# Mathémagie III

Michel Rigo

http://www.discmath.ulg.ac.be/
http://orbi.ulg.ac.be/handle/2268/154362

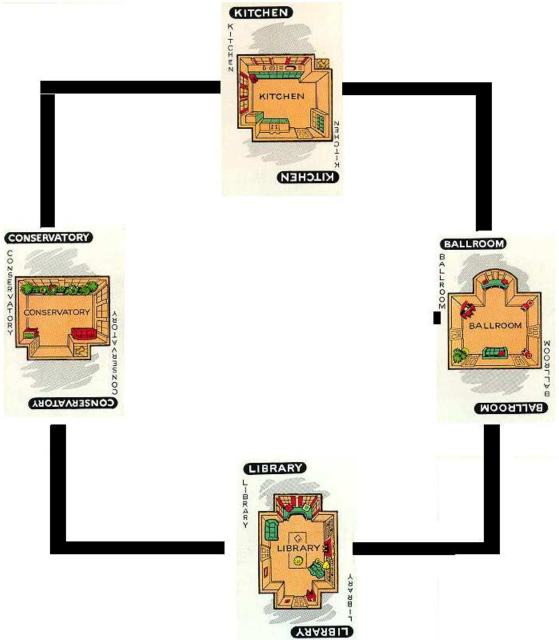
# Table des tours

- Cluedo
- Le tour de Mazzino
- La divination de Leonardo de Pisa
- Universal Cycle !
- Mélange de Gilbreath
- Encore le mélange de Josephus ?
- Un petit dernier pour la route !

*A Mathematician is a conjuror*
*who gives away his secrets*
— John H. Conway.

# Cluedo

David Copperfield – Jean Doyen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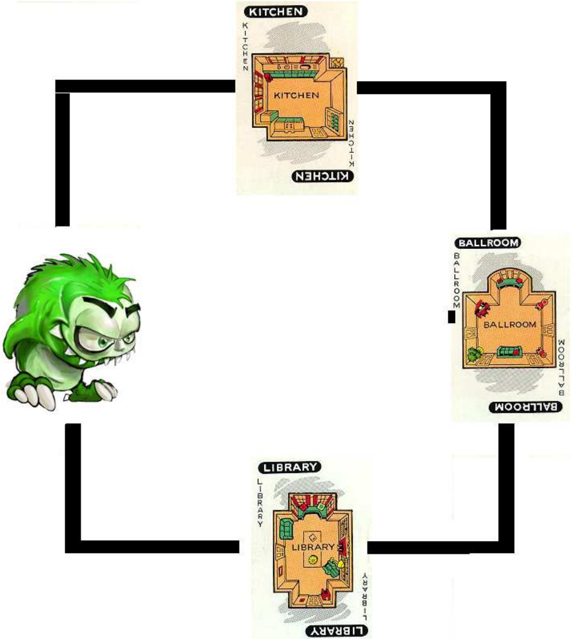
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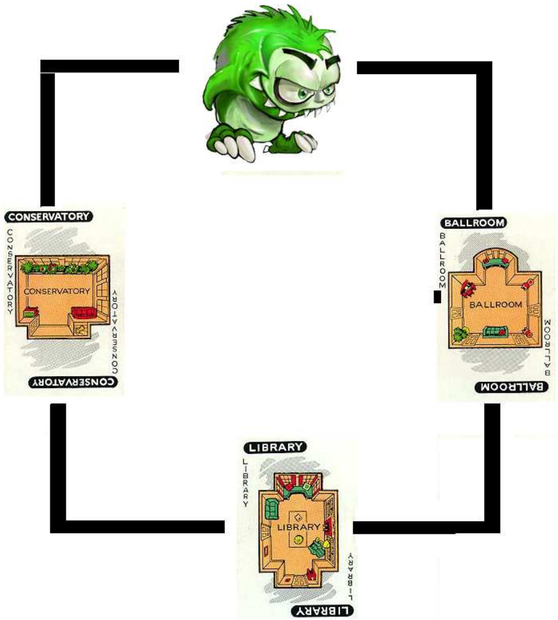
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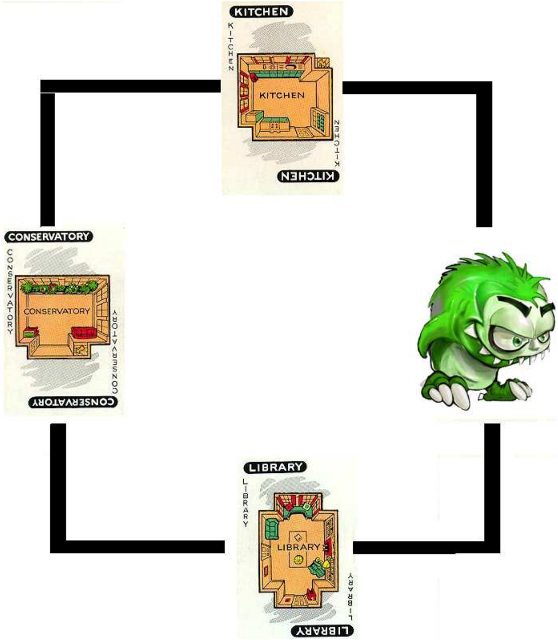
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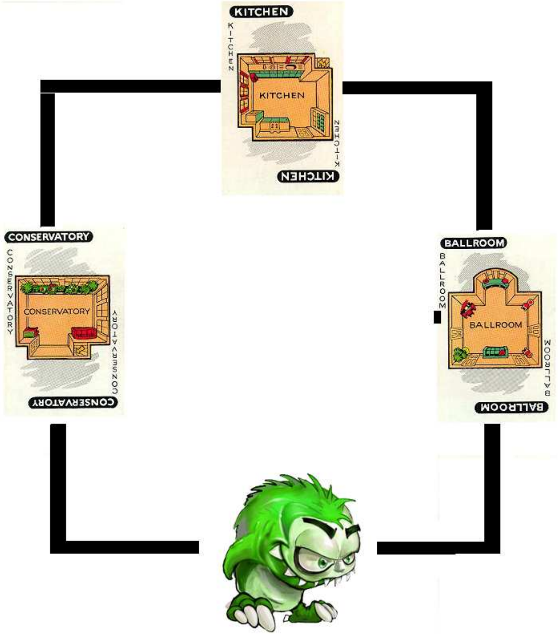
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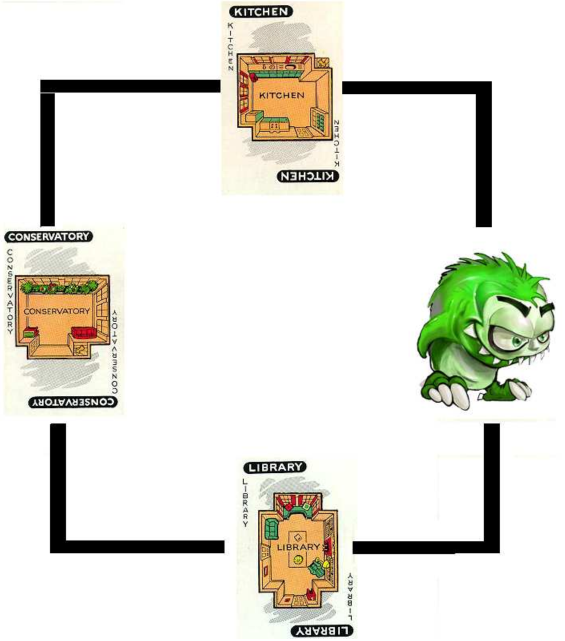
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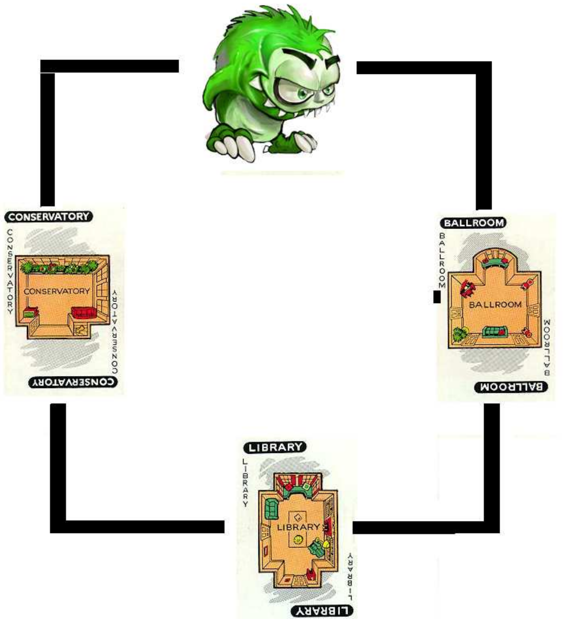
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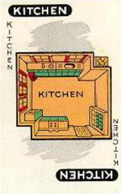
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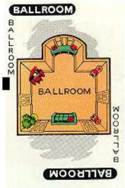
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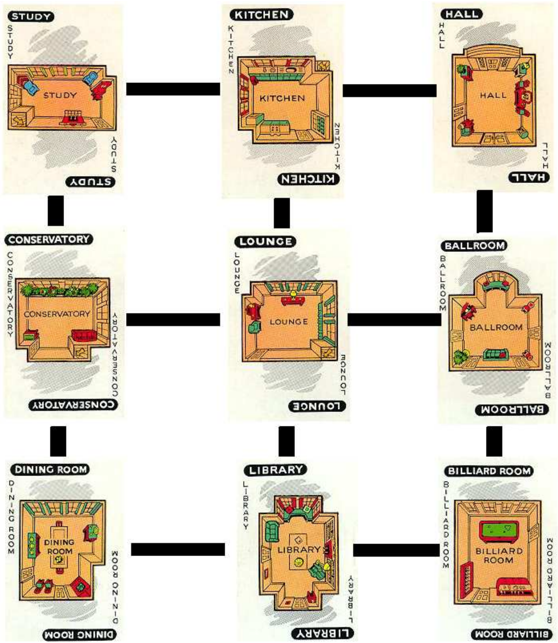
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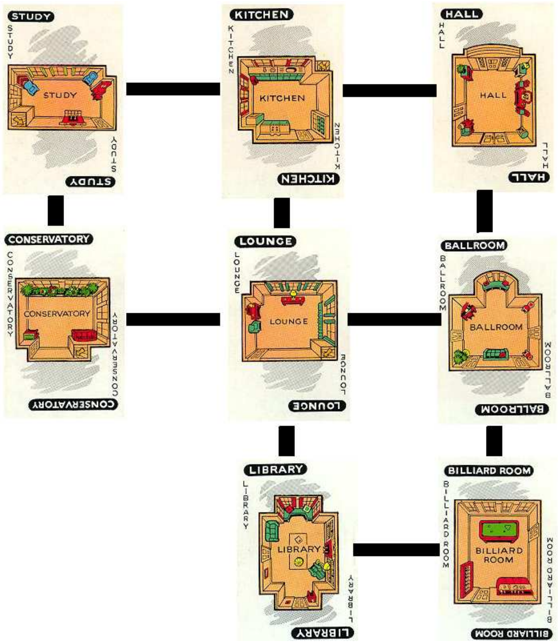
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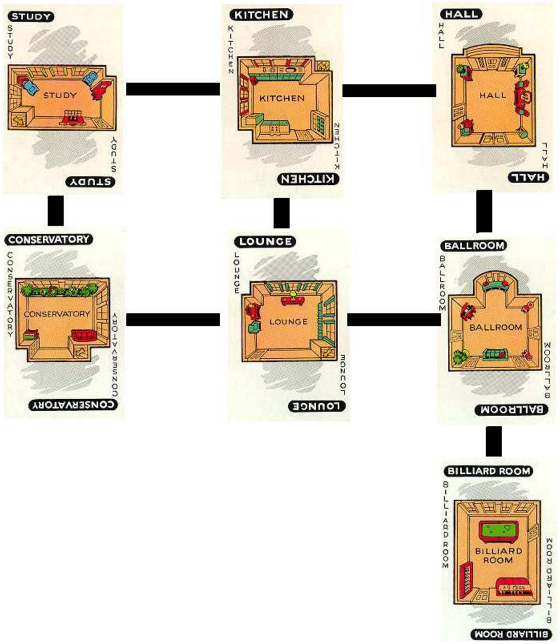
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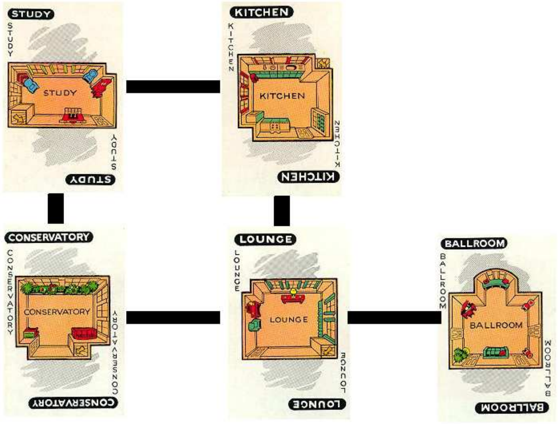
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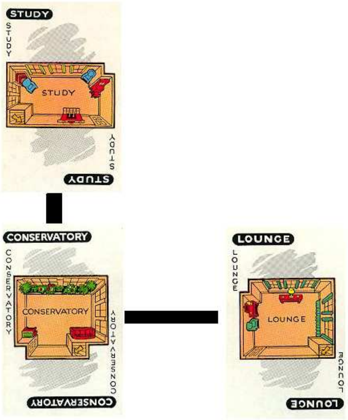
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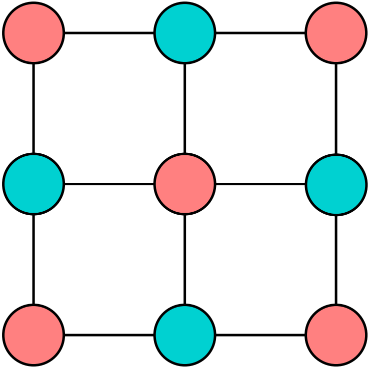


# Applications

- Problème d'*assignation optimale*
- Ouvriers et tâches à accomplir
- Donneurs d'organes et malades en attente d'une greffe
- Attribution d'adresses sur le réseau

**Théorème Djoković (1973)**

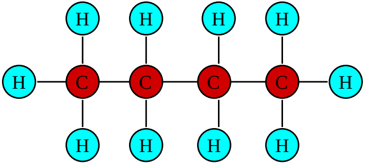
Les sommets d'un graphe simple peuvent recevoir une adresse binaire telle que la distance de Hamming entre ces adresses corresponde à la distance dans le graphe SSI le graphe est biparti + *condition technique*...

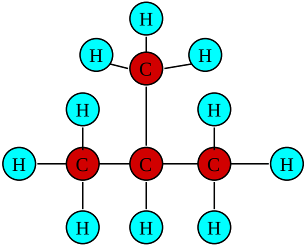
**Théorème**

Un graphe est $2$-colorable si et seulement si il est biparti.

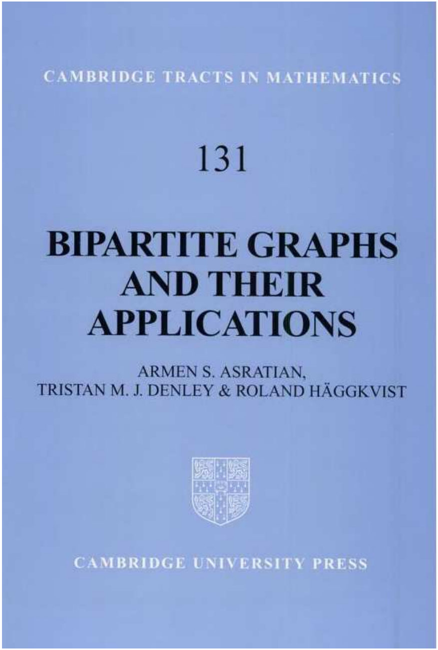
# Applications

- En chimie (organique), plusieurs substances chimiques peuvent avoir la même "formule" mais des structures moléculaires différentes, les *isomères*. Quelles configurations sont *combinatoirement* possibles ?




butane $C_4H_{10}$ et isobutane (2-methylpropane)


CAMBRIDGE TRACTS IN MATHEMATICS
131
BIPARTITE GRAPHS AND THEIR APPLICATIONS
ARMEN S. ASRATIAN, TRISTAN M. J. DENLEY & ROLAND HÄGGKVIST
CAMBRIDGE UNIVERSITY PRESS

# Le tour de Mazzino

30+2 cartes

$\heartsuit - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

$\spadesuit - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

on coupe le jeu. . .

$\heartsuit - 1 - 2 - 3 - 4 - 5 \,||\, 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

$\spadesuit - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

$\heartsuit - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

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on coupe le jeu. . .

$\heartsuit - 1 - 2 - 3 - 4 - 5 \,||\, 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

$\spadesuit - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15$

$$6-7-8-9-10-11-12-13-14-15-\spadesuit-1-2-3-4-5$$

$$6-7-8-9-10-11-12-13-14-15-\heartsuit-1-2-3-4-5$$

on fait deux tas. . .

$$6-8-10-12-14-\spadesuit-2-4-6-8-10-12-14-\heartsuit-2-4$$

$$7-9-11-13-15-1-3-5-7-9-11-13-15--1-3--5$$

$6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15 - \spadesuit - 1 - 2 - 3 - 4 - 5$

$6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15 - \heartsuit - 1 - 2 - 3 - 4 - 5$

on fait deux tas...

$6 - 8 - 10 - 12 - 14 - \spadesuit - 2 - 4 - 6 - 8 - 10 - 12 - 14 - \heartsuit - 2 - 4$

$7 - 9 - 11 - 13 - 15 - 1 - 3 - 5 - 7 - 9 - 11 - 13 - 15 - -1 - 3 - -5$

$6 - 8 - 10 - 12 - 14 - \spadesuit - 2 - 4 - 6 - 8 - 10 - 12 - 14 - \heartsuit - 2 - 4$

on fait deux tas...

$6 - 10 - 14 - 2 - 6 - 10 - 14 - 2$

$8 - 12 - \spadesuit - 4 - 8 - 12 - \heartsuit - 4$

$$6 - 8 - 10 - 12 - 14 - \spadesuit - 2 - 4 - 6 - 8 - 10 - 12 - 14 - \heartsuit - 2 - 4$$

on fait deux tas...

$$6 - 10 - 14 - 2 - 6 - 10 - 14 - 2$$

$$8 - 12 - \spadesuit - 4 - 8 - 12 - \heartsuit - 4$$

$$8 - 12 - \spadesuit - 4 - 8 - 12 - \heartsuit - 4$$

on fait deux tas...

$$8 - \spadesuit - 8 - \heartsuit$$

$$12 - 4 - 12 - 4$$

on fait deux tas...

$$\spadesuit - \heartsuit$$

$$8 - 8$$

$$8 - 12 - \spadesuit - 4 - 8 - 12 - \heartsuit - 4$$

on fait deux tas...

$$8 - \spadesuit - 8 - \heartsuit$$

$$12 - 4 - 12 - 4$$

on fait deux tas...

$$\spadesuit - \heartsuit$$

$$8 - 8$$

$$8 - 12 - \color{blue}{\spadesuit} - 4 - 8 - 12 - \color{red}{\heartsuit} - 4$$

on fait deux tas...

$$8 - \color{blue}{\spadesuit} - 8 - \color{red}{\heartsuit}$$

$$12 - 4 - 12 - 4$$

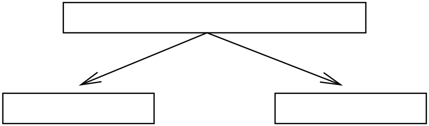
on fait deux tas...

$$\color{blue}{\spadesuit} - \color{red}{\heartsuit}$$

$$8 - 8$$

# Applications

Philosophie du “diviser pour régner”.



Un tri “efficace” comme le *quicksort* $\mathcal{O}(n \log n)$ dans le pire cas.

# Divination

Leonardo de Pisa

## Choisir un nombre entre 1 et 54...

$1 - 4 - 6 - 9 - 12 - 14 - 17 - 19 - 22 - 25 - 27$

$30 - 33 - 35 - 38 - 40 - 43 - 46 - 48 - 51 - 53$

$2 - 7 - 10 - 15 - 20 - 23 - 28 - 31 - 36 - 41 - 44 - 49 - 54$

$3 - 4 - 11 - 12 - 16 - 17 - 24 - 25 - 32 - 33 - 37 - 38 - 45 - 46 - 50 - 51$

$5 - 6 - 7 - 18 - 19 - 20 - 26 - 27 - 28 - 39 - 40 - 41 - 52 - 53 - 54$

## Choisir un nombre entre 1 et 54...

$1 - 4 - 6 - 9 - 12 - 14 - 17 - 19 - 22 - 25 - 27$

$30 - 33 - 35 - 38 - 40 - 43 - 46 - 48 - 51 - 53$

$2 - 7 - 10 - 15 - 20 - 23 - 28 - 31 - 36 - 41 - 44 - 49 - 54$

$3 - 4 - 11 - 12 - 16 - 17 - 24 - 25 - 32 - 33 - 37 - 38 - 45 - 46 - 50 - 51$

$5 - 6 - 7 - 18 - 19 - 20 - 26 - 27 - 28 - 39 - 40 - 41 - 52 - 53 - 54$

## Choisir un nombre entre 1 et 54...

$1 - 4 - 6 - 9 - 12 - 14 - 17 - 19 - 22 - 25 - 27$

$30 - 33 - 35 - 38 - 40 - 43 - 46 - 48 - 51 - 53$

$2 - 7 - 10 - 15 - 20 - 23 - 28 - 31 - 36 - 41 - 44 - 49 - 54$

$3 - 4 - 11 - 12 - 16 - 17 - 24 - 25 - 32 - 33 - 37 - 38 - 45 - 46 - 50 - 51$

$5 - 6 - 7 - 18 - 19 - 20 - 26 - 27 - 28 - 39 - 40 - 41 - 52 - 53 - 54$

## Choisir un nombre entre 1 et 54...

$1 - 4 - 6 - 9 - 12 - 14 - 17 - 19 - 22 - 25 - 27$

$30 - 33 - 35 - 38 - 40 - 43 - 46 - 48 - 51 - 53$

$2 - 7 - 10 - 15 - 20 - 23 - 28 - 31 - 36 - 41 - 44 - 49 - 54$

$3 - 4 - 11 - 12 - 16 - 17 - 24 - 25 - 32 - 33 - 37 - 38 - 45 - 46 - 50 - 51$

$5 - 6 - 7 - 18 - 19 - 20 - 26 - 27 - 28 - 39 - 40 - 41 - 52 - 53 - 54$

Choisir un nombre entre 1 et 54...

$1 - 4 - 6 - 9 - 12 - 14 - 17 - 19 - 22 - 25 - 27$

$30 - 33 - 35 - 38 - 40 - 43 - 46 - 48 - 51 - 53$

$2 - 7 - 10 - 15 - 20 - 23 - 28 - 31 - 36 - 41 - 44 - 49 - 54$

$3 - 4 - 11 - 12 - 16 - 17 - 24 - 25 - 32 - 33 - 37 - 38 - 45 - 46 - 50 - 51$

$5 - 6 - 7 - 18 - 19 - 20 - 26 - 27 - 28 - 39 - 40 - 41 - 52 - 53 - 54$

$8 - 9 - 10 - 11 - 12 - 29 - 30 - 31 - 32 - 33 - 42 - 43 - 44 - 45 - 46$

$13 - 14 - 15 - 16 - 17 - 18 - 19 - 20 - 47 - 48 - 49 - 50 - 51 - 52 - 53 - 54$

$21 - 22 - 23 - 24 - 25 - 26 - 27 - 28 - 29 - 30 - 31 - 32 - 33$

J'hésite entre deux nombres ? $< 34$ ou $\geq 34$ ?

$8 - 9 - 10 - 11 - 12 - 29 - 30 - 31 - 32 - 33 - 42 - 43 - 44 - 45 - 46$

$13 - 14 - 15 - 16 - 17 - 18 - 19 - 20 - 47 - 48 - 49 - 50 - 51 - 52 - 53 - 54$

$21 - 22 - 23 - 24 - 25 - 26 - 27 - 28 - 29 - 30 - 31 - 32 - 33$

J'hésite entre deux nombres ? $< 34$ ou $\geq 34$ ?

$8 - 9 - 10 - 11 - 12 - 29 - 30 - 31 - 32 - 33 - 42 - 43 - 44 - 45 - 46$

$13 - 14 - 15 - 16 - 17 - 18 - 19 - 20 - 47 - 48 - 49 - 50 - 51 - 52 - 53 - 54$

$21 - 22 - 23 - 24 - 25 - 26 - 27 - 28 - 29 - 30 - 31 - 32 - 33$

J'hésite entre deux nombres ? $< 34$ ou $\geq 34$ ?

$8 - 9 - 10 - 11 - 12 - 29 - 30 - 31 - 32 - 33 - 42 - 43 - 44 - 45 - 46$

$13{-}14{-}15{-}16{-}17{-}18{-}19{-}20{-}47{-}48{-}49{-}50{-}51{-}52{-}53{-}54$

$21 - 22 - 23 - 24 - 25 - 26 - 27 - 28 - 29 - 30 - 31 - 32 - 33$

J'hésite entre deux nombres ? $< 34$ ou $\geq 34$ ?

Suite de Fibonacci $F_{i+2} = F_{i+1} + F_i$, $F_0 = 1$, $F_1 = 2$

$$\ldots, 610, 377, 233, 144, 89, 55, 34, 21, 13, 8, 5, 3, 2, 1$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 15 | 100010 | 29 | 1010000 | 43 | 10010001 |
| 2 | 10 | 16 | 100100 | 30 | 1010001 | 44 | 10010010 |
| 3 | 100 | 17 | 100101 | 31 | 1010010 | 45 | 10010100 |
| 4 | 101 | 18 | 101000 | 32 | 1010100 | 46 | 10010101 |
| 5 | 1000 | 19 | 101001 | 33 | 1010101 | 47 | 10100000 |
| 6 | 1001 | 20 | 101010 | 34 | 10000000 | 48 | 10100001 |
| 7 | 1010 | 21 | 1000000 | 35 | 10000001 | 49 | 10100010 |
| 8 | 10000 | 22 | 1000001 | 36 | 10000010 | 50 | 10100100 |
| 9 | 10001 | 23 | 1000010 | 37 | 10000100 | 51 | 10100101 |
| 10 | 10010 | 24 | 1000100 | 38 | 10000101 | 52 | 10101000 |
| 11 | 10100 | 25 | 1000101 | 39 | 10001000 | 53 | 10101001 |
| 12 | 10101 | 26 | 1001000 | 40 | 10001001 | 54 | 10101010 |
| 13 | 100000 | 27 | 1001001 | 41 | 10001010 | | |
| 14 | 100001 | 28 | 1001010 | 42 | 10010000 | | |

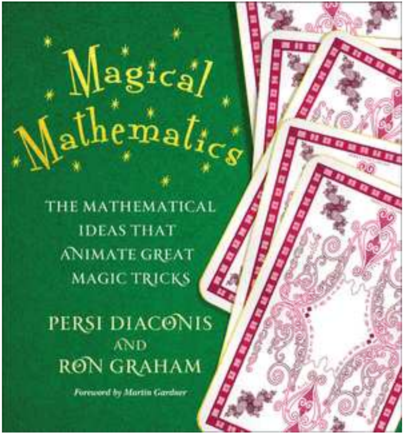
# Applications

Représentation non-standard de nombres

- Codages
- Gauss et algorithme d'Avizienis pour l'addition
- NAF (non-adjacent form) et courbes elliptiques
- Logique, vérification, théorie des jeux,. . .

# Universal Cycle !
## Persi Diaconis – Ron Graham

*Magical Mathematics:*
*The Mathematical Ideas That Animate Great Magic Tricks*
*Princeton University Press*



On coupe le jeu – on distribue 5 cartes consécutives –
que les 3 plus hautes cartes se fassent connaître. . .

# Vos cartes...

| | | |
|---|---|---|
| $1\clubsuit, V\spadesuit, 7\diamondsuit, 4\heartsuit, 6\clubsuit$<br>$1\heartsuit, D\diamondsuit, 6\heartsuit, 7\clubsuit, 4\diamondsuit$<br>$R\heartsuit, V\clubsuit, 8\clubsuit, 7\heartsuit, 10\clubsuit$ | $D\spadesuit, 1\clubsuit, V\spadesuit, 7\diamondsuit, 4\heartsuit$<br>$D\clubsuit, R\spadesuit, 9\clubsuit, V\heartsuit, 2\diamondsuit$<br>$9\clubsuit, V\heartsuit, 2\diamondsuit, 3\heartsuit, 4\clubsuit$ | $V\diamondsuit, 7\spadesuit, 1\spadesuit, 5\clubsuit, 3\diamondsuit$<br>$D\heartsuit, 3\spadesuit, R\heartsuit, V\clubsuit, 8\clubsuit$<br>$\cdot$ |
| $R\spadesuit, 9\clubsuit, V\heartsuit, 2\diamondsuit, 3\heartsuit$<br>$10\clubsuit, 4\spadesuit, 9\spadesuit, 8\diamondsuit, 3\clubsuit$<br>$\cdot$ | $4\spadesuit, 9\spadesuit, 8\diamondsuit, 3\clubsuit, 2\spadesuit$<br>$3\spadesuit, R\heartsuit, V\clubsuit, 8\clubsuit, 7\heartsuit$<br>$5\spadesuit, 1\heartsuit, D\diamondsuit, 6\heartsuit, 7\clubsuit$ | $\cdot$<br>$6\diamondsuit, D\spadesuit, 1\clubsuit, V\spadesuit, 7\diamondsuit$<br>$5\heartsuit, D\clubsuit, R\spadesuit, 9\clubsuit, V\heartsuit$ |
| $V\clubsuit, 8\clubsuit, 7\heartsuit, 10\clubsuit, 4\spadesuit$<br>$1\diamondsuit, 5\heartsuit, D\clubsuit, R\spadesuit, 9\clubsuit$<br>$1\spadesuit, 5\clubsuit, 3\diamondsuit, 10\heartsuit, 8\spadesuit$ | $\cdot$<br>$3\diamondsuit, 10\heartsuit, 8\spadesuit, 9\heartsuit, 2\heartsuit$<br>$7\heartsuit, 10\clubsuit, 4\spadesuit, 9\spadesuit, 8\diamondsuit$ | $9\diamondsuit, 5\spadesuit, 1\heartsuit, D\diamondsuit, 6\heartsuit$<br>$6\clubsuit, 8\heartsuit, R\clubsuit, 9\diamondsuit, 5\spadesuit$<br>$3\clubsuit, 2\spadesuit, R\diamondsuit, 10\diamondsuit, 6\diamondsuit$ |
| $V\spadesuit, 7\diamondsuit, 4\heartsuit, 6\clubsuit, 8\heartsuit$<br>$D\diamondsuit, 6\heartsuit, 7\clubsuit, 4\diamondsuit, 10\spadesuit$<br>$V\heartsuit, 2\diamondsuit, 3\heartsuit, 4\clubsuit, 5\diamondsuit$ | $7\spadesuit, 1\spadesuit, 5\clubsuit, 3\diamondsuit, 10\heartsuit$<br>$2\spadesuit, R\diamondsuit, 10\diamondsuit, 6\diamondsuit, D\spadesuit$<br>$10\spadesuit, 1\diamondsuit, 5\heartsuit, D\clubsuit, R\spadesuit$ | $8\clubsuit, 7\heartsuit, 10\clubsuit, 4\spadesuit, 9\spadesuit$<br>$4\diamondsuit, 10\spadesuit, 1\diamondsuit, 5\heartsuit, D\clubsuit$<br>$5\clubsuit, 3\diamondsuit, 10\heartsuit, 8\spadesuit, 9\heartsuit$ |

| | |
|---|---|
| $\cdot$<br>$\cdot$<br>$R\clubsuit, 9\diamondsuit, 5\spadesuit, 1\heartsuit, D\diamondsuit$ | $9\spadesuit, 8\diamondsuit, 3\clubsuit, 2\spadesuit, R\diamondsuit$<br>$10\heartsuit, 8\spadesuit, 9\heartsuit, 2\heartsuit, D\heartsuit$<br>$R\diamondsuit, 10\diamondsuit, 6\diamondsuit, D\spadesuit, 1\clubsuit$ |
| $8\spadesuit, 9\heartsuit, 2\heartsuit, D\heartsuit, 3\spadesuit$<br>$2\clubsuit, V\diamondsuit, 7\spadesuit, \spadesuit, 5\clubsuit$<br>$2\heartsuit, D\heartsuit, 3\spadesuit, R\heartsuit, V\clubsuit$ | $\cdot$<br>$8\heartsuit, R\clubsuit, 9\diamondsuit, 5\spadesuit, 1\heartsuit$<br>$\cdot$ |
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## Définition (cycle universel)

Soient $n, k$ des entiers.
Ordonner, selon un cycle, $n$ cartes de telle façon que chaque suite de $k$ cartes consécutives possède un “codage/type” unique.

$$5\heartsuit - V\heartsuit - \underbrace{\boxed{R\clubsuit \;-\; 10\spadesuit \;-\; 6\diamondsuit \;-\; 3\clubsuit \;-\; 5\spadesuit}}_{T=(1,2,3)} - 7\diamondsuit - D\heartsuit - 8\clubsuit$$

$k = 5$ et on code “les positions des trois plus grandes cartes”
nombre de codes distincts : $5 \times 4 \times 3 = 60$ donc $n \leq 60$

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$$5\heartsuit - V\heartsuit - R\clubsuit - \underbrace{\boxed{10\spadesuit \;-\; 6\diamondsuit \;-\; 3\clubsuit \;-\; 5\spadesuit \;-\; 7\diamondsuit}}_{T=(1,5,2)} - D\heartsuit - 8\clubsuit$$

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## Définition (cycle universel)

Soient $n, k$ des entiers.
Ordonner, selon un cycle, $n$ cartes de telle façon que chaque suite de $k$ cartes consécutives possède un “codage/type” unique.

$$5\heartsuit - V\heartsuit - R\clubsuit - 10\spadesuit - \underbrace{\boxed{6\diamondsuit - 3\clubsuit - 5\spadesuit - 7\diamondsuit - D\heartsuit}}_{T=(5,4,3)} - 8\clubsuit$$

$k = 5$ et on code “les positions des trois plus grandes cartes”
nombre de codes distincts : $5 \times 4 \times 3 = 60$ donc $n \leq 60$

# Construction aléatoire

(A) Initialiser

- `tas`=$\{6,\dots,13,1,\dots,13,1,\dots,13,1,\dots,13\}$
- `suite`=$\{1,2,3,4,5\}$, `type`=$\{\{3,2,1\}\}$

Tant que tas $\neq \emptyset$

(B) Prendre au hasard une carte $c$ de `tas`

Si les éléments $s_{n-3}, s_{n-2}, s_{n-1}, s_n$ de `suite` et $c$ satisfont

1. $s_{n-3}, s_{n-2}, s_{n-1}, s_n, c$ sont 2 à 2 distincts
2. le type $T$ de $(s_{n-3}, s_{n-2}, s_{n-1}, s_n, c) \notin$ `type`

Alors, ajouter $T$ à `type`, ajouter $c$ à `suite`, retirer $c$ de `tas`

Sinon, si moins de 50 tentatives, revenir en (B)

Sinon, si plus de 50 tentatives, revenir en (A)

Test final (pour cycle): $(s_{49}, \dots, s_{52}, s_1), \dots, (s_{52}, s_1, s_2, s_3, s_4)$ satisfont-ils 1. et 2. ? Si, non, revenir en (A).

$n = 44$

$$1, 2, 3, 4, 5, 1, 13, 10, 7, 9, 11, 4, 6, 1, 12, 2, 9, 5, 11, 13, 4, 8, 10, 2, 9,$$
$$\to 13, 11, 12, 10, 3, 4, 13, 5, 2, 6, 3, 12, 1, 5, 9, 7, 12, 8, 6.$$

$n = 45$

$$1, 2, 3, 4, 5, 1, 11, 10, 2, 4, 5, 13, 12, 1, 3, 2, 7, 13, 4, 10, 9, 6, 13, 8,$$
$$\to 11, 5, 7, 1, 13, 6, 3, 8, 7, 12, 2, 6, 11, 4, 5, 12, 9, 7, 6, 10, 11.$$

$n = 46$

$$1, 2, 3, 4, 5, 1, 9, 12, 2, 11, 5, 9, 1, 12, 4, 7, 8, 6, 2, 12, 13, 9, 11, 8, 2,$$
$$\to 6, 4, 7, 3, 12, 10, 11, 13, 7, 6, 10, 3, 9, 8, 7, 13, 4, 6, 10, 8, 5.$$

$n = 47$

$$1, 2, 3, 4, 5, 1, 2, 13, 7, 4, 10, 2, 11, 9, 7, 6, 8, 11, 10, 7, 6, 12, 8, 3, 13,$$

$$\to 5, 11, 8, 3, 6, 5, 9, 13, 1, 7, 5, 12, 11, 13, 10, 8, 3, 1, 9, 12, 4, 10.$$

$n = 48$ (2,2 secondes)

$$1, 2, 3, 4, 5, 1, 9, 13, 3, 5, 12, 10, 6, 2, 11, 8, 3, 4, 12, 13, 5, 1, 3, 2, 7, 5,$$

$$\to 10, 4, 9, 11, 6, 10, 7, 8, 4, 6, 12, 9, 10, 11, 8, 1, 9, 7, 2, 8, 13, 11.$$

$n = 49$ (11,6 secondes)

$$1, 2, 3, 4, 5, 1, 10, 3, 13, 8, 12, 5, 4, 11, 6, 8, 9, 5, 2, 7, 3, 4, 11, 10, 7, 6,$$

$$\to 12, 9, 13, 1, 8, 7, 2, 11, 5, 1, 7, 12, 9, 3, 6, 2, 12, 10, 4, 9, 11, 13, 10.$$

$n = 50$ (15,4 secondes)

$$1, 2, 3, 4, 5, 1, 13, 7, 6, 8, 4, 1, 3, 12, 9, 2, 5, 6, 12, 10, 13, 11, 6, 3, 4, 1,$$

$$\to 7, 11, 5, 9, 10, 7, 6, 13, 9, 11, 2, 8, 7, 3, 12, 2, 9, 8, 11, 4, 12, 5, 8, 13.$$

$n = 51$ (6,9 secondes)

$$1, 2, 3, 4, 5, 1, 2, 6, 12, 3, 1, 11, 6, 12, 10, 7, 2, 11, 8, 9, 7, 4, 5, 1, 13, 7,$$

$$\to 2, 12, 3, 10, 6, 9, 13, 7, 11, 4, 12, 5, 8, 9, 3, 11, 13, 5, 10, 9, 4, 8, 13, 10, 6.$$

$n = 52$ (69 secondes)

$$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$$

$$\to 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$$

# Construction de la suite

$$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$$
$$\to 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3, 2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2), (2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1), (4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5), (1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4), (4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3), (3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3), (5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4), (1, 3, 2), (2, 1, 5), (1, 5, 4)

# Construction de la suite

$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$
$\to 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3,
2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2),
(2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1),
(4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5),
(1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4),
(4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3),
(3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3),
(5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4),
(1, 3, 2), (2, 1, 5), (1, 5, 4)

# Construction de la suite

$$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$$
$$\to 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3,
2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2),
(2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1),
(4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5),
(1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4),
(4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3),
(3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3),
(5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4),
(1, 3, 2), (2, 1, 5), (1, 5, 4)

# Construction de la suite

$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$
$\rightarrow 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3, 2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2), (2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1), (4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5), (1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4), (4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3), (3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3), (5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4), (1, 3, 2), (2, 1, 5), (1, 5, 4)

# CONSTRUCTION DE LA SUITE

$$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$$
$$\to 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3, 2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2), (2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1), (4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5), (1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4), (4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3), (3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3), (5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4), (1, 3, 2), (2, 1, 5), (1, 5, 4)

# Construction de la suite

$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$
$\rightarrow 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3,
2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2),
(2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1),
(4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5),
(1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4),
(4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3),
(3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3),
(5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4),
(1, 3, 2), (2, 1, 5), (1, 5, 4)

# Construction de la suite

$$1, 2, 3, 4, 5, 1, 10, 6, 13, 4, 2, 9, 7, 8, 1, 11, 2, 12, 10, 7, 6, 9, 3, 8, 7, 2, 1,$$
$$\to 12, 9, 5, 11, 13, 10, 6, 3, 5, 7, 12, 8, 4, 13, 11, 5, 6, 3, 9, 13, 4, 11, 12, 8, 10.$$

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3,
2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

(5, 4, 3), (4, 3, 2), (5, 3, 2), (4, 5, 2), (5, 3, 4), (4, 2, 3), (3, 1, 2),
(2, 5, 1), (1, 4, 5), (3, 5, 4), (2, 4, 3), (5, 1, 3), (4, 2, 1), (5, 3, 1),
(4, 2, 5), (3, 1, 4), (2, 3, 4), (1, 2, 5), (1, 4, 2), (3, 5, 1), (2, 4, 5),
(1, 3, 4), (2, 3, 1), (5, 1, 2), (4, 5, 1), (3, 4, 5), (2, 5, 3), (5, 1, 4),
(4, 3, 5), (3, 2, 4), (2, 1, 3), (1, 2, 3), (1, 5, 2), (5, 4, 1), (4, 5, 3),
(3, 4, 2), (5, 2, 3), (4, 1, 5), (3, 4, 1), (2, 3, 5), (1, 2, 4), (1, 5, 3),
(5, 4, 2), (4, 3, 1), (3, 5, 2), (2, 5, 4), (1, 4, 3), (3, 2, 5), (2, 1, 4),
(1, 3, 2), (2, 1, 5), (1, 5, 4)

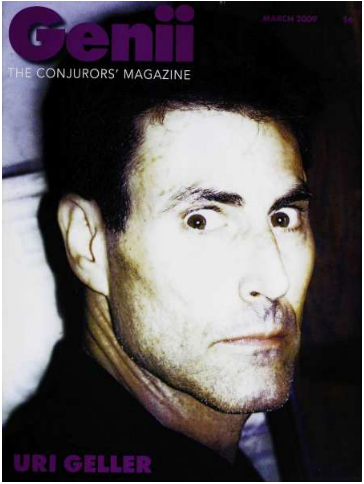
# Construction de la suite

2, 3, 4, 5, 6, 2, V, 7, A, 5, 3, 10, 8, 9, 2, D, 3, R, V, 8, 7, 10, 4, 9, 8, 3, 2, R, 10, 6, D, A, V, 7, 4, 6, 8, R, 9, 5, A, D, 6, 7, 4, 10, A, 5, D, R, 9, V

# Questions mathématiques

- Plutôt que l'heuristique, y-a-t-il un moyen de construction explicite (e.g., graphe de de Bruijn) ?
- Peut-on énumérer ces suites ?
- Applications à d'autres objets combinatoires qu'un jeu de cartes.
- R. Johnson, Universal cycles for permutations, *Discrete Mathathematics* **309** (2009), 5264–5270.

# Mélange de Norman Gilbreath
Mimosa – Alain Zalmanski, *Tangente* **152**




Genii, mars 2009.

$$\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}$$

$$\underbrace{1234512345123}_{b}\,||\,\underbrace{4512345123451234 5}_{a}$$

$$\overrightarrow{a} = 4512345123451234 5$$

$$\boxed{\overleftarrow{b} = 3215432154321}$$

$$34521\ 12354\ 54312\ 32451\ 15234\ 32145$$

$$\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}$$

$$\underbrace{1234512345123}_{b}\,||\,\underbrace{{\color{red}451234512345 12345}}_{a}$$

$$\overrightarrow{a} = 45123451234512345$$

$$\boxed{\overleftarrow{b} = 3215432154321}$$

34521 12354 54312 32451 15234 32145

$$\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}$$

$$\underbrace{1234512345123}_{b}\,||\,\underbrace{\color{red}{451234512345123 45}}_{a}$$

$$\overrightarrow{a} = \color{red}{4512345123451234 5}$$

$$\boxed{\overleftarrow{b} = 3215432154321}$$

$$34521\ 12354\ 54312\ 32451\ 15234\ 32145$$

$$\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}\,\overbrace{12345}$$

$$\underbrace{1234512345123}_{b}\,||\,\underbrace{\color{red}{4512345123451234 5}}_{a}$$

$$\overrightarrow{a} = \color{red}{4512345123451234 5}$$

$$\boxed{\overleftarrow{b} = 3215432154321}$$

$$3\color{red}{45}21\ \color{red}{123}54\ \color{red}{5}43\color{red}{12}\ \color{red}{3}2\color{red}{451}\ 15\color{red}{23}4\ 321\color{red}{45}$$

$$\underbrace{34521}\ 12354\ 54312\ 32451\ 15234\ 32145$$

$$a = \overrightarrow{45}|\,123451234512345$$

$$b = \overleftarrow{321}|\,5432154321$$

$$\underbrace{34521}\ \underbrace{12354}\ 54312\ 32451\ 15234\ 32145$$

$$a = \overrightarrow{45} | \overrightarrow{1234} | 51234512345$$

$$b = \overleftarrow{321} | \overleftarrow{5} | 432154321$$

$$\underbrace{34521}\ \underbrace{12354}\ \underbrace{54312}\ 32451\ 15234\ 32145$$

$$a = \overrightarrow{45}|\,\overrightarrow{1234}|\,\overrightarrow{512}|\,34512345$$

$$b = \overleftarrow{321}|\,\overleftarrow{5}\,|\,\overleftarrow{43}|\,2154321$$

$$\underbrace{34521}\ \underbrace{12354}\ \underbrace{54312}\ \underbrace{32451}\ 15234\ 32145$$

$$a = \overrightarrow{45} \,|\, \overrightarrow{1234} \,|\, \overrightarrow{512} \,|\, \overrightarrow{3451} \,|\, 2345$$

$$b = \overleftarrow{321} \,|\, \overleftarrow{5} \,|\, \overleftarrow{43} \,|\, \overleftarrow{2} \,|\, 154321$$

$$\underbrace{3\,45\,21}\ \underbrace{123\,5\,4}\ \underbrace{5\,43\,12}\ \underbrace{3\,2\,451}\ \underbrace{15\,23\,4}\ 321\,45$$

$$a = \overrightarrow{45} | \overrightarrow{1234} | \overrightarrow{512} | \overrightarrow{3451} | \overrightarrow{23} | 45$$

$$b = \overleftarrow{321} | \overleftarrow{5} | \overleftarrow{43} | \overleftarrow{2} | \overleftarrow{154} | 321$$

$$\underbrace{34521}\ \underbrace{12354}\ \underbrace{54312}\ \underbrace{32451}\ \underbrace{15234}\ \underbrace{32145}$$

$$a = \overrightarrow{45} | \overrightarrow{1234} | \overrightarrow{512} | \overrightarrow{3451} | \overrightarrow{23} | \overrightarrow{45}$$

$$b = \overleftarrow{321} | \overleftarrow{5} | \overleftarrow{43} | \overleftarrow{2} | \overleftarrow{154} | \overleftarrow{321}$$

$$123456789$$

$$a = 56789 \qquad \overleftarrow{b} = 4321$$

$$456378219$$

**Théorème**

Pour un mélange des cartes $\{1, \ldots, N\}$, les conditions suivantes sont équivalentes :

- On effectue un mélange de Gilbreath.
- Pour tout $j \leq N$, les $j$ premières cartes après mélange sont des entiers consécutifs.
- Pour tout $j \leq N$, les $j$ premières cartes après mélange sont distinctes modulo $j$.

# Encore le mélange de Josephus ?

Persi Diaconis – Ron Graham

http://images.math.cnrs.fr/Un-peu-de-mathemagie-avec-Flavius.html

$\mu(n)$ position de la $n$-ième carte après mélange

$$\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 8 & 4 & 7 & 2 & 6 & 3 & 5 \end{pmatrix}$$

Mélange de Monge[1] "au-dessus/en-dessous" (nombre pair de cartes)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | 8 |
| 2 | | | | | | 6 | 6 | 6 |
| 3 | | | | 4 | 4 | 4 | 4 | 4 |
| 4 | | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | | | 3 | 3 | 3 | 3 | 3 | 3 |
| 7 | | | | | 5 | 5 | 5 | 5 |
| 8 | | | | | | | 7 | 7 |

$\nu(n)$ position de la $n$-ième carte après mélange au-dessus/en-dessous

$$\nu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 6 & 3 & 7 & 2 & 8 & 1 \end{pmatrix}$$

[1] Gaspard Monge (1746–1818) donna les premières propriétés de ces mélanges en 1773.

Mélange "Milk shuffle" ou "Klondike shuffle"

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | 5 |
| 2 | | | | | | | 4 | 4 |
| 3 | | | | | | 6 | 6 | 6 |
| 4 | | | | | 3 | 3 | 3 | 3 |
| 5 | | | | 7 | 7 | 7 | 7 | 7 |
| 6 | | | 2 | 2 | 2 | 2 | 2 | 2 |
| 7 | | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

$\lambda(n)$ position de la $n$-ième carte après le "Milk shuffle"

$$\lambda = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 4 & 2 & 1 & 3 & 5 & 7 \end{pmatrix}$$

$$\nu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 6 & 3 & 7 & 2 & 8 & 1 \end{pmatrix}$$

$$\lambda = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 4 & 2 & 1 & 3 & 5 & 7 \end{pmatrix}$$

Un petit dernier pour la route !

*Les dossiers de La Recherche* Août–Septembre 2013

$$1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - V - D - R$$

$$\clubsuit = 6 \quad \diamondsuit = 7 \quad \heartsuit = 8 \quad \spadesuit = 9$$

- Choisissez une carte
- Ajoutez la valeur de la carte à celle qui suit immédiatement
- Multipliez le résultat par $5$
- Ajouter la valeur de la couleur

$$((v + v + 1) \times 5) + c = 10v + 5 + c = 10(v + 1) + c - 5$$

http://reflexions.ulg.ac.be/Mathmusantes