Equivalent Static Wind Loads for structures with non-proportional damping

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University of Cape Town, South Africa
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Analysis of structures under random excitations

Structures

View from the sky, Cape town

are subjected to wind excitations

Wembley Square, Cape town (2009)

Vista High school, Cape town (2009)

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Dynamic analysis of large structures

Modal basis

Normal modes of vibration — Modal damping matrix

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi} = \mathbf{I} \qquad \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi} = \mathbf{D} \qquad \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi} = \mathbf{\Omega}$$

Modal stiffness matrix (diagonal)

Rayleigh Damping

$$\mathbf{C} = \alpha\mathbf{K} + \beta\mathbf{M} \longrightarrow \mathbf{D} = \mathbf{D}_d$$ (diagonal)

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Sources of non-proportionality

- damping devices (TMD, TLCD), aerodynamic damping and...

D is not diagonal

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Coupled system of equation of motion

Modal amplitudes

$$\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{\Omega}\mathbf{q} = \mathbf{g}$$

Generalized forces

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Dynamic analysis of large structures

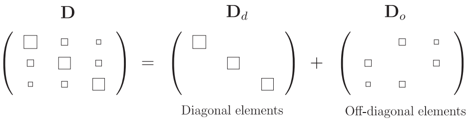
- Split damping matrix

$$\underbrace{\begin{pmatrix} \Box & \square & \square \\ \square & \Box & \square \\ \square & \square & \Box \end{pmatrix}}_{\displaystyle \overset{\mathbf{D}}{}} = \underset{\text{Diagonal elements}}{\overset{\mathbf{D}_d}{\begin{pmatrix} \Box & & \\ & \Box & \\ & & \Box \end{pmatrix}}} + \underset{\text{Off-diagonal elements}}{\overset{\mathbf{D}_o}{\begin{pmatrix} & \square & \square \\ \square & & \square \\ \square & \square & \end{pmatrix}}}$$

[1] Rayleigh. (1877). The Theory of Sound.Vol. 1. New-York : Dover Publication

Dynamic analysis of large structures

- Split damping matrix



- Decoupling approximation[1]

$$\mathbf{H}_d = (-\mathbf{I}\omega^2 + j\omega\mathbf{D}_d + \mathbf{\Omega})^{-1} \longrightarrow$$

Inversion of a diagonal matrix only
Decoupled system

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Dynamic analysis of large structures

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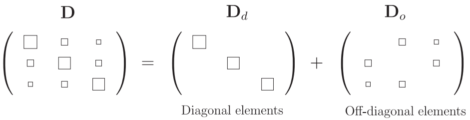

Decoupling approximation[1]

$$\mathbf{H}_d = \left(-\mathbf{I}\omega^2 + j\omega\mathbf{D}_d + \mathbf{\Omega}\right)^{-1}$$

→ Inversion of a diagonal matrix only
Decoupled system

Full matrix inversion

$$\mathbf{H} = \left(-\mathbf{I}\omega^2 + j\omega\mathbf{D} + \mathbf{\Omega}\right)^{-1}$$

$$\mathbf{H} = \left(\mathbf{I} + j\omega\mathbf{H}_d\mathbf{D}_o\right)^{-1}\mathbf{H}_d$$

→ Full matrix inversion
Coupled system

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Asymptotic expansion method

- Key-idea[1]

$$\mathbf{H} = (\mathbf{I} + j\omega \mathbf{H}_d \mathbf{D}_o)^{-1} \mathbf{H}_d$$

$$(\mathbf{I}+\mathbf{X})^{-1} \simeq \mathbf{I} - \mathbf{X} + \mathbf{X}^2 - \ldots = \mathbf{I} + \sum_{i=1}^{k} (-\mathbf{X})^i$$

Condition: $r(\mathbf{X}) = ||\boldsymbol{\lambda}||_\infty < 1$

Eigenvalues of $\mathbf{X}$

[1] Denoël and Degée. (2009). Asymptotic expansion of slightly coupled modal dynamic transfer functions non-proportional damping. *Journal of Sound and Vibration* 328, 1-2, 1-8

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Condition: $r(\mathbf{X}) = ||\boldsymbol{\lambda}||_\infty < 1$

Eigenvalues of $\mathbf{X}$

Approximation of H

Decoupling approximation (diagonal)

$$\mathbf{H}_k = \mathbf{H}_d + \sum_{i=1}^{k} (-j\omega)^i \left(\mathbf{H}_d \mathbf{D}_o\right)^i \mathbf{H}_d$$

Corrections terms (non-diagonal)

No full matrix inversion

Inversion of a diagonal matrix only

Approximate the coupled system

[1] Denoël and Degée. (2009). Asymptotic expansion of slightly coupled modal dynamic transfer functions non-proportional damping. *Journal of Sound and Vibration* 328, 1-2, 1-8

Stochastic modal analysis

- Exact solution

$$\mathbf{S}^{(q)} = \mathbf{H}\mathbf{S}^{(g)}\mathbf{H}^*$$

$\mathbf{S}^{(g)}$: PSD matrix of generalized forces

$\mathbf{S}^{(q)}$: PSD matrix of modal displacements

[1] Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291

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- Stochastic modal analysis[1]

$$\mathbf{S}^{(q_k)} = \mathbf{H}_k\mathbf{S}^{(g)}\mathbf{H}_k^*$$

$$\mathbf{S}^{(q_k)} = \mathbf{S}^{(q_d)} + \sum_{i=1}^{k} \mathbf{\Delta S}^{(q_i)}$$

$\mathbf{S}^{(q_d)}$: Solution in the uncoupled system

$\sum_{i=1}^{k} \mathbf{\Delta S}^{(q_i)}$: Corrections terms due to non-proportionality damping

[1] Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291

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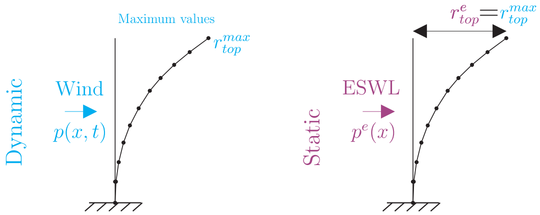
Equivalent static wind loads



- Chen & Kareem formulation[1]

$$\mathbf{p}_j^e = g_j \sum_{m=1}^{M} W_{jm} \boldsymbol{\psi}_m$$

- **Objective** :

Approximate formulation $\mathbf{p}_j^{e,k}$ in case of non-proportional damping

[1] Chen, and Kareem. (2009). Equivalent static wind loads for buffeting response of bridges by mass and liquid dampers. *Journal of Structural Engineering-Asce* 127, 12, 1467-1475

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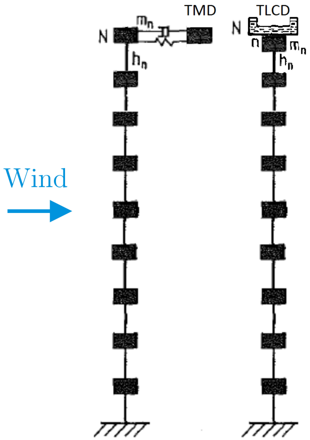
306 m Tall building



- 10-lumped-mass cantiliver beam model
- Random excitation : **wind**
  - 1-D Gaussian velocity field
- Structural and aerodynamic data from[1]
- Two studied cases :
  - Tuned Mass Damper
  - Tuned Liquid Column Damper

[1] Xu, Samali, and Kwok. (2009). Control of along-wind response of structures by mass and liquid dampers. *Journal of Engineering Mechanics* 118, 1, 20-39

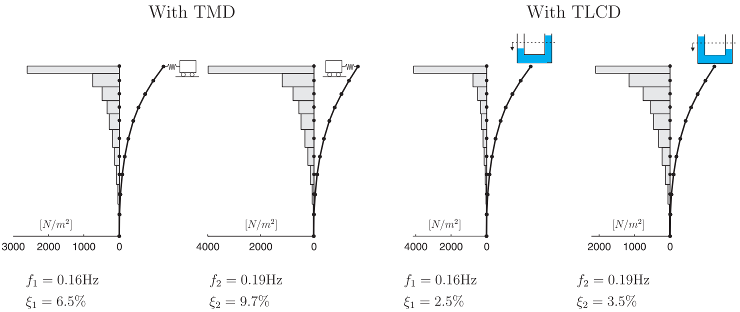
Modal properties

- Inertial forces per unit surface

$$\Psi_m = \mathbf{K}\Phi_m$$

(Ψ_m: mth inertial force; Φ_m: mth modal shape)

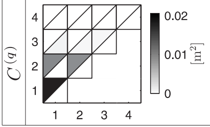
- First two modes (five modes considered for the analysis)



Structural analysis

Covariance matrix of modal displacements

$$\int_{-\infty}^{+\infty} \mathbf{S}^{(q_k)}\, d\omega = \mathbf{C}^{(q_d)} + \sum_{i=1}^{k} \mathbf{\Delta C}^{(q_i)}$$

		Max. relative errors		
	Exact solution (k=∞)	Decoupled (k=0)	k^{th} approximation of $\mathbf{H}$ (k=1)	(k=2)
With TMD		23%	14%	6%

		Max. relative errors		
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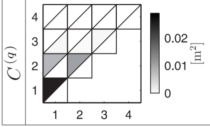
With TLCD		7%	2%	≃0%

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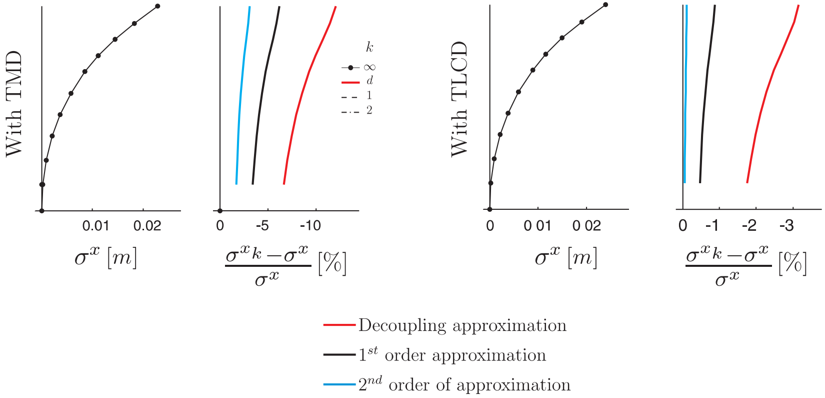
Structural analysis

- Standard deviations of nodal displacements



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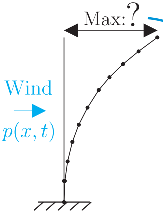
Structural design

- Envelope values (min and max) of the structural responses
  - Extreme value theory



$$r_j^{k,max} = g_j^{r_k} \sigma_j^{r_k}$$

peak factor

standard deviation

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Asymptotic expansion method

- Weighted combinations of the inertial forces

$$\mathbf{p}_j^{e,k} = g_j^k \sum_m^M W_{jm}^k \boldsymbol{\psi}_m$$

k^{th} approximation of the weighting coefficients

$$W_{jm}^k = \alpha_j^k W_{jm}^d + \sum_{i=1}^k \Delta W_{jm}^i$$

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- Definition of the ESWL

kth approximation of the ESWL

$$\mathbf{p}_j^{e,k} = \alpha_j^k \mathbf{p}_j^{e,d} + \Delta \mathbf{p}_j^{e,k}$$

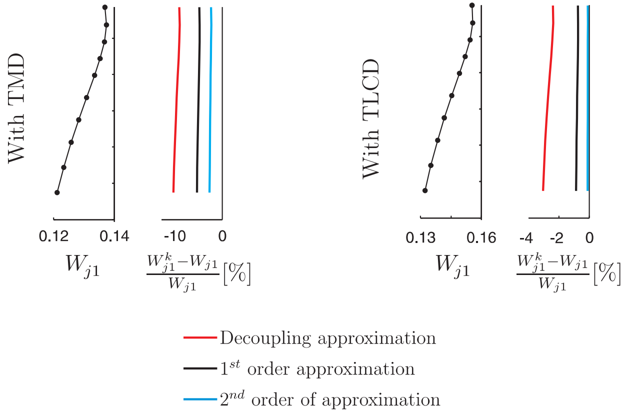
correction resulting from the non-proportionality of damping

ESWL for the uncoupled system

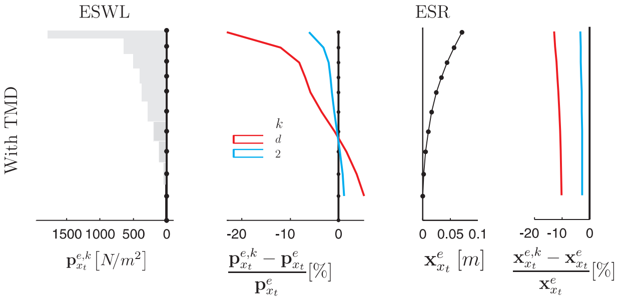
scaled coefficients

Weighting coefficients

- First inertial force



ESWL-Horizontal displacement at the top



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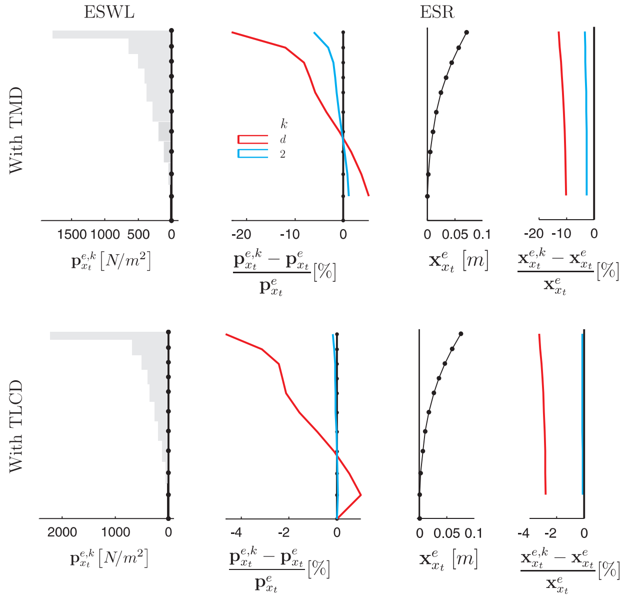

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- Perspective
  - Dynamic system with non-linear terms

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The team...

Thomas Canor

Vincent Denoël

...thanks you for your kind attention

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Questions ?

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