Evolution of Power Spectrum of Solar-like Oscillations During the Ascending Phase on the Red Giant Branch

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Abstract. CoRoT (Convection Rotation and planetary Transits) and Kepler observations of red giants reveal rich spectra of non-radial solar-like oscillations allowing their internal structures to be probed. We investigate an evolutionary sequence along the Red Giant Branch to determine when during the evolution of these stars g-dominated mixed modes are more likely to be detectable. We found that radiative damping in the core plays an important role in giving a theoretical limit on the evolutionary sequence for the detectability of g-dominated mixed modes.

1. Introduction

In this study, we consider three stellar models (Table 1, Figure 1) on the Red Giant Branch (RGB) of 1.5 $M_\odot$ (typical of CoRoT and Kepler targets). They are computed with the code ATON (Ventura et al. 2008) using Mixing Length Theory (MLT) for the treatment of convection with $\alpha_{\text{MLT}} = 1.9$. The initial chemical composition is $X = 0.7$ and $Z = 0.02$.

To compute the mode lifetimes, we used the non-adiabatic pulsation code MAD (Dupret et al. 2002) with a non-local time-dependent treatment of the convection (TDC) (Grigahcène et al. 2005). The amplitudes are computed using a stochastic excitation model (Samadi & Goupil 2001) with solar parameters for the description of the turbulence in the upper part of the convective envelope.

Table 1. Main characteristics of our models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mass ($M_\odot$)</th>
<th>Radius ($R_\odot$)</th>
<th>$T_{\text{eff}}$(K)</th>
<th>$\log(L/L_\odot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.5</td>
<td>5.17</td>
<td>4809</td>
<td>1.11</td>
</tr>
<tr>
<td>B</td>
<td>1.5</td>
<td>7.31</td>
<td>4668</td>
<td>1.36</td>
</tr>
<tr>
<td>C</td>
<td>1.5</td>
<td>11.9</td>
<td>4455</td>
<td>1.70</td>
</tr>
</tbody>
</table>
2. Energetic Aspects of Oscillations

2.1. Radiative Damping

The damping rate $\eta$ of a mode is given by the expression:

$$\eta = - \frac{\int_V dW}{2\sigma I |\xi_r(R)|^2 M^*},$$  \hspace{1cm} (1)

where $\sigma$ is the angular frequency of the mode, $I$ the dimensionless mode inertia, $\xi_r$ the radial displacement and $R$ and $M$ the total radius and mass of the star. Here, the mode inertia $I$ is defined by

$$I = \int_0^M |\xi|^2 dm / (M |\xi_r(R)|^2).$$ \hspace{1cm} (2)

The work performed by the gas during one oscillation cycle $\left(\int_V dW\right)$ takes into account both radiative and convective zone contributions (see Sect. 2.2 for the convective contribution). In the g-mode cavity of red giants, the high density contrast between the core and the envelope leads to a high radial wave-number $k_r = \sqrt{l(l+1)} r^{-1} N / \sigma$, where $N$ is the BruntVäisälä frequency, and $\sigma$ the angular frequency of the mode. This entails
large variations of the temperature gradient with loss of heat at the hot phase, leading to radiative damping. As the star climbs the RGB, the density contrast between the core and the envelope increases and so does the radiative damping.

2.2. Convection – Oscillation Interaction

For solar-like oscillations in red giants, the transition region (where the thermal time scale is of the same order as the oscillation period) occurs in the upper part of the convective envelope. In this region, the time-scale of most energetic turbulent eddies is also of the same order as the oscillation period. Hence it is important for the estimation of the damping to take into account the interaction between convection and oscillations. This is made by using a non-local, TDC which takes into account the variations of the convective flux and of the turbulent pressure due to the oscillations [see Grigahcène et al. (2005) and Dupret et al. (2006) for the description of this treatment and the explanation of the parameters given in Table 2].

The TDC treatment involves a complex parameter $\beta$ in the closure term of the perturbed energy equation. It is adjusted so that the depression of the damping rates occur at $\nu_{\text{max}}$ as suggested by Belkacem et al. (2012).

Table 2. Values of the non-local parameters (a,b) and of the parameter $\beta$ used for all our models.

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>b</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19</td>
<td>5.7</td>
<td>($-0.5 - 1.30i$)</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>5.7</td>
<td>($-0.5 - 1.32i$)</td>
</tr>
<tr>
<td>C</td>
<td>19</td>
<td>5.7</td>
<td>($-0.5 - 1.41i$)</td>
</tr>
</tbody>
</table>

2.3. Stochastic Excitation

The estimation of the power ($P$) injected into the oscillation by the turbulent convection is made through a stochastic excitation model (Samadi & Goupil 2001). It takes into account the contribution of the turbulent Reynolds stress and of the entropy. Associated with the damping rates from non-adiabatic computations we can compute the amplitude of the mode given by:

$$V^2 = \frac{P}{2\eta M I}.$$  (3)

To compute the height ($H$) of a mode in the power spectrum we have to distinguish between resolved and unresolved modes. For resolved modes (those with $\tau < T_{\text{obs}}/2$):

$$H = V^2(R) \times \tau.$$  (4)

For unresolved modes (those with $\tau \geq T_{\text{obs}}/2$):

$$H = V^2(R) \times T_{\text{obs}}/2,$$  (5)

where $V(R)$ is the amplitude of the oscillation at the surface, $\tau$ the lifetime of the mode (given by the inverse of the damping rate) and $T_{\text{obs}}$ the duration of observations. We used $T_{\text{obs}} = 150$ days, which corresponds to a CoRoT long run.
3. Results

Figure 2. Lifetimes of modes $\ell = 0$ (red), $\ell = 1$ (blue), $\ell = 2$ (green). Models A (top), B(middle), C (bottom).
Figure 3. Theoretical power spectrum of modes $\ell = 0$ (red), $\ell = 1$ (blue), $\ell = 2$ (green). Models A (top), B (middle), C (bottom).

All our results (Figs. 2 and 3) extend the tendencies found by Dupret et al. (2009) to lower stellar masses more representative of the mass range of CoRot and Kepler targets.
Model A: For this model at the bottom of the RGB, the lifetimes present a clear modulation, due to the variation of the inertia. The radiative damping is not important and dipolar mixed-modes are detectable. Quadrupolar mixed-modes either have high lifetimes (higher than 500 days) and could not be resolved or they are hidden in the mode trapped in the envelope. We also noticed that, as already seen in adiabatic computations (Montalbán et al. 2012), modes have a large period spacing.

Model B: In this intermediate model on the RGB, the radiative damping becomes non-negligible. Lifetimes are still modulated by inertia except for the $\ell = 2$ low frequency $g$-type modes. This damping is still low enough to have detectable dipolar mixed-modes in the power spectrum. As in the adiabatic computations, modes have a smaller period-spacing because of a higher density contrast between the core and the envelope.

Model C: This model, higher on the RGB has a much higher density contrast. The modulation of the lifetimes by the inertia has disappeared because of a very important radiative damping for all modes except those strongly trapped in the envelope. This implies that no mixed-modes are detectable in this model.

4. Conclusions

On the RGB mixed-modes are detectable until the radiative damping becomes too important. For $1.5 M_\odot$ stars on the RGB and with 150 days of observations, mixed-modes are predicted to be detectable for stars with $\nu_{\text{max}} \gtrsim 100 \mu$Hz and $\Delta \nu \gtrsim 8 \mu$Hz. For more massive stars, this limit appears at lower $\nu_{\text{max}}$ and $\Delta \nu$ [model B in the work of Dupret et al. (2009)].

References

Dupret, M.-A., Goupil, M.-J., Samadi, R., Grigahcène, A., & Gabriel, M. 2006, in Proc. of SOHO 18/GONG 2006/HELAS I, Beyond the spherical Sun, ESA SP-624, p.78.1