Equivalent Static Wind Loads for structures with non-proportional damping

N. Blaise, T. Canor & V. Denoël

University of Liège (Belgium)

University of Cape Town, South Africa
2 September 2013
Introduction

Illustrative example

Equivalent static wind loads

Conclusion
Analysis of structures under random excitations

Structures

are subjected to wind excitations

Wembley Square, Cape town (2009)

Vista High school, Cape town (2009)
Dynamic analysis of large structures

■ Modal basis

\[ \Phi^T M \Phi = I \quad \Phi^T C \Phi = D \quad \Phi^T K \Phi = \Omega \]

Normal modes of vibration

Modal damping matrix

Modal stiffness matrix (diagonal)

□ Rayleigh Damping

\[ C = \alpha K + \beta M \quad \Rightarrow \quad D = D_d \quad (\text{diagonal}) \]
Dynamic analysis of large structures

- **Modal basis**
  
  
  $\Phi^T M \Phi = I$  \quad $\Phi^T C \Phi = D$  \quad $\Phi^T K \Phi = \Omega$

  Modal stiffness matrix (diagonal)

- **Rayleigh Damping**
  
  $C = \alpha K + \beta M$  \quad $D = D_d$ (diagonal)

- **Sources of non-proportionality**
  
  - damping devices (TMD, TLCD), aerodynamic damping and...

  $D$ is not diagonal
Dynamic analysis of large structures

- Modal basis

\[ \Phi^T M \Phi = I \quad \Phi^T C \Phi = D \quad \Phi^T K \Phi = \Omega \]

- Modal stiffness matrix (diagonal)

- Rayleigh Damping

\[ C = \alpha K + \beta M \quad \rightarrow \quad D = D_d \quad \text{(diagonal)} \]

- Sources of non-proportionality
  - damping devices (TMD, TLCD), aerodynamic damping and...

\[ D \text{ is not diagonal} \]

- Coupled system of equation of motion

\[ \ddot{\mathbf{q}} + D \dot{\mathbf{q}} + \Omega \mathbf{q} = \mathbf{g} \quad \text{Generalized forces} \]
Dynamic analysis of large structures

Split damping matrix

\[ D = D_d + D_o \]

\[ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} = \begin{pmatrix} \square \\ \square \end{pmatrix} \quad \text{Diagonal elements} \quad \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix} \quad \text{Off-diagonal elements} \]

Dynamic analysis of large structures

Split damping matrix

\[ \mathbf{D} = \mathbf{D}_d + \mathbf{D}_o \]

Decoupling approximation\(^1\)

\[ \mathbf{H}_d = (-\mathbf{I} \omega^2 + j \omega \mathbf{D}_d + \Omega)^{-1} \quad \text{Inversion of a diagonal matrix only} \]

Decoupled system

Dynamic analysis of large structures

- Split damping matrix

\[
\begin{pmatrix}
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\end{pmatrix}
= \begin{pmatrix}
\Box & \Box \\
\Box & \Box \\
\Box & \Box \\
\Box & \Box \\
\end{pmatrix}
+ \begin{pmatrix}
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\end{pmatrix}
\]

Diagonal elements

Off-diagonal elements

- Decoupling approximation\(^1\)

\[
H_d = (-\mathbf{I}\omega^2 + j\omega \mathbf{D}_d + \Omega)^{-1}
\]

Inversion of a diagonal matrix only

Decoupled system

- Full matrix inversion

\[
\begin{align*}
\mathbf{H} &= (-\mathbf{I}\omega^2 + j\omega \mathbf{D} + \Omega)^{-1} \\
\mathbf{H} &= (\mathbf{I} + j\omega \mathbf{H}_d \mathbf{D}_o)^{-1} \mathbf{H}_d
\end{align*}
\]

Full matrix inversion

Coupled system

Asymptotic expansion method

Key-idea\(^1\)

\[
H = (I + j\omega H_d D_o)^{-1} H_d
\]

\[
(I + X)^{-1} \approx I - X + X^2 - \ldots = I + \sum_{i=1}^{k} (-X)^i
\]

**Condition:** \( r(X) = \|\lambda\|_{\infty} < 1 \)

Asymptotic expansion method

Key-idea
\[ H = (I + j\omega H_dD_o)^{-1} H_d \]

\[ (I + X)^{-1} \approx I - X + X^2 - ... = I + \sum_{i=1}^{k} (-X)^i \]

Condition: \[ r(X) = \|\lambda\|_{\infty} < 1 \]

Approximation of \( H \)
Decoupling approximation (diagonal)
\[ H_k = H_d + \sum_{i=1}^{k} (-j\omega)^i (H_dD_o)^i H_d \]

No full matrix inversion
Inversion of a diagonal matrix only
Approximate the coupled system

---

Stochastic modal analysis

- Exact solution

\[ S^{(q)} = H S^{(g)} H^* \]

PSD matrix of generalized forces

PSD matrix of modal displacements

\[ {\text{Canor, Blaise and Deno"el. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. }} \text{Journal of Sound and Vibration 331, 24, 5283-5291} \]
Stochastic modal analysis

- Exact solution

\[ S^{(q)} = HS^{(g)}H^* \]

- PSD matrix of generalized forces
- PSD matrix of modal displacements

- Decoupling approximation

\[ S^{(q_d)} = H_dS^{(g)}H_d^* \]
Stochastic modal analysis

- **Exact solution**

\[ S^{(q)} = H S^{(g)} H^* \]

- PSD matrix of generalized forces
- PSD matrix of modal displacements

- **Decoupling approximation**

\[ S^{(q_d)} = H_d S^{(g)} H_d^* \]

- **Stochastic modal analysis\(^1\)**

\[ S^{(q_k)} = H_k S^{(g)} H_k^* \]

- Solution in the uncoupled system

\[ S^{(q_k)} = S^{(q_d)} + \sum_{i=1}^{k} \Delta S^{(q_i)} \]

- Corrections terms due to non-proportionality damping

---

\(^1\) Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291
Chen & Kareem formulation

\[ p_j^e = g_j \sum_{m=1}^{M} W_{jm} \psi_m \]

Objective:

Approximate formulation \( p_j^{e,k} \) in case of non-proportional damping

---

Introduction

Illustrative example

Equivalent static wind loads

Conclusion
306 m Tall building

10-lumped-mass cantiliver beam model

Random excitation: wind
- 1-D Gaussian velocity field

Structural and aerodynamic data from

Two studied cases:
- Tuned Mass Damper
- Tuned Liquid Column Damper

---

Modal properties

■ Inertial forces per unit surface

\[ \Psi_m = K \Phi_m \]

\( m^{th} \) inertial force

\( m^{th} \) modal shape

■ First two modes (five modes considered for the analysis)

With TMD:

\[ f_1 = 0.16 \text{Hz} \]

\[ \zeta_1 = 6.5\% \]

\[ f_2 = 0.19 \text{Hz} \]

\[ \zeta_2 = 9.7\% \]

With TLCD:

\[ f_1 = 0.16 \text{Hz} \]

\[ \zeta_1 = 2.5\% \]

\[ f_2 = 0.19 \text{Hz} \]

\[ \zeta_2 = 3.5\% \]
# Structural analysis

- **Covariance matrix of modal displacements**

\[
\int_{-\infty}^{+\infty} S(q_k) \, dq = C^{(qd)} + \sum_{i=1}^{k} \Delta C^{(q_i)}
\]

<table>
<thead>
<tr>
<th>With TMD</th>
<th>Exact solution ((k=\infty))</th>
<th>Decoupled ((k=0))</th>
<th>(k^{th}) approximation of (H) ((k=1))</th>
<th>(k^{th}) approximation of (H) ((k=2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C(q))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. relative errors</td>
<td></td>
<td></td>
<td>23%</td>
<td>14%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>With TLCD</th>
<th>Exact solution ((k=\infty))</th>
<th>Decoupled ((k=0))</th>
<th>(k^{th}) approximation of (H) ((k=1))</th>
<th>(k^{th}) approximation of (H) ((k=2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C(q))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. relative errors</td>
<td></td>
<td></td>
<td>7%</td>
<td>2%</td>
</tr>
</tbody>
</table>
Standard deviations of nodal displacements

- Decoupling approximation
- 1st order approximation
- 2nd order of approximation
Structural design

- Envelope values (min and max) of the structural responses
  - Extreme value theory

\[ r_{j}^{k,max} = g_{j}^{r_{k}} \sigma_{j}^{r_{k}} \]

- Wind load \( p(x,t) \)
  - Max: ?
  - Peak factor
  - Standard deviation
Introduction

Illustrative example

Equivalent static wind loads

Conclusion
Asymptotic expansion method

- **Weighted combinations of the inertial forces**

\[
p_{j}^{e,k} = g_{j}^{k} \sum_{m}^{M} W_{jm}^{k} \psi_{m}
\]

\( k^{th} \) approximation of the weighting coefficients

\[
W_{jm}^{k} = \alpha_{j}^{k} W_{jm}^{d} + \sum_{i=1}^{k} \Delta W_{jm}^{i}
\]
Asymptotic expansion method

- Weighted combinations of the inertial forces

\[ p_j^{e,k} = g_j^k \sum_m^M W_{jm}^k \psi_m \]

\[ k^{th} \text{ approximation of the weighting coefficients} \]

\[ W_{jm}^k = \alpha_j^k W_{jm}^d + \sum_{i=1}^k \Delta W_{jm}^i \]

- Definition of the ESWL

\[ p_j^{e,k} = \alpha_j^k p_j^{e,d} + \Delta p_j^{e,k} \]

\[ k^{th} \text{ approximation of the ESWL} \]

- correction resulting from the non-proportionality of damping

- ESWL for the uncoupled system

- scaled coefficients
Weighting coefficients

First inertial force

With TMD

\[ W_{j1} \]

\[ \frac{W_{j1}^k - W_{j1}}{W_{j1}} \text{ [%]} \]

With TLCD

\[ W_{j1} \]

\[ \frac{W_{j1}^k - W_{j1}}{W_{j1}} \text{ [%]} \]

- Red: Decoupling approximation
- Black: 1st order approximation
- Blue: 2nd order of approximation
ESWL - Horizontal displacement at the top

With TMD

$P_{x,t}^{e,k} \text{ [N/m}^2\text{]}$

$\frac{P_{x,t}^{e,k} - P_{x,t}^{e}}{P_{x,t}^{e}} \text{ [%]}$

$k$

$d$

$2$

$X_{x,t}^{e}$ [m]

$\frac{X_{x,t}^{e,k} - X_{x,t}^{e}}{X_{x,t}^{e}} \text{ [%]}$
ESWL-Horizontal displacement at the top

Introduction
Illustrative example
Equivalent static wind loads
Conclusion
Introduction

Illustrative example

Equivalent static wind loads

Conclusion
Asymptotic expansion of the modal transfer matrix enables to avoid full transfer matrix inversion.
Asymptotic expansion of the modal transfer matrix enables to avoid full transfer matrix inversion

New method for the establishment of ESWL for structures with non-proportional damping analysed in the modal basis
- Asymptotic expansion of the modal transfer matrix enables to avoid full transfer matrix inversion
- New method for the establishment of ESWL for structures with non-proportional damping analysed in the modal basis
- Studied case: 306 m Tall building
  - Second order approximation of $H$ is sufficient
  - ESWL obtained with the new method correctly fit the real ones
- Asymptotic expansion of the modal transfer matrix enables to avoid full transfer matrix inversion
- **New method** for the establishment of ESWL for structures with non-proportional damping analysed in the modal basis
- **Studied case**: 306 m Tall building
  - Second order approximation of $\mathbf{H}$ is sufficient
  - ESWL obtained with the new method correctly fit the real ones
- **Applications**
  - Equivalent static design
  - Structural optimization using ESWL
Asymptotic expansion of the modal transfer matrix enables to avoid full transfer matrix inversion.

New method for the establishment of ESWL for structures with non-proportional damping analysed in the modal basis.

Studied case: 306 m Tall building
- Second order approximation of $H$ is sufficient
- ESWL obtained with the new method correctly fit the real ones

Applications
- Equivalent static design
- Structural optimization using ESWL

Perspective
- Dynamic system with non-linear terms
The team...

Thomas Canor

Vincent Denoël

...thanks you for your kind attention

Read out more about us on : www.orbi.ulg.ac.be

Contact me at : N.Blaise@ulg.ac.be
Questions?