

Assessing the probability of training image-based geological scenarios using geophysical data

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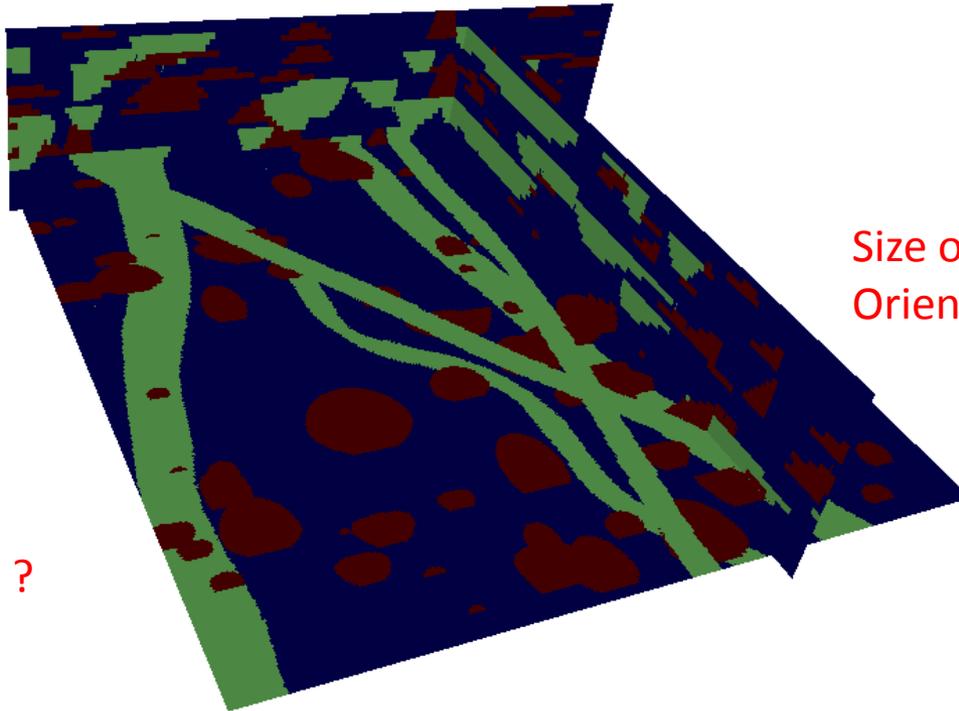
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Multiple-point statistics (MPS) uses a training image (TI) to depict the expected geological heterogeneity

Training image representing an alluvial aquifer with gravel channels and clay lenses

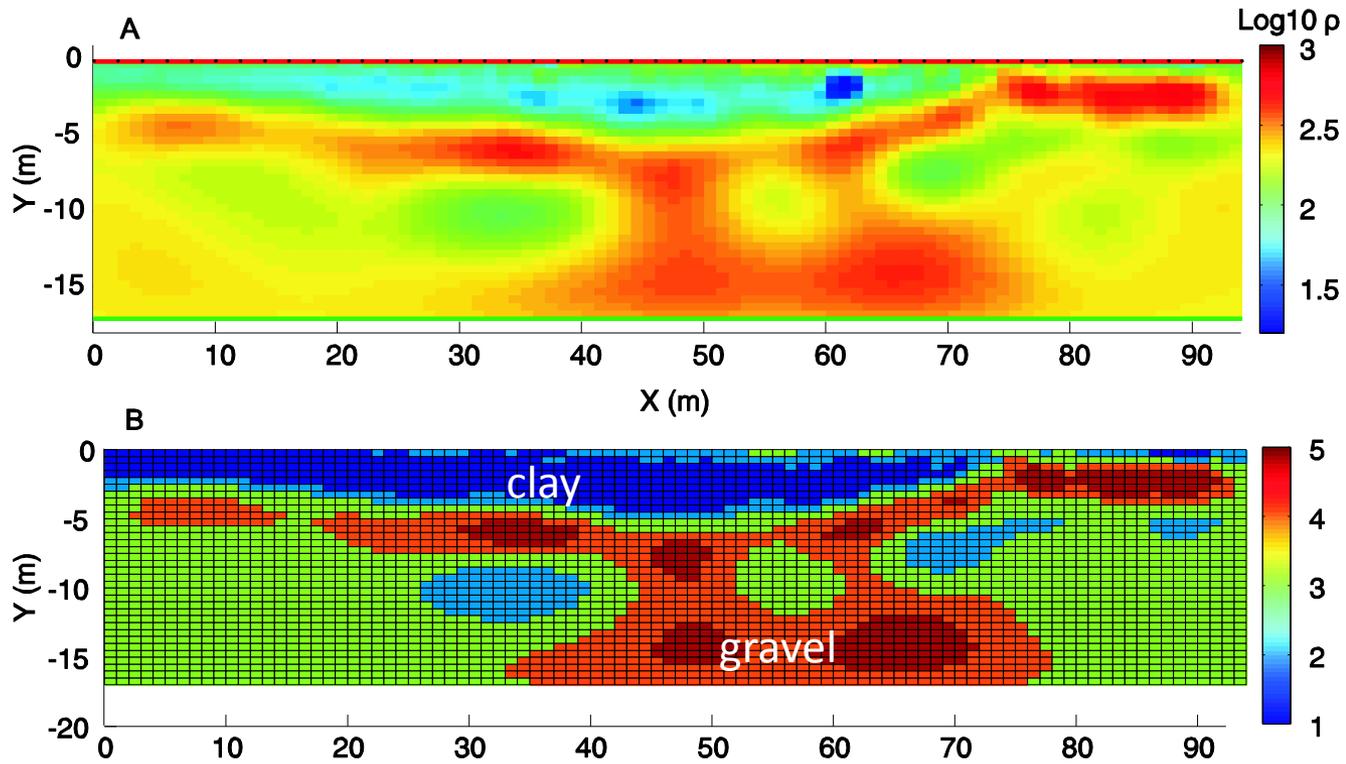


Size of channels ?
Orientation ?

Size of lenses ?

Considerable uncertainty often remains concerning the geometry and architecture of facies elements.

Geophysical data may provide spatial information on subsurface properties

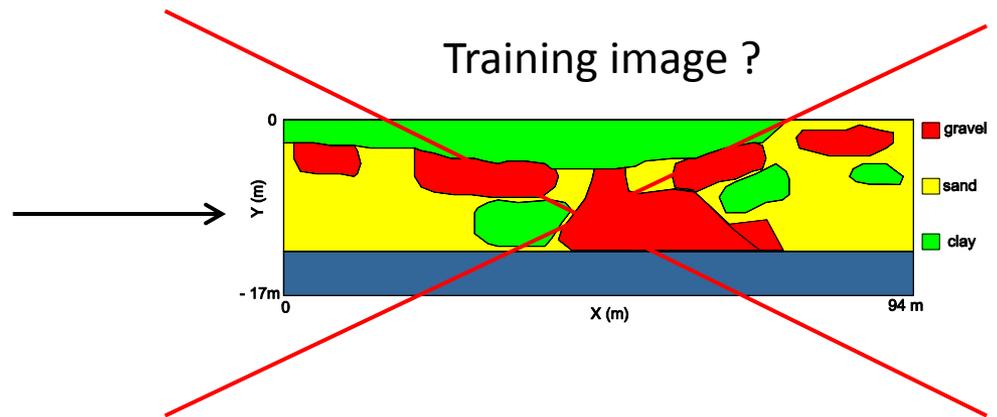
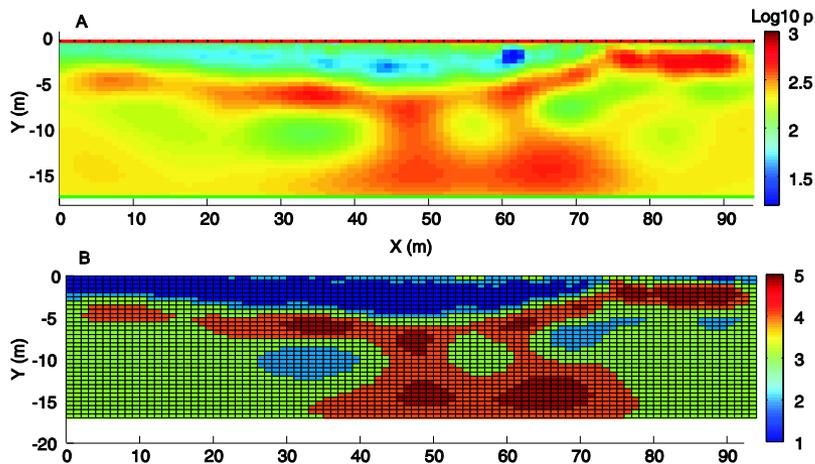


Classification based on electrical resistivity

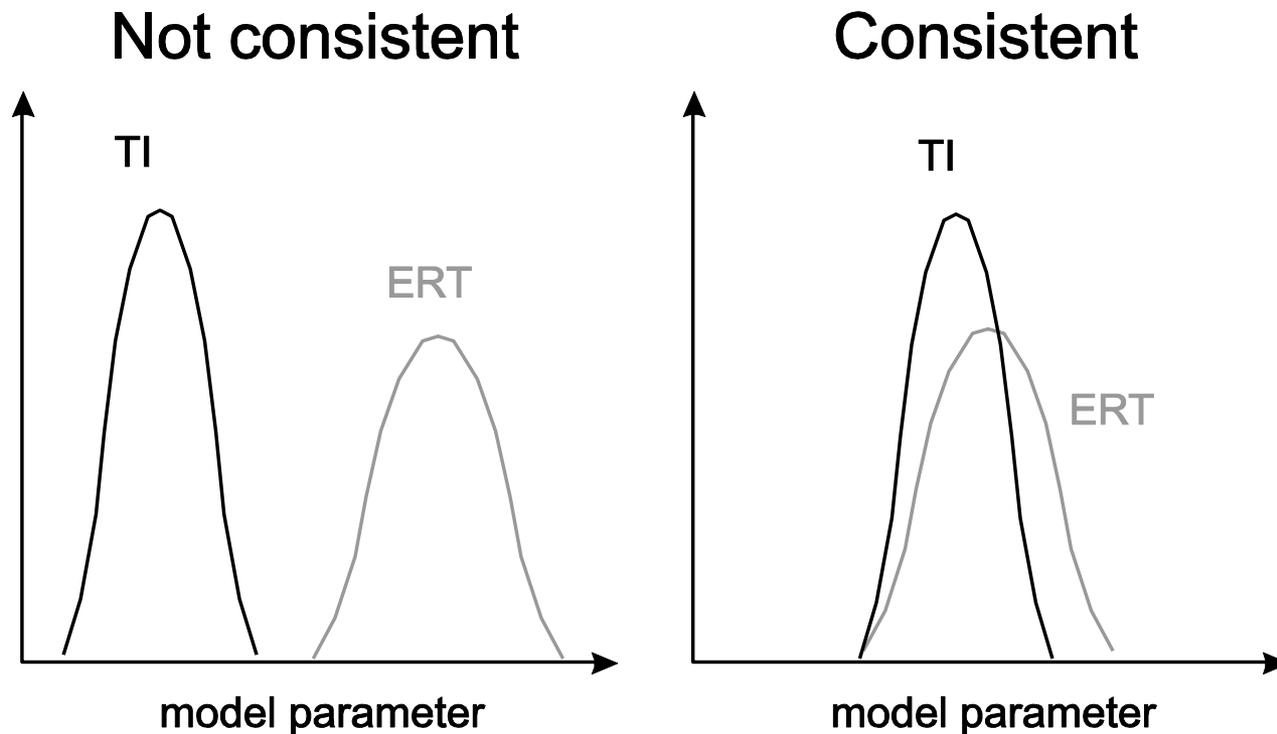
Electrical resistivity tomography (ERT) yields a model of subsurface resistivity, high values being linked with gravel and low values with clay.

The use of geophysical image to extract TI is limited due to the resolution of geophysics and the philosophy of MPS

1. Geophysical methods are not able to differentiate unequivocally different facies: small scale features may be hidden by larger elements
2. Geophysical methods may be used to constraint spatially MPS simulations, TI and soft data should be independent



Still, we would like to ensure that training images and geophysical models are somehow compatible



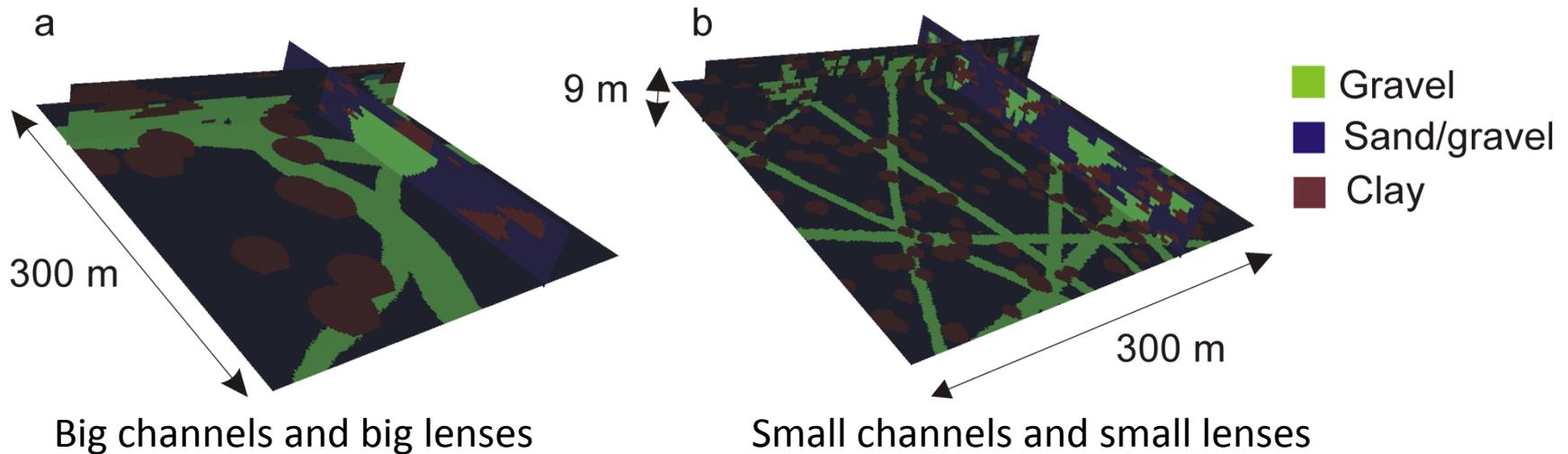
This would ensure that conditioning with soft data will not create conflict with features of the TI.

1. Creating training image-based scenarios
2. Methodology to verify the consistency using MDS
3. Analysis of the results
4. Conclusion and future works

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We are working in the Meuse River alluvial aquifer (Belgium), very few outcrops and sedimentological studies are available to build TI

Based on our knowledge of the deposits, we propose to represent channels composed of gravel and lenses composed of clay within a sand/gravel background

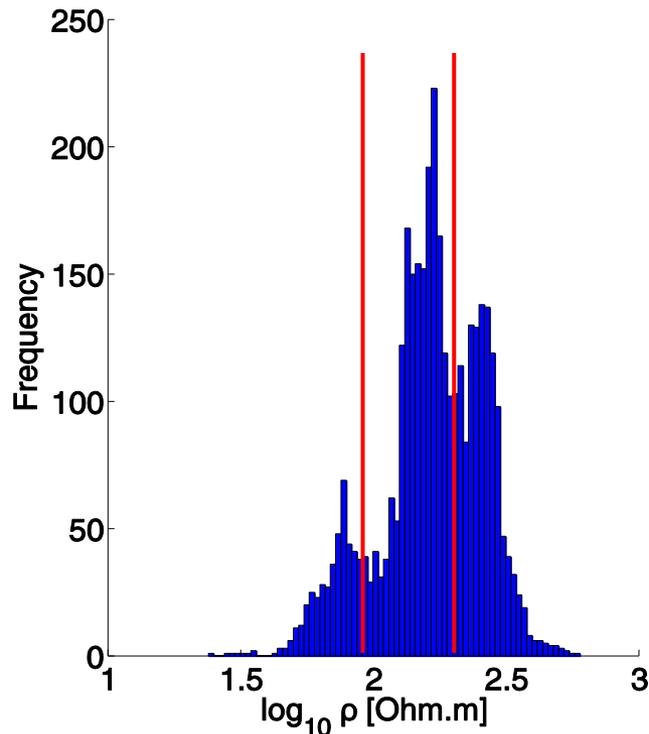


However, very few data exists on the size of these elements, we take this uncertainty into account by creating various TI-based scenarios.

6 scenarios are simulated with three different sizes for gravel channels and two sizes for clay lenses

In each training image, 12 sections are randomly selected to be compared with true ERT field sections, leading to 72 different models.

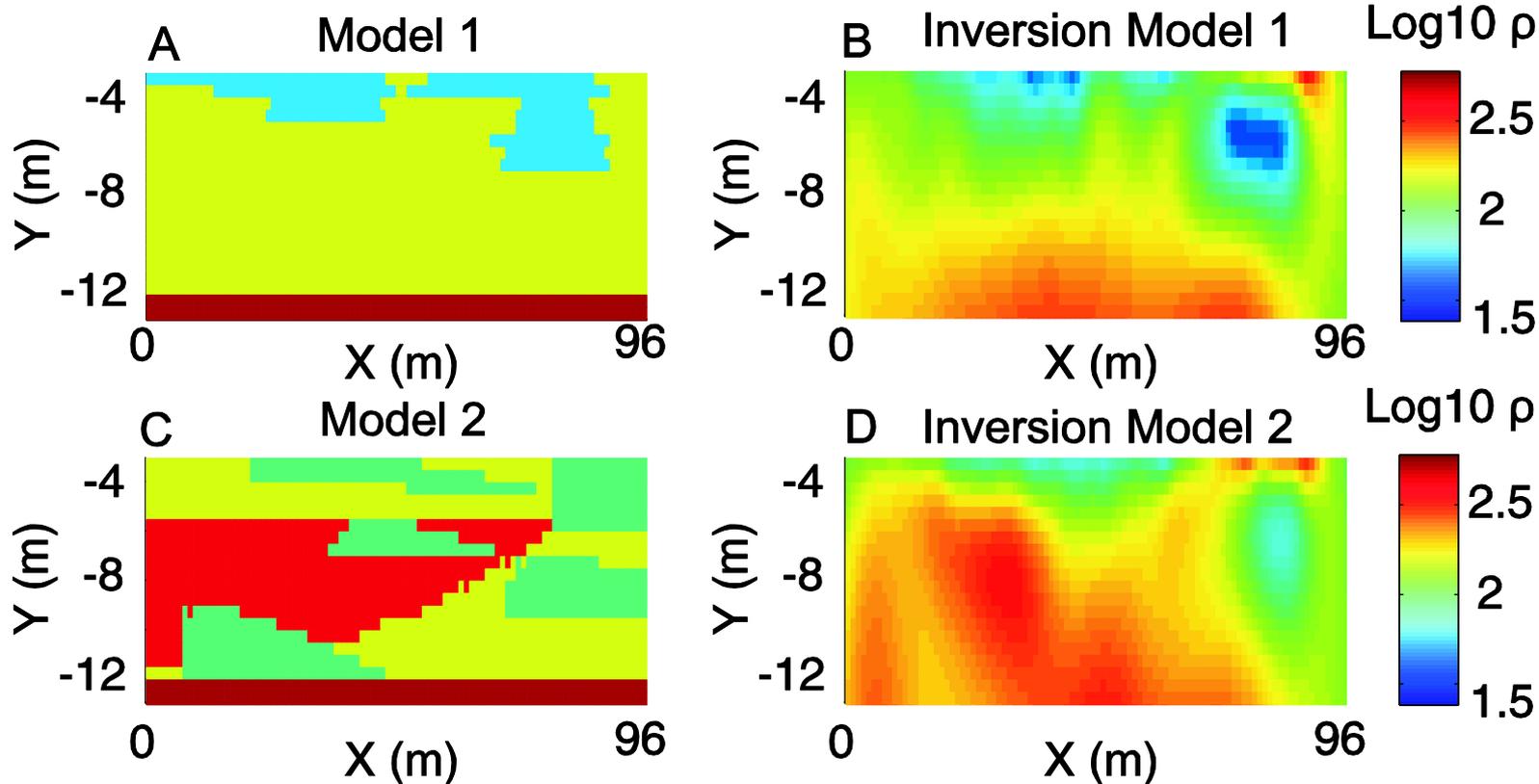
For each model, facies are assigned a value of electrical resistivity. Three alternative scenarios are considered, based on field knowledge, to take variability into account.



1. Clay = 60 Ohm-m, Sand = 160 Ohm-m, Gravel = 300 Ohm-m
2. Clay = 60 Ohm-m, Sand = 160 Ohm-m, Gravel = 450 Ohm-m
3. Clay = 90 Ohm-m, Sand = 160 Ohm-m, Gravel = 450 Ohm-m

Total of 216 synthetic resistivity models

ERT data are simulated for each synthetic model and those data are then inverted



Visual inspection illustrate the limited resolution of ERT but suggests that TI may be able to explain the observed resistivity distribution.

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Synthetic inverted sections and true field sections are compared using multi-dimensional scaling (MDS)

1. The Euclidean distance between any two inverted models (electrical resistivity of each cell) is calculated

$$d_{ij} = \sqrt{(\rho_i - \rho_j)^T (\rho_i - \rho_j)}$$

2. This defines a metric space in which the L models are defined in terms of distance to each other.
3. A change of variable is done such that

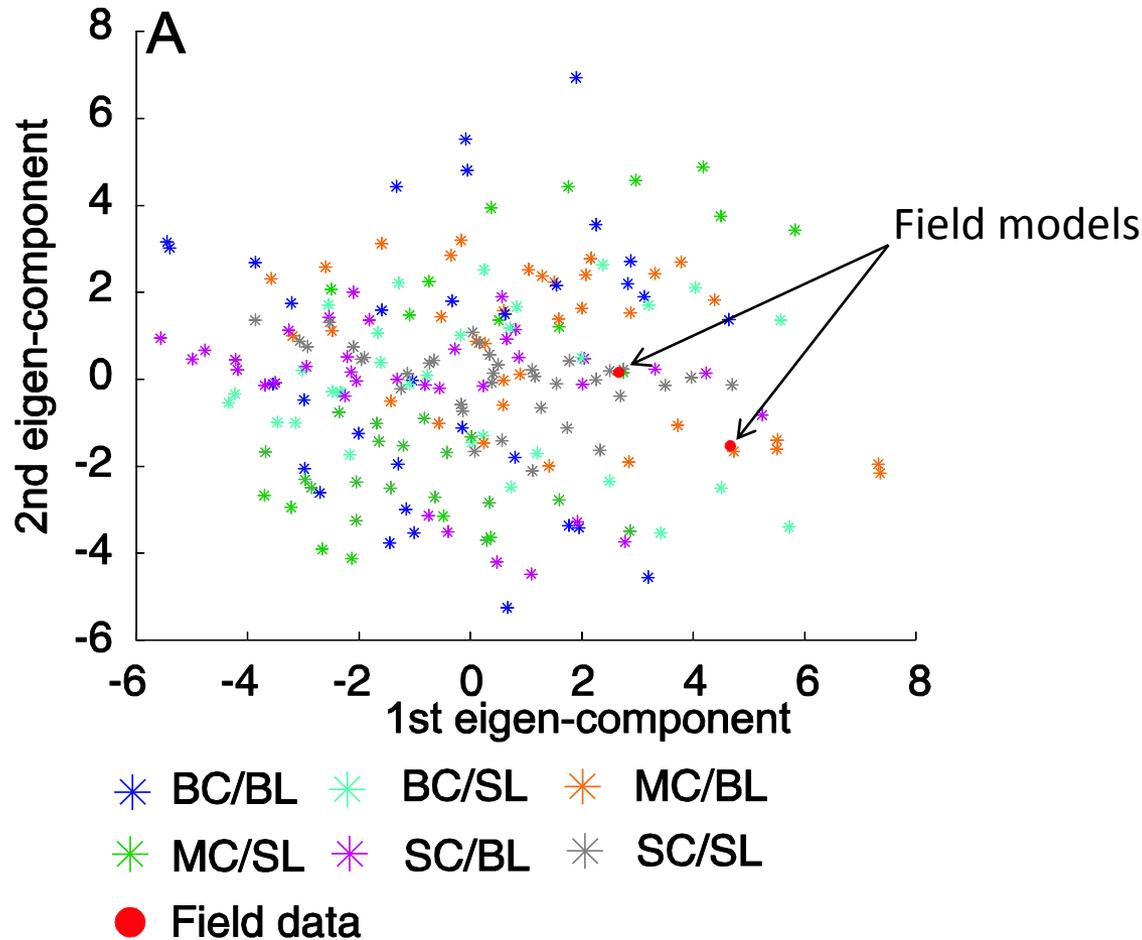
$$\mathbf{B} = \mathbf{H}\mathbf{A}\mathbf{H} \quad \text{Size } L \times L \text{ with} \quad a_{ij} = -\frac{1}{2}d_{ij}^2 \quad \text{and} \quad \mathbf{H} = \mathbf{I} - \frac{1}{L}\mathbf{1}\mathbf{1}^T$$

4. The eigenvalue decomposition of \mathbf{B} offers a way to project the models into a d -dimensions map, using the first d eigenvalues and eigenvectors.

$$\mathbf{B} = \mathbf{V}_B \mathbf{\Lambda}_B \mathbf{V}_B^T \quad \xrightarrow{\text{MDS}} \quad \mathbf{X}_d = \mathbf{V}_{B,d} \mathbf{\Lambda}_{B,d}^{1/2}$$

5. \mathbf{X}_d are the coordinates of the model in the MDS map of d -dimensions

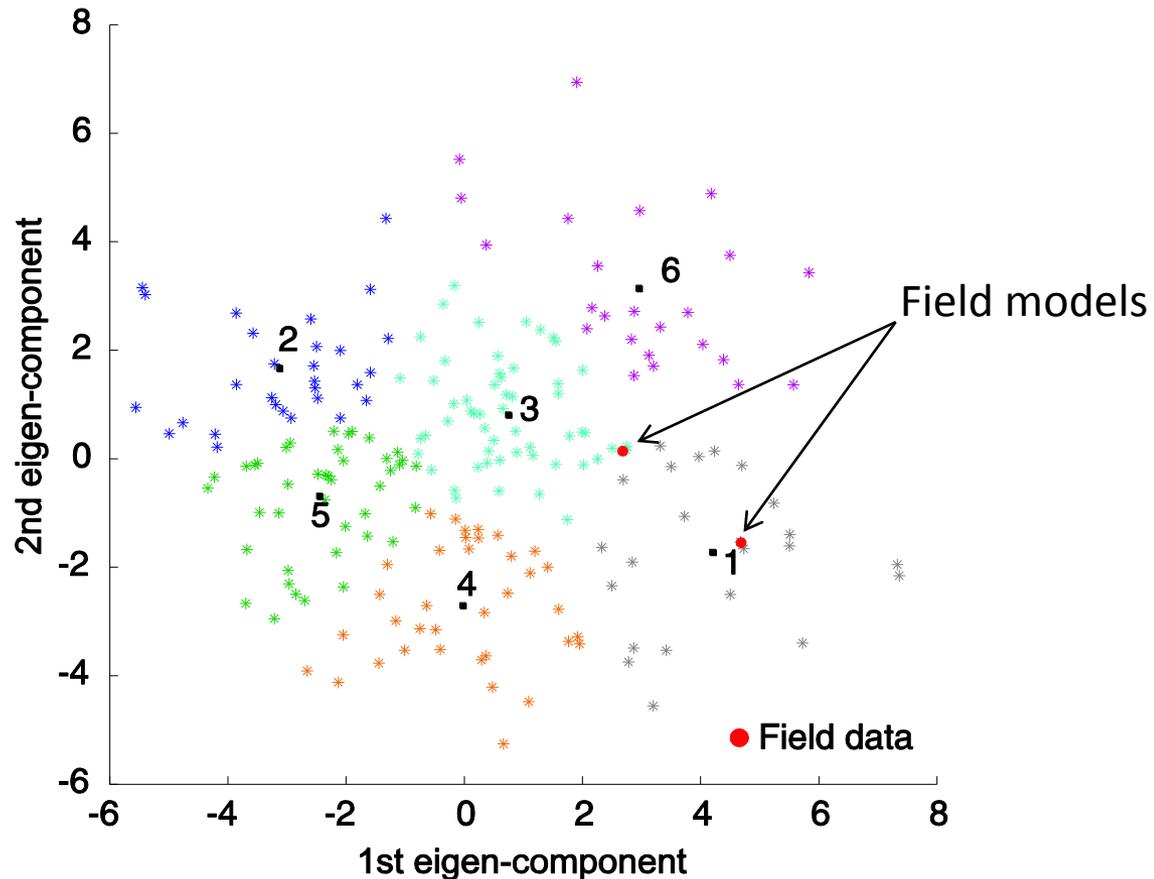
If the TI-based scenarios are consistent with ERT, field models should fall in the distribution of synthetic models



The procedure was applied on two field models that were compared to the 216 synthetic models, they fall into the 2D distribution.

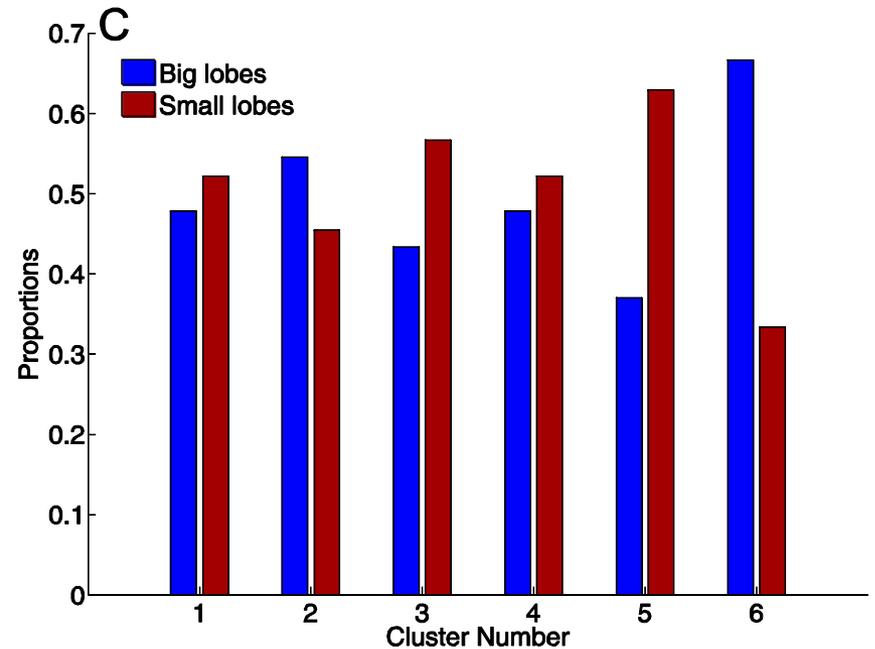
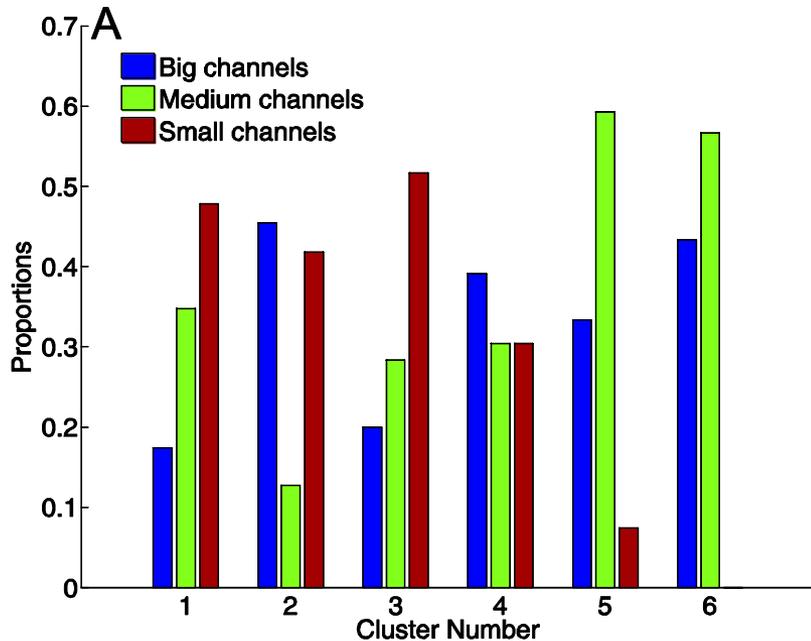
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We can try to analyze the distribution of models to have more insights on the tested scenarios



A k-mean clustering approach is used to group the models and see how they are distributed in the 2D space, field data being in clusters 1 and 3

The analysis shows that the size of the channel is a more sensitive parameter than the size of lenses



In cluster 1 and 3, models with small channels are more abundant, suggesting that small channels scenarios are more probable

The proportion of big and small lenses is almost similar (50%)

The probability analysis of different scenarios can be seen as a conditional probability problem

(Park et al., 2013)

$$P(\text{TI} = \text{ti}_k \mid \mathbf{D} = \mathbf{d}_{\text{obs}}) = \frac{f(\mathbf{d}_{\text{obs}} \mid \text{ti}_k)P(\text{TI} = \text{ti}_k)}{\sum_k f(\mathbf{d}_{\text{obs}} \mid \text{ti}_k)P(\text{TI} = \text{ti}_k)}$$

\mathbf{d}_{obs} is the projection of the response from each synthetic model indicating a location on the MDS map

We can estimate $f(\mathbf{d}_{\text{obs}} \mid \text{ti}_k)$ using the adaptive kernel density estimation technique with a bivariate normal function applied on the MDS map

$$f_X(x_1, x_2) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{1}{2}\left[\frac{(x_1 - \mu_x)^2}{\sigma_x^2} + \frac{(x_2 - \mu_y)^2}{\sigma_y^2}\right]\right)$$

x_1 and x_2 are the coordinates of the models, μ_x and μ_y correspond to the coordinates of the reference value (the field data for example), σ_x and σ_y are bandwidth parameters depending on the cluster

The probability of each scenario according to field data is computed and the conclusion are similar to the cluster analysis

All scenarios have the same prior probability $P = 0.1667$

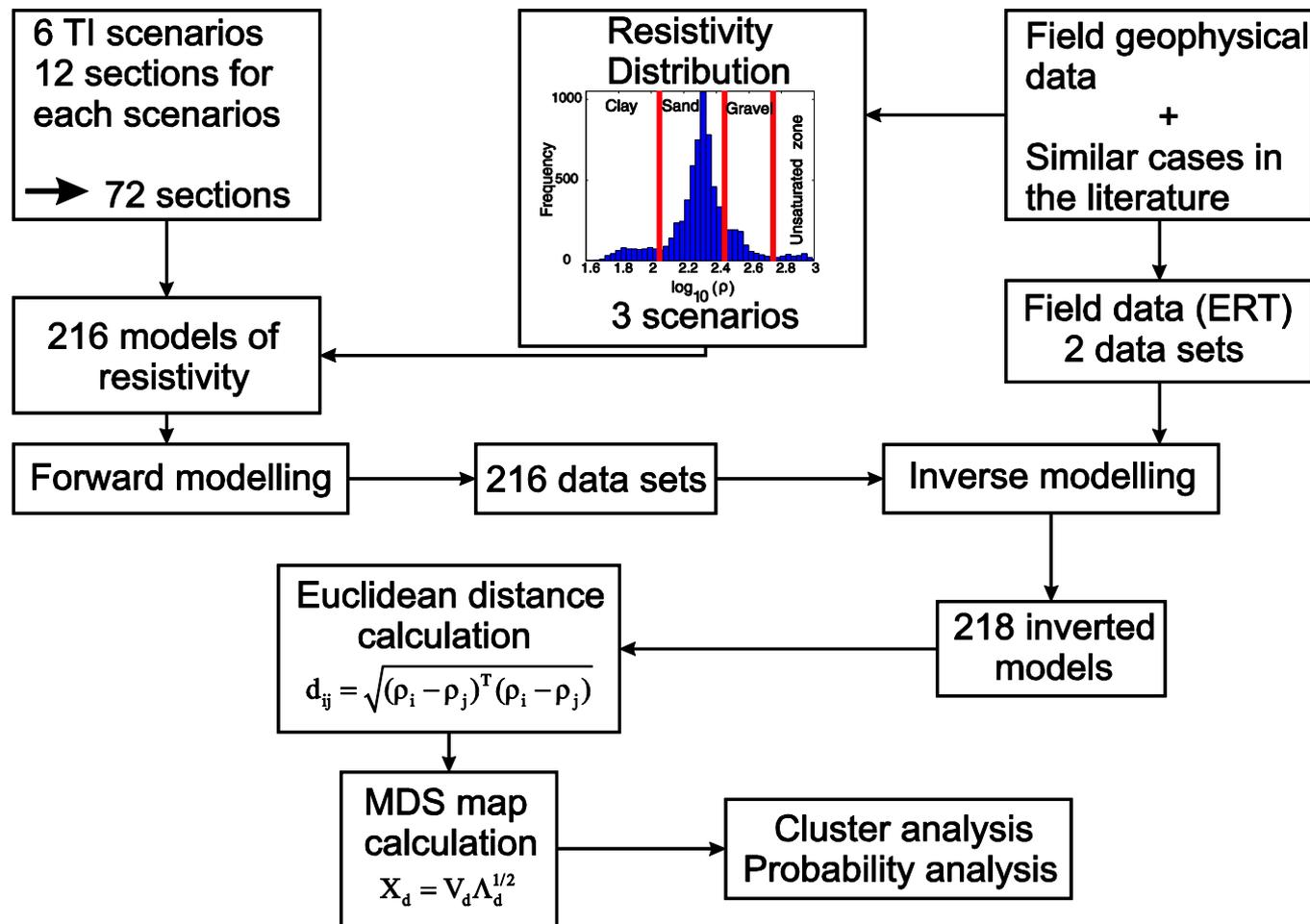
Training Images conditional probabilities	BC/BL	BC/SL	MC/BL	MC/SL	SC/BL	SC/SL
Field data 1	0.0472	0.1061	0.1618	0.0613	0.1951	0.4285
Field data 2	0.0157	0.1497	0.4259	0.0252	0.1624	0.2211

BC=big channels MC=medium channels SC=small channels
BL=big lenses SL=small lenses

Scenarios with big channels have low probability of occurrence, small and medium channels are more likely

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The proposed methodology enables to verify the « consistency » of training-image based scenarios with geophysical data using synthetic benchmarks



This method is a way to justify and validate the choice of a training image and to propose alternatives for uncertainty analysis and stochastic simulations

Important remarks and limitations

1. The choice of resistivity distribution plays an important role, the geophysicist must ensure that reliable values are chosen
2. The choice of the Euclidean distance is locally dependent which may be a drawback in MPS since training image are not locally dependent
3. It would be possible to use other type of distance measurements or other attributes than resistivity
4. The choice of the dimensions of the map is often 2D or 3D for visual inspection, here it represents about 60% of the total variance, higher dimensions are possible
5. 3D geophysical surveys are more costly and time consuming, but the methodology can be extended to 3D models as well, it would be useful to distinguish between channels and elongated bars
6. The methodology shows that there is some consistency between TI and geophysical data, but not that the TI is perfect or that other scenarios are impossible

Thank you very much for your attention