Contact Model between Superelements in Dynamic Multibody Systems

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Contact modeling

Contact between two superelements
- Flexibility accounted for with reasonable CPU time and memory
- Contact forces transferred to load directly the modal variables ➔ very compact formulation

Contact between rigid bodies
- low memory and CPU requirements
- rigidity assumption

Contact between FE models
- Flexibility accurately represented
- CPU time and memory highly expensive

• 2 possible strategies for the contact formulation: penalty versus LCP
Motivation

Gear pair modeling

- Global model [Cardona, 1995]
  - Kinematic joints defined between 2 nodes (wheel centers).
  - Gear wheels = rigid body.
  - Spring-damper along the normal pressure line.
  - Gross modeling of meshing defaults:
    (backlash, load transmission error, friction,…).

- Contact condition between FE models
  - Deformation of gear teeth and gear web accurately taken into account.
  - Meshing defaults naturally modeled.
  - Short time simulation of 2 gear wheels.

- Contact model between superelements [Ziegler & Eberhard, 2011]
  - Gear wheel flexible behavior globally accounted for.
  - Determination of actual contact points by means of 3D gear wheel geometry.
  - Study of misalignment, backlash, gear hammering,…
Outline

• Corotational formulation of a superelement
• Contact detection algorithm
• Contact force formulation
• Numerical results:
  • Cam system
  • Gear pair simulation
• Ongoing work: dual approach for superelement formulation
Superelement formulation

Craig-Bampton method: substructuring technique for linear elastic model

1. Superelement
2. Contact formulation
3. Cam system
4. Gear pair
5. Ongoing work

- Static boundary modes

\[ \Psi_B = -K_{II}^{-1} K_{IB} \]

- Internal vibration modes

\[ (K_{II} - \omega^2 M_{II}) \Psi_I = 0_{n_I \times n_I} \]

- Reduction basis (mode matrix)

\[ \Psi = \begin{bmatrix} I & 0 \\ \Psi_B & \Psi_I \end{bmatrix} \]

- Reduced stiffness and mass matrices

\[ K = \Psi^T K \Psi = \begin{bmatrix} K_{BB} & 0 \\ 0 & \mu \omega^2 \end{bmatrix} \]
\[ M = \Psi^T M \Psi = \begin{bmatrix} M_{BB} & M_{BI} \\ M_{IB} & \mu \end{bmatrix} \]
1. Superelement

2. Contact formulation

3. Cam system

4. Gear pair

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Corotational formulation of a superelement

- Vector of generalized coordinates

\[ \eta = \begin{bmatrix} u_B \\ \gamma_B \\ \eta_I \end{bmatrix} \quad q = \begin{bmatrix} x_0 \\ \alpha_0 \\ x_B \\ \alpha_B \\ \eta_I \end{bmatrix} \quad \delta \eta = P(q) \delta q \]

- Elastic forces in the absolute inertial frame

\[ g_{elastic} = P^T \bar{K} \eta \]

- Constraints to determine the corotational frame position

\[ \Phi(q) \equiv \tau_{rig}^T \bar{M}_B \eta_B(q) = 0 \]

- Kinematics of a superelement

\[ \begin{align*}
    x_P &= x_0 + R_0 (X_P + u_P) \\
    R_P &= R_0 R(\gamma_P)
\end{align*} \]

Rigid body modes

\[ \tau_{rig} = \begin{bmatrix} \tau_{rig,1} \\
    \vdots \\
    \tau_{rig,i} \\
    \vdots \\
    \tau_{rig,nb} \end{bmatrix} \]

\[ \tau_{rig,i} = \begin{bmatrix} I_{3\times3} & -\tilde{X}_{Bi} \\
    0_{3\times3} & I_{3\times3} \end{bmatrix} \]
Boundary nodes vs. contact nodes

- If all candidate contact nodes are boundary nodes
  ➔ huge number of generalized coordinates

\[ \mathbf{q} = \begin{pmatrix} \mathbf{x}_0 \\ \alpha_0 \\ \mathbf{x}_B \\ \alpha_B \\ \eta_I \end{pmatrix} \]

- Solution:
  - A few number of boundary nodes.
  - The position of candidate contact nodes are computed from modal variables and the corotational frame position.

\[
\mathbf{x}_{C_i} = \mathbf{x}_0^s + \mathbf{R}_0^s (X_{C_i} + \mathbf{\Psi}_{C_i} \eta^s ) \\
\mathbf{x}_{N_i} = \mathbf{x}_0^m + \mathbf{R}_0^m (X_{N_i} + \mathbf{\Psi}_{N_i} \eta^m )
\]

- Direct loading of the modal variables (static and dynamic).

➔ very compact formulation
Contact detection algorithm

1. Superelement
2. Contact formulation
3. Cam system
4. Gear pair model
5. Ongoing work

Projection point outside the surface element \(\Rightarrow\) inactive contact

Projection point inside the surface element \(\Rightarrow\) active contact
Contact force

- Contact law: penalty method with a stiffness and a damping contribution

\[
f(\ell, \dot{\ell}) = \begin{cases} 
S_c^* \left( k_p \ell^m + c \ell^m \dot{\ell} \right) & \text{if } \ell > 0 \text{ active contact} \\
0 & \text{if } \ell < 0 \text{ no contact}
\end{cases}
\]

penetration length:

\[
\ell = n^T(x_{N1} - x_{Ci})
\]

\[
\dot{\ell} = n^T(\dot{x}_{N1} - \dot{x}_{Ci}) + (x_{N1} - x_{Ci})^T \dot{n}
\]

- Contact force vector

\[
f_c = w f n
\]

participation factor
Contact force

- Each force applied on a contact node is transformed in order to load the modal variables of the superelement:

$$\delta W_{Ci}^{con} = \delta x_{Ci}^T f_c = \delta q^T g_{Ci}^{int,con}$$

with

$$\delta x_{Ci} = \delta x_0 - R_0 (X_{Ci} + u_{Ci}) \delta \Theta_0 + R_0 \delta u_{Ci}$$

$$= \delta x_0 - R_0 (X_{Ci} + \Psi_{Ci} \eta) \delta \Theta_0 + R_0 \Psi_{Ci} P \delta q$$

- The internal force vector due to a contact force is expressed as:

$$g_{Ci}^{int,con} = P^T \Psi_{Ci}^T R_0^T f_c + \begin{pmatrix} f_c \\ (X_{Ci} + \Psi_{Ci} \eta) R_0^T f_c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- Analytical computation of its contribution to the iteration matrix
Total number of DOFs: 142
• 18+9 boundary nodes
• 20+20 vibrations nodes
<< 107532 Dofs of full FE model

Eigenfrequencies of internal vibration modes (Hz)

<table>
<thead>
<tr>
<th></th>
<th>roller</th>
<th>cam</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>2.5E4</td>
<td>4.5E3</td>
</tr>
<tr>
<td>$f_{20}$</td>
<td>6.6E4</td>
<td>2.2E4</td>
</tr>
</tbody>
</table>
Numerical results
Numerical results

Modal intensities: cam

Modal intensities: roller
1. Superelement
2. Contact formulation
3. Cam system
4. Gear pair model
5. Ongoing work

Gear pair modeling: various steps

1) CAD modeling (CATIA V5)

2) FE modeling and model reduction (SAMCEF)

3) Simulation of unilateral contact between superelements (MATLAB)

Selection strategy of slave-master flank pairs
Model description

<table>
<thead>
<tr>
<th></th>
<th>pinion</th>
<th>gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth [-]</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>Pitch diameter [mm]</td>
<td>73,2</td>
<td>109,8</td>
</tr>
<tr>
<td>Outside diameter [mm]</td>
<td>82,64</td>
<td>118,64</td>
</tr>
<tr>
<td>Root diameter [mm]</td>
<td>62,5</td>
<td>98,37</td>
</tr>
<tr>
<td>Addendum coef. [-]</td>
<td>0,196</td>
<td>0,125</td>
</tr>
<tr>
<td>Tooth width [mm]</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Pressure angle [deg]</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Module [mm]</td>
<td>4,5</td>
<td></td>
</tr>
</tbody>
</table>

(Lundvall, Strömberg, Klarbring, 2004)

- 1 boundary node per tooth flank
- 100 internal vibrations modes
  ➔ 695 DOFS << 480171 for FEM
- Parallel rotation axis ➔ no misalignment
- Large center distance ➔ significant backlash
- At $t=0s$, $\omega_1 = -1000$ rpm, $\omega_2 = 667$ rpm
- For $t > 0s$: Viscous torque: $T_1 = -1 \omega_1 ; \omega_2 = 667$ rpm
- Time step: $h=1E.-6s$

### Eigenfrequencies of internal vibration modes (Hz)

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<tr>
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<th>gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>19520</td>
<td>10402</td>
</tr>
<tr>
<td>$f_{100}$</td>
<td>146068</td>
<td>115469</td>
</tr>
</tbody>
</table>
Numerical results

Contact force between tooth flanks [N]

- tooth 8 / flank 2
- tooth 9 / flank 2
- tooth 10 / flank 2
- tooth 11 / flank 2

Zoom
Numerical results

Displacement of boundary nodes [m]

- tooth 8 / flank 2
- tooth 8 / flank 1
- tooth 9 / flank 2
- tooth 9 / flank 1
- tooth 10 / flank 2
- tooth 10 / flank 1
- tooth 11 / flank 2
- tooth 11 / flank 1
- other boundary nodes

Time [s]
1. Superelement  
2. Contact formulation  
3. Cam system  
4. Gear pair model  
5. Ongoing work

**Numerical results**

- Modal amplitudes [-]

- Rotation velocity [rpm]
Ongoing work: dual approach

- Dual Craig-Bampton [Rixen, 2004]
  - Subset of free-free vibration modes

- Attachment modes to have a correct static response at interface nodes
  Unit displacements $\rightarrow$ unit loads

Filtering with respect to the elastic modes
$\Rightarrow$ **residual** attachment modes

- Mode matrix
  \[
  \overline{\Psi} = \begin{bmatrix} \overline{\Psi}_f & \overline{\Psi}_r \end{bmatrix}
  \]
Ongoing work: dual approach

- Residual attachment modes can be orthogonalized in order to get full diagonal matrices
  \[
  \bar{K} = \begin{bmatrix}
  \omega_f^2 & \mu_f & 0 \\
  0 & \omega_r^2 & \mu_r
  \end{bmatrix}, \quad \bar{M} = \begin{bmatrix}
  \mu_f & 0 \\
  0 & \mu_r
  \end{bmatrix}
  \]

- Floating frame to describe the rigid body motion of the superlement instead of corotational frame
  \[
  q = \begin{bmatrix}
  x_0 \\
  \alpha_0 \\
  \eta_f \\
  r
  \end{bmatrix}, \quad u \cong \bar{\Psi}_f \eta_f + \Psi_r r
  \]

- Main difference with respect to MacNeal and Rubin methods:
  \rightarrow assembly with interface forces rather than interface displacements

- Contact element unchanged

SE generalized coordinates  \rightarrow \text{Position of candidate contact nodes}  \rightarrow \text{Contact detection algorithm}  \rightarrow \text{Contact force}  \rightarrow \text{Loading of SE coordinates}
Conclusion

• Summary
  • Reduction of model size by 1 to several orders of magnitude
  • Direct loading of the modal generalized variables

• Perspectives:
  • Improvement of the contact detection algorithm
  • Dynamic management of contact zones (mode switching)
  • Contact law with algebraic constraint and nonsmooth time integration
  • Friction forces
  • Testing in various configurations (e.g. misalignment,…)
  • Implementation in a commercial software
  • Simulation of a full TORSEN differential
Thank you for your attention!

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