Dynamics of Elasto-Inertial Turbulence in Flows with Polymer Additives

V. E. Terrapon
Y. Dubief
J. Soria

TSFP-8
Poitiers, 30 August 2013
Acknowledgements

Collaborators

Yves Dubief  
University of Vermont, USA

Julio Soria  
Monash University, Australia  
King Abdulaziz University, Kingdom of Saudi Arabia

Financial support

• Marie Curie FP7 CIG
• Vermont Advanced Computing Center
• US National Institutes of Health
• Australian Research Council
• Center for Turbulence Research Summer Program
Polymers and turbulence

Turbulent drag reduction

- Up to 80% friction drag reduction, even at low concentration
- No significant effect on drag in laminar flows
- Bounded by Maximum Drag Reduction (MDR) asymptote

Fire hoses with and without polymer additives

• Pipeline
Elastic turbulence

- Existence of elastic turbulence in flows with curved streamlines
- Observed at low Reynolds number
- Strong increase in mixing properties

Blood flow
Micro-channel flow

Turbulent drag reduction
Early turbulence

Transition to turbulence promoted by polymers

Transition to turbulence around an ogival head with ventilated cavity (Hoyt, 1977)

Elastic turbulence

Turbulent drag reduction

• Transition to turbulence promoted by polymers

• Biofluids
Polymers and turbulence

Newtonian | Viscoelastic
---|---

Elastic turbulence

Elastic turbulence

Early turbulence

Drag reduction

Re

10^1

10^3

10^5

Transition

Laminar

Turbulent

TSFP 8 - Terrapon, Dubief & Soria - Poitiers, 30 August 2013
Polymers and turbulence

Newtonian  Viscoelastic

Laminar

Transition

Turbulent

Re

$10^1$

$10^3$

$10^5$

Elastic turbulence

Early turbulence

Drag reduction

Elasto-Inertial Turbulence (EIT)

- State of small-scale turbulence
- Contributions from both elastic and inertial instabilities
- Observed over a wide range of Reynolds numbers
- Possibly state characterizing MDR
Polymers and turbulence

Newtonian

Viscoelastic

Elastic turbulence

Early turbulence

Drag reduction

Key questions

• Is drag reduction
  – a viscous and large-scale effect (Lumley)
  – an elastic and small-scale effect (de Gennes)

• What is the nature of EIT?
  – Relative contributions of elastic and inertial instabilities?
  – Characteristics of MDR?
  – Dynamical interactions between flow and polymers?
Viscoelastic NSE – FENE-P model

Continuity
\[ \nabla \cdot \mathbf{u} = 0 \]

Momentum
\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{\beta}{\text{Re}} \nabla^2 \mathbf{u} + \frac{1 - \beta}{\text{Re}} \nabla \cdot \mathbf{T} + \frac{dP}{dx} \mathbf{e}_x \]

Polymer stress
\[ \mathbf{T} = \frac{1}{\text{Wi}} \left( \frac{C}{1 - \text{tr}C/L^2} - \mathbf{I} \right) \]

Conformation tensor
\[ \frac{\partial \mathbf{C}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{C} = \nabla \mathbf{u} \cdot \mathbf{C} + \mathbf{C} \cdot \nabla \mathbf{u}^T - \mathbf{T} \]

\[ \beta \quad \text{Ratio of solvent viscosity to zero-shear viscosity of solution} \]
\[ L \quad \text{Maximum polymer extension} \]
\[ \text{Re} \quad \text{Reynolds number} \]
\[ \text{Wi} \quad \text{Weissenberg number} \]
\[ E = \frac{\text{Wi}}{\text{Re}} \quad \text{Elasticity} \]
Numerical approach

Space
- Structured grid
- 2nd order FD for velocity
- Non-dissipative 4th order compact scheme for polymer stress
- Compact upwind scheme for advection terms of conformation tensor

Time
- Semi-implicit fractional step
- 2nd order Crank-Nicolson/3rd order Runge-Kutta
- Implicit scheme for trace of C to ensure bounded trace

Artificial dissipation
- Local artificial dissipation (LAD)
- Only used when determinant of tensor C becomes negative

Min et al. (2001), Vaithianathan & Collins (2003), Dubief et al. (2005), Dallas et al. (2010)

• Important to rely on accurate numerical method
• Global dissipation ($S_{eff} \sim 1$) damps all small scales
• Capturing small polymer scales is critical to represent the correct physics
**Configuration**

### Periodic channel flow

- Mean pressure gradient in $x$
- Periodic in $x$ and $z$
- Wall (no-slip) at $y=\pm h$
- Size: $10h \times 2h \times 5h$
- Grid: $256 \times 151 \times 256$

---

<table>
<thead>
<tr>
<th>$\text{Re}=\frac{U_b H}{v}$</th>
<th>$\text{Wi}=\frac{\lambda U_b}{h}$</th>
<th>$\text{Wi}^+ = \lambda \dot{\gamma}$</th>
<th>$\beta$</th>
<th>$L$</th>
<th>$h^+$</th>
<th>$\Delta x^+$</th>
<th>DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>8</td>
<td>24</td>
<td>0.9</td>
<td>200</td>
<td>40</td>
<td>1.5</td>
<td>+7.0%</td>
</tr>
<tr>
<td>1000</td>
<td>60</td>
<td>180</td>
<td></td>
<td>40</td>
<td>1.4</td>
<td>+3.5%</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>8</td>
<td>96</td>
<td></td>
<td>200</td>
<td>130</td>
<td>5.0</td>
<td>-56%</td>
</tr>
<tr>
<td>6000</td>
<td>60</td>
<td>720</td>
<td></td>
<td>120</td>
<td>4.6</td>
<td>-61%</td>
<td></td>
</tr>
</tbody>
</table>
Transitional viscoelastic flows

Channel flow simulations

Friction factor

- Departure from laminar state at $Re \sim 800$
- Smooth transition from laminar to MDR state
- Flow dynamics controlled by elastic and inertial instabilities
Transitional viscoelastic flows

Channel flow simulations

Friction factor

Re=1000, Wi^+=24
• Not laminar
• Elastic contributions

Re=6000, Wi^+=96
• Inertial & elastic contributions
• Turbulent?
• New state?

Isosurface of Q_a invariant
Transitional viscoelastic flows

Pipe flow experiment with PAAm solution

Friction factor

Results of numerical simulations are confirmed by experimental measurements

Samanta et al., *PNAS* 110(26), 2013
Qualitative flow behavior

Second invariant of the velocity gradient tensor: $Q_a = \frac{1}{2} (\Omega^2 - S^2)$
Qualitative flow behavior

Polymer extension \((C_{ii} L^2)^{1/2}\)

Re = 1000
Wi\(^+\) = 24
Qualitative flow behavior

Second invariant of the velocity gradient tensor: $Q_0 = \frac{1}{2} (\Omega^2 - S^2)$
Qualitative flow behavior

\[ \text{Polymer extension } \left( \frac{C_{ii}}{L^2} \right)^{1/2} \]

\[ tU_b/h = 0.0 \]

\[ \text{Re} = 6000 \]

\[ \text{Wi}^+ = 96 \]
Typical structures

- Train of cylindrical $Q_a$ structures of alternating sign
- On each side of sheet
- Associated with polymeric part of pressure
- Correlated with polymer body force $f_p$
Flow topology

EIT flow: JPDF

- Change from shear flow ($R_a = Q_a = 0$) to mixed flow
- At low $Re$, symmetric distribution around 2D flow ($R_a = 0$)
- At higher $Re$, “teardrop” shape similar to Newtonian turbulence
Energy transfers

Turbulent kinetic energy budget

\[ \int P \, dV - \int \varepsilon \, dV - \int \Pi_e \, dV = 0 \]

Production

Dissipation

Transfer between elastic energy and turbulent kinetic energy

Transfers of turbulent kinetic energy

Re=1000, Wi^+=24

Energy transfer from polymers to flow!
Our current understanding

### Hyperbolic transport equation

\[ \partial_t C + (\mathbf{u} \cdot \nabla) C = \ldots \]

- Formation of very thin sheets
- Trains of cylindrical structures

### Pressure Poisson equation

\[ \nabla^2 p = 2Q_a - \frac{1 - \beta}{\text{Re}} \nabla \cdot (\nabla \cdot \mathbf{T}) \]

- Elliptical pressure redistribution of energy
- Excitation of extensional sheet flow

### Mixed extensional-shear flow

\[ \ldots = C(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T C - \mathbf{T} \]

- Increase of extensional viscosity (anisotropic)
- Anisotropic polymer body force
Conclusion and future work

Key take-away messages

• EIT is a new state of small-scale turbulence driven by both elastic and inertial instabilities
• EIT could characterize MDR regime
• EIT explains seemingly contradictory phenomena in viscoelastic turbulence
• EIT provides support to de Gennes’ theory

Next steps

• Further characterize EIT
  – Two-dimensionality
  – Energy transfers and backscatter
• Understand the exact mechanisms during transition process

Samanta et al., “Elasto-inertial turbulence”, PNAS 110(26), 2013