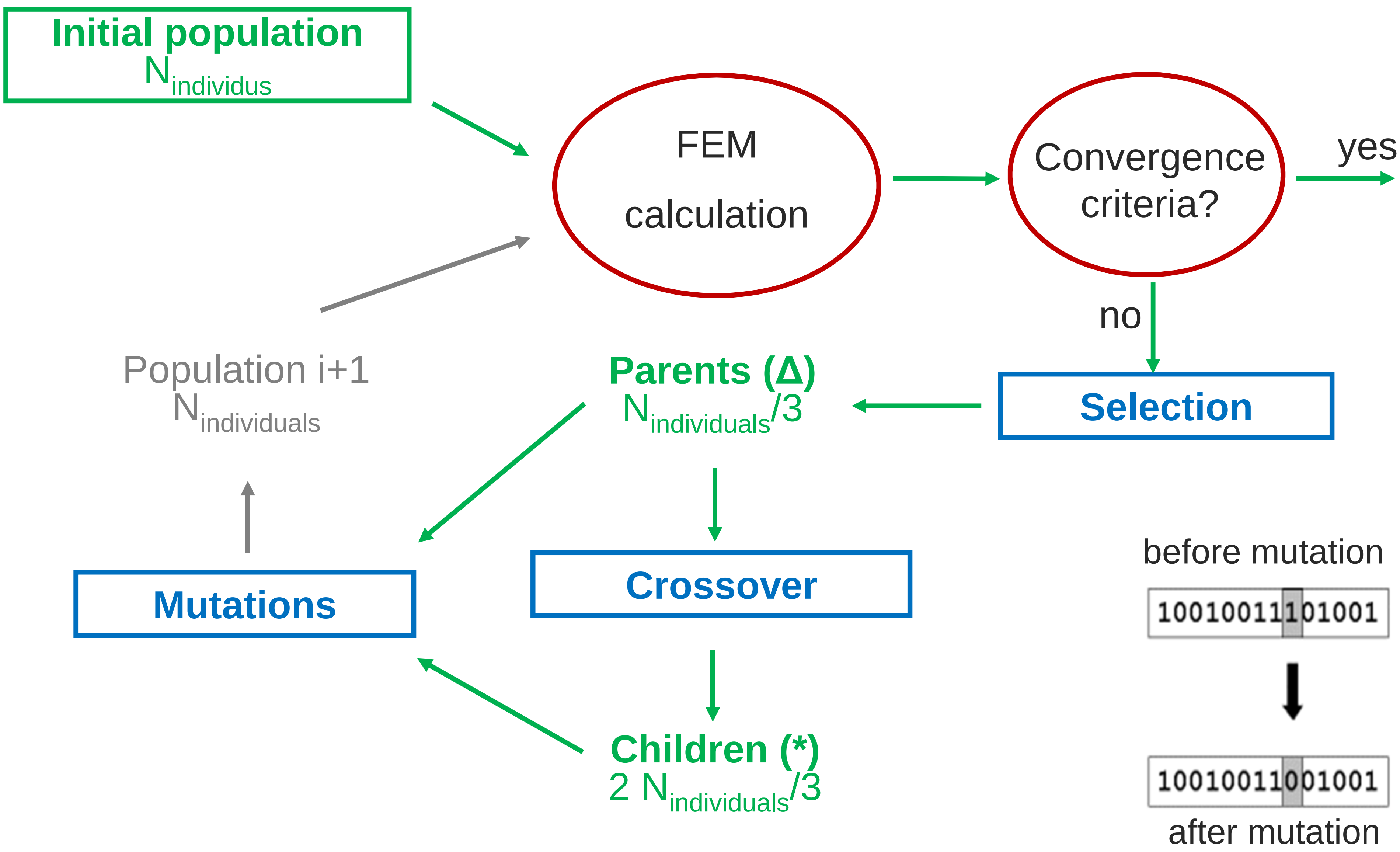


AI_Lagamine – Genetic algorithm optimization

Initial population
$N_{individus}$

FEM calculation

Convergence criteria?

yes

no

Selection

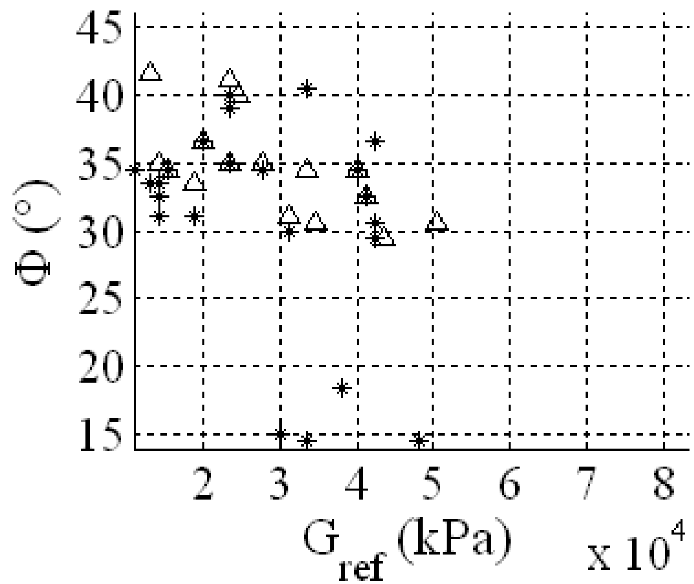
Parents (Δ)
$N_{individuals}/3$

Population i+1
$N_{individuals}$

Crossover

Mutations

Children (*)
$2\ N_{individuals}/3$

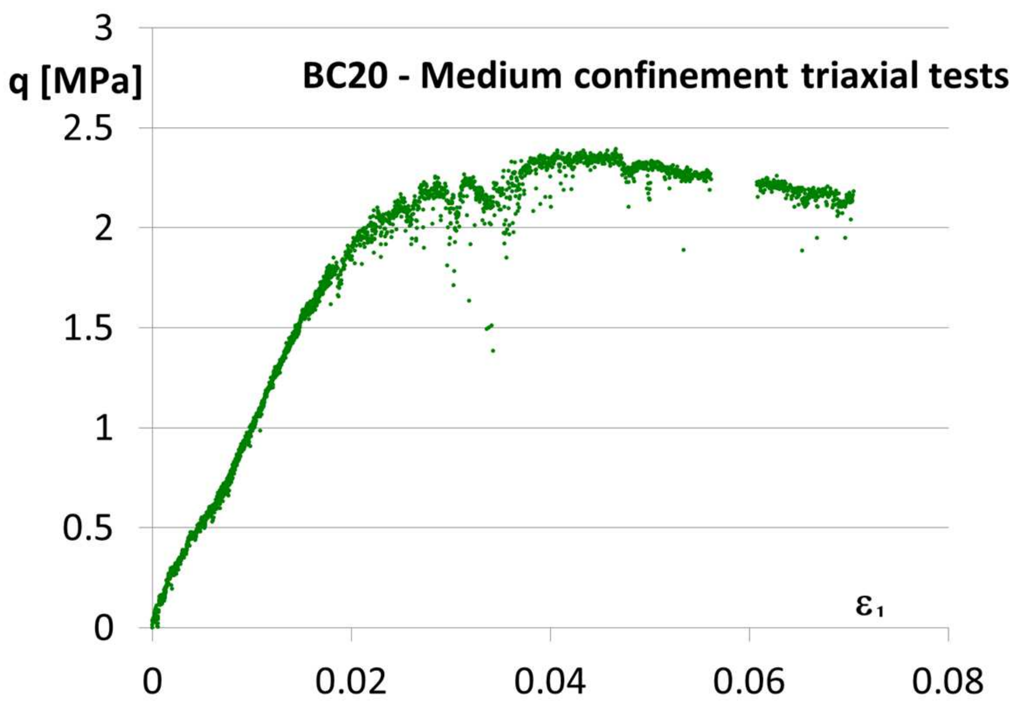


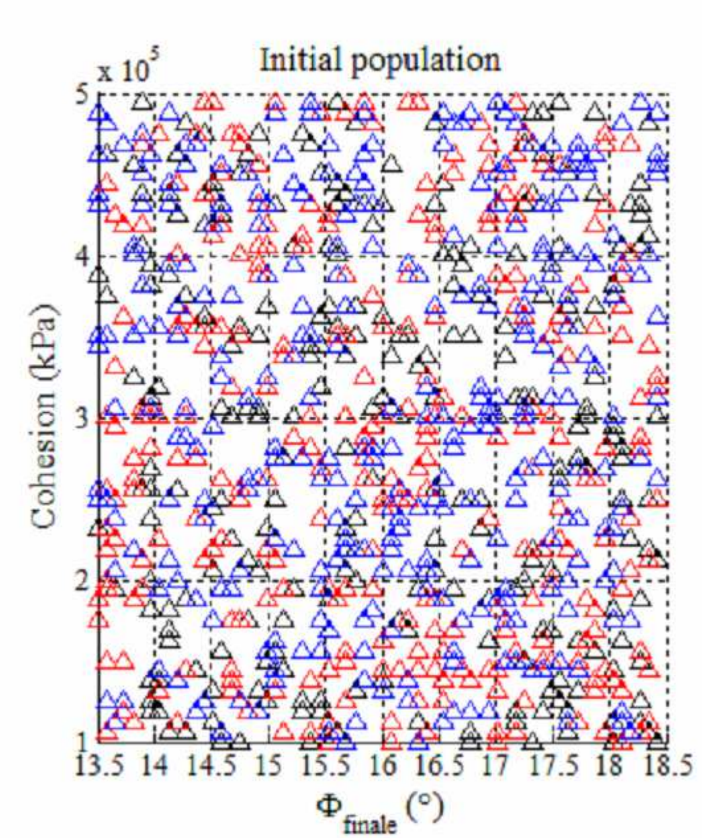
Al_Lagamine – Example – Boom Clay triaxial test

- Calibration of triaxial test performed by Coll (2005) – p'_0 = 2.3MPa

Elastoplastic model with Drucker-Prager criterion and friction angle hardening

Calibration of cohesion c and final friction angle ϕ_{final}



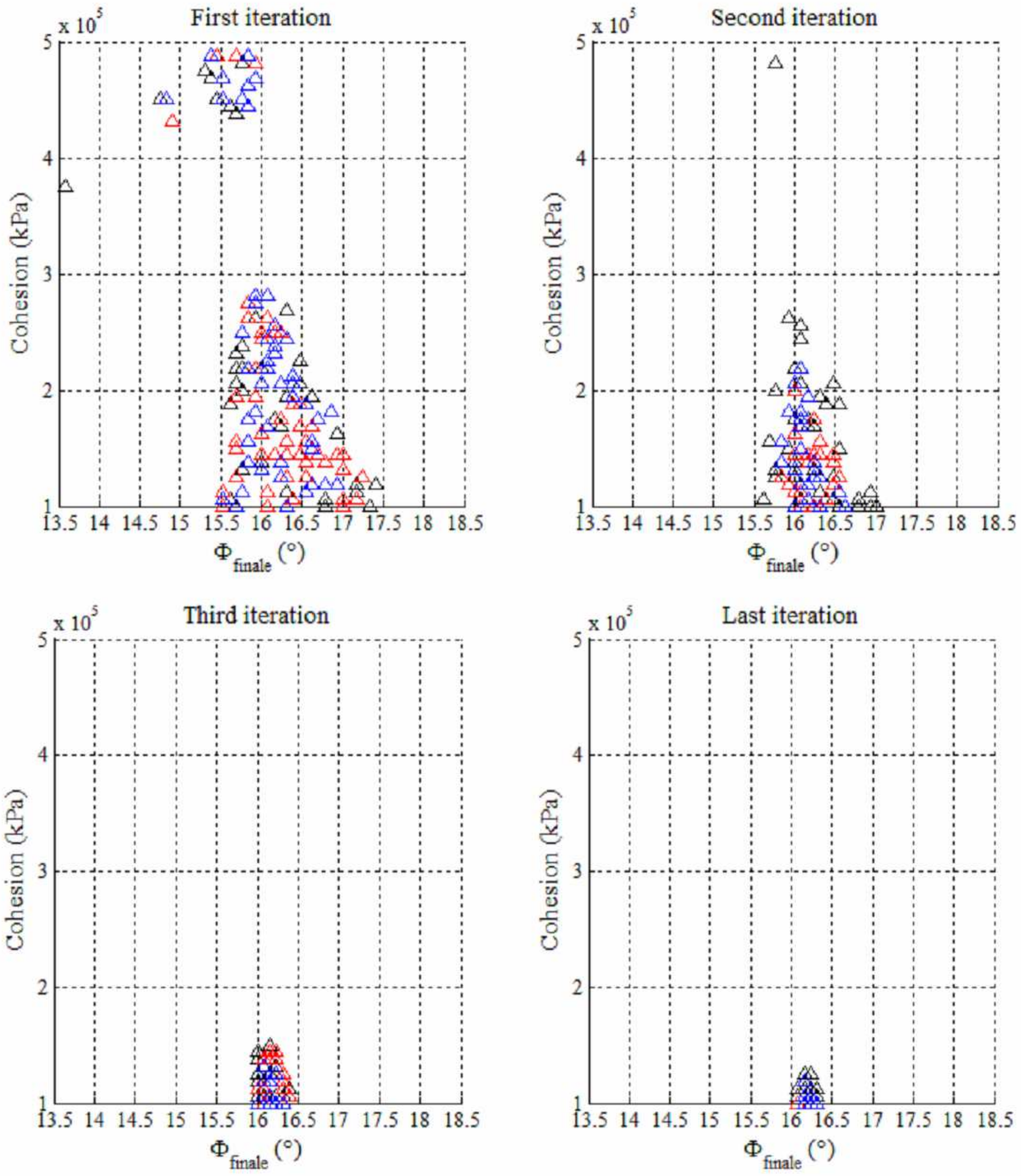


E = 100MPa; ν = 0.2;

$\phi_{initial}$ = 11°; ψ = 10°; B_p = 0.002

GA run 3 times on (ϕ_{final}, c) research space

AI_Lagamine – Example – Boom Clay triaxial test



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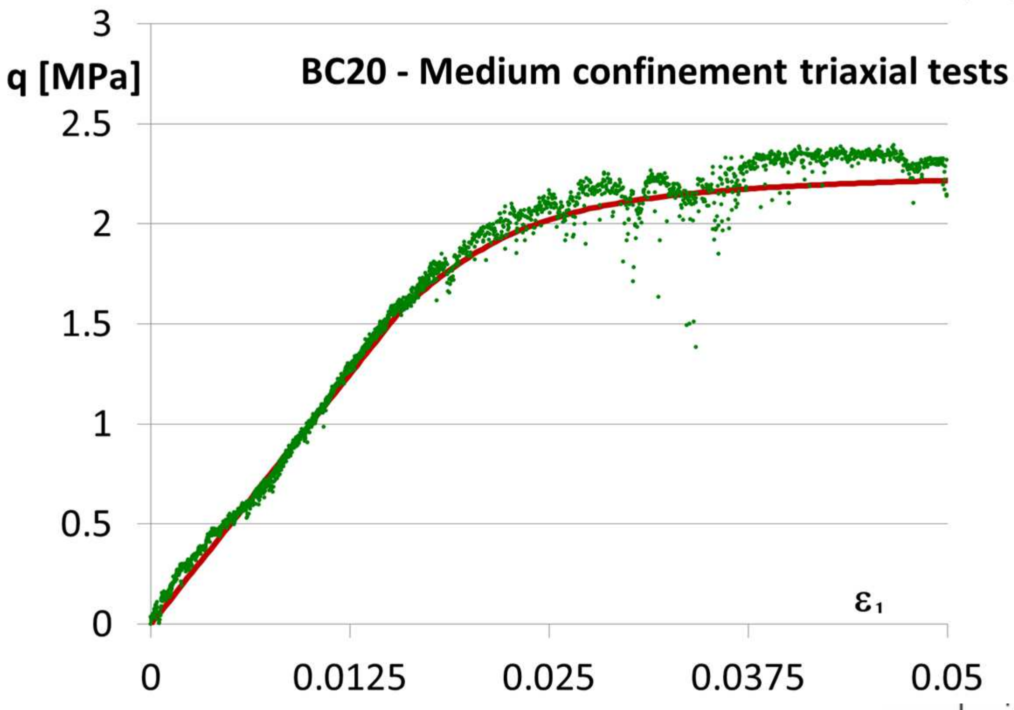

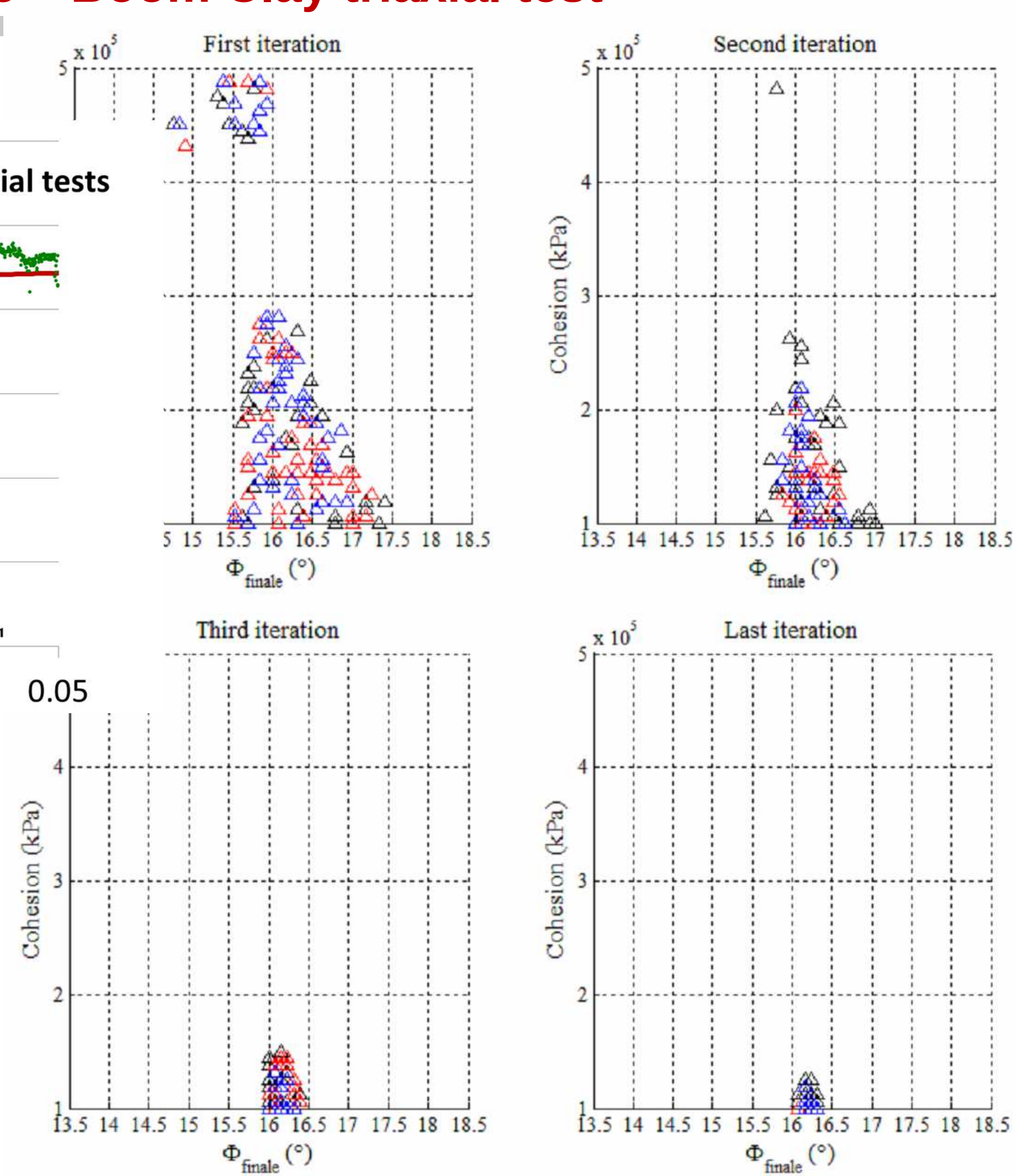

Good fit with

$16 < \phi_{final} < 16.5$

$100kPa < \quad c \quad < 120kPa$

Al_Lagamine – Comments

- The range of variation for each parameter must be defined, however:
- The method is efficient even for disperse measurements
- Same solutions are identified whatever are the initial sets of parameters randomly chosen (no local minimum)
- Quick convergence when tests are sensitive to the parameters, otherwise identification of relations between these parameters
- Possibility of
  - identifying a large number of parameters simultaneously
  - fitting several data simultaneously

  ⇨ Estimation of averaged parameter sets satisfying all data
  - choosing complex tests inducing heterogeneous stress and strain fields close to the ones reached during the real process (but CPU !!!)

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Optim

Levenberg-Marquardt optimization

AI_Lagamine

Genetic algorithm optimization

Automatic strategy to estimate material parameters
(automatic pre- and post-analysis)

Applicable on all parameters of all constitutive laws

Possibility of fitting several types of data simultaneously

Optim:

- The tests chosen must be sensitive to the parameters
- The efficiency of the method is linked to the initial set of data, so several initial sets of data are to be tested to avoid local minima
- More efficient for homogeneous materials and well-posed problems

AI_Lagamine:

- If tests are not enough sensitive to the parameters then identification of relations between these parameters
- Same solutions are identified whatever are the initial sets of parameters randomly chosen (no local minimum)
- More efficient for heterogeneous materials and ill-posed problems (with lot of uncertainties)

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But be careful, these tools can not replace

any physical interpretation!

Some references on optimization methods

Geotechnics (laboratory or in situ measurements):

- Levasseur S., Malécot Y., Boulon M., Flavigny E. (2008) Soil parameter identification using a genetic algorithm. *Int. J. Numer. Anal. Meth. Geomech.*, vol. 32(2): 189-213.
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Mechanic of materials:

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