Asteroseismology of β Cephei stars

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- This is an informal talk!
- Only β Cephei stars today
- Not an exhaustive review
- Not a "theory" talk

O stars: very massive; winds (mass loss)
a few pulsators known
asteroseismology in its infancy **X**

See talk at the conference

B stars:

- Be stars: fast rotators, emission lines, pulsators - complicated
- SPB stars: B2 - B9

$M \sim 4 - 7 \, M_\odot$

multiperiodic pulsators

P = 0.5 – 5 days

High-order g modes in asymptotic regime

- β Cephei stars:

B0 - B3

$M \sim 8 - 18 \, M_\odot$

multiperiodic pulsators; slow rotators

P = 2 – 8 hours

Sparse spectrum of low-order p and/or g modes

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- SPB stars: B2 - B9

  $M \sim 4 - 7\ M_{\odot}$

  multiperiodic pulsators

  P = 0.5 – 5 days

  High-order g modes in asymptotic regime

- β **Cephei stars:**

  B0 - B3
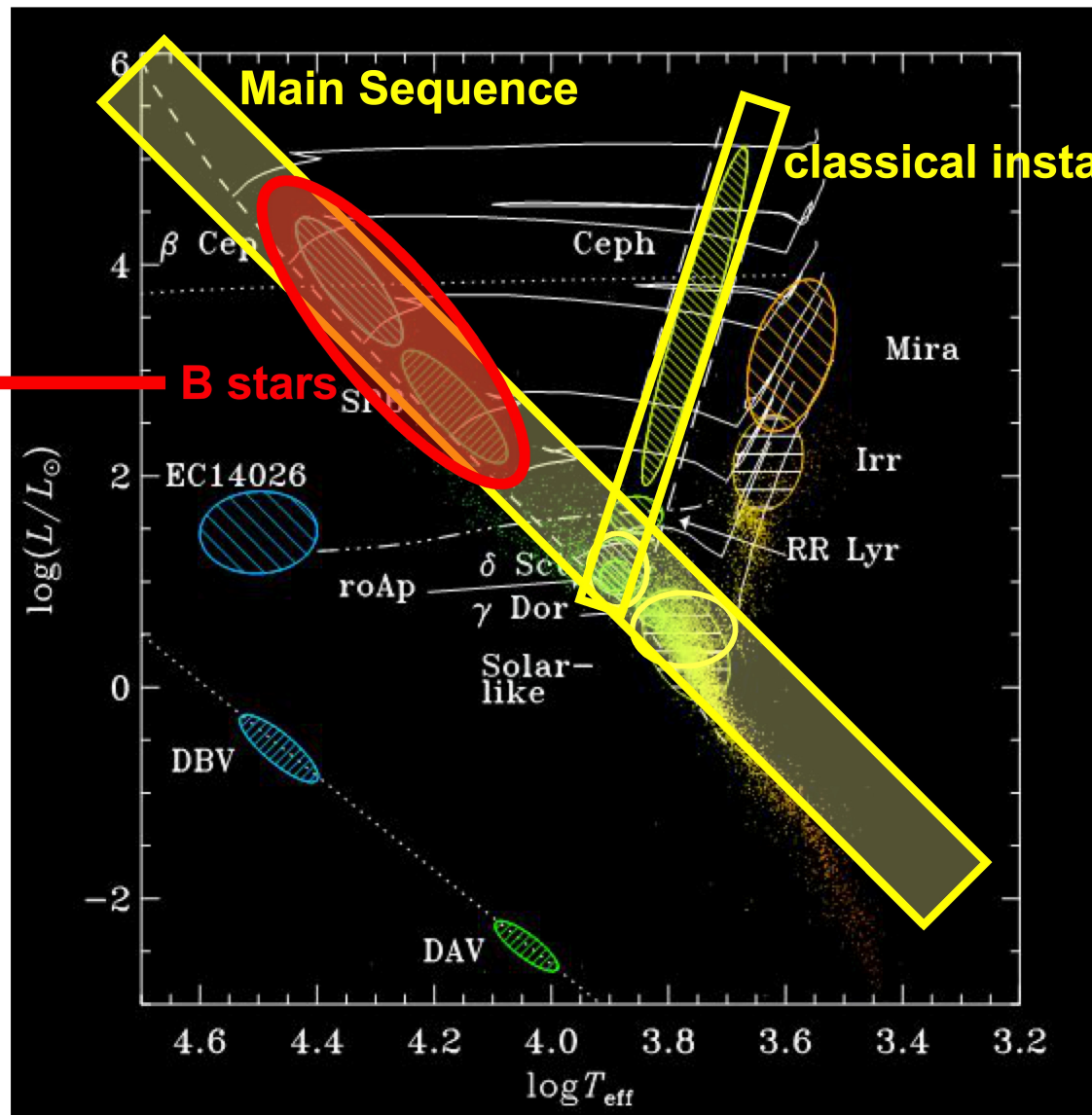
  $M \sim 8 - 18\ M_{\odot}$

  multiperiodic pulsators; slow rotators

  P = 2 – 8 hours

  Sparse spectrum of low-order p and/or g modes
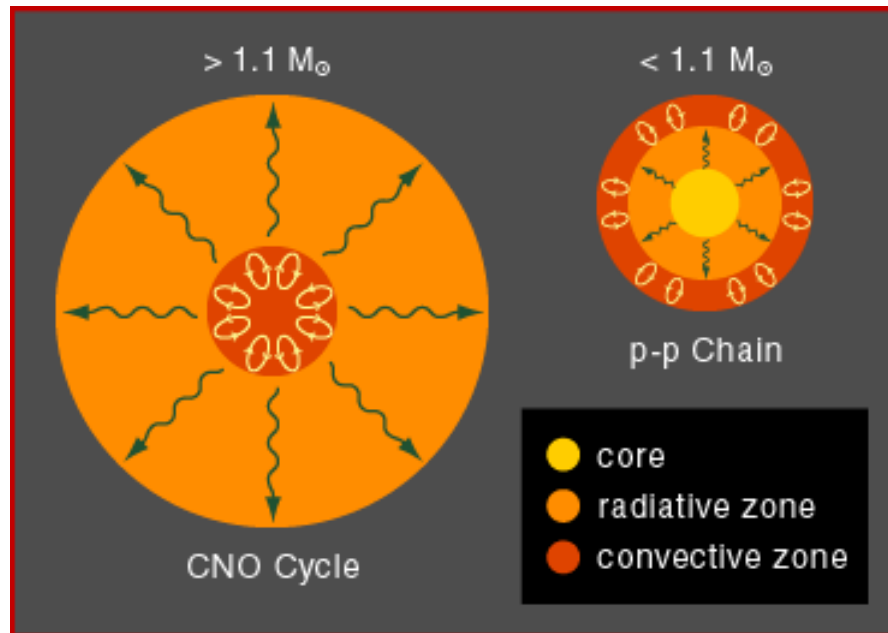
logT$_{eff}$ ~ 4.2 - 4.4
logL ~ 2 - 4

Main sequence B stars

Solar-type stars

CNO burning

massive convective core

radiative outer zone

Convective or radiative core

Radiative zone

Convective envelope

> 1.1 M☉   < 1.1 M☉

p-p Chain

CNO Cycle

● core
● radiative zone
● convective zone

different structures ➔ very different pulsation spectra

Why are seismic studies of massive main sequence stars interesting?

Because their spectra include modes that probe their deep structure, in particular the boundary of the convective core

➔ info on the overshooting/mixing at the core boundary
➔ info on the internal rotation profile

solar-type stars → superficial convective layer → stochastically excited modes

δ Scuti stars → κ mechanism in the He ionization zone

massive stars → κ mechanism in the Fe partial ionization zone (opacity bump)

κ **mechanism:**
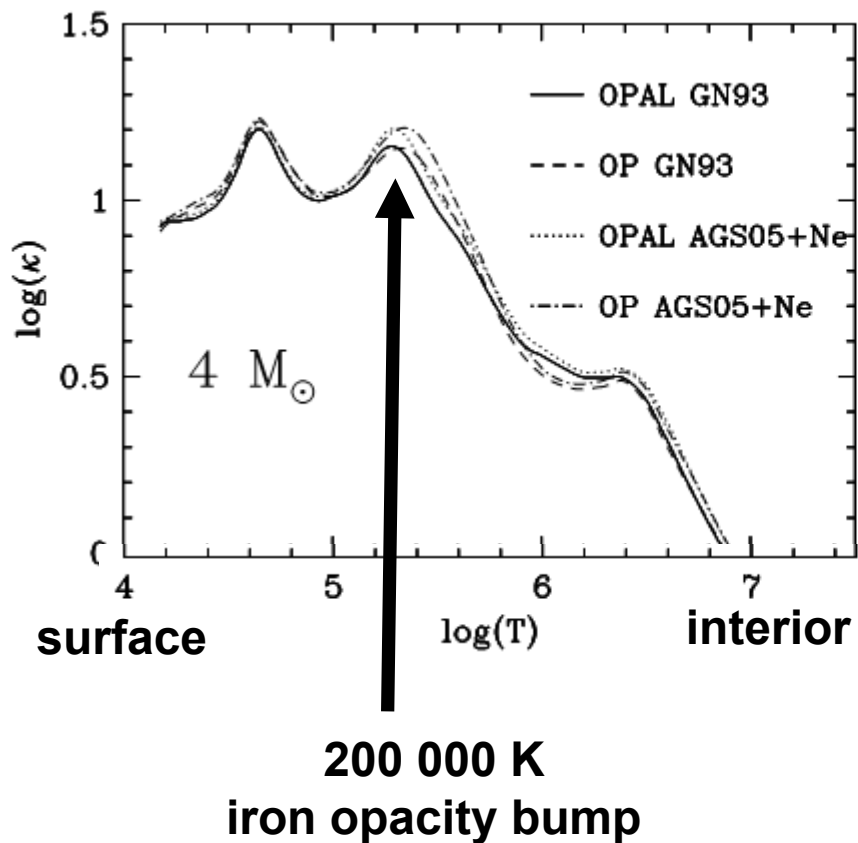
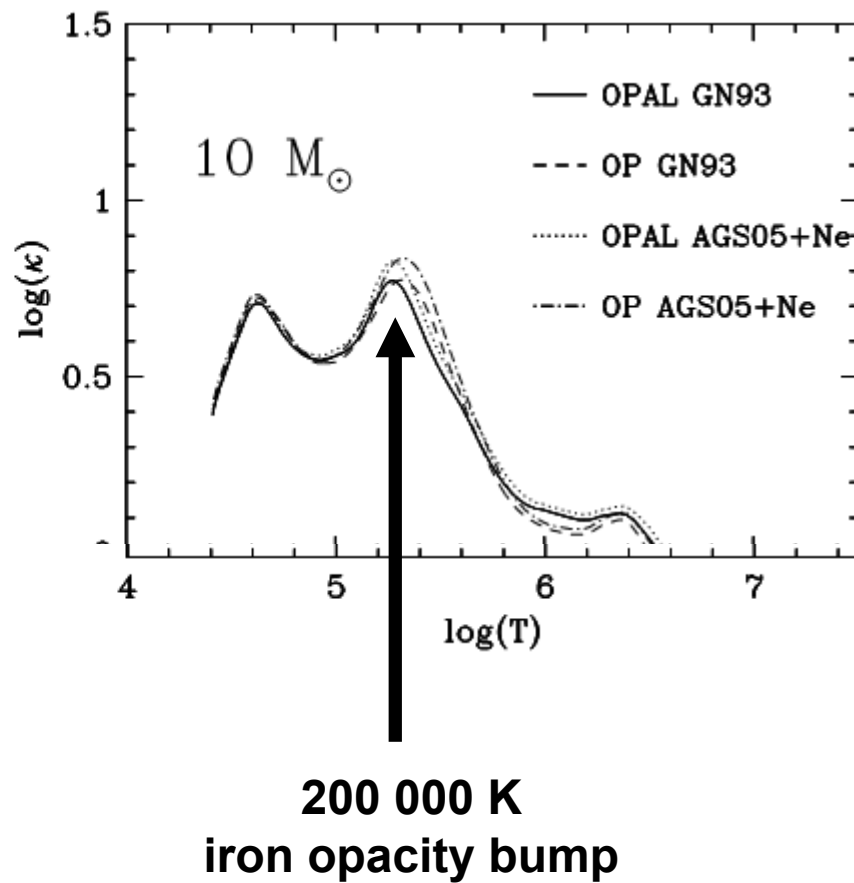**occurs IF opacity bump (due to Helium or iron-group elements partial ionization) COINCIDES with the transition zone (between adiabatic and non-adiabatic regions)**
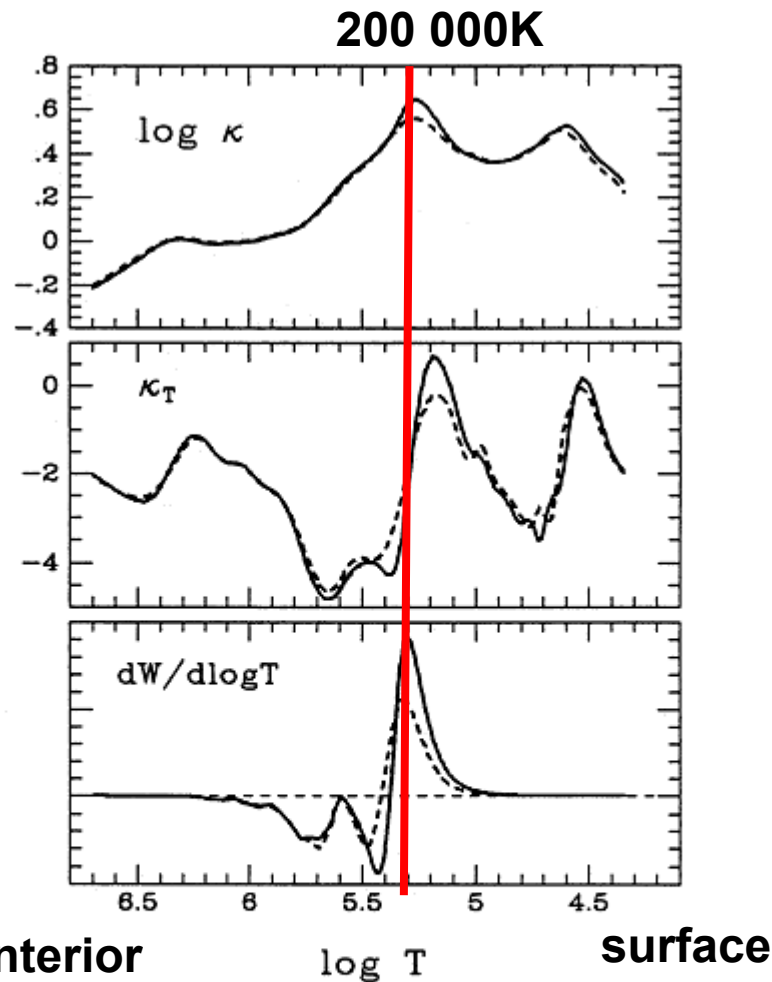
opacities

SPB

β **cephei**

200 000 K
iron opacity bump

200 000 K
iron opacity bump
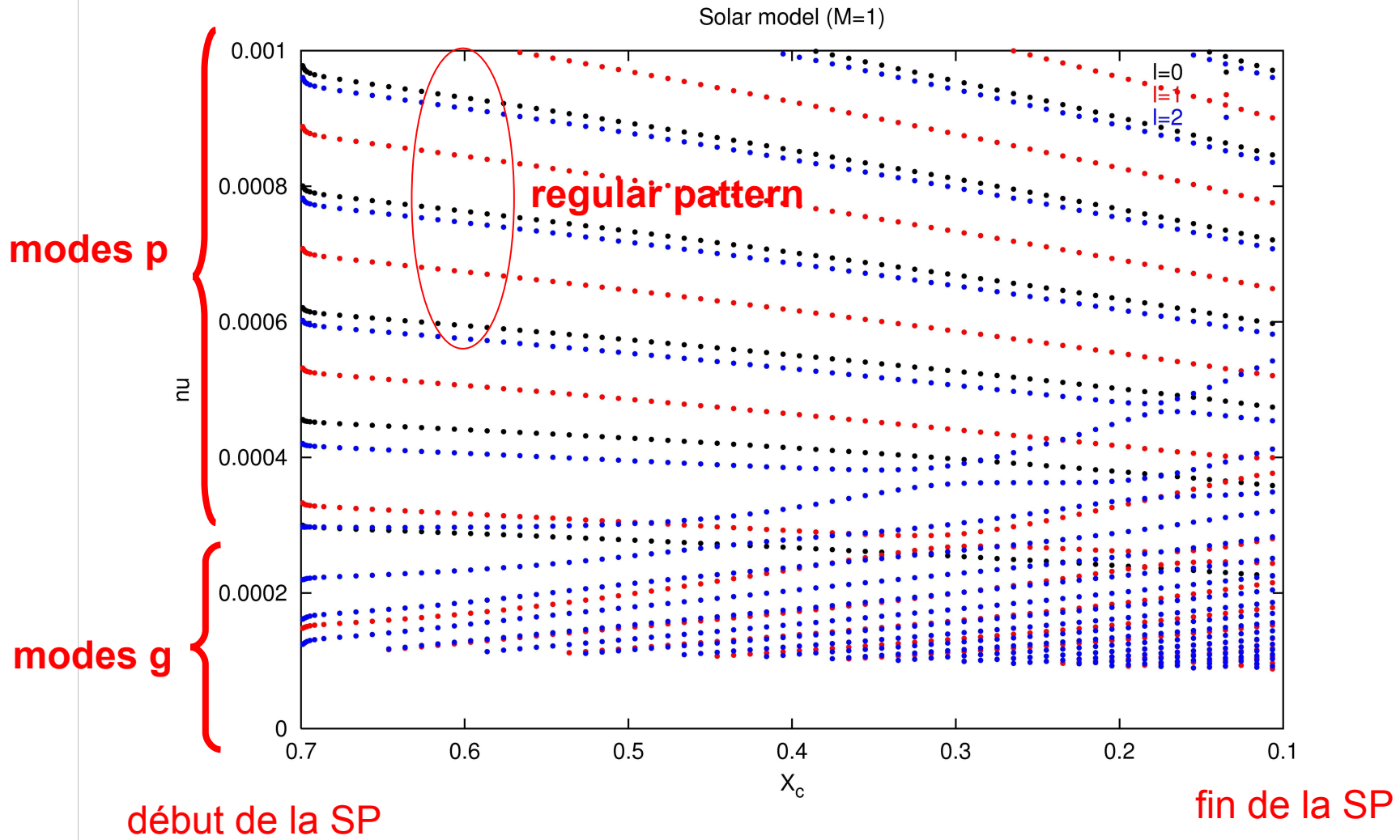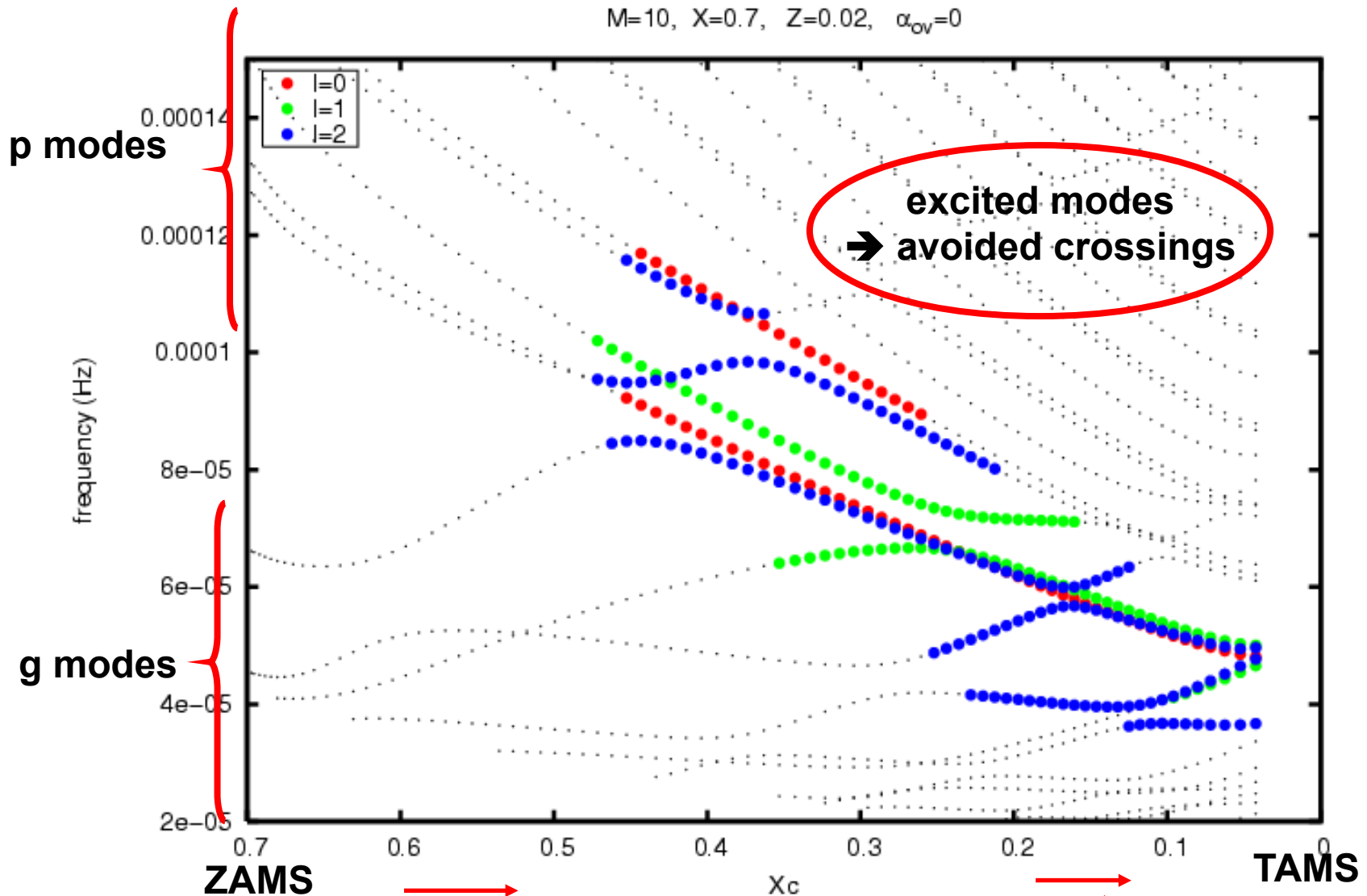
M=12

200 000K

interior   log T   surface

Figure 1. Opacity, κ, opacity derivative, $\kappa_T = (\partial \ln \kappa / \partial \ln T)_\rho$, and the differential work integral for the fundamental mode of radial pulsation, $dW/d\log T$ (in arbitrary units), plotted against temperature in a model of a β Cep star. The model parameters are: $M = 12$ M$_\odot$, $\log L/L_\odot = 4.22$, $\log T_{eff}/K = 4.368$, $X = 0.7$, $Z = 0.03$. Driving occurs in zones where $dW/d\log T > 0$. Continuous and dashed lines correspond to the newer and older OPAL opacity tables, respectively.

frequencies of solar-type stars: many high-order p modes

Solar model (M=1)

modes p

modes g

regular pattern

l=0
l=1
l=2

nu

X_c

début de la SP

fin de la SP

radial and non radial modes in β Cephei models

M=10, X=0.7, Z=0.02, α_{ov}=0

p modes

g modes

frequency (Hz)

l=0
l=1
l=2

**excited modes
→ avoided crossings**

ZAMS

TAMS

Xc

age ↗

T_eff ↘

excited modes: in a given range of dimensionless frequencies
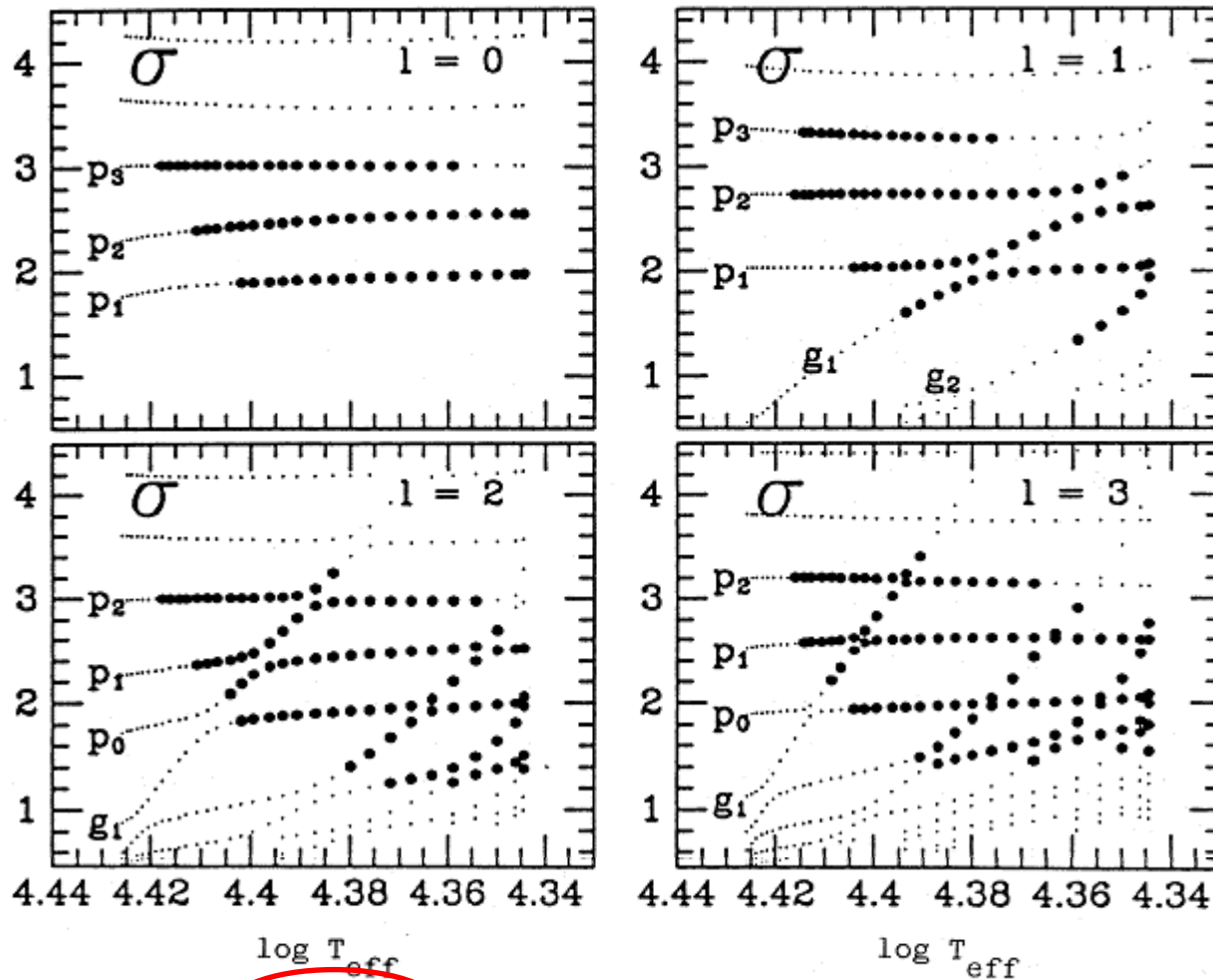
Figure 5. Dimensionless frequencies, $\sigma = \omega/\sqrt{4\pi G\langle\rho\rangle}$, of low-order and low-degree (l) modes for models of a 12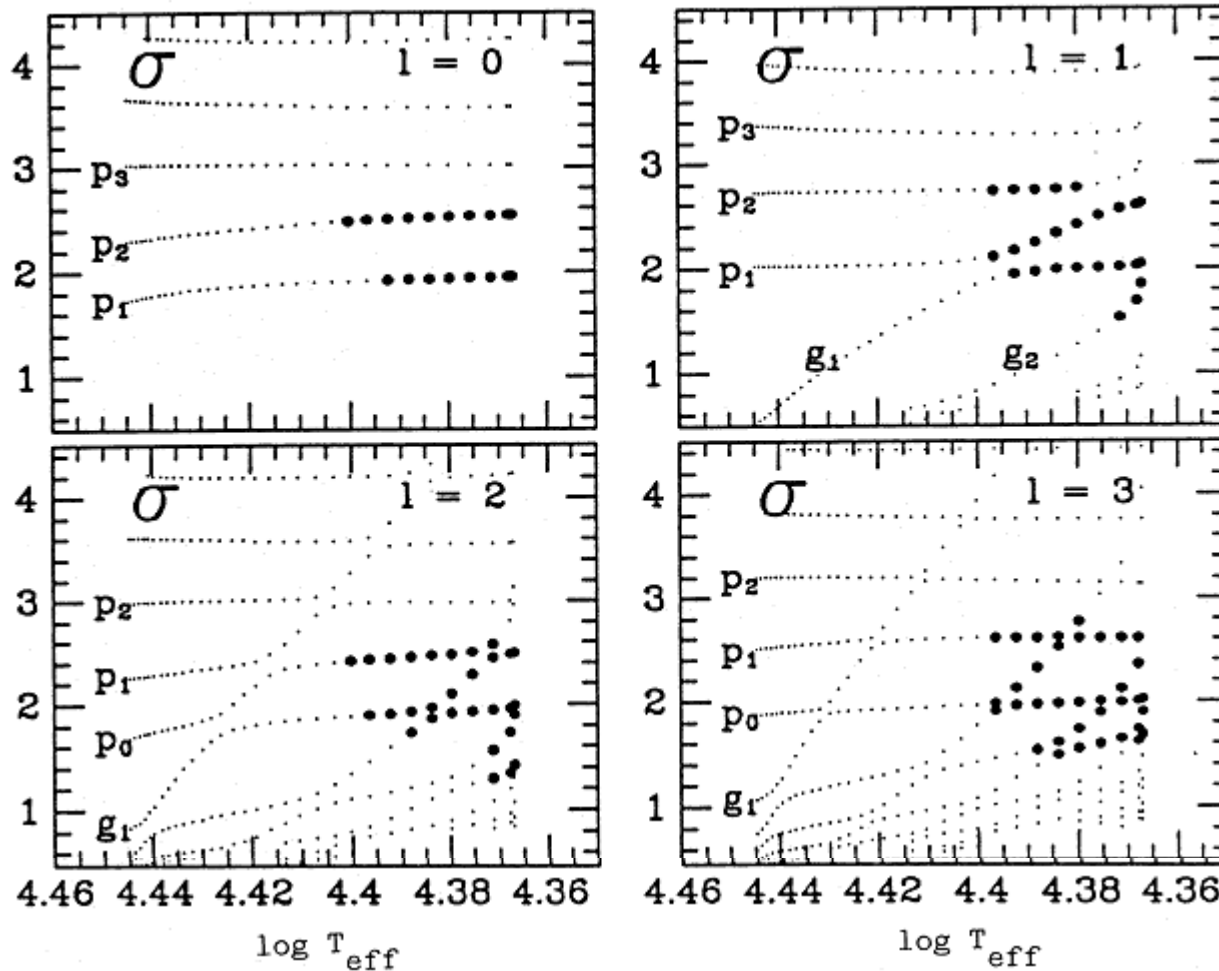-$M_{\odot}$, $Z = 0.03$ star in its MS evolutionary phase. The small and the large dots correspond to stable and unstable modes, respectively.
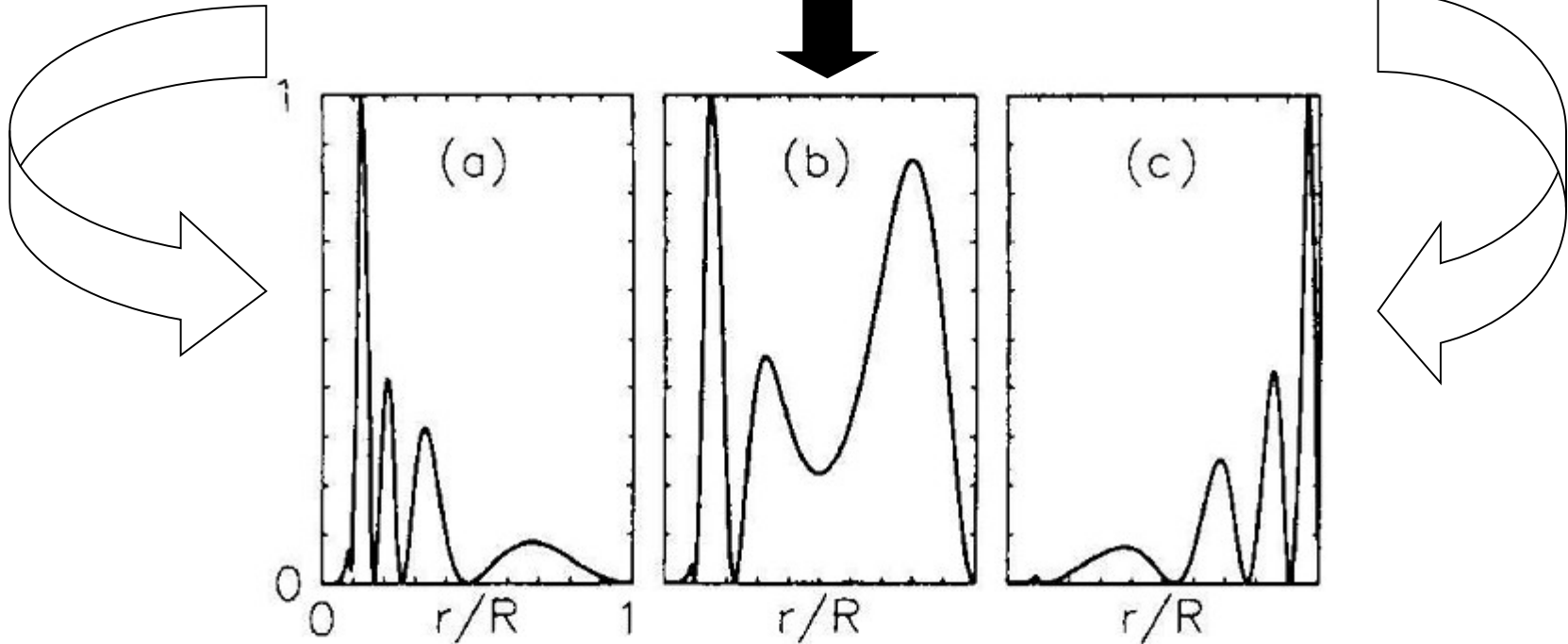
Lower Z ➔ fewer excited modes

Figure 6. Dimensionless frequencies, $\sigma = \omega/\sqrt{4\pi G\langle\rho\rangle}$, of low-order and low-degree (l) modes for models of a 12-$M_\odot$, $Z=0.02$ star in its MS evolutionary phase. The small and the large dots correspond to stable and unstable modes, respectively.

g modes have a large amplitude near the core of the star
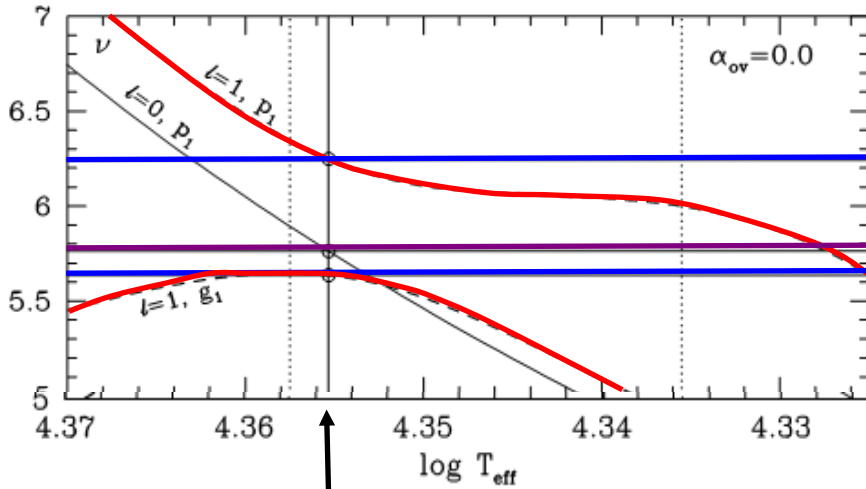
mixed mode
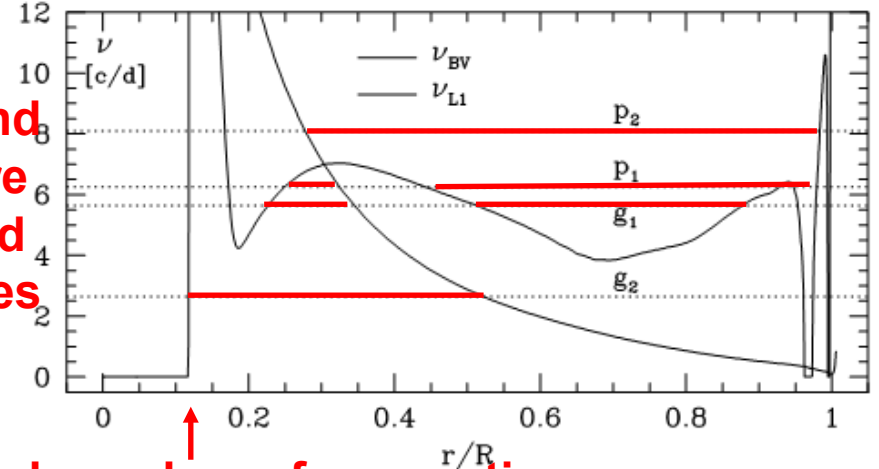
p modes have large amplitudes near the surface

Figure 1.4: Typical variation of the kinetic energy density as a function of depth in the star for a *g*-mode (left), a mixed mode (middle) and a *p*-mode (right) in an evolved star of 2 $M_\odot$. The different modes sample different parts of the star. (figure taken from Roxburgh et al. 2000)

avoided crossing ➔ mixed modes ➔ info on core boundary

fitted model of ν Eridani (3 frequencies)

Lamb frequency $S_l^2 \equiv \dfrac{[l(l+1)]c^2}{r^2}$

Brunt-Väisälä frequency

$$N^2 = g\left(\frac{1}{\Gamma_1 p}\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)$$

$$N^2 \simeq \frac{g^2\rho}{p}(\nabla_{\mathrm{ad}} - \nabla + \nabla_\mu)$$

oscillation energy distribution
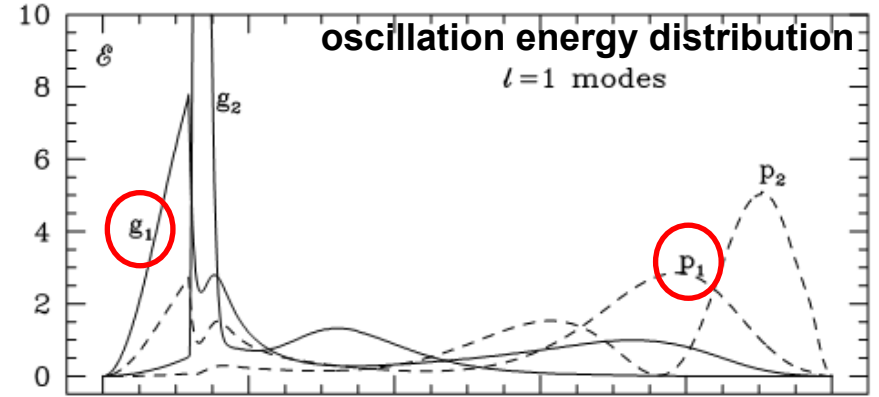
p1 and g1 are mixed modes

boundary of convective core

$\omega_{n,l,m} >$ S_l, N : **propagation of sound waves**
$\omega_{n,l,m} <$ S_l, N : **propagation of gravity waves**
otherwise, evanescent

➔ EACH MODE BRINGS AN INFORMATION ABOUT A DIFFERENT LAYER OF THE STAR !

Rotation splittings and rotation
FOR SLOW ROTATORS

degeneracy: 2m+1 frequencies for each mode l

$$\sigma_{k\ell m} - \sigma_{k\ell 0} = m \int K_{k\ell}(r)\Omega(r)\,dr$$

kernel constructed from the eigenfunctions of the modes in the absence of rotation

rotation profile inside the star

each frequency (each couple l,k) = one linear condition on Ω

$$\int K_i(r)\Omega(r)\,dr = w_i, \quad i = 1, \ldots, N.$$

HD129929

Detection of 6 frequencies:

$f1 = 6.461699 \ c/d \ (l=2, m=?)$
$f2 = 6.978305 \ c/d \ (l=1, m=0)$
$f3 = 6.449590 \ c/d \ (l=2, m=?)$
$f4 = 6.990431 \ c/d \ (l=1, m=1)$
$f5 = 6.590940 \ c/d \ (l=0, m=0)$
$f6 = 6.966172 \ c/d \ (l=1, m=-1)$

2 consecutive members of quintuplet

one triplet
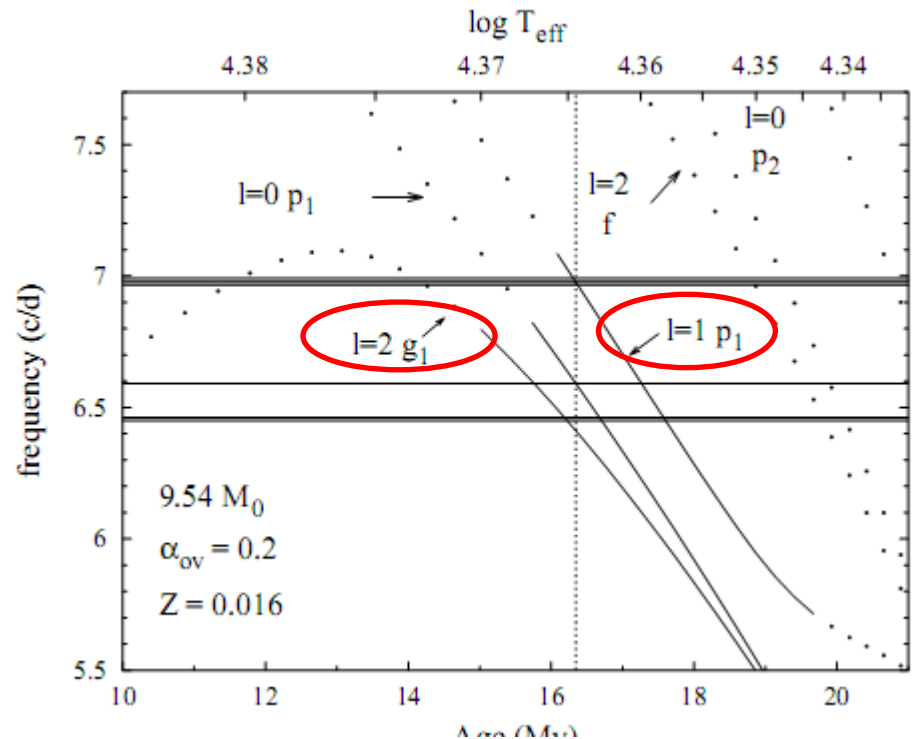
04/10/2011   KIT

**kernels are very different for the two modes
➔ give information about the rotation at different depths
(of the radiative envelope)!**

HD129929

core
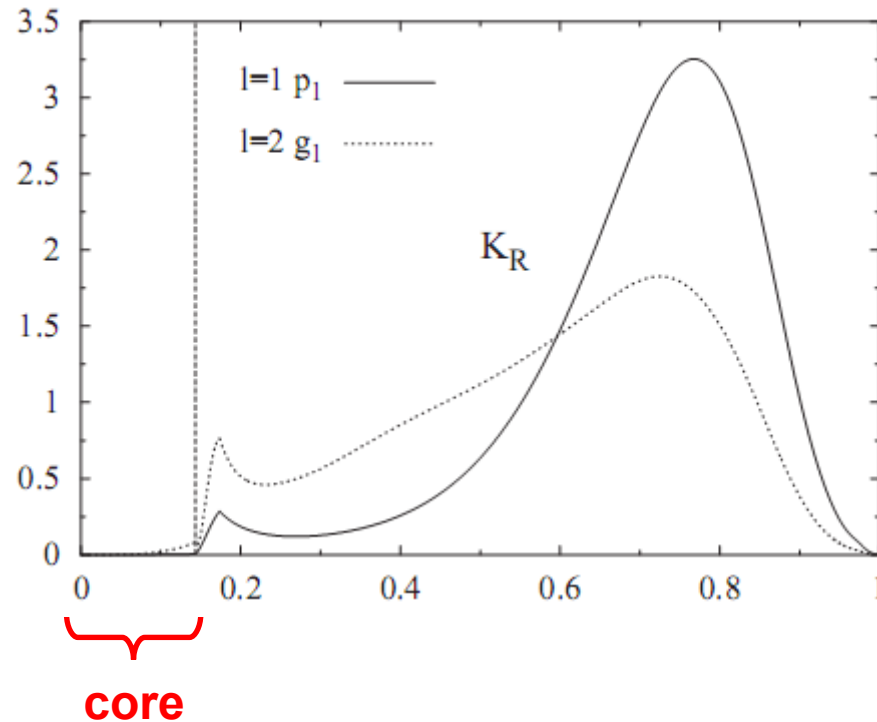
HD129929 - A β cephei star
Périodogramme

sparse spectrum
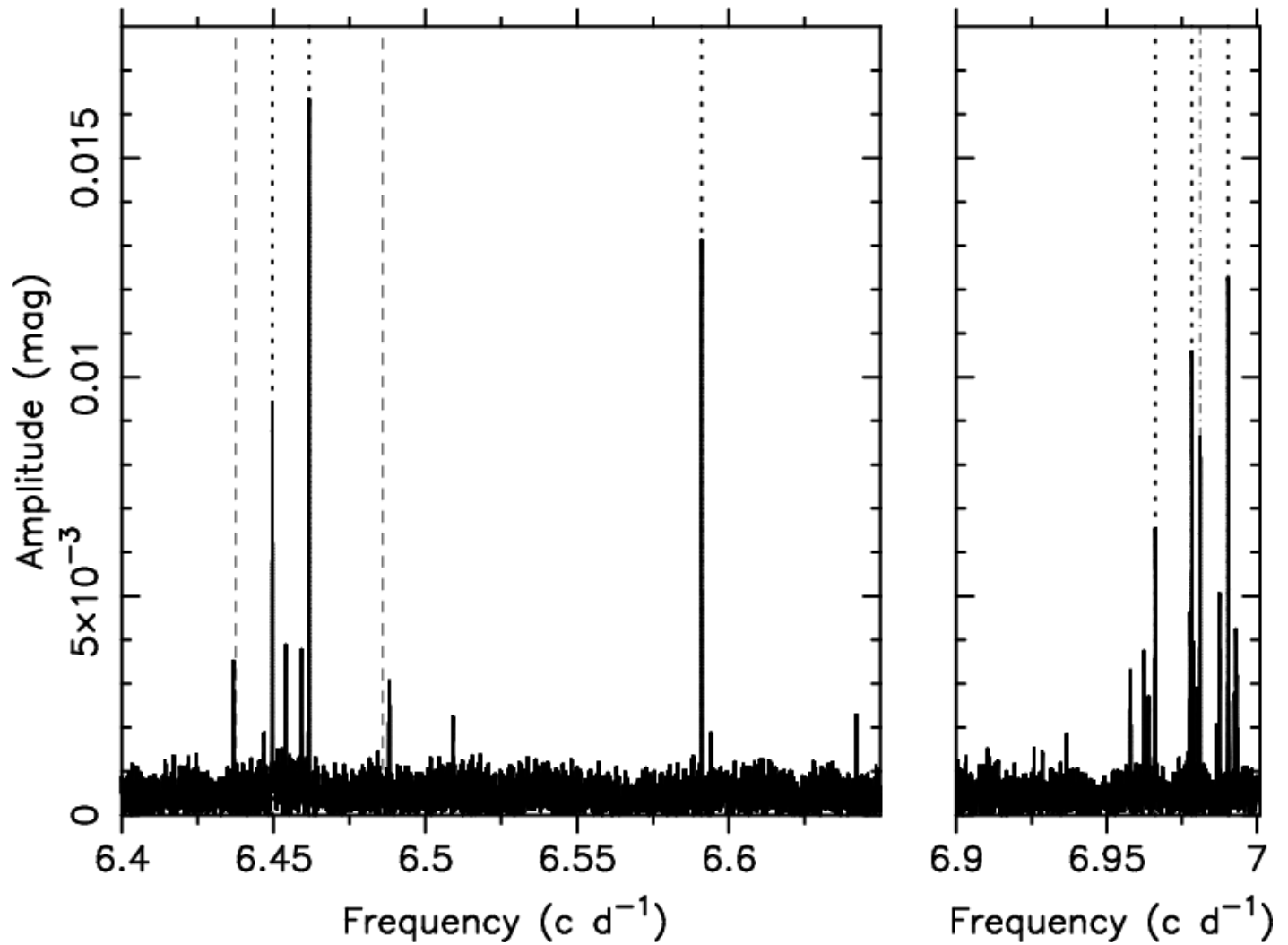
Methods of observation:

photometric : time series – direct measure of intensity variations

spectroscopic : measure of the changes of the surface velocity

The two methods sample the SAME pulsations, but in different ways

modes observed: low degree l

Intensity variations and Doppler shift of spectral lines are weighted averages of the pulsation amplitude over the τ=2/3 surface
➔reduced sensitivity to modes with high degree l

Doppler observations: projection of the velocity over the line of sight
➔slightly better observations of modes of moderate degree l
Example: l=3 modes detected in velocity observations, not in intensity observations.

Low frequencies → long-term monitoring is necessary
especially true for high-order g-mode pulsators (SPB)

Ground-based observations:
networks of small and medium size telescopes (Whole Earth Telescope,
Delta Scuti Network, STEllar PHotometry International network)
large multisite campaigns on dedicated stars

Space missions:
non-dedicated: WIRE, Kepler
dedicated: MOST, CoRoT

Mode identification → need for multi-site multicolour photometry
<u>and</u> high-resolution spectroscopy

example of a β Cephei star: 12 Lacertae

mode identification:
photometric
spectroscopic

Table 2. Frequencies and amplitudes of the first moment of the Si III λ4553 Å line together with their S/N ratio (we refer to the text for explanation). Error estimates (Montgomery & O'Donoghue 1999) for the independent frequencies range from ±0.000002 for f_1 to ±0.00002 for f_9. The error on the amplitude is 0.01 km s^{-1}.

ID	Frequency [d^{-1}]	[μHz]	Amplitude [km s^{-1}]	S/N
f_1	5.178964	59.941713	14.50	99.6
f_2	5.334224	61.738704	7.70	52.2
f_3	5.066316	58.637917	6.26	42.7
f_4	5.490133	63.543206	2.61	17.5
f_9	0.342841	3.968067	1.34	7.1
f_5	4.256966	49.270440	0.92	6.8
f_6	5.218075	60.394387	0.84	5.8
f_7	6.702318	77.573125	0.62	4.4
f_8	7.407162	85.731042	0.67	5.1
f_9	5.84511	67.65184	0.79	5.2
$2f_1$	10.35814	119.88590	0.63	4.5
$f_2 + f_3$	10.40056	120.37693	0.72	3.8
$f_1 + f_2$	10.51319	121.68044	2.59	19.7
$2f_1 + f_3$	15.42400	178.51860	0.72	6.6
$f_1 + f_2 + f_3$	15.57950	180.31838	0.71	6.4

Table 5. Final results for the mode identifications for 12 Lac from our spectroscopic analysis together with the results from the photometric amplitude ratios (Handler et al. 2006).

ID	Frequency [d^{-1}]	ℓ Spectr	ℓ Phot.	n
f_1	5.178964	1	1	1
f_2	5.334224	0	0	0
f_3	5.066316	1	1	0
f_4	5.490133	2	2	1
f_9	0.342841	-	1,2,4	0,-1
f_5	4.256966	-	2	[+]
f_6	5.218075	-	2,4	?
f_7	6.702318	-	1	?
f_8	7.407162	-	1,2	?
f_9	5.84511	-	1,2	?
f_p	5.30912	-	1,2	?

about 10 frequencies

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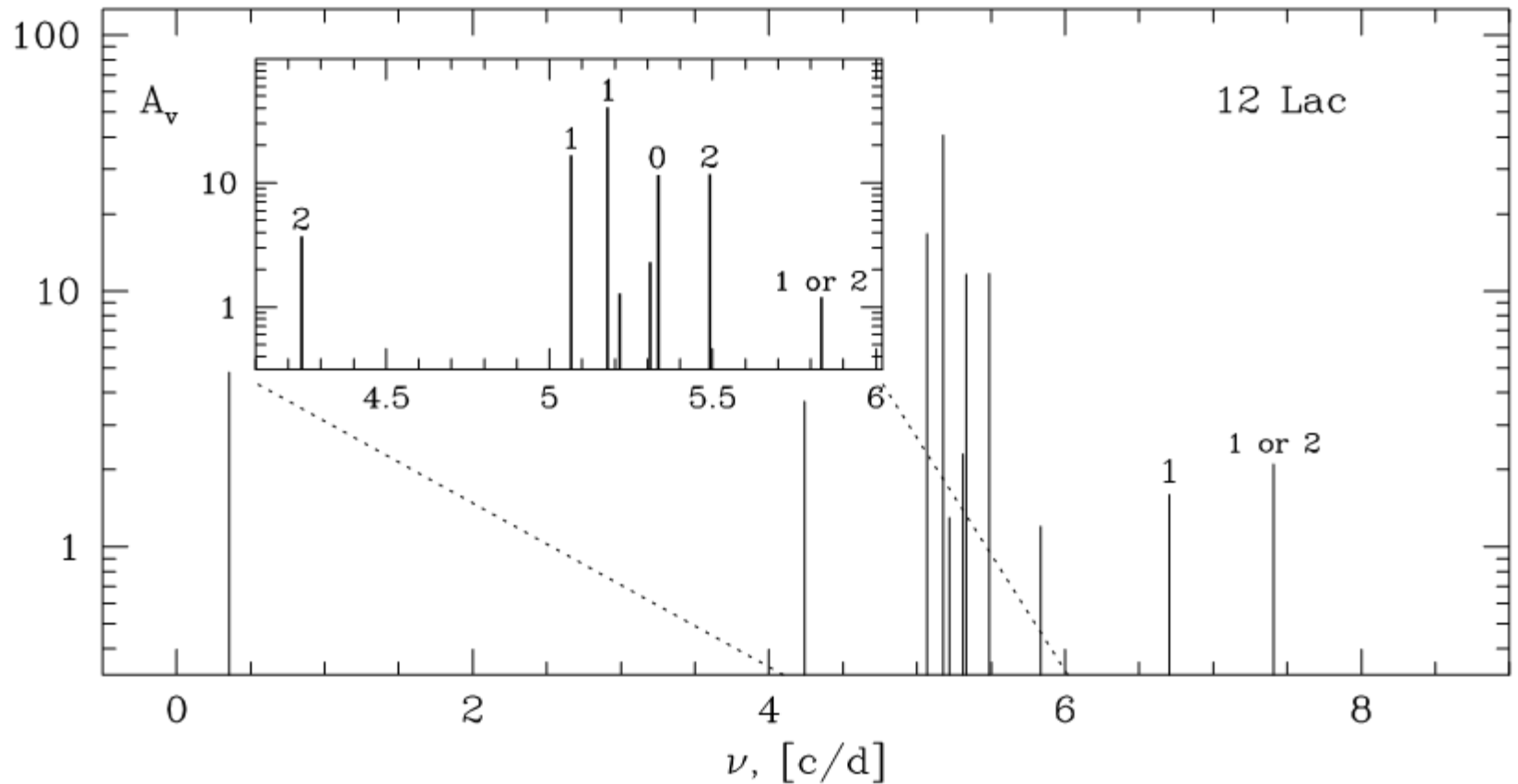# example of a β Cephei star: 12 Lacertae

Figure 1. Oscillation spectra of ν Eri and 12 Lac based on Jerzykiewicz et al. (2005) and Handler et al. (2006) data, respectively. The numbers above the bars are the most likely ℓ-values, as inferred from data on amplitudes in four passbands of Strömgren photometry.

Asteroseismology of B stars ≠ asterosismic studies of solar-type stars

solar-type stars

short-lives modes

comb-like spectrum of low degree high-order p modes

mode identification relatively easy with echelle diagram

asymptotic regime fit large and small separations

B stars

long-lived modes

**sparse spectrum of low order, low-degree p and g modes(β Cephei)
or
high-order g-modes (SPB)**

mode identification is difficult need multicolour photometry and spectroscopy

fit exactly each frequency

Main-sequence B stars:
mode identification

multicolour photometry: amplitude and phase behavior of an oscillation mode are different in different filters

➔ **degree l can be determined**

line-profile variations: its shape is entirely determined by the parameters in all the pulsationnal velocities, including **l and m**

Asteroseismology of main-sequence B stars: Method

β **Cephei stars**: sparse spectrum in non-asymptotic regime
DIRECT FITTING

- run forward stellar models
- fit each axisymmetric frequency
- analyze the rotational splittings
- get constraints on the stellar parameters, the other observables
  (log T_{eff}, log g, [M/H]$_{surf}$, L, R), and the physics
- improve the physics

➔ **NICE RESULTS OBTAINED**

Main-sequence B stars
Theoretical modelling

« Standard » stellar evolution code + « interesting » physics

modeling parameters:
Mass M
Initial Hydrogen mass fraction X
Initial Metallicity Z
   (or initial Helium mass fraction Y)
Overshooting parameter $\diamond_{ov}$

initial chemical composition
opacities: opal / OP / ?
« mixing »: overshooting, rotation, ?
convection theory
diffusion and gravitational settling
radiative accelerations
magnetic fields
mass loss
departures from spherical symmetry
...

Asteroseismology:
what we would like to learn

Basic stellar parameters

Mixing

Internal rotation

Diffusion, gravitational settling

Convective overshooting

Opacities, equation of state

Evolution of the chemical abundances

Mass loss

Magnetic fields

Examples of well-studied stars
or
« A few * success * stories »

Before Corot

- 16 Lacertae

- HD129929

- ν Eridani

- θ Ophiuchi

- 12 Lacertae

Corot stars

- HD 180642

- HD 50230

- HD 51756

- HD 170580

Examples of well-studied stars

- **16 Lacertae** : **3 frequencies observed and identified, 2 axisymmetric modes; no multiplet**

  ➔ **very precise values of M, T_{eff}, L, age**

- HD129929

- ν Eridani

- θ Ophiuchi

- 12 Lacertae

- HD 180642

Examples of well-studied stars

- 16 Lacertae :   3 frequencies observed and identified, 2 axisymmetric modes; no multiplet
  ➔ very precise values of M, T_{eff}, L, age

- **HD129929** : **6 freq. observed and identified, 2 axisymmetric modes, 1 triplet,**
  (movie)   **2 members of a quintuplet**
  ➔ **precise values for M, Z, T_{eff}, L, age**
      + evidence for core overshooting   **Note: bad frequencies**
      + evidence for non-rigid rotation   **= no solution!!!**

- ν Eridani

- θ Ophiuchi

- 12 Lacertae

Examples of well-studied stars

- 16 Lacertae :   3 frequencies observed and identified, 2 axisymmetric modes; no multiplet
  ➔ very precise values of M, T_{eff}, L, age

- HD129929 : 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet
  ➔ precise values for M, Z, T_{eff}, L, age
  + evidence for core overshooting and non-rigid rotation

- **ν Eridani: 12 frequencies observed, 7 identified,**
  2 axisymmetric modes, one triplet
  + 2 low-frequency modes

- θ Ophiuchi

- 12 Lacertae

ν Eridani observations

Large multisite photometric campaign 2002-2003: 11 telescopes, 148 clear nights

+

Large multisite spectroscopic campaign: 11 observatories, 2294 high-resolution spectra
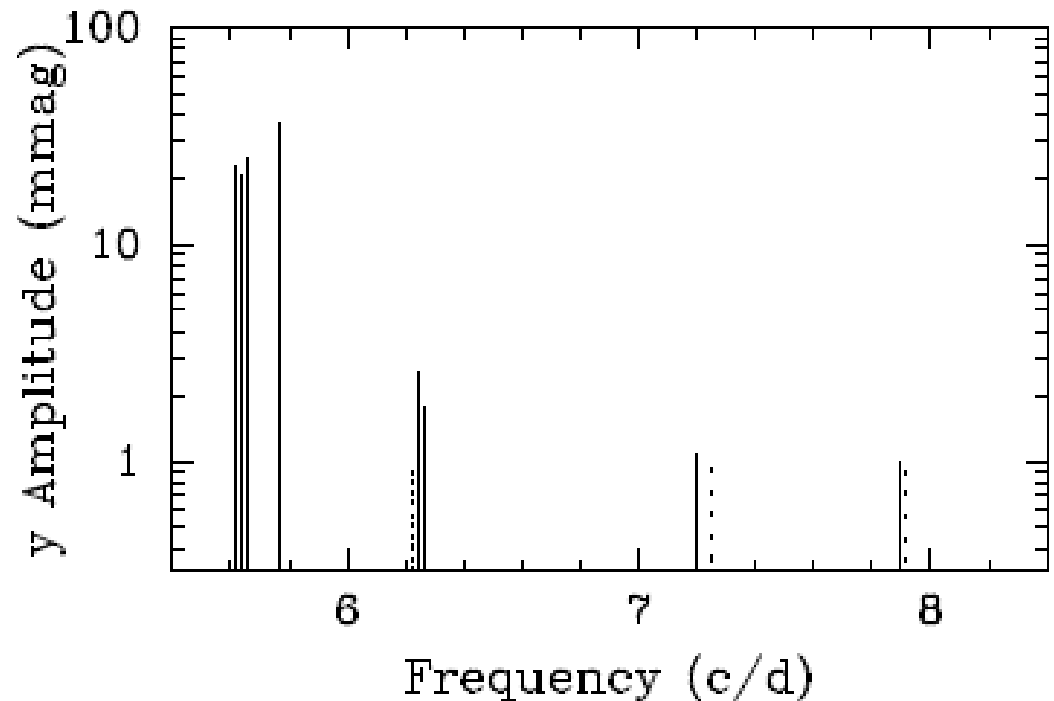
+

Large multisite photometric campaign 2003-2004: 5 telescopes, 142 clear nights

ν Eridani pulsations

first photometric campaign ➔ 8 independent frequencies detected + 1 low-frequency mode

| ID	Freq. (cd^{-1})
f_1	5.76327
f_9	5.62006
f_4	5.63716
f_2	5.65393
f_6	6.24408
f_7	6.26205
f_8	7.19994
f_3	7.89780
f_A	0.43218

Handler et al. 2004

ν Eridani pulsations

**spectroscopic campaign ➔ 7 independent frequencies detected
+ no low-frequency mode**

Number	Identification	Freq. (cd^{-1})
1	f_1	5.763298
2	f_2	5.654014
3	f_3	5.620097
4	f_4	5.637432
5	$f_1 + f_2$	11.417312
6	$f_1 + f_3$	11.383395
7	$f_1 + f_4$	11.400730
8	$f_1 + f_2 + f_3$	17.037409
9	f_9	6.242458
10	f_{10}	7.898225
11	$2f_1 + f_2 + f_3$	22.800707
12	$2f_1 + f_3$	17.146693
13	f_{13}	6.221429
14	$2f_1 + f_2$	17.180610
15	$2f_1 + f_4$	17.163392
16	$f_1 + f_{10}$	13.661523
17	$3f_1$	17.289894
18	$3f_4$	16.912296
19	$1 + f_2 + f_3$	12.274111
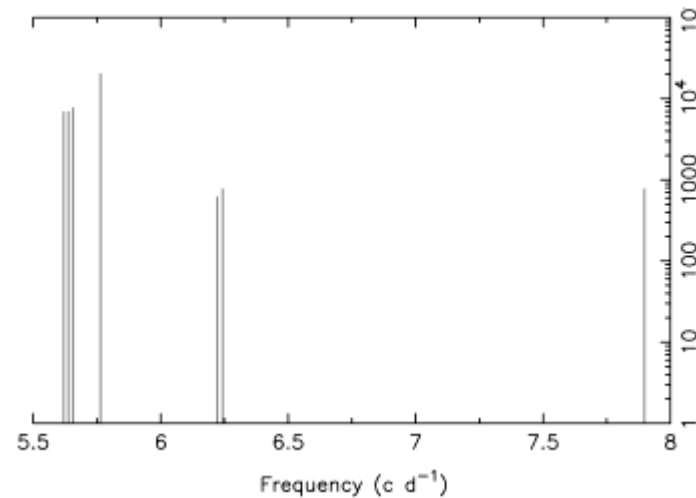
Aerts et al. 2004

ν Eridani pulsations

second photometric campaign ➜ 10 independent frequencies detected + 2 low-frequency modes

| ID	Frequency [d^{-1}]
f_1	5.763256 ± 0.000012
f_2	5.653897 ± 0.000020
f_3	5.619979 ± 0.000021
f_4	5.637215 ± 0.000025
f_5	7.89859 ± 0.00022
f_7	6.26225 ± 0.00025
f_A	0.43257 ± 0.00028
f_6	6.24468 ± 0.00034
f_B	0.61411 ± 0.00033
f_{10}	7.9296 ± 0.0005
f_9	7.9132 ± 0.0005
f_8	7.2006 ± 0.0005

Jerzykiewicz et al. 2005

ν Eridani pulsations

combined first + second photometric campaign ➔ 12 independent frequencies detected + 2 low-frequency modes

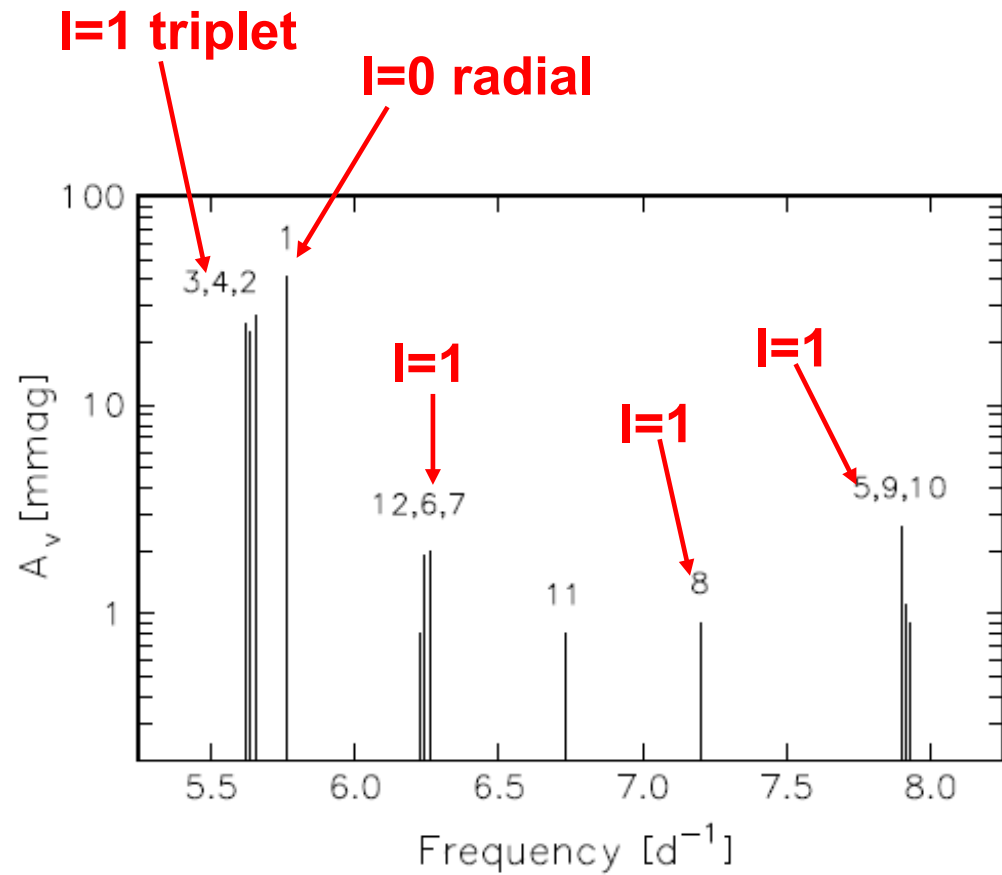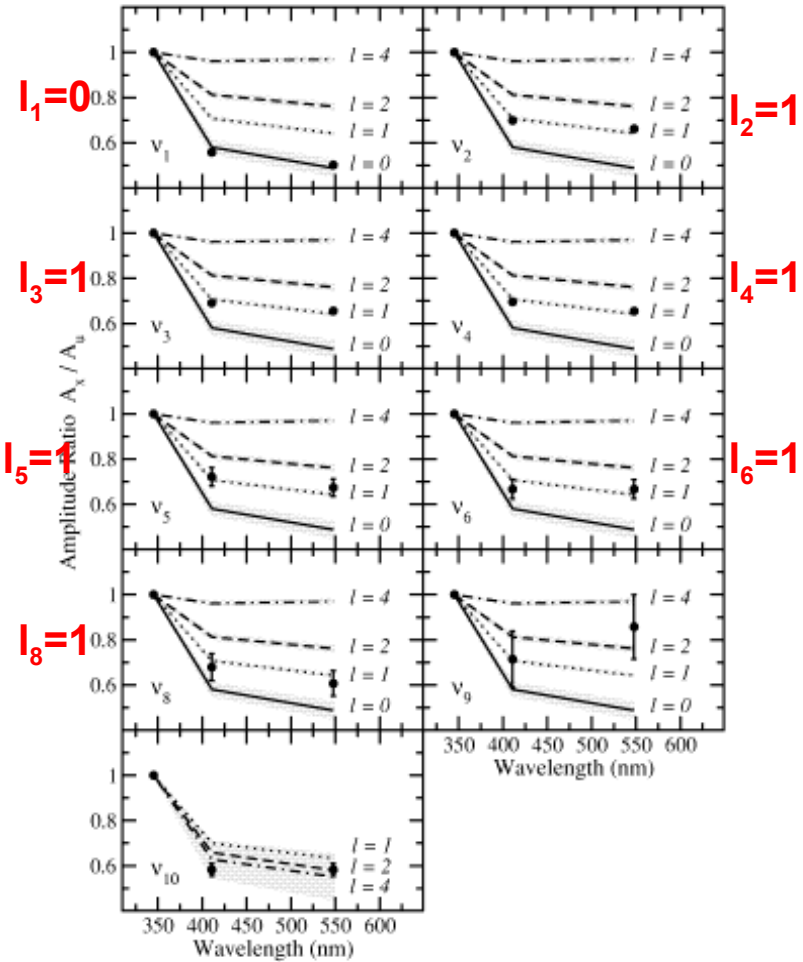| ID	Frequency [d^{-1}]
f_1	5.7632828 ± 0.0000019
f_2	5.6538767 ± 0.0000030
f_3	5.6200186 ± 0.0000031
f_4	5.6372470 ± 0.0000038
f_5	7.898200 ± 0.000032
→ f_A	0.432786 ± 0.000032
f_7	6.262917 ± 0.000044
f_6	6.243847 ± 0.000042
→ f_B	0.61440 ± 0.00005
f_9	7.91383 ± 0.00008
f_{10}	7.92992 ± 0.00010
f_8	7.20090 ± 0.00009
f_{11}	6.73223 ± 0.00012
f_{12}	6.22360 ± 0.00012

HYBRID β CEPHEI/SPB star!
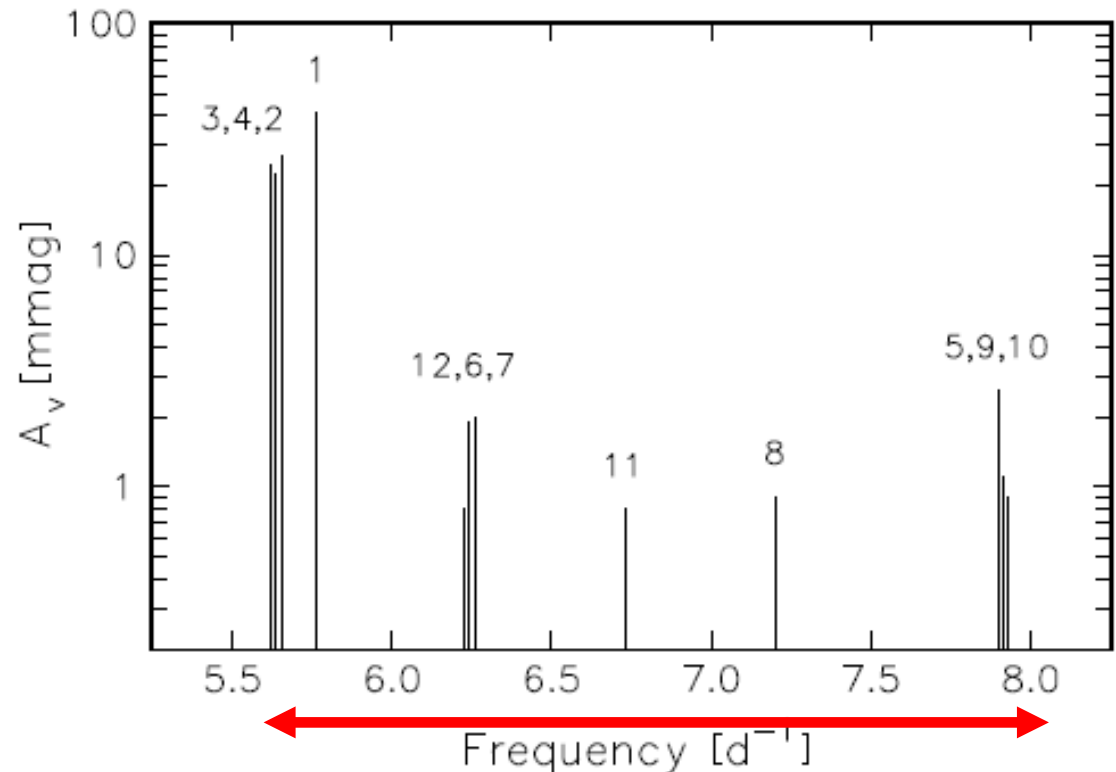
ν Eridani
mode identification

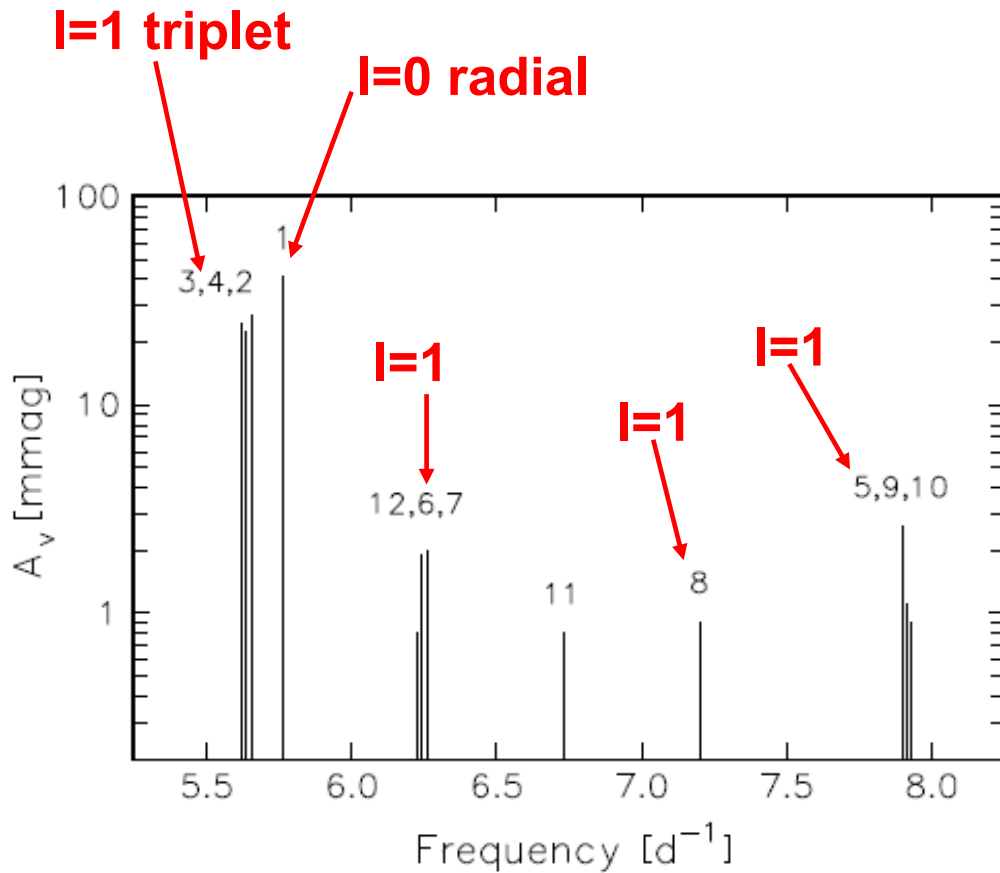# ν Eridani modelling

2 axisymmetric modes identified + complete triplet + additional frequencies
➔ asteroseismic modelling possible

For each set of model parameters (M, X, Z, α_{ov}), fitting the radial mode gives the **age of the star (T_{eff})**
For a given set of model parameters (X, Z, α_{ov}), fitting the second axisymmetric frequency fixes **the age (T_{eff}) AND the mass M**

ν Eridani modelling

l=1 triplet

l=0 radial

l=1

l=1

l=1

➔ f_1 is a p1 mode
➔ f_4 is a l=1 g1 mode

f_6 can be fitted by the l=1 p1 mode, without additional constraints on the parameters (for each α_{ov}, M and Z are now fixed)

ν Eridani modelling

for all the models that fit f_1 and f_4, look at the values of the f_6 (l=1, p1) and f_5 (l=1, p2) frequencies
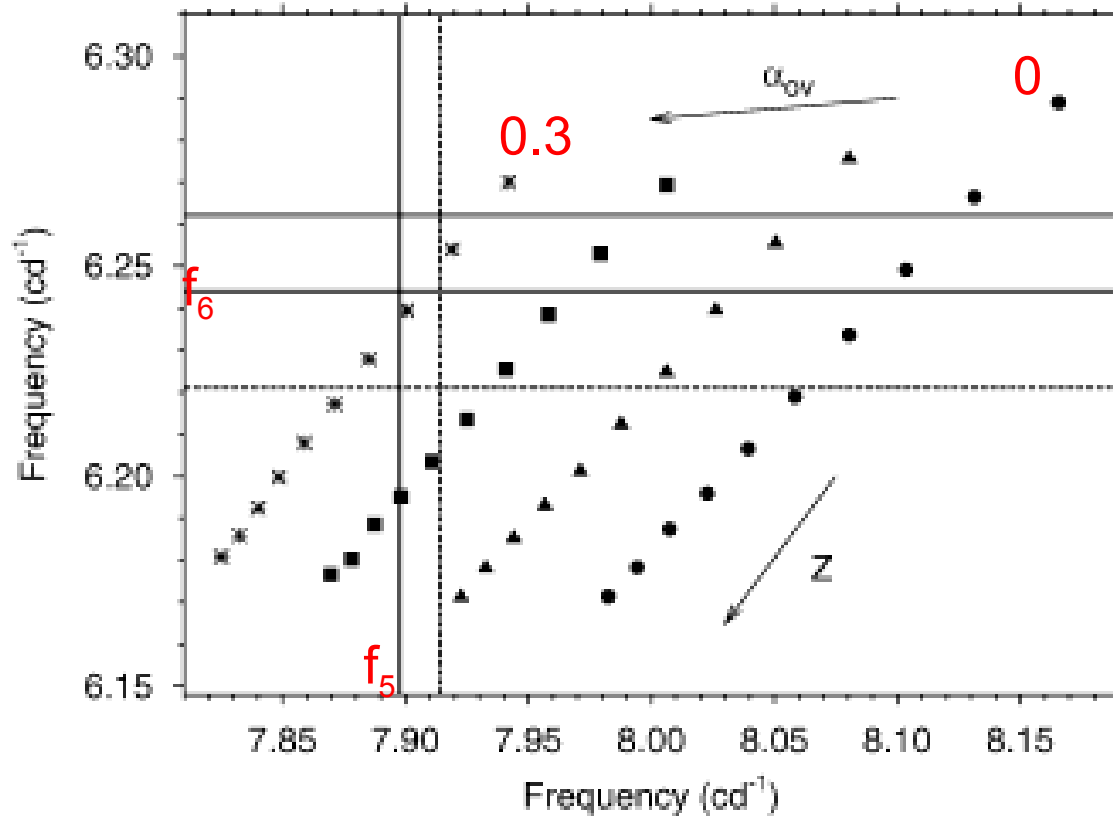
➔ « high » overshooting: α_{ov} ~ 0.3

and

« low » metallicity: Z~ 0.015

≅ Eridani modelling

Modelling seems to be OK
also evidence for non-rigid rotation

BUT

No excited frequencies in the range of observed frequencies!!!

➔**Non-standard models (higher Fe or lower X)**
or
Ad-hoc enhancement of iron in the driving region

➔ **higher frequency modes can be excited,**
but NOT the low frequency g modes

ν Eridani modelling

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or
Ad-hoc enhancement of iron in the driving region**

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but NOT the low frequency g modes**

Abundances and opacities

Chemical composition (in particular Fe)
and
opacity tables (κ mechanism)

➔ very important to determine excitation

Abundances
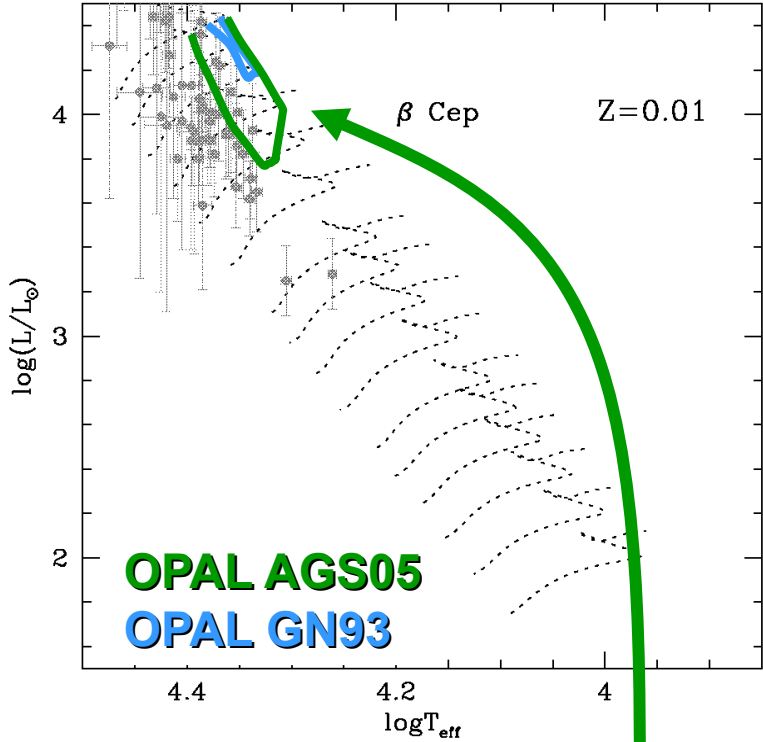
Z=0.01 Kolaczkowski et al. 06;
Morel et al. 06

AGS05 $Z_\odot$=0.012
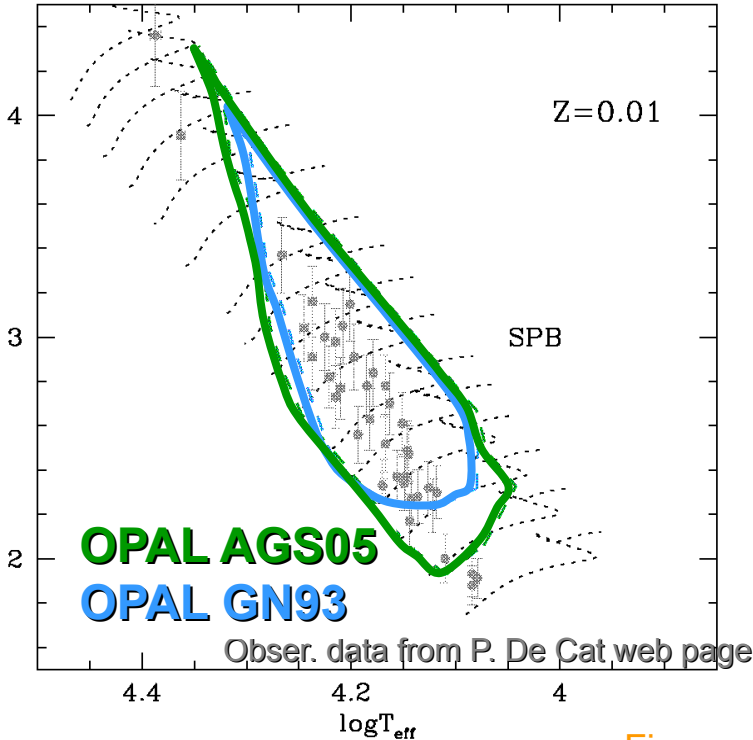
AGS05 ➘ 20% more Fe !

Figures courtesy of Montalban&Miglio

Opacities

OP   GN93
OPAL GN93

β Cep

Z=0.01

OP   GN93
OPAL GN93

SPB

Z=0.01

Obser. data from P. De Cat web page

Figures courtesy of
Montalban&Miglio

Abstract and opacities

OP AGS05
OP GN93
OPAL AGS05
OPAL GN93
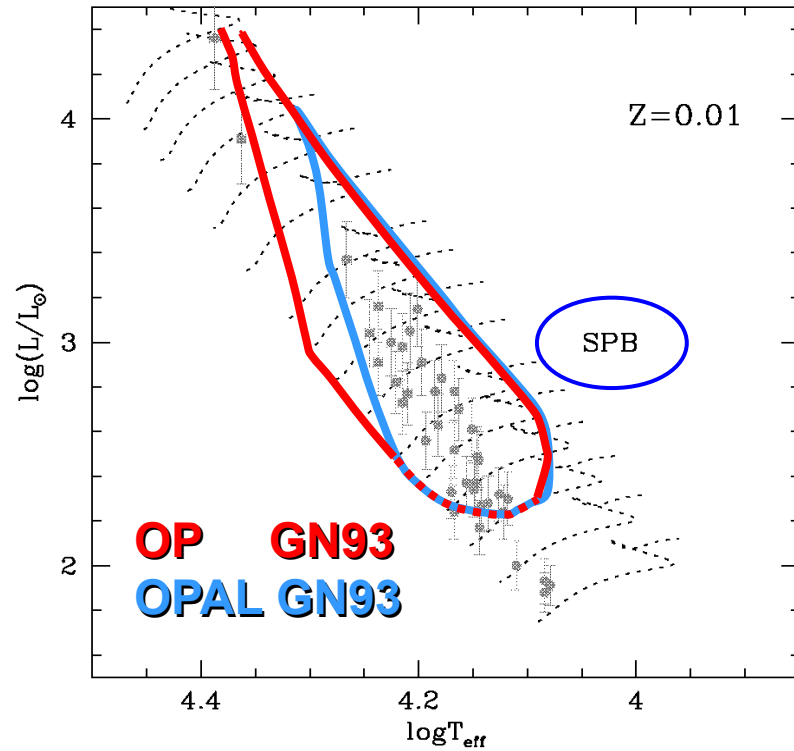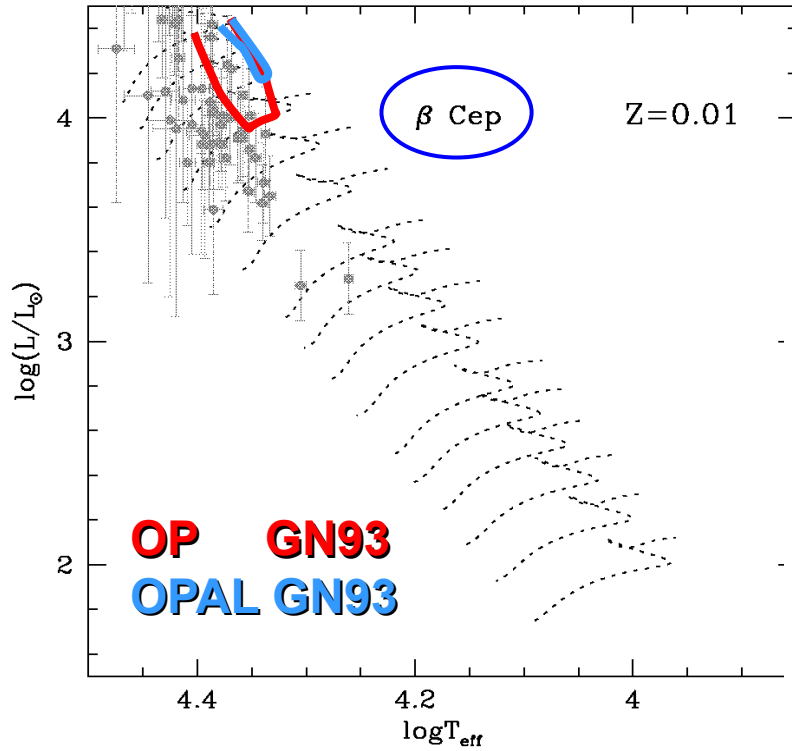
Obser. data from P. De Cat web page

Figures courtesy of
Montalban&Miglio
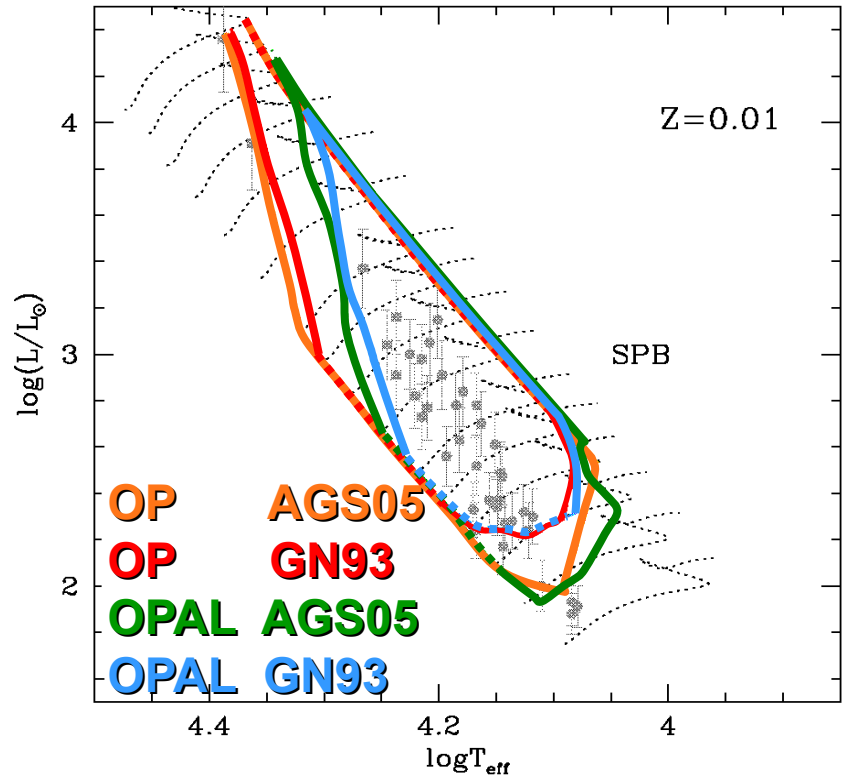
Abundances and opacities

AGS05 and OP

⬇

bluer border of SPB and
β Cephei instability strips

larger number of hybrid
SPB-β Cephei pulsators

more β Cephei modes
excited at Z=0.01
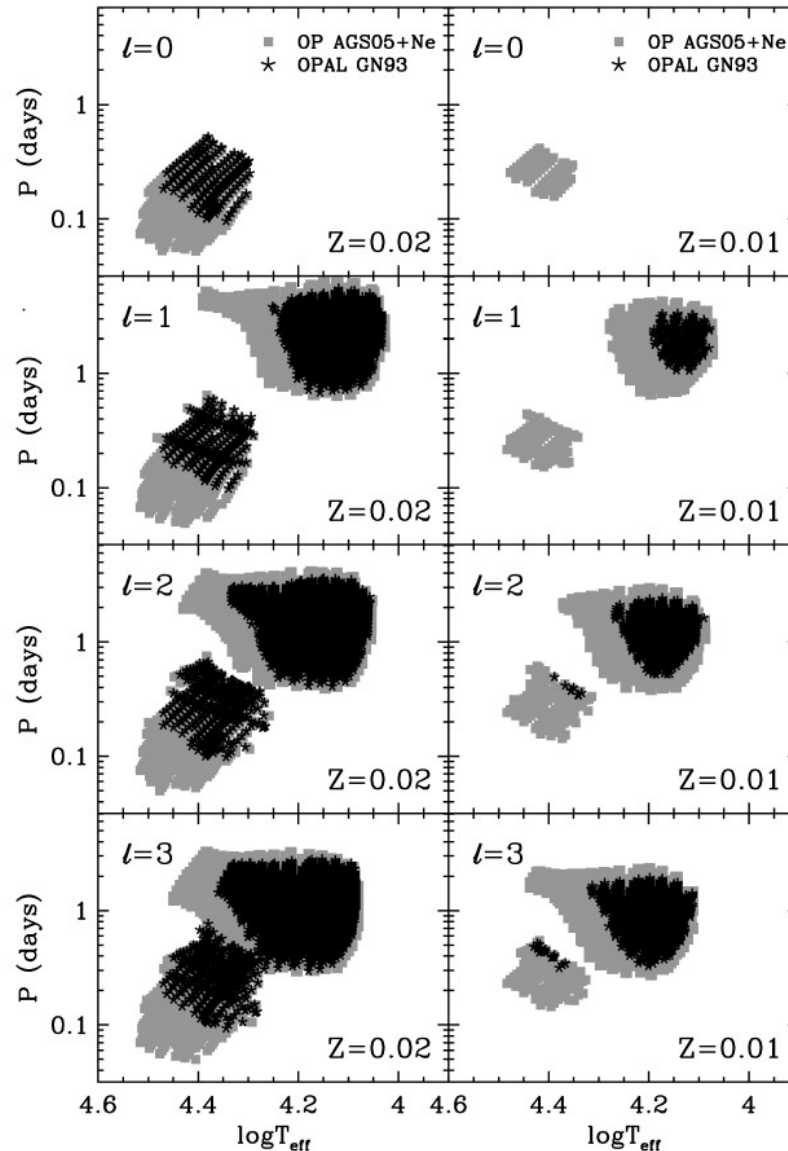
ν Eridani modelling

Previous results: GN93 and OPAL
Now: AGS05 and OP

fit 4 frequencies

⇩

- **X=0.7211**
  Z=0.021
  α_{ov}=0.22
  M=9.1

- **X=0.70**
  Z=0.022
  α_{ov}=0.22
  M=8.9

04/10/2011   KITP - A.Thoul   52
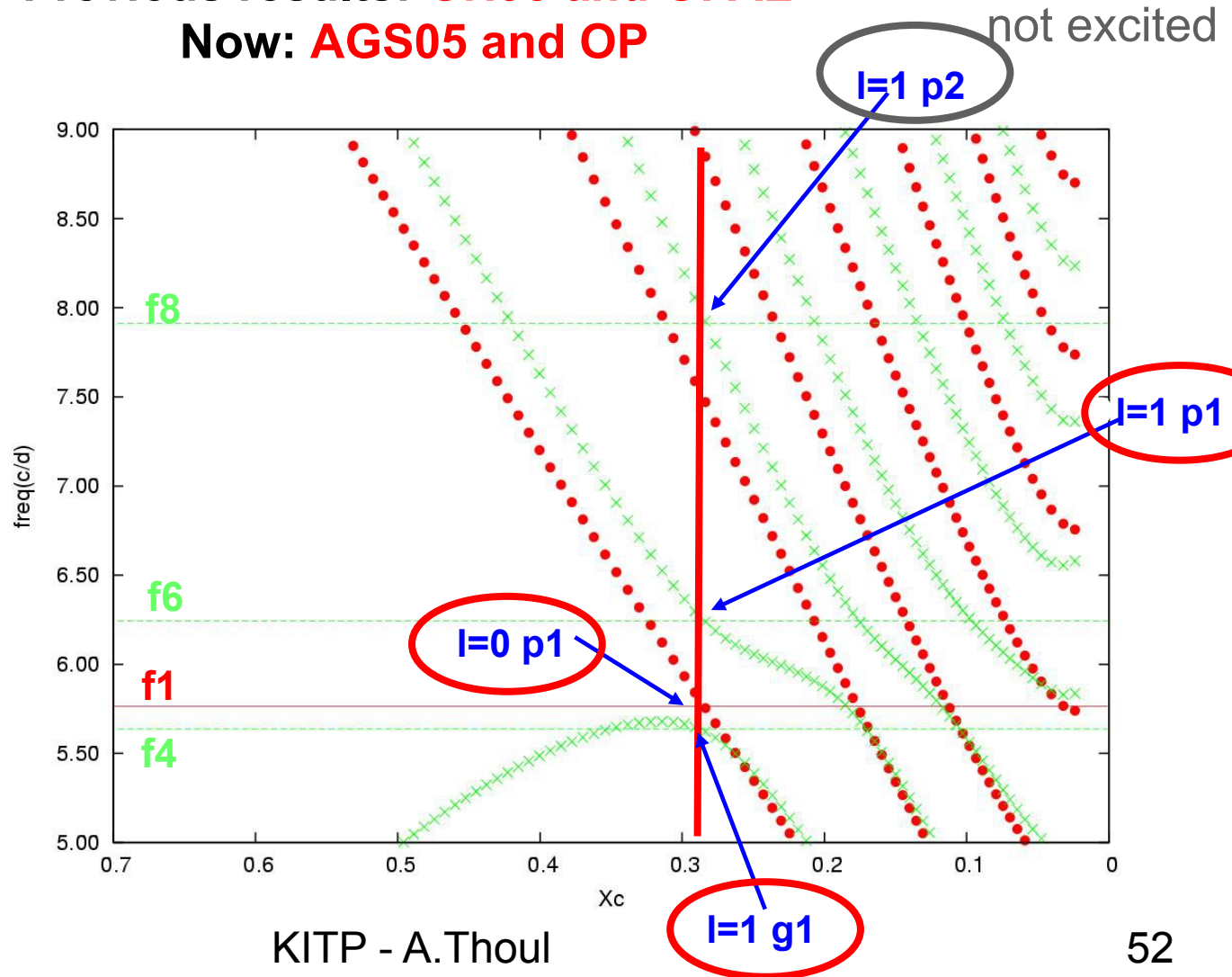
ν Eridani modelling

ID	Frequency [d^{-1}]
f_1	5.7632828 ± 0.0000019
f_2	5.6538767 ± 0.0000030
f_3	5.6200186 ± 0.0000031
f_4	5.6372470 ± 0.0000038
f_5	7.898200 ± 0.000032
f_A	0.432786 ± 0.000032
f_7	6.262917 ± 0.000044
f_6	6.243847 ± 0.000042
f_B	0.61440 ± 0.00005
f_9	7.91383 ± 0.00008
f_{10}	7.92992 ± 0.00010
f_8	7.20090 ± 0.00009
f_{11}	6.73223 ± 0.00012
f_{12}	6.22360 ± 0.00012

low-frequency high order g modes :

excited in range 0.55-0.91 c/d

➔ f_B is excited, but not f_A

ν Eridani

- **AGS05 abundances and OP opacities help solve the problems of νEri.**

- **BUT some problems remain:**
    - **Z higher than observed; independent of X!**
    - **highest frequency mode not excited**
    - **range of excited high-order g modes**

Examples of well-studied stars

- 16 Lacertae :   3 frequencies observed and identified, 2 axisymmetric modes; no multiplet
  ➔ very precise values of M, T_{eff}, L, age

- HD129929 : 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet
  ➔ precise values for M, Z, T_{eff}, L, age
  + evidence for core overshooting and non-rigid rotation

- ν **Eridani:** **12 frequencies observed, 7 identified, 2 axisymmetric modes, one triplet + 2 low-frequency modes = HYBRID β Cephei/SPB pulsator**
  ➔ **precise values for M, Z, T_{eff}, log g, age, α_{ov}, non-rigid rotation**
  + need for updated abundances AGS05 and OP opacities
  + problems not completely solved

- θ Ophiuchi

- 12 Lacertae

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rigid rotation possible

non-axisymmetry in splitting explained by second-order effects of rotation

+ problem with spectroscopic error box...

• 12 Lacertae

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  - ➔ very precise values for M, T_{eff}, log g, age, α_{ov}, ...
  - **evidence for high overshooting**
  - **rigid rotation possible**
  - **non-axisymmetry in splitting explained by second-order effects of rotation**
  - + problems with spectroscopic error box

- **12 Lacertae: 10 frequencies observed, 6 identified, problematic identification...**

12 (DD) Lacertae
A long history...

Observed as a **variable star** since **1915!**

$$P = \text{period}, \cdot 193089 \text{ days}, = 4^{\text{h}} 38^{\text{m}} 3^{\text{s}}$$

$1/P = 5{,}17896$ c/d

Present value: 5.179034 c/d

Observed as a **multiperiodic** variable star since **1957!**

« Thus the star 12 (DD) Lacertae has a principal period of 4h. 38m. and a secondary period of 4h. 44m., while recently a third component of 3h. 45m. has been found, and a fourth one of 3.9h. has been suggested. The periods occur in the variation of both the light and the radial velocity. »

Two frequencies known with **good precision** since **1961!**

$$P_1 = 0^{\text{d}}19308858$$
$$P_2 = 0^{\text{d}}1973685$$

$f_1 = 5{,}17897$ c/d
$f_2 = 5{,}06666$ c/d

Present values:
5.179034c/d
5.066346 c/d

12 (DD) Lacertae
a long history...

1978: 6 frequencies, including an l=3 triplet

«   As a result of a frequency analysis of the published observations of 12 Lacertae, six short-period sine-wave components are found in the star's light variation. The component frequencies have the following values: 32.5426 ± 0.0007, 31.8341 ± 0.0022, 34.4951 ± 0.0024, 33.5189 ± 0.0013, 66.0615 ± 0.0015, $26.644 \pm 0.009 \, \text{rad}/\text{d}$, and the »

«   From the position of the four strongest components in the frequency spectrum it is concluded that 12 Lacertae is a slowly rotating nonradial oscillator in which two spherical harmonic modes of different degree are simultaneously excited. If use is made of the line profile observations, the triplet frequencies can be identified as corresponding to $l = 3$, $m = -1, -2, -3$ oscillations »

f_1 = 5,17930 c/d ●

f_2 = 5.0665 c/d

f_3 = 5.4901 c/d   ●   l=3

f_4 = 5.3347 c/d   ●

f_5 = 10.5140 c/d

f_6 = 4.2405 c/d

$f_4 - f_1$ = 0.1554 c/d

$f_3 - f_4$ = 0.1554 c/d

Present values:

f_1 = 5,179034 c/d

f_2 = 5.066346 c/d

f_3 = 5.490167 c/d

f_4 = 5.334357 c/d

f_5 = - c/d

f_6 = 4.24062 c/d

12 (DD) Lacertae
a complicated history...

1980: radial mode + different identification: triplet is l=2, NOT l=3!

« Line profile observations of the Si III λ4567 line in the β Cephei star 12 Lacertae have been successfully fitted over three observing runs with a four-mode solution consistent with periods determined earlier by Jerzykiewicz. It is found that the variations in line shape can be fitted only with a radial mode of amplitude 13 km s^{-1} and with three nonradial modes of amplitude 40 km s^{-1} whose states are described by $l = 2$, $m = 0$, -1, and -2. The latter three modes are evidently equipartitioned in energy. The $l = 2$ identification is also compatible with the observed light and color amplitudes for these modes, but $l \geq 3$ fails to meet these tests. Two independent »

1994: try to discriminate between proposed identifications using the moment method BUT impossible to identify the modes!

1998: try to identify the modes through stellar modelling BUT impossible to identify the modes!

Abstract

Five pulsation modes have been detected in this well-known beta Cephei star. Three of them, including the strongest one, form an equidistant frequency triplet. We consider identifications of the observed pulsation frequencies with computed eigenfrequencies of low degree modes (l <= 2) in a series of stellar models covering the range of the effective temperature and surface gravity consistent with best available data. We show that the existing determinations of the degree of even the strongest observed mode are discrepant and therefore do little to constrain the problem. Finally, we discuss the difficulties posed by the observed equidistant frequency triplet.

12 (DD) Lacertae
a very complicated history...

⭐ **1999: discrepant determinations of the degree of the VERY equidistant triplet**
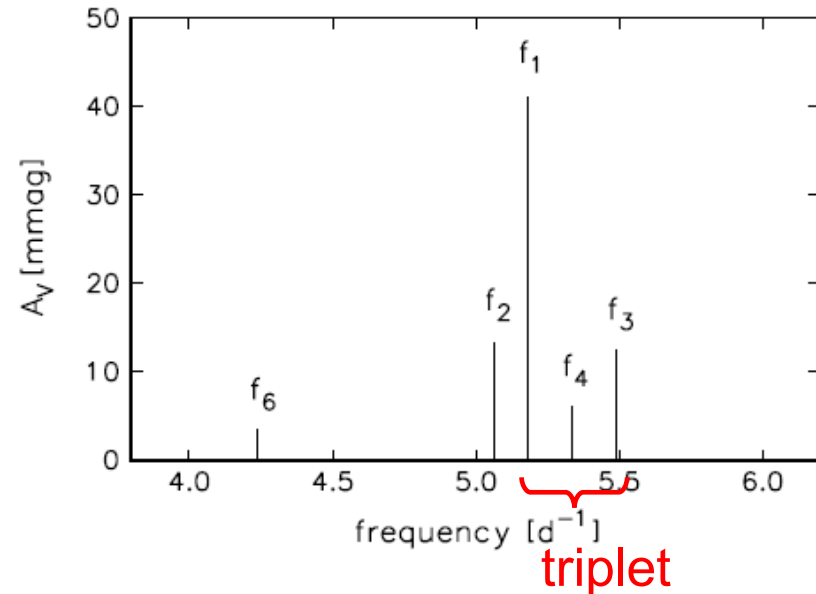
triplet

Abstract. Five pulsation modes are simultaneously excited in this well-known β Cephei star. Three of them, including the one with the largest light and radial-velocity amplitudes, form a triplet. The triplet is equidistant in frequency to within the errors of measurement, that is, $0.0003 \, \mathrm{d}^{-1}$.

Explaining why the triplet should be so nearly equidistant turns out to be a real challenge to the theory. We investigate the following three options: (1) rotational splitting, (2) an oblique magnetic pulsator, and (3) nonlinear phase lock. Unfortunately, apart from the frequencies, the data are meager. Photometric indices yield the effective temperature and surface gravity of rather low accuracy. In addition, the existing determinations of the spherical harmonic degree of even the strongest observed mode are discrepant. Consequently, the model parameters are not well constrained.

We show that of the three above-mentioned options, the oblique pulsator model is unlikely because it would require excessively strong dipolar field or a special field geometry. The rotational splitting is a possibility, but only for an $\ell = 2, p_0$ mode in a model with specific values of the effective temperature and surface gravity. Finally, we note that the nonlinear phase lock
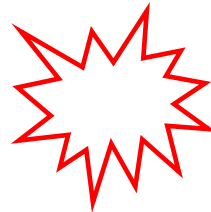
12 (DD) Lacertae
the story continues.... and....
SURPRISE!!!!!!

2006: Large **multisite campaign ➜ 10 independant frequencies.**
The equidistant triplet **not a triplet at all**: different values of l !!!

ABSTRACT

We report a multisite photometric campaign for the β Cephei star 12 Lacertae. 750 hours of high-quality differential photoelectric Strömgren, Johnson and Geneva time-series photometry were obtained with 9 telescopes during 190 nights. Our frequency analysis results in the detection of 23 sinusoidal signals in the light curves. Eleven of those correspond to independent pulsation modes, and the remainder are combination frequencies. We find some slow aperiodic variability such as that seemingly present in several β Cephei stars. We perform mode identification from our colour photometry, derive the spherical degree ℓ for the five strongest modes unambiguously and provide constraints on ℓ for the weaker modes. We find a mixture of modes of $0 \leqslant \ell \leqslant 4$. In particular, we prove that the previously suspected rotationally split triplet within the modes of 12 Lac consists of modes of different ℓ; their equal frequency splitting must thus be accidental.

12 (DD) Lacertae
the latest observations and identifications

Table 2. Multifrequency solution for our time-resolved photometry of 12 Lac. Formal error estimates (following Montgomery & O'Donoghue 1999) for the independent frequencies range from ± 0.000007 cd⁻¹ for f_1 to ± 0.00023 cd⁻¹ for f_{10}. Formal errors on the amplitudes are ± 0.2 mmag in u and ± 0.1 mmag in v and V. The S/N ratio, computed following Breger et al. (1993), is for the V filter data.

ID	Freq. (cd⁻¹)	u Ampl. (mmag)	v Ampl. (mmag)	V Ampl. (mmag)	S/N
f_1	5.179034	56.4	40.7	38.1	178.6
f_2	5.066346	23.3	16.7	16.0	74.6
f_3	5.490167	14.2	11.7	11.1	52.4
f_4	5.334357	21.9	11.6	10.0	47.3
f_5	4.24062	4.4	3.7	3.6	15.8
f_A	0.35529	7.2	4.8	5.0	14.4
f_6	7.40705	2.8	2.1	2.0	9.7
f_7	5.30912	2.7	2.3	2.0	9.5
f_8	5.2162	1.3	1.3	1.3	6.2
f_9	6.7023	2.2	1.6	1.3	6.3
f_{10}	5.8341	1.8	1.2	1.3	6.1
$f_1 + f_4$	10.513392	8.3	5.9	5.5	32.9
$f_3 + f_A$	5.84546	2.3	1.6	1.8	8.7
$f_2 + f_4$	10.400704	2.3	1.7	1.7	10.3
$2f_1$	10.358069	1.9	1.2	1.2	6.9
$f_1 + f_2$	10.245381	1.7	1.3	1.2	6.9
$2f_8$	10.4324	1.5	1.4	1.0	6.1
$2f_4$	10.668715	1.0	0.8	0.7	4.3
$f_3 + f_4$	10.824524	0.8	0.7	0.6	3.9
$f_2 + f_3$	10.556514	1.1	0.8	0.7	4.0
$2f_1 + f_2$	15.424415	0.6	0.6	0.5	4.3
$f_1 + f_2 + f_4$	15.579738	1.0	0.6	0.5	4.5
$2f_1 + f_4$	15.692426	1.0	0.6	0.5	4.3

identification:

Table 3. Mode identifications for 12 Lac from our analysis of the photometric amplitude ratios.

ID	Freq. (cd⁻¹)	ℓ
f_1	5.179034	1
f_2	5.066346	1
f_3	5.490167	2
f_4	5.334357	0
f_5	4.24062	2
f_A	0.35529	1, 2 or 4
f_6	7.40705	1 or 2
f_7	5.30912	2 or 1 or 3
f_8	5.2162	4 or 2
f_9	6.7023	1
f_{10}	5.8341	1 or 2
$f_3 + f_A$	5.84546	2 or 1
$2f_8$	10.4324	1 or 2 or 3

hybrid β Cephei/ SPB?

$f_1 - f_2 = 0{,}112688$ c/d
$f_4 - f_1 = 0{,}155323$ c/d
$f_3 - f_4 = 0{,}15581$ c/d

Not a triplet: coincidence!!!

12 (DD) Lacertae
a very interesting story indeed!

As mentioned in the Introduction, two hypotheses to explain the pulsation spectrum of 12 Lac seemed promising before our multisite campaign took place: first, the presence of a rotationally split triplet consisting of the modes (f_1, f_3, f_4) and second, the presence of the fundamental and first radial overtones (modes f_5, f_3). Our mode identification allows us to judge these hypotheses: neither is correct.

The suspected rotationally split structure consists of modes of $\ell = 1, 0, 2$, and the suspected radial modes both turned out to be $\ell = 2$. Consequently, all previous attempts to understand the pulsation spectrum of 12 Lac were not correct.

12 (DD) Lacertae
present status of the observations

Summary of the evolution of the mode identification
for the frequencies of 12Lac:

(l,m)	Stamford & Watson (1977)	Jerzykiewicz (1978)	Smith (1980)	Mathias et al (1994)	Aerts et al (1996)	Dziembowski & Jerzykiewicz (1999)	Handler et al (2006)	**Desmet (2008)**
f_1 = 5.179 c/d	(2,-2)	(3,-1)	(2,0)	(2,$\cong$ 1)	(2,-1)	(1, ?)	(1, 1)	**(1,1)**
f_2 = 5.066 c/d	(2,0)	(1,-1)	(0,0)	(?,0)	(2 or 3, ?)	(1 or 2,?)	(1, 0)	**(1,0)**
f_3 = 5.490 c/d		(3,-3)	(2,-2)		(2,-2)	(1, ?)	(2, ?)	**(2,1)**
f_4 = 5.334 c/d		(3,-2)	(2,-1)	(2,0)	(3,1)		(0, 0)	**(0,0)**
f_5 = 4.241 c/d							(2, ?)	**(2,?)**
f_6 = 7.407 c/d							(1 or 2, ?)	

12 (DD) Lacertae modelling

assumptions:

$X = 0.70$, $Z = 0.015$, composition A04, opacity OP,

$\log T_{eff} = [4.355, 4.395]$, $\log L = [4.0, 4.35]$,

Handler et al. 2006 frequencies, Desmet et al. 2008 identifications

no overshooting

Fit f_4 as l=0 p_1 mode (radial fundamental)

Fit f_2 as l=1 g_1 mode (n=-1)

Constraints on T_{eff} and L ➔ **no other radial orders possible for f_2 and f_4**

➔ $M = 11.77$ M_0 $\log T_{eff} = 4.39$ $\log L = 4.19$

➔ f_3 is (l,m,n) = (2,1,-1)

➔ f_5 is 0.3 c/d from nearest quadrupole (l,m,n) = (2,2,-2)

➔ with some effects of rotation, f_5 is (2,2,-2) or (2,1,?)

➔ all modes unstable except low-frequency mode

12 (DD) Lacertae
BUT the story continues!!!!!!!

assumptions:
X = 0.72, Z = 0.015, composition AGS05, opacity OP,
$\log T_{eff}$=4.389±0.018, log g = 3.65±0.15,
Handler et al. 2006 frequencies, Desmet et al. 2008 identifications

We rule out f_4 as the radial fundamental, because f_9 is identified as l=1

➔f_4 is the first overtone (l,n)=(0,2)

➔**NO PROBLEM WITH THE ERROR BOX**

➔f_2 is l=1, p1 mode
  f_3 could be a l=2, n=0 mode with m>0
  f_5 could be a l=2 g2 mode
  f_6 cannot be a l=1 mode, and has to be l=2
  f_7 cannot be a l=1 mode, and could be part of an
       l=2 quintuplet with f3 or an l=3
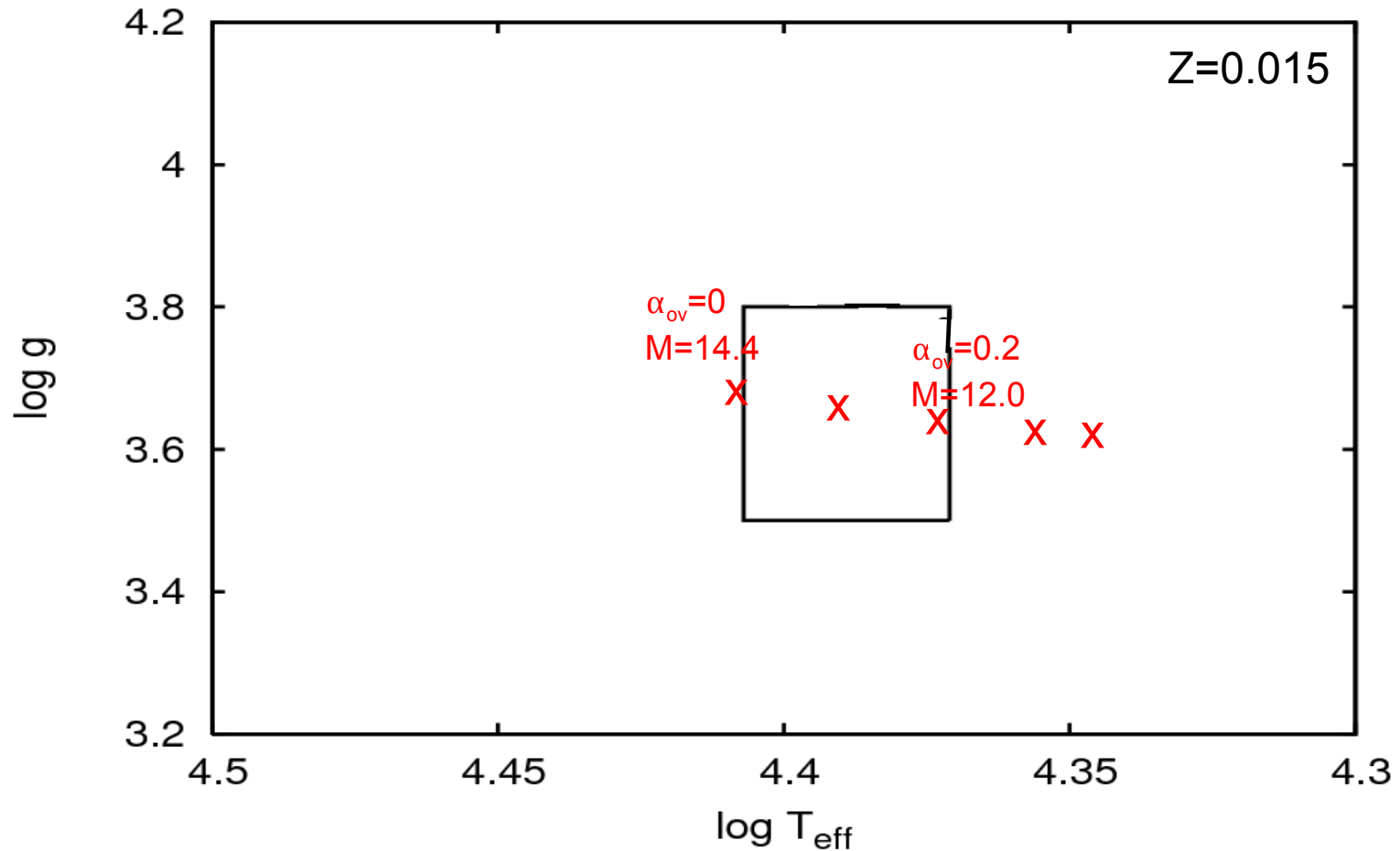 f_8 cannot be a l=4
  f_9 is l=1 p3 mode
  f_{10} cannot be a l=2, and could be l=2

Table 3. Mode identifications for 12 Lac from our analysis of the photometric amplitude ratios.

ID	Freq. (cd^{-1})	ℓ
f_1	5.179034	1
f_2	5.066346	1
f_3	5.490167	2
f_4	5.334357	0
f_5	4.24062	2
f_A	0.35529	1, 2 or 4
f_6	7.40705	1 or 2
f_7	5.30912	2 or 1 or 3
f_8	5.2162	4 or 2
f_9	6.7023	1
f_{10}	5.8341	1 or 2
$f_3 + f_A$	5.84546	2 or 1
$2f_8$	10.4324	1 or 2 or 3

12 (DD) Lacertae
the story continues...

12 (DD) Lacertae
## the story continues...

range of unstable modes

**FOR Z=0.015
BUT Z_{obs}=0.010 !!**

Examples of well-studied stars

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  → **very precise values of M, T_{eff}, L, age**

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  → precise values for M, Z, T_{eff}, L, age
  + evidence for core overshooting and non-rigid rotation

- ν Eridani: 12 frequencies observed, 7 identified, 2 axisymmetric modes, one triplet + 2 low- frequency modes
  → precise values for M, Z, T_{eff}, log g, age, α_{ov}, non-rigid rotation
  + need for updated abundances AGS05 and OP opacities
  + problems not completely solved

- θ Ophiuchi: 7 frequencies observed, all identified, 2 axisymmetric modes, one triplet, 3 members of a quintuplet
  → very precise values for M, T_{eff}, log g, age, α_{ov}, ...
  evidence for high overshooting
  rigid rotation possible
  non-axisymmetry in splitting explained by second-order effects of rotation

- **12 Lacertae:** **10 frequencies observed, 6 identified, problematic identification...**
  → very precise values for M, T_{eff}, log g, age, α_{ov}, ...
  → **reliable mode identification is crucial**
  → **problem of excitation not entirely solved**

Corot B stars
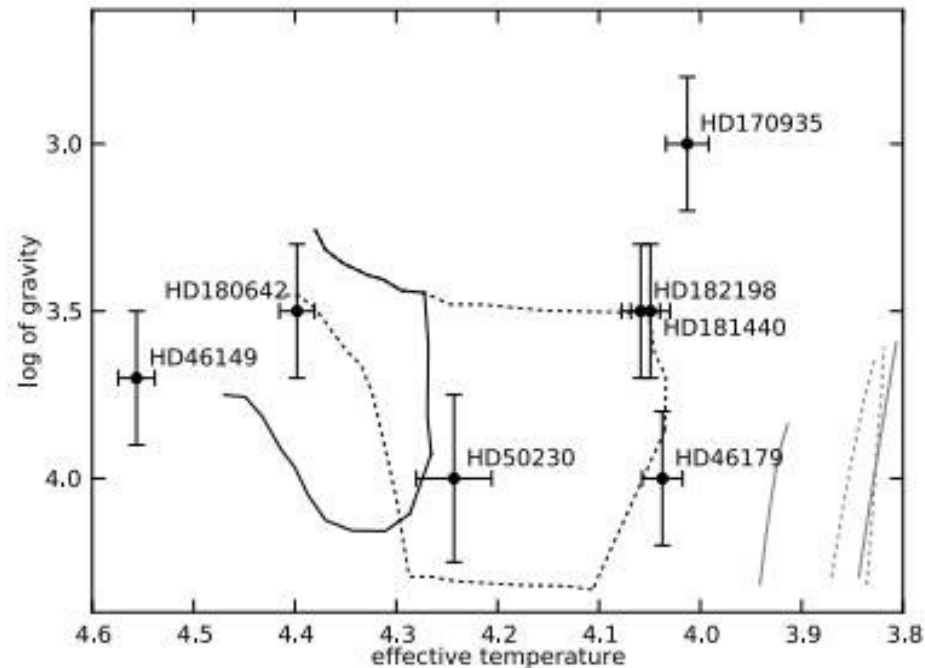
Fig. 1 Observational $\log T_{\mathrm{eff}}$-$\log g$ diagram with the two predicted instability strips (SPB, dashed line, β Cep, solid line) and a selection of CoRoT targets, using OPAL opacities and a metallicity of $Z = 0.02$. For reference, also the γ Dor/δ Sct instability strips are shown (grey dashed/solid lines).
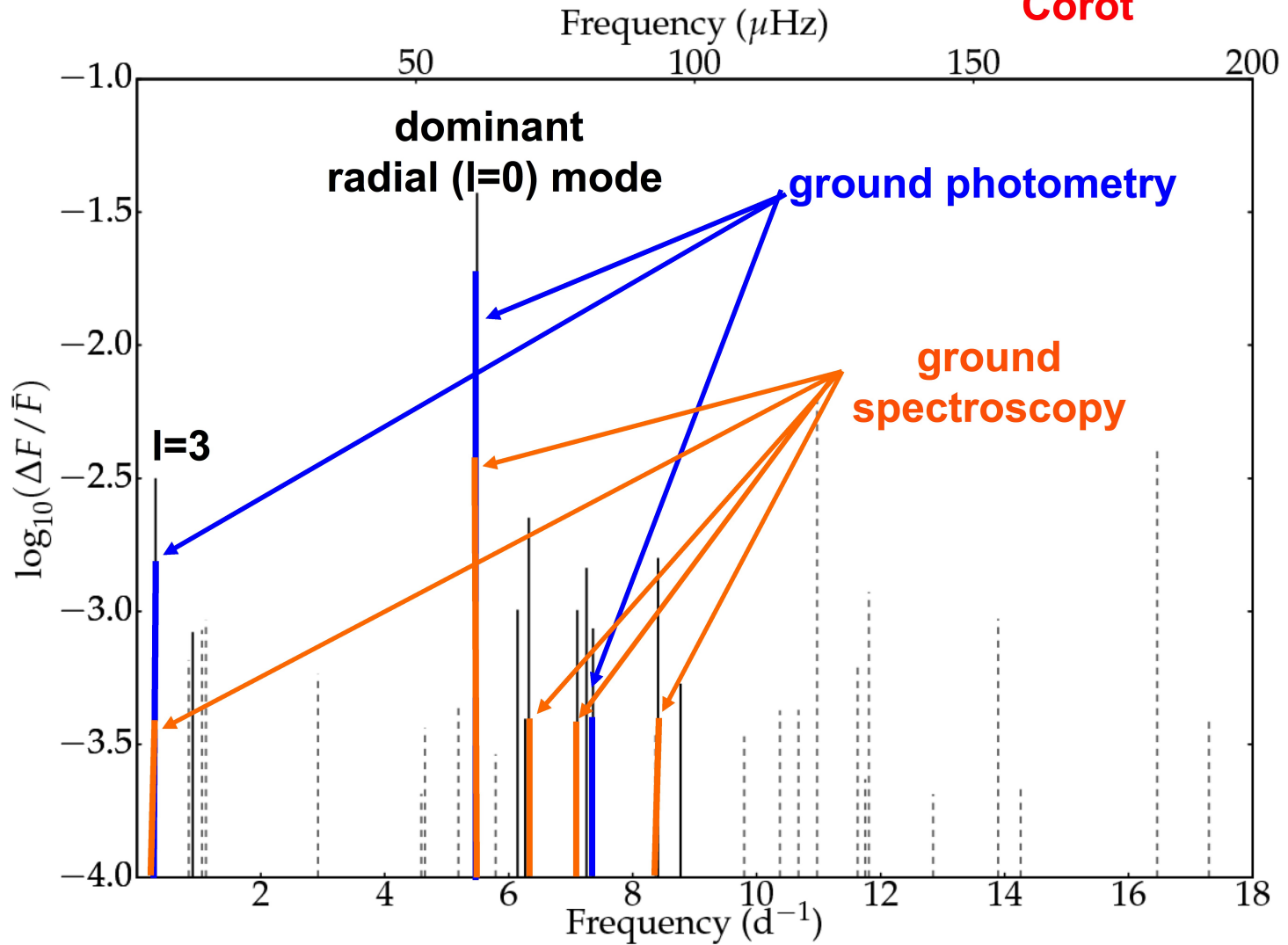
Examples of well-studied stars

- **HD 180642: (Corot main target + ground): 11 independent frequencies**
  + 3 harmonics + 19 combination freq. (locked phases)
  1 identified (high amplitude radial mode, no clear multiplet
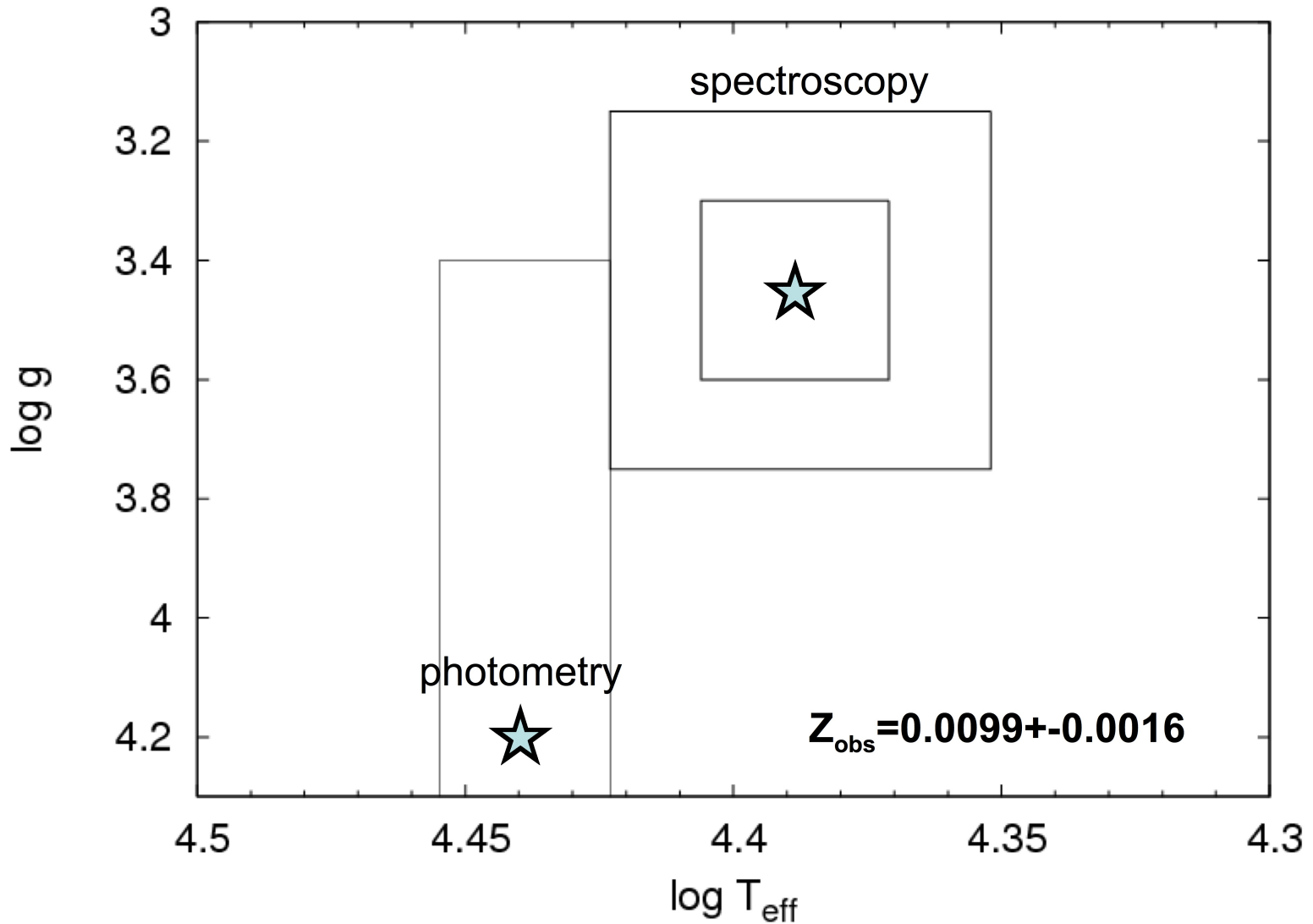
- HD 50230

- HD 51756

- HD 170580

HD180642

11 independent frequencies from Corot

HD180642

$Z_{obs}=0.0099+-0.0016$

HD180642

Modelling:

Fit the radial mode, check excitation, then fit remaining modes

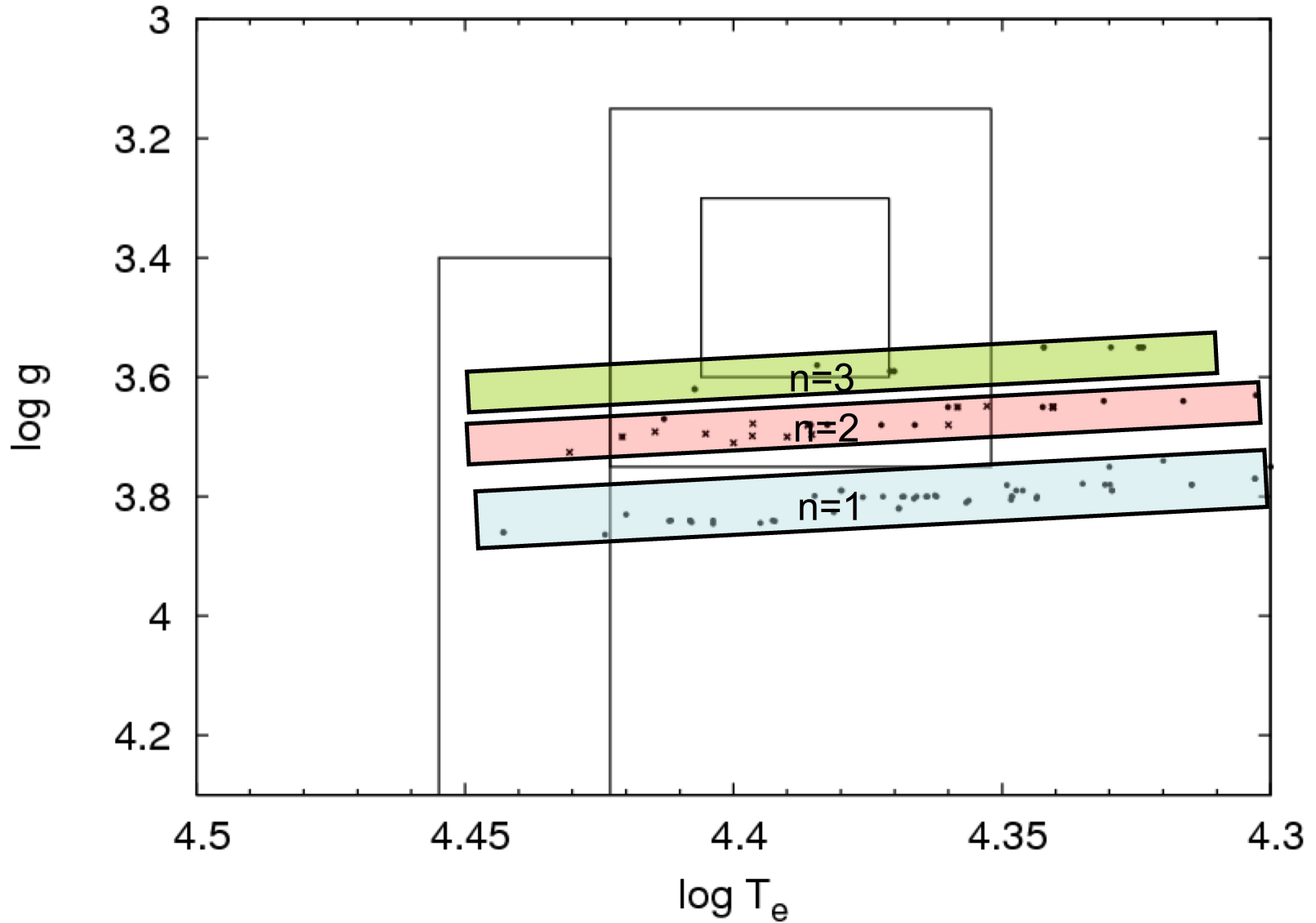### "Free parameters": X, Z, α_{ov}, M

Range of parameters explored:
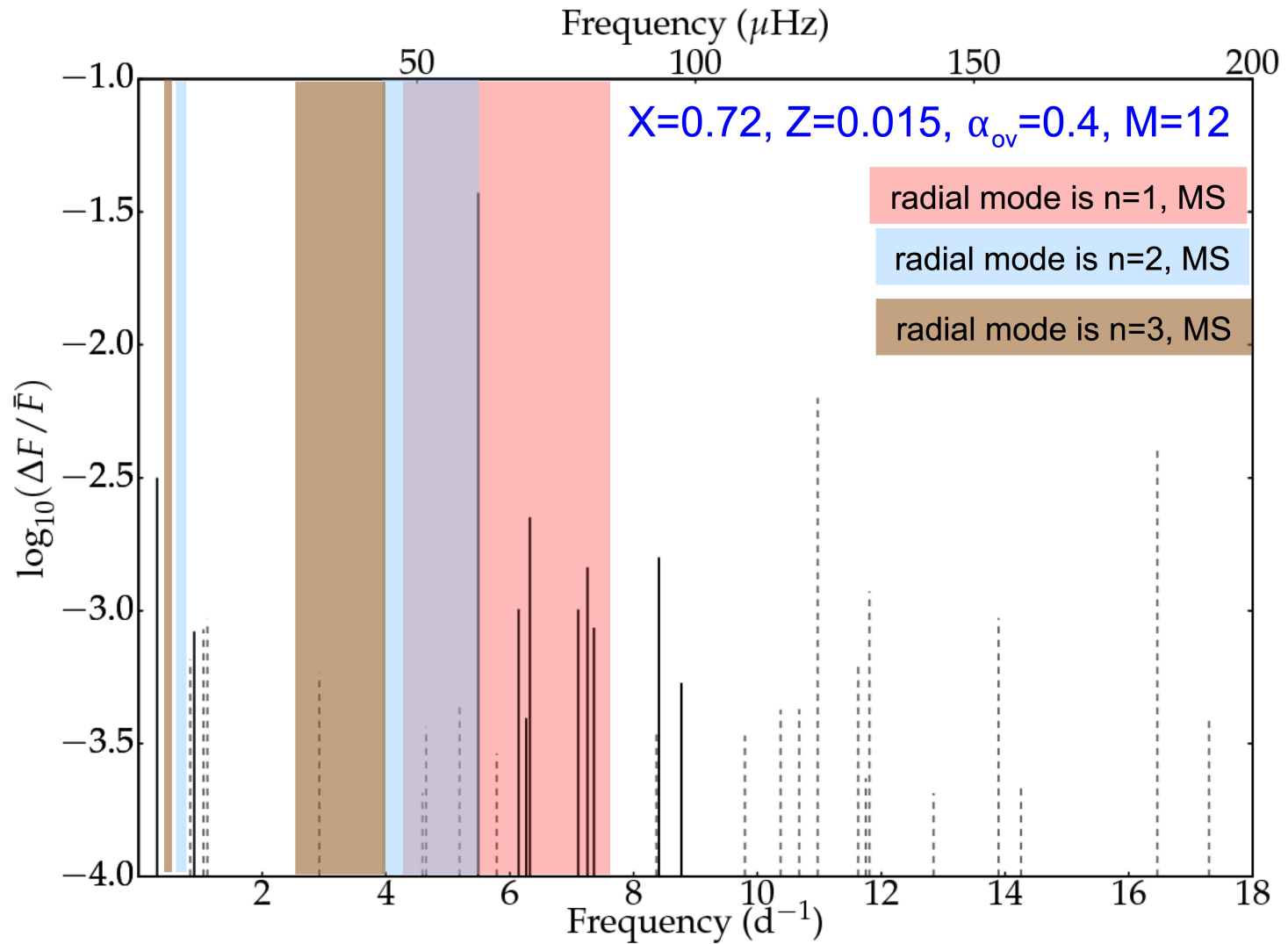### X = 0.68 – 0.74
Z = 0.010 – 0.018
M = 7.6 – 20
α_{ov} = 0 - 0.5

HD180642

HD180642

Examples of well-studied stars

- **HD 180642:** (Corot main target + ground): 11 independent frequencies + 3 harmonics
  + 19 combination freq. (locked phases)
  1 identified (high amplitude radial mode, no clear multiplet

  ➔ precise values for M, T_{eff}, log g, age, α_{ov}, ...

  ➔**Very low overshooting**

  ➔ **Discrepancy between spectroscopic and seismic log g:**
**pulsational broadening not taken into account correctly when deducing
log g from wings of spectral lines**

- HD 50230

- HD 51756

- HD 170580

Examples of well-studied stars

• **HD 180642:** (Corot main target + ground): 11 independent frequencies + 3 harmonics
   + 19 combination freq. (locked phases)
   1 identified (high amplitude radial mode, no clear multiplet

➔ precise values for M, T_{eff}, log g, age, α_{ov}, ...

➔ Very low overshooting

➔ Discrepancy between spectroscopic and seismic log g: pulsational broadening not taken into account correctly when deducing log g from wings of spectral lines

• **HD 50230: hundreds of gravity modes and tens of p modes**
   non-uniform period spacings ➝ α_{ov} >0.2 and smooth
   gradient of chemical composition at core boundary

• HD 51756

• HD 170580

Examples of well-studied stars

- HD 180642: (Corot main target + ground): 11 independent frequencies + 3 harmonics
      + 19 combination freq. (locked phases)
       1 identified (high amplitude radial mode, no clear multiplet

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- **HD 170580: many high order g modes, one p mode**
  ... in progress...

Kepler B stars
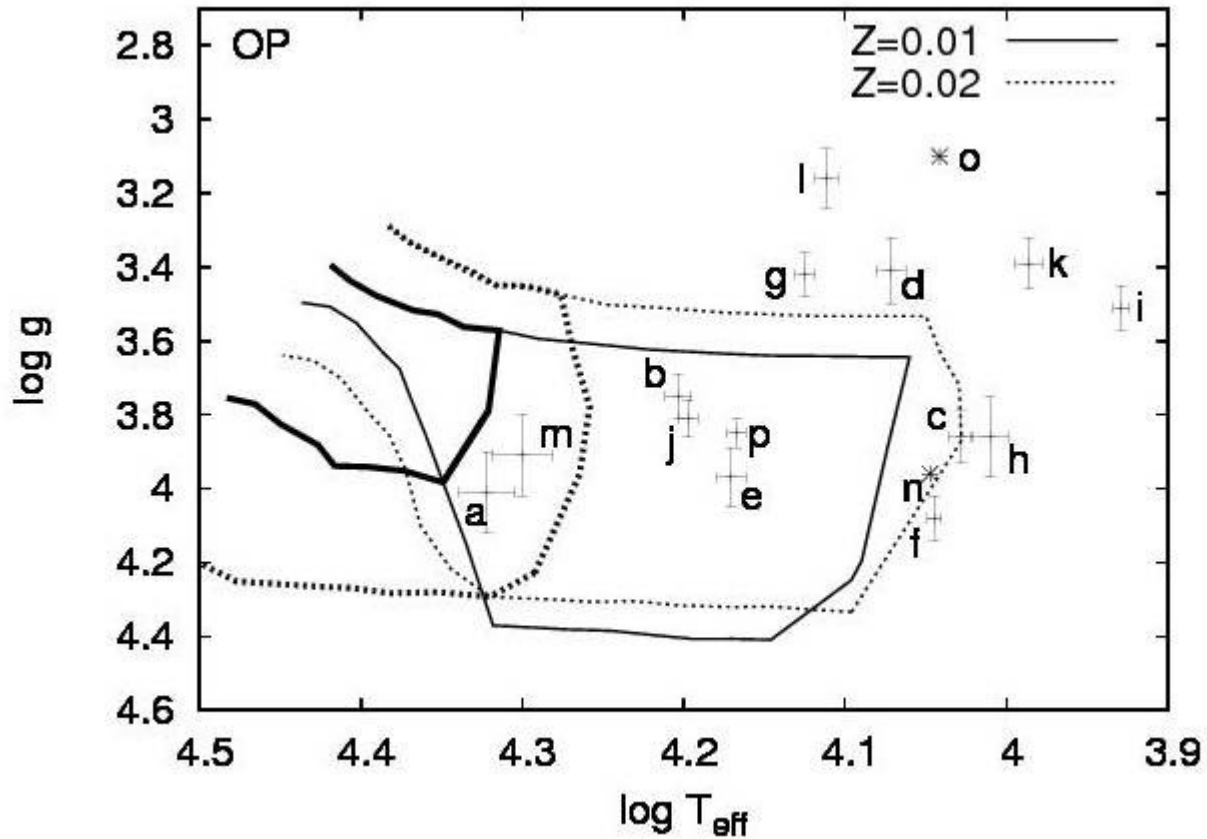
What we have learned so far from the asteroseismology of β Cephei stars

very precise values of the basic stellar parameters (M, T_{eff}, log g, L, age)

evidence for varying core overshooting (0 to 0.4)

evidence for non-rigid rotation in some β Cephei stars

need for updated abundances AGS05 and OP opacities (ν Eri, θ Oph, 12Lac)

non-axisymmetry in splitting can be explained by second-order effects of rotation (θ Oph)

reliable mode identification is crucial (story of 12Lac)

Challenges (observational)

- **good frequency determination**: need to resolve the individual frequencies ➔ long data sets with very good coverage
Note: for classical pulsators, no inherent problem, since modes are phase coherent over periods of years or more
(in contrast to solar-like pulsators)
BUT a real challenge for SPB stars due to long periods of high-order g-modes

- **reliable mode identification: necessary!** (see 12Lac)
More difficult for classical pulsators than for solar-like oscillations (no equal spacings)
Multiplets (rotational splittings) can overlap with the spectrum of axisymmetric frequencies (see 12Lac)

- need **good determination of basic stellar parameters** (Teff, log g, [M/H]) to rule out some models

Challenges (observational)

➔ **lots of photometric data will come from space observations**

BUT

need ground-based follow-up with
multicolour photometry
and
high-resolution spectroscopy

BECAUSE

detailed modelling is impossible without reliable frequencies
AND mode identification
as well as a precise position in the HR diagram

Challenges (modelling)

- explain the range of modes excitation
- and the presence of β Cephei and SPB stars in low Z environments

- explain mode selection

Challenges (modelling)

➔ **Improve physics** in models:
rotation, mixing, convection, magnetic field...

• **Good opacities** are crucial!

• Correct initial **chemical composition**
+ **its evolution** (diffusion, radiative accelerations)

• Stellar modelling of fast rotators

Before Corot

- 16 Lacertae   2003

- HD129929   2003

- ν Eridani   2004

- θ Ophiuchi   2007

- 12 Lacertae   2009

Corot stars

- HD 180642   2011

- HD 51756   2011