Asteroseismology of β Cephei stars

Anne Thoul
Chercheur FNRS
Université de Liège, Belgium
• This is an informal talk!
• Only $\beta$ Cephei stars today
• Not an exhaustive review
  • Not a “theory” talk
O stars: very massive; winds (mass loss)
a few pulsators known
asteroseismology in its infancy

B stars:
- Be stars: fast rotators, emission lines, pulsators - complicated
- SPB stars: B2 - B9
  \[ M \sim 4 - 7 \, M_\odot \]
multiperiodic pulsators
  \[ P = 0.5 - 5 \text{ days} \]
  High-order g modes in asymptotic regime
- \( \beta \) Cephei stars:
  B0 - B3
  \[ M \sim 8 - 18 \, M_\odot \]
multiperiodic pulsators; slow rotators
  \[ P = 2 - 8 \text{ hours} \]
  Sparse spectrum of low-order p and/or g modes

See talk at the conference
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See talk at the conference
\[ \log T_{\text{eff}} \approx 4.2 - 4.4 \]
\[ \log L \approx 2 - 4 \]
Main sequence B stars

- CNO burning
- massive convective core
- radiative outer zone

Solar-type stars

- Convective or radiative core
- Radiative zone
- Convective envelope

Different structures $\Rightarrow$ very different pulsation spectra
Why are seismic studies of massive main sequence stars interesting?

Because their spectra include modes that probe their deep structure, in particular the boundary of the convective core

- info on the overshooting/mixing at the core boundary
- info on the internal rotation profile
solar-type stars → superficial convective layer → stochastically excited modes

δ Scuti stars → mechanism in the He ionization zone

massive stars → mechanism in the Fe partial ionization zone (opacity bump)
κ mechanism:

occurs IF opacity bump (due to Helium or iron-group elements partial ionization) COINCIDES with the transition zone (between adiabatic and non-adiabatic regions)
The diagrams illustrate the iron opacity bump in SPB and β cephei stars.

- **SPB**
  - 4 $M_{\odot}$
  - Iron opacity bump at 200,000 K

- **β cephei**
  - 10 $M_{\odot}$
  - Iron opacity bump at 200,000 K
Figure 1. Opacity, $\kappa$, opacity derivative, $\kappa_T = (\partial \ln \kappa / \partial \ln T)_\rho$, and the differential work integral for the fundamental mode of radial pulsation, $dW/d \log T$ (in arbitrary units), plotted against temperature in a model of a $\beta$ Cep star. The model parameters are: $M = 12\ M_\odot$, $\log L/L_\odot = 4.22$, $\log T_{\text{eff}}/K = 4.368$, $X = 0.7$, $Z = 0.03$. Driving occurs in zones where $dW/d \log T > 0$. Continuous and dashed lines correspond to the newer and older OPAL opacity tables, respectively.
Cas 1: $M=1$

frequencies of solar-type stars: many high-order $p$ modes

modes $p$

modes $g$

début de la SP

fin de la SP
radial and non radial modes in β Cephei models

\[ M = 10, \ X = 0.7, \ Z = 0.02, \ \alpha_{\text{ov}} = 0 \]

- p modes
- g modes

excited modes

\[ \Rightarrow \text{avoided crossings} \]

age

\[ T_{\text{eff}} \]

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excited modes: in a given range of dimensionless frequencies

Figure 5. Dimensionless frequencies, $\sigma = \omega / \sqrt{4\pi G(\rho)}$, of low-order and low-degree ($l$) modes for models of a 12-M$_{\odot}$, Z = 0.03 star in its MS evolutionary phase. The small and the large dots correspond to stable and unstable modes, respectively.
Figure 6. Dimensionless frequencies, $\sigma = \omega/\sqrt{4\pi G(\rho)}$, of low-order and low-degree ($l$) modes for models of a 12-$M_\odot$, $Z=0.02$ star in its MS evolutionary phase. The small and the large dots correspond to stable and unstable modes, respectively.
g modes have a large amplitude near the core of the star

p modes have large amplitudes near the surface

**Figure 1.4**: Typical variation of the kinetic energy density as a function of depth in the star for a $g$-mode (left), a mixed mode (middle) and a $p$-mode (right) in an evolved star of $2 \, M_\odot$. The different modes sample different parts of the star. (figure taken from Roxburgh et al. 2000)
avoided crossing $\rightarrow$ mixed modes $\rightarrow$ info on core boundary

fitted model of $\nu$ Eridani (3 frequencies)

Lamb frequency $S_l^2 \equiv \frac{[l(l+1)]c^2}{r^2}$

Brunt-Väisälä frequency

$$N^2 = g \left( \frac{1}{\Gamma_1 p} \frac{dp}{dr} - \frac{1}{\rho} \frac{d\rho}{dr} \right)$$

$$N^2 \approx \frac{g^2 \rho}{p} (\nabla_{ad} - \nabla + \nabla_{\mu})$$

boundary of convective core

$\omega_{n,l,m} > S_l$, $N$ : propagation of sound waves

$\omega_{n,l,m} < S_l$, $N$ : propagation of gravity waves

otherwise, evanescent
EACH MODE BRINGS AN INFORMATION ABOUT A DIFFERENT LAYER OF THE STAR!
Rotation splittings and rotation
FOR SLOW ROTATORS

degeneracy: 2m+1 frequencies for each mode l

\[ \sigma_{k\ell m} - \sigma_{k\ell 0} = m \int K_{k\ell}(r) \Omega(r) \, dr \]

kernel constructed from the eigenfunctions of the modes in the absence of rotation

rotation profile inside the star

each frequency (each couple l,k) = one linear condition on \( \omega \)

\[ \int K_i(r) \Omega(r) \, dr = w_i, \quad i = 1, \ldots, N. \]
Detection of 6 frequencies:

- $f_1 = 6.461699 \, \text{c/d (} l=2, m=? \text{)}$
- $f_2 = 6.978305 \, \text{c/d (} l=1, m=0 \text{)}$
- $f_3 = 6.449590 \, \text{c/d (} l=2, m=? \text{)}$
- $f_4 = 6.990431 \, \text{c/d (} l=1, m=1 \text{)}$
- $f_5 = 6.590940 \, \text{c/d (} l=0, m=0 \text{)}$
- $f_6 = 6.966172 \, \text{c/d (} l=1, m=-1 \text{)}$

2 consecutive members of quintuplet

one triplet

- $l=0 \ p_1$
- $l=2 \ p_2$
- $l=2 \ g_1$
- $l=1 \ p_1$

9.54 M$_0$

$\alpha_{ov} = 0.2$
$Z = 0.016$

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kernels are very different for the two modes ➔ give information about the rotation at different depths (of the radiative envelope)!

HD129929
HD129929 - A β cephei star
Périodogramme

sparse spectrum
Methods of observation:

photometric: time series – direct measure of intensity variations

spectroscopic: measure of the changes of the surface velocity

The two methods sample the SAME pulsations, but in different ways

modes observed: low degree \( l \)

Intensity variations and Doppler shift of spectral lines are weighted averages of the pulsation amplitude over the \( \tau = 2/3 \) surface

\( \Rightarrow \) reduced sensitivity to modes with high degree \( l \)

Doppler observations: projection of the velocity over the line of sight

\( \Rightarrow \) slightly better observations of modes of moderate degree \( l \)

Example: \( l = 3 \) modes detected in velocity observations, not in intensity observations.
Low frequencies $\rightarrow$ long-term monitoring is necessary especially true for high-order g-mode pulsators (SPB)

Ground-based observations:
networks of small and medium size telescopes (Whole Earth Telescope, Delta Scuti Network, STEllar PHotometry International network)
large multisite campaigns on dedicated stars

Space missions:
non-dedicated: WIRE, Kepler
dedicated: MOST, CoRoT

Mode identification $\rightarrow$ need for multi-site multicolour photometry and high-resolution spectroscopy
example of a $\beta$ Cephei star: 12 Lacertae

Table 2. Frequencies and amplitudes of the first moment of the SiIII $\lambda$4553 Å line together with their S/N ratio (we refer to the text for explanation). Error estimates (Montgomery & O'Donoghue 1999) for the independent frequencies range from ±0.000002 for $f_1$ to ±0.000002 for $f_9$. The error on the amplitude is 0.01 km s$^{-1}$.

<table>
<thead>
<tr>
<th>ID</th>
<th>Frequency $[d^{-1}]$</th>
<th>Amplitude $[\mu Hz]$</th>
<th>S/N</th>
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<tr>
<td>$f_1$</td>
<td>5.178964</td>
<td>59.941713</td>
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<td>$f_2$</td>
<td>5.334224</td>
<td>61.738704</td>
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<td>$f_3$</td>
<td>5.066316</td>
<td>58.637917</td>
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<tr>
<td>$f_4$</td>
<td>5.490133</td>
<td>63.543206</td>
<td>2.61</td>
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<td>$f_5$</td>
<td>3.432841</td>
<td>3.968067</td>
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<tr>
<td>$f_6$</td>
<td>4.256966</td>
<td>49.270440</td>
<td>0.92</td>
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<td>$f_7$</td>
<td>5.218075</td>
<td>60.394387</td>
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<td>$f_8$</td>
<td>6.702318</td>
<td>77.53125</td>
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<td>7.407162</td>
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Mode identification: photometric spectroscopic

Table 5. Final results for the mode identifications for 12 Lac from our spectroscopic analysis together with the results from the photometric amplitude ratios (Handler et al. 2006).

<table>
<thead>
<tr>
<th>ID</th>
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<td>1</td>
<td>1</td>
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<tr>
<td>$f_2$</td>
<td>5.334224</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$f_3$</td>
<td>5.066316</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$f_4$</td>
<td>5.490133</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$f_5$</td>
<td>3.432841</td>
<td>-</td>
<td>1,2,4</td>
<td>0,-1</td>
</tr>
<tr>
<td>$f_6$</td>
<td>4.256966</td>
<td>-</td>
<td>2</td>
<td>[+]</td>
</tr>
<tr>
<td>$f_7$</td>
<td>5.218075</td>
<td>-</td>
<td>2,4</td>
<td>?</td>
</tr>
<tr>
<td>$f_8$</td>
<td>6.702318</td>
<td>-</td>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>$f_9$</td>
<td>7.407162</td>
<td>-</td>
<td>1,2</td>
<td>?</td>
</tr>
<tr>
<td>$f_p$</td>
<td>5.30912</td>
<td>-</td>
<td>1,2</td>
<td>?</td>
</tr>
</tbody>
</table>

about 10 frequencies
example of a $\beta$ Cephei star: 12 Lacertae

Figure 1. Oscillation spectra of $\nu$ Eri and 12 Lac based on Jerzykiewicz et al. (2005) and Handler et al. (2006) data, respectively. The numbers above the bars are the most likely $\ell$-values, as inferred from data on amplitudes in four passbands of Strömgren photometry.
Asteroseismology of B stars ≠ asterosismic studies of solar-type stars

**solar-type stars**
- short-lives modes
- comb-like spectrum of low degree
- high-order p modes
- mode identification relatively easy
  - with echelle diagram
- asymptotic regime
  - fit large and small separations

**B stars**
- long-lived modes
- sparse spectrum of low order, low-degree p and g modes (β Cephei)
- or
- high-order g-modes (SPB)
- mode identification is difficult
- need multicolour photometry and spectroscopy
- fit exactly each frequency
Main-sequence B stars: mode identification

**multicolour photometry**: amplitude and phase behavior of an oscillation mode are different in different filters

⇒ **degree l** can be determined

**line-profile variations**: its shape is entirely determined by the parameters in all the pulsationnal velocities, **including l and m**
Asteroseismology of main-sequence B stars: Method

β Cephei stars: sparse spectrum in non-asymptotic regime

DIRECT FITTING
- run forward stellar models
- fit each axisymmetric frequency
- analyze the rotational splittings
- get constraints on the stellar parameters, the other observables
  (log $T_{\text{eff}}$, log $g$, [$M/H$]$_{\text{surf}}$, $L$, $R$), and the physics
- improve the physics

⇒ NICE RESULTS OBTAINED
Main-sequence B stars
Theoretical modelling

« Standard » stellar evolution code + « interesting » physics

modeling parameters:
Mass M
Initial Hydrogen mass fraction X
Initial Metallicity Z
(or initial Helium mass fraction Y)
Overshooting parameter $\alpha_{ov}$

initial chemical composition
opacities: opal / OP / ?
« mixing »: overshooting, rotation, ?
convection theory
diffusion and gravitational settling
radiative accelerations
magnetic fields
mass loss
departures from spherical symmetry
...

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Asteroseismology: what we would like to learn

- Basic stellar parameters
- Mixing
- Internal rotation
- Diffusion, gravitational settling
- Convective overshooting
- Evolution of the chemical abundances
- Magnetic fields
- Opacities, equation of state
- Mass loss
Examples of well-studied stars or
« A few * success * stories »

**Before Corot**

- 16 Lacertae
- HD129929
- ν Eridani
- θ Ophiuchi
- 12 Lacertae

**Corot stars**

- HD 180642
- HD 50230
- HD 51756
- HD 170580
Examples of well-studied stars

• **16 Lacertae**: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet ➔ very precise values of M, T\text{eff}, L, age

• HD129929

• ν Eridani

• θ Ophiuchi

• 12 Lacertae

• HD 180642
Examples of well-studied stars

• 16 Lacertae: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet  
  ➔ very precise values of $M$, $T_{\text{eff}}$, $L$, age

• HD129929: 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet,  
  2 members of a quintuplet  
  ➔ precise values for $M$, $Z$, $T_{\text{eff}}$, $L$, age  
  + evidence for core overshooting  
  + evidence for non-rigid rotation

  (movie)

• $\nu$ Eridani

• $\theta$ Ophiuchi

• 12 Lacertae

Note: bad frequencies = no solution!!!
Examples of well-studied stars

• 16 Lacertae: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet → very precise values of M, $T_{\text{eff}}$, L, age

• HD129929: 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet → precise values for M, Z, $T_{\text{eff}}$, L, age
  + evidence for core overshooting and non-rigid rotation

• ν Eridani: 12 frequencies observed, 7 identified, 2 axisymmetric modes, one triplet
  + 2 low-frequency modes

• θ Ophiuchi

• 12 Lacertae
v Eridani observations

Large multisite photometric campaign 2002-2003: 11 telescopes, 148 clear nights

Large multisite spectroscopic campaign: 11 observatories, 2294 high-resolution spectra

Large multisite photometric campaign 2003-2004: 5 telescopes, 142 clear nights
$\nu$ Eridani pulsations

first photometric campaign $\Rightarrow$ 8 independent frequencies detected + 1 low-frequency mode

<table>
<thead>
<tr>
<th>ID</th>
<th>Freq. (cd$^{-1}$)</th>
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<tbody>
<tr>
<td>$f_1$</td>
<td>5.76327</td>
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<td>$f_2$</td>
<td>5.62006</td>
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<tr>
<td>$f_4$</td>
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<td>7.19994</td>
</tr>
<tr>
<td>$f_A$</td>
<td>7.89780</td>
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Handler et al. 2004
ν Eridani pulsations

spectroscopic campaign → 7 independent frequencies detected
+ no low-frequency mode

<table>
<thead>
<tr>
<th>Number</th>
<th>Identification</th>
<th>Freq. (cd$^{-1}$)</th>
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<tr>
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<td>$f_1 + f_3$</td>
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Aerts et al. 2004
ν Eridani pulsations

second photometric campaign ➔ 10 independent frequencies detected + 2 low-frequency modes

<table>
<thead>
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<th>ID</th>
<th>Frequency [d⁻¹]</th>
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<tbody>
<tr>
<td>( f_1 )</td>
<td>5.763256 ± 0.000012</td>
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<tr>
<td>( f_2 )</td>
<td>5.653897 ± 0.000020</td>
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<tr>
<td>( f_3 )</td>
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<td>( f_4 )</td>
<td>5.637215 ± 0.000025</td>
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<td>( f_B )</td>
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<tr>
<td>( f_{10} )</td>
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<tr>
<td>( f_9 )</td>
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<td>( f_8 )</td>
<td>7.2006 ± 0.0005</td>
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Jerzykiewicz et al. 2005
ν Eridani pulsations
combined first + second photometric campaign ➔ 12 independent frequencies detected + 2 low-frequency modes

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<th>ID</th>
<th>Frequency [d⁻¹]</th>
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<tr>
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<td>5.7632828 ± 0.0000019</td>
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<td>f₂</td>
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<td>f₃</td>
<td>5.6200186 ± 0.0000031</td>
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<td>f₄</td>
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<td>f₅</td>
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<td>f₆</td>
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<td>f₇</td>
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<td>f₈</td>
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<td>f₉</td>
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<td>f₁₀</td>
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<td>f₁₁</td>
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<td>f₁₂</td>
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<td>f₁₃</td>
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<tr>
<td>f₁₄</td>
<td>6.22360 ± 0.00012</td>
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HYBRID β CEPHEI/SPB star!
\( \nu \) Eridani
mode identification

- Mode identification:
  - \( l_1 = 0 \)
  - \( l_2 = 1 \)
  - \( l_3 = 1 \)
  - \( l_4 = 1 \)
  - \( l_5 = 1 \)
  - \( l_6 = 1 \)
  - \( l_8 = 1 \)

- Frequency and amplitude plot:
  - \( l=0 \) radial
  - \( l=1 \) triplet

- Wavelength vs. Amplitude ratio (\( A'/A \))
  - Peaks at:
    - \( 3, 4, 2 \)
    - \( 12, 6, 7 \)
    - \( 11 \)
    - \( 8 \)
    - \( 5, 9, 10 \)
ν Eridani modelling

2 axisymmetric modes identified + complete triplet + additional frequencies
⇒ asteroseismic modelling possible

For each set of model parameters \((M,X,Z,\alpha_{ov})\), fitting the radial mode gives the age of the star \((T_{\text{eff}})\).

For a given set of model parameters \((X, Z, \alpha_{ov})\), fitting the second axisymmetric frequency fixes the age \((T_{\text{eff}})\) AND the mass \(M\).
ν Eridani modelling

- $f_1$ is a p1 mode
- $f_4$ is a l=1 g1 mode

$f_6$ can be fitted by the l=1 p1 mode, without additional constraints on the parameters (for each $\alpha_{ov}$, M and Z are now fixed)
ν Eridani modelling

for all the models that fit $f_1$ and $f_4$, look at the values of the $f_6$ ($l=1$, $p1$) and $f_5$ ($l=1$, $p2$) frequencies

\[ \begin{align*}
\text{« high » overshooting:} & \quad \alpha_{ov} \sim 0.3 \\
\text{and} & \\
\text{« low » metallicity:} & \quad Z \sim 0.015
\end{align*} \]
Eridani modelling

Modelling seems to be OK
also evidence for non-rigid rotation

BUT

No excited frequencies in the range of observed frequencies!!!

- Non-standard models (higher Fe or lower X)
  or
  Ad-hoc enhancement of iron in the driving region

- higher frequency modes can be excited,
  but NOT the low frequency g modes
v Eridani
modelling

Modelling seems to be OK
also evidence for non-rigid rotation

BUT

No excited frequencies in the range of observed frequencies!!!

- Non-standard models (higher Fe or lower X)
  or
Ad-hoc enhancement of iron in the driving region

- higher frequency modes can be excited,
  but NOT the low frequency g modes
Abundances and opacities

Chemical composition (in particular Fe) and opacity tables ($\kappa$ mechanism)

→ very important to determine excitation
Abundances

Z=0.01 Kolaczkowski et al. 06; Morel et al. 06

AGS05 Z$_{\odot}$=0.012

Figures courtesy of Montalban&Miglio

Obser. data from P. De Cat web page

20% more Fe!

04/10/2011

KITP - A.Thoul
Figures courtesy of Montalban & Miglio

Obser. data from P. De Cat web page
Abundances and opacities

Figures courtesy of Montalban&Miglio

Obser. data from P. De Cat web page
Abundances and opacities

AGS05 and OP

bluer border of SPB and $\beta$ Cephei instability strips

larger number of hybrid SPB-$\beta$ Cephei pulsators

more $\beta$ Cephei modes excited at $Z=0.01$
ν Eridani modelling

Previous results: GN93 and OPAL
Now: AGS05 and OP

fit 4 frequencies

- $X=0.7211$
  $Z=0.021$
  $\alpha_{ov}=0.22$
  $M=9.1$

- $X=0.70$
  $Z=0.022$
  $\alpha_{ov}=0.22$
  $M=8.9$

04/10/2011
Eridani modelling

low-frequency high order g modes:

excited in range $0.55$-$0.91$ c/d

$\Rightarrow f_B$ is excited, but not $f_A$

<table>
<thead>
<tr>
<th>ID</th>
<th>Frequency [d$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>5.7632828 ± 0.0000019</td>
</tr>
<tr>
<td>$f_2$</td>
<td>5.6538767 ± 0.0000030</td>
</tr>
<tr>
<td>$f_3$</td>
<td>5.6200186 ± 0.0000031</td>
</tr>
<tr>
<td>$f_4$</td>
<td>5.6372470 ± 0.0000038</td>
</tr>
<tr>
<td>$f_5$</td>
<td>7.898200 ± 0.000032</td>
</tr>
<tr>
<td>$f_A$</td>
<td>0.432786 ± 0.000032</td>
</tr>
<tr>
<td>$f_7$</td>
<td>6.262917 ± 0.000044</td>
</tr>
<tr>
<td>$f_6$</td>
<td>6.243847 ± 0.000042</td>
</tr>
<tr>
<td>$f_B$</td>
<td>0.61440 ± 0.00005</td>
</tr>
<tr>
<td>$f_9$</td>
<td>7.91383 ± 0.00008</td>
</tr>
<tr>
<td>$f_{10}$</td>
<td>7.92992 ± 0.00010</td>
</tr>
<tr>
<td>$f_8$</td>
<td>7.20090 ± 0.00009</td>
</tr>
<tr>
<td>$f_{11}$</td>
<td>6.73223 ± 0.00012</td>
</tr>
<tr>
<td>$f_{12}$</td>
<td>6.22360 ± 0.00012</td>
</tr>
</tbody>
</table>
ν Eridani

- AGS05 abundances and OP opacities help solve the problems of νEri.

- BUT some problems remain:
  - Z higher than observed; independent of X!
  - highest frequency mode not excited
  - range of excited high-order g modes
Examples of well-studied stars

• 16 Lacertae: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet ➔ very precise values of $M$, $T_{\text{eff}}$, $L$, age

• HD129929: 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet ➔ precise values for $M$, $Z$, $T_{\text{eff}}$, $L$, age + evidence for core overshooting and non-rigid rotation

• ν Eridani: 12 frequencies observed, 7 identified, 2 axisymmetric modes, one triplet + 2 low-frequency modes = HYBRID β Cephei/SPB pulsator ➔ precise values for $M$, $Z$, $T_{\text{eff}}$, $\log g$, age, $\alpha_{\text{ov}}$, non-rigid rotation + need for updated abundances AGS05 and OP opacities + problems not completely solved

• θ Ophiuchi

• 12 Lacertae
Examples of well-studied stars

• 16 Lacertae: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet
  ➔ very precise values of M, T$_{\text{eff}}$, L, age

• HD129929: 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet
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  + need for updated abundances AGS05 and OP opacities
  + problems not completely solved

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• 12 Lacertae
Examples of well-studied stars

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  + problems not completely solved

• θ Ophiuchi: 7 frequencies observed, all identified, 2 axisymmetric modes, one triplet, 3 members of a quintuplet
  ➔ very precise values for M, T_{eff}, log g, age, α_{ov}, ...
  evidence for high overshooting
  rigid rotation possible
  non-axisymmetry in splitting explained by second-order effects of rotation
  + problem with spectroscopic error box...

• 12 Lacertae
Examples of well-studied stars

• 16 Lacertae: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet
  ➔ very precise values of M, $T_{\text{eff}}$, L, age

• HD129929: 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet
  ➔ precise values for M, Z, $T_{\text{eff}}$, L, age
  + evidence for core overshooting and non-rigid rotation

• η Eridani: 12 frequencies observed, 7 identified, 2 axisymmetric modes, one triplet + 2 low-frequency modes
  ➔ precise values for M, Z, $T_{\text{eff}}$, log g, age, $\alpha_{\text{ov}}$, non-rigid rotation
  + need for updated abundances AGS05 and OP opacities
  + problems not completely solved

• θ Ophiuchi: 7 frequencies observed, all identified, 2 axisymmetric modes, one triplet, 3 members of a quintuplet
  ➔ very precise values for M, $T_{\text{eff}}$, log g, age, $\alpha_{\text{ov}}$, ...
  evidence for high overshooting
  rigid rotation possible
  non-axisymmetry in splitting explained by second-order effects of rotation
  + problems with spectroscopic error box

• 12 Lacertae: 10 frequencies observed, 6 identified, problematic identification...
12 (DD) Lacertae
A long history...

Observed as a **variable star** since **1915**!

\[
P = \text{period, } \cdot 193089 \text{ days, } = 4^h 38^m 3^s
\]

\[
\frac{1}{P} = 5.17896 \text{ c/d}
\]

Present value: **5.179034 c/d**

Observed as a **multiperiodic** variable star since **1957**!

« *Thus the star 12 (DD) Lacertae has a principal period of 4h. 38m. and a secondary period of 4h. 44m., while recently a third component of 3h. 45m. has been found, and a fourth one of 3.9h. has been suggested. The periods occur in the variation of both the light and the radial velocity.* »

Two frequencies known with **good precision** since **1961**!

\[
P_1 = 0^d19308858
\]

\[
P_2 = 0^d1973685
\]

\[
f_1 = 5.17897 \text{ c/d}
\]

\[
f_2 = 5.06666 \text{ c/d}
\]

Present values: \(5.179034\text{ c/d}\)
\(5.066346 \text{ c/d}\)

04/10/2011 KITP - A.Thoul
12 (DD) Lacertae
a long history...

1978: 6 frequencies, including an l=3 triplet

As a result of a frequency analysis of the published observations of 12 Lacertae, six short-period sine-wave components are found in the star’s light variation. The component frequencies have the following values: 32.5426 ± 0.0007, 31.8341 ± 0.0022, 34.4951 ± 0.0024, 33.5189 ± 0.0013, 66.0615 ± 0.0015, 26.644 ± 0.009 rad/d, and the

From the position of the four strongest components in the frequency spectrum it is concluded that 12 Lacertae is a slowly rotating nonradial oscillator in which two spherical harmonic modes of different degree are simultaneously excited. If use is made of the line profile observations, the triplet frequencies can be identified as corresponding to \( l = 3 \), \( m = -1, -2, -3 \) oscillations.

\[
\begin{align*}
f_1 &= 5.17930 \text{ c/d} \\ f_2 &= 5.0665 \text{ c/d} \\ f_3 &= 5.4901 \text{ c/d} \\ f_4 &= 5.3347 \text{ c/d} \\ f_5 &= 10.5140 \text{ c/d} \\ f_6 &= 4.2405 \text{ c/d} \\
\end{align*}
\]

Present values:

\[
\begin{align*}
f_1 &= 5.179034 \text{ c/d} \\ f_2 &= 5.066346 \text{ c/d} \\ f_3 &= 5.490167 \text{ c/d} \\ f_4 &= 5.334357 \text{ c/d} \\ f_5 &= - \text{ c/d} \\ f_6 &= 4.24062 \text{ c/d} \\
\end{align*}
\]
12 (DD) Lacertae
a complicated history...

1980: radial mode + different identification: triplet is \( l=2 \), NOT \( l=3 \)!

Line profile observations of the Si \( \text{III} \lambda 4567 \) line in the \( \beta \) Cephei star 12 Lacertae have been successfully fitted over three observing runs with a four-mode solution consistent with periods determined earlier by Jerzykiewicz. It is found that the variations in line shape can be fitted only with a radial mode of amplitude \( 13 \text{ km s}^{-1} \) and with three nonradial modes of amplitude \( 40 \text{ km s}^{-1} \) whose states are described by \( l = 2, m = 0, -1, \) and \(-2\). The latter three modes are evidently equipartitioned in energy. The \( l = 2 \) identification is also compatible with the observed light and color amplitudes for these modes, but \( l > 3 \) fails to meet these tests. Two independent...

1994: try to discriminate between proposed identifications using the moment method BUT impossible to identify the modes!

1998: try to identify the modes through stellar modelling BUT impossible to identify the modes!

Abstract

Five pulsation modes have been detected in this well-known beta Cephei star. Three of them, including the strongest one, form an equidistant frequency triplet. We consider identifications of the observed pulsation frequencies with computed eigenfrequencies of low degree modes \( (l \leq 2) \) in a series of stellar models covering the range of the effective temperature and surface gravity consistent with best available data. We show that the existing determinations of the degree of even the strongest observed mode are discrepant and therefore do little to constrain the problem. Finally, we discuss the difficulties posed by the observed equidistant frequency triplet.
A very complicated history...

1999: discrepant determinations of the degree of the VERY equidistant triplet

Abstract. Five pulsation modes are simultaneously excited in this well-known β Cephei star. Three of them, including the one with the largest light and radial-velocity amplitudes, form a triplet. The triplet is equidistant in frequency to within the errors of measurement, that is, 0.0003 d⁻¹.

Explaining why the triplet should be so nearly equidistant turns out to be a real challenge to the theory. We investigate the following three options: (1) rotational splitting, (2) an oblique magnetic pulsator, and (3) nonlinear phase lock. Unfortunately, apart from the frequencies, the data are meager. Photometric indices yield the effective temperature and surface gravity of rather low accuracy. In addition, the existing determinations of the spherical harmonic degree of even the strongest observed mode are discrepant. Consequently, the model parameters are not well constrained.

We show that of the three above-mentioned options, the oblique pulsator model is unlikely because it would require excessively strong dipolar field or a special field geometry. The rotational splitting is a possibility, but only for an \( \ell = 2 \), \( p_0 \) mode in a model with specific values of the effective temperature and surface gravity. Finally, we note that the nonlinear phase lock
12 (DD) Lacertae
the story continues.... and....
SURPRISE!!!!!!

2006: Large multisite campaign ➔ 10 independant frequencies.
The equidistant triplet not a triplet at all: different values of l !!!

ABSTRACT
We report a multisite photometric campaign for the β Cephei star 12 Lacertae. 750 hours of high-quality differential photoelectric Strömgren, Johnson and Geneva time-series photometry were obtained with 9 telescopes during 190 nights. Our frequency analysis results in the detection of 23 sinusoidal signals in the light curves. Eleven of those correspond to independent pulsation modes, and the remainder are combination frequencies. We find some slow aperiodic variability such as that seemingly present in several β Cephei stars. We perform mode identification from our colour photometry, derive the spherical degree ℓ for the five strongest modes unambiguously and provide constraints on ℓ for the weaker modes. We find a mixture of modes of 0 ≤ ℓ ≤ 4. In particular, we prove that the previously suspected rotationally split triplet within the modes of 12 Lac consists of modes of different ℓ; their equal frequency splitting must thus be accidental.
**12 (DD) Lacertae**

the latest observations and identifications

Table 2. Multifrequency solution for our time-resolved photometry of 12 Lac. Formal error estimates (following Montgomery & O’Donoghue 1999) for the independent frequencies range from \(\pm 0.000007 \text{ cd}^{-1}\) for \(f_1\) to \(\pm 0.00023 \text{ cd}^{-1}\) for \(f_{10}\). Formal errors on the amplitudes are \(\pm 0.2 \text{ mmag}\) in \(u\) and \(\pm 0.1 \text{ mmag}\) in \(v\) and \(V\). The S/N ratio, computed following Breger et al. (1993), is for the \(V\) filter data.

<table>
<thead>
<tr>
<th>ID</th>
<th>Freq. (cd(^{-1}))</th>
<th>(u) Ampl. (mmag)</th>
<th>(v) Ampl. (mmag)</th>
<th>(V) Ampl. (mmag)</th>
<th>S/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>5.179034</td>
<td>56.4</td>
<td>40.7</td>
<td>38.1</td>
<td>178.6</td>
</tr>
<tr>
<td>(f_2)</td>
<td>5.066346</td>
<td>23.3</td>
<td>16.7</td>
<td>16.0</td>
<td>74.6</td>
</tr>
<tr>
<td>(f_3)</td>
<td>5.490167</td>
<td>14.2</td>
<td>11.7</td>
<td>11.1</td>
<td>52.4</td>
</tr>
<tr>
<td>(f_4)</td>
<td>5.334357</td>
<td>21.9</td>
<td>11.6</td>
<td>10.0</td>
<td>47.3</td>
</tr>
<tr>
<td>(f_5)</td>
<td>4.24062</td>
<td>4.4</td>
<td>3.7</td>
<td>3.6</td>
<td>15.8</td>
</tr>
<tr>
<td>(f_A)</td>
<td>0.35529</td>
<td>7.2</td>
<td>4.8</td>
<td>5.0</td>
<td>14.4</td>
</tr>
<tr>
<td>(f_B)</td>
<td>7.40705</td>
<td>2.8</td>
<td>2.1</td>
<td>2.0</td>
<td>9.7</td>
</tr>
<tr>
<td>(f_C)</td>
<td>3.50912</td>
<td>2.7</td>
<td>2.3</td>
<td>2.0</td>
<td>9.5</td>
</tr>
<tr>
<td>(f_D)</td>
<td>5.2162</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>6.2</td>
</tr>
<tr>
<td>(f_E)</td>
<td>6.7023</td>
<td>2.2</td>
<td>1.6</td>
<td>1.3</td>
<td>6.3</td>
</tr>
<tr>
<td>(f_{10})</td>
<td>5.8341</td>
<td>1.8</td>
<td>1.2</td>
<td>1.3</td>
<td>6.1</td>
</tr>
<tr>
<td>(f_1 + f_4)</td>
<td>10.513392</td>
<td>8.3</td>
<td>5.9</td>
<td>5.5</td>
<td>32.9</td>
</tr>
<tr>
<td>(f_3 + f_A)</td>
<td>5.84546</td>
<td>2.3</td>
<td>1.6</td>
<td>1.8</td>
<td>8.7</td>
</tr>
<tr>
<td>(f_2 + f_4)</td>
<td>10.400704</td>
<td>2.3</td>
<td>1.7</td>
<td>1.7</td>
<td>10.3</td>
</tr>
<tr>
<td>(2f_1)</td>
<td>10.358069</td>
<td>1.9</td>
<td>1.2</td>
<td>1.2</td>
<td>6.9</td>
</tr>
<tr>
<td>(f_1 + f_2)</td>
<td>10.245381</td>
<td>1.7</td>
<td>1.3</td>
<td>1.2</td>
<td>6.9</td>
</tr>
<tr>
<td>(2f_8)</td>
<td>10.4324</td>
<td>1.5</td>
<td>1.4</td>
<td>1.0</td>
<td>6.1</td>
</tr>
<tr>
<td>(2f_4)</td>
<td>10.668715</td>
<td>1.0</td>
<td>0.8</td>
<td>0.7</td>
<td>4.3</td>
</tr>
<tr>
<td>(f_3 + f_4)</td>
<td>10.824524</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>3.9</td>
</tr>
<tr>
<td>(f_2 + f_3)</td>
<td>10.556514</td>
<td>1.1</td>
<td>0.8</td>
<td>0.7</td>
<td>4.0</td>
</tr>
<tr>
<td>(2f_1 + f_2)</td>
<td>15.424415</td>
<td>0.6</td>
<td>0.6</td>
<td>0.5</td>
<td>4.3</td>
</tr>
<tr>
<td>(f_1 + f_2 + f_4)</td>
<td>15.579738</td>
<td>1.0</td>
<td>0.6</td>
<td>0.5</td>
<td>4.5</td>
</tr>
<tr>
<td>(2f_1 + f_4)</td>
<td>15.692426</td>
<td>1.0</td>
<td>0.6</td>
<td>0.5</td>
<td>4.3</td>
</tr>
</tbody>
</table>

**identification:**

Table 3. Mode identifications for 12 Lac from our analysis of the photometric amplitude ratios.

<table>
<thead>
<tr>
<th>ID</th>
<th>Freq. (cd(^{-1}))</th>
<th>(\ell)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>5.179034</td>
<td>1</td>
</tr>
<tr>
<td>(f_2)</td>
<td>5.066346</td>
<td>1</td>
</tr>
<tr>
<td>(f_3)</td>
<td>5.490167</td>
<td>2</td>
</tr>
<tr>
<td>(f_4)</td>
<td>5.334357</td>
<td>0</td>
</tr>
<tr>
<td>(f_5)</td>
<td>4.24062</td>
<td>2</td>
</tr>
<tr>
<td>(f_A)</td>
<td>0.35529</td>
<td>1, 2 or 4</td>
</tr>
<tr>
<td>(f_6)</td>
<td>7.40705</td>
<td>1 or 2</td>
</tr>
<tr>
<td>(f_7)</td>
<td>5.30912</td>
<td>2 or 1 or 3</td>
</tr>
<tr>
<td>(f_8)</td>
<td>5.2162</td>
<td>4 or 2</td>
</tr>
<tr>
<td>(f_9)</td>
<td>6.7023</td>
<td>1</td>
</tr>
<tr>
<td>(f_{10})</td>
<td>5.8341</td>
<td>1 or 2</td>
</tr>
<tr>
<td>(f_3 + f_A)</td>
<td>5.84546</td>
<td>2 or 1</td>
</tr>
<tr>
<td>(2f_8)</td>
<td>10.4324</td>
<td>1 or 2 or 3</td>
</tr>
</tbody>
</table>

**Not a triplet: coincidence!!!**

\(f_1 - f_2 = 0.112688 \text{ c/d}\)

\(f_4 - f_3 = 0.155323 \text{ c/d}\)

\(f_3 - f_4 = 0.15581 \text{ c/d}\)
As mentioned in the Introduction, two hypotheses to explain the pulsation spectrum of 12 Lac seemed promising before our multisite campaign took place: first, the presence of a rotationally split triplet consisting of the modes \((f_1, f_3, f_4)\) and second, the presence of the fundamental and first radial overtones (modes \(f_5, f_3\)). Our mode identification allows us to judge these hypotheses: neither is correct.

The suspected rotationally split structure consists of modes of \(\ell = 1, 0, 2\), and the suspected radial modes both turned out to be \(\ell = 2\). Consequently, all previous attempts to understand the pulsation spectrum of 12 Lac were not correct.
### 12 (DD) Lacertae

**present status of the observations**

Summary of the evolution of the mode identification for the frequencies of 12Lac:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1 = 5.179$ c/d</td>
<td>$(2,-2)$</td>
<td>$(3,-1)$</td>
<td>$(2,0)$</td>
<td>$(2,\equiv 1)$</td>
<td>$(2,-1)$</td>
<td>$(1, ?)$</td>
<td>$(1, 1)$</td>
<td>$(1,1)$</td>
</tr>
<tr>
<td>$f_2 = 5.066$ c/d</td>
<td>$(2,0)$</td>
<td>$(1,-1)$</td>
<td>$(0,0)$</td>
<td>$(?,0)$</td>
<td>$(2$ or $3, ?)$</td>
<td>$(1$ or $2,?)$</td>
<td>$(1, 0)$</td>
<td>$(1,0)$</td>
</tr>
<tr>
<td>$f_3 = 5.490$ c/d</td>
<td>$(3,-3)$</td>
<td>$(2,-2)$</td>
<td>$(2,-2)$</td>
<td>$(1, ?)$</td>
<td>$(2, ?)$</td>
<td>$(2, ?)$</td>
<td>$(2,1)$</td>
<td></td>
</tr>
<tr>
<td>$f_4 = 5.334$ c/d</td>
<td>$(3,-2)$</td>
<td>$(2,-1)$</td>
<td>$(2,0)$</td>
<td>$(3,1)$</td>
<td>$(0, 0)$</td>
<td>$(0,0)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_5 = 4.241$ c/d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(2, ?)$</td>
<td>$(2,?)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_6 = 7.407$ c/d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(1$ or $2, ?)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12 (DD) Lacertae modelling

assumptions:
X = 0.70, Z = 0.015, composition A04, opacity OP,
log $T_{\text{eff}}$ = [4.355, 4.395], log $L$ = [4.0, 4.35],
Handler et al. 2006 frequencies, Desmet et al. 2008 identifications
no overshooting

Fit $f_4$ as $l=0$ $p_1$ mode (radial fundamental)
Fit $f_2$ as $l=1$ $g_1$ mode ($n$=-1)
Constraints on $T_{\text{eff}}$ and $L$ ➔ no other radial orders possible for $f_2$ and $f_4$

$\Rightarrow M=11.77$ M$_0$  $\quad$ log $T_{\text{eff}} = 4.39$  $\quad$ log $L = 4.19$
$\Rightarrow f_3$ is ($l,m,n$) = (2,1,-1)
$\Rightarrow f_5$ is 0.3 c/d from nearest quadrupole ($l,m,n$) = (2,2,-2)
$\Rightarrow$ with some effects of rotation, $f_5$ is (2,2,-2) or (2,1,?)
$\Rightarrow$ all modes unstable except low frequency mode
12 (DD) Lacertae
BUT the story continues!!!!!!!

assumptions:
X = 0.72, Z = 0.015, composition AGS05, opacity OP,
logT_{eff}=4.389±0.018, log g = 3.65±0.15,
Handler et al. 2006 frequencies, Desmet et al. 2008 identifications

We rule out $f_4$ as the radial fundamental, because $f_9$ is identified as $l=1$
$\Rightarrow f_4$ is the first overtone $(l,n)=(0,2)$
$\Rightarrow$ NO PROBLEM WITH THE ERROR BOX

$\Rightarrow f_2$ is $l=1$, $p1$ mode
$\Rightarrow f_3$ could be a $l=2$, $n=0$ mode with $m>0$
$\Rightarrow f_5$ could be a $l=2$ $g2$ mode
$\Rightarrow f_6$ cannot be a $l=1$ mode, and has to be $l=2$
$\Rightarrow f_7$ cannot be a $l=1$ mode, and could be part of an
$l=2$ quintuplet with $f_3$ or an $l=3$
$\Rightarrow f_8$ cannot be a $l=4$
$\Rightarrow f_9$ is $l=1$ $p3$ mode
$\Rightarrow f_{10}$ cannot be a $l=2$, and could be $l=2$

<table>
<thead>
<tr>
<th>ID</th>
<th>Freq. (cd^{-1})</th>
<th>$\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>5.17934</td>
<td>1</td>
</tr>
<tr>
<td>$f_2$</td>
<td>5.06634</td>
<td>1</td>
</tr>
<tr>
<td>$f_3$</td>
<td>5.490167</td>
<td>2</td>
</tr>
<tr>
<td>$f_4$</td>
<td>5.334357</td>
<td>0</td>
</tr>
<tr>
<td>$f_5$</td>
<td>4.24062</td>
<td>2</td>
</tr>
<tr>
<td>$f_A$</td>
<td>0.35529</td>
<td>1, 2 or 4</td>
</tr>
<tr>
<td>$f_6$</td>
<td>7.40705</td>
<td>1 or 2</td>
</tr>
<tr>
<td>$f_7$</td>
<td>5.30012</td>
<td>2 or 1 or 3</td>
</tr>
<tr>
<td>$f_8$</td>
<td>5.21624</td>
<td>4 or 2</td>
</tr>
<tr>
<td>$f_9$</td>
<td>6.7023</td>
<td>1 or 2</td>
</tr>
<tr>
<td>$f_{10}$</td>
<td>5.8341</td>
<td>1 or 2</td>
</tr>
<tr>
<td>$f_9 + f_A$</td>
<td>5.84546</td>
<td>2 or 1</td>
</tr>
<tr>
<td>$2f_8$</td>
<td>10.4324</td>
<td>1 or 2 or 3</td>
</tr>
</tbody>
</table>
12 (DD) Lacertae
the story continues...

\[ \alpha_{ov} = 0 \]
\[ M = 14.4 \]
\[ Z = 0.015 \]

\[ \alpha_{ov} = 0.2 \]
\[ M = 12.0 \]
12 (DD) Lacertae
the story continues...

range of unstable modes

FOR $Z=0.015$
BUT $Z_{\text{obs}}=0.010$ !!
Examples of well-studied stars

• 16 Lacertae: 3 frequencies observed and identified, 2 axisymmetric modes; no multiplet
  ➔ very precise values of $M$, $T_{\text{eff}}$, $L$, age

• HD129929: 6 freq. observed and identified, 2 axisymmetric modes, 1 triplet, 2 members of a quintuplet
  ➔ precise values for $M$, $Z$, $T_{\text{eff}}$, $L$, age
  + evidence for core overshooting and non-rigid rotation

• $\varpi$ Eridani: 12 frequencies observed, 7 identified, 2 axisymmetric modes, one triplet + 2 low-frequency modes
  ➔ precise values for $M$, $Z$, $T_{\text{eff}}$, log $g$, age, $\alpha_{\text{ov}}$, non-rigid rotation
  + need for updated abundances AGS05 and OP opacities
  + problems not completely solved

• $\theta$ Ophiuchi: 7 frequencies observed, all identified, 2 axisymmetric modes, one triplet, 3 members of a quintuplet
  ➔ very precise values for $M$, $T_{\text{eff}}$, log $g$, age, $\alpha_{\text{ov}}$, ...
  evidence for high overshooting
  rigid rotation possible
  non-axisymmetry in splitting explained by second-order effects of rotation

• 12 Lacertae: 10 frequencies observed, 6 identified, problematic identification...
  ➔ very precise values for $M$, $T_{\text{eff}}$, log $g$, age, $\alpha_{\text{ov}}$, ...
  ➔ reliable mode identification is crucial
  ➔ problem of excitation not entirely solved
Corot B stars

Fig. 1  Observational $\log T_{\text{eff}}$-$\log g$ diagram with the two predicted instability strips (SPB, dashed line, $\beta$ Cep, solid line) and a selection of CoRoT targets, using OPAL opacities and a metallicity of $Z = 0.02$. For reference, also the $\gamma$ Dor/$\delta$ Sct instability strips are shown (grey dashed/solid lines).
Examples of well-studied stars

• HD 180642: (Corot main target + ground): 11 independent frequencies + 3 harmonics + 19 combination freq. (locked phases) 1 identified (high amplitude radial mode, no clear multiplet

• HD 50230

• HD 51756

• HD 170580
HD180642

11 independent frequencies from Corot

dominant radial (l=0) mode

ground photometry

ground spectroscopy

04/10/2011

KITP - A.Thoul
HD180642

Modelling:

Fit the radial mode, check excitation, then fit remaining modes

“Free parameters”: $X$, $Z$, $\alpha_{ov}$, $M$

Range of parameters explored:

$X = 0.68 - 0.74$

$Z = 0.010 - 0.018$

$M = 7.6 - 20$

$\alpha_{ov} = 0 - 0.5$
HD180642

X=0.72, Z=0.015, $\alpha_{ov}$=0.4, M=12

radial mode is n=1, MS
radial mode is n=2, MS
radial mode is n=3, MS
Examples of well-studied stars

- **HD 180642**: (Corot main target + ground): 11 independent frequencies + 3 harmonics + 19 combination freq. (locked phases)
  - 1 identified (high amplitude radial mode, no clear multiplet)
  - Precise values for M, \( T_{\text{eff}} \), log g, age, \( \alpha_{\text{ov}} \), ...
  - Very low overshooting
  - Discrepancy between spectroscopic and seismic log g: pulsational broadening not taken into account correctly when deducing log g from wings of spectral lines

- HD 50230
- HD 51756
- HD 170580
Examples of well-studied stars

• **HD 180642**: (Corot main target + ground): 11 independent frequencies + 3 harmonics + 19 combination freq. (locked phases)
  1 identified (high amplitude radial mode, no clear multiplet
  ➔ precise values for $M$, $T_{\text{eff}}$, $\log g$, age, $\alpha_{ov}$, ...
  ➔ Very low overshooting
  ➔ Discrepancy between spectroscopic and seismic $\log g$: pulsational broadening not taken into account correctly when deducing $\log g$ from wings of spectral lines

• **HD 50230**: hundreds of gravity modes and tens of $p$ modes
  non-uniform period spacings ➔ $\alpha_{ov} > 0.2$ and smooth gradient of chemical composition at core boundary

• **HD 51756**

• **HD 170580**
Examples of well-studied stars

• HD 180642: (Corot main target + ground): 11 independent frequencies + 3 harmonics + 19 combination freq. (locked phases)
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• HD 50230: hundreds of gravity modes and tens of p modes
  non-uniform period spacings
  $\alpha_{\text{ov}}>0.2$ and smooth gradient of chemical composition at core boundary

• HD 51756: STABLE

• HD 170580

04/10/2011  KITP - A.Thoul
Examples of well-studied stars

- **HD 180642**: (Corot main target + ground): 11 independent frequencies + 3 harmonics + 19 combination freq. (locked phases)
  - 1 identified (high amplitude radial mode, no clear multiplet)
  - Precise values for M, T\text{eff}, \log g, age, \alpha_{ov}, ...
  - Very low overshooting
  - Discrepancy between spectroscopic and seismic \log g: pulsational broadening not taken into account correctly when deducing \log g from wings of spectral lines

- **HD 50230**: hundreds of gravity modes and tens of p modes
  - Non-uniform period spacings
  - \alpha_{ov} > 0.2 and smooth gradient of chemical composition at core boundary

- **HD 51756**: STABLE

- **HD 170580**: many high order g modes, one p mode
  - ... in progress...
Kepler B stars

![Graph showing mass vs. log T_{eff} for Kepler B stars with labels a to o.](image)
What we have learned so far from the asteroseismology of \( \beta \) Cephei stars

very precise values of the basic stellar parameters
\((M, T_{\text{eff}}, \log g, L, \text{age})\)

evidence for varying core overshooting (0 to 0.4)

evidence for non-rigid rotation in some \( \beta \) Cephei stars

need for updated abundances AGS05 and OP opacities
\((\nu \text{ Eri, } \theta \text{ Oph, } 12\text{Lac})\)

non-axisymmetry in splitting can be explained by second-order effects of rotation (\( \theta \) Oph)

reliable mode identification is crucial (story of 12Lac)
Challenges (observational)

- **good frequency determination**: need to resolve the individual frequencies \(\Rightarrow\) long data sets with very good coverage

Note: for classical pulsators, no inherent problem, since modes are phase coherent over periods of years or more  
(in contrast to solar-like pulsators)

BUT a real challenge for SPB stars due to long periods of high-order g-modes

- **reliable mode identification: necessary!** (see 12Lac)

More difficult for classical pulsators than for solar-like oscillations  
(no equal spacings)

Multiplets (rotational splittings) can overlap with the spectrum of axisymmetric frequencies (see 12Lac)

- **need good determination of basic stellar parameters**  
  \((\text{Teff, log g, } [\text{M/H}])\) to rule out some models
Challenges (observational)

- lots of photometric data will come from space observations

BUT

need ground-based follow-up with
multicolour photometry
and
high-resolution spectroscopy

BECAUSE

detailed modelling is impossible without reliable frequencies
AND mode identification
as well as a precise position in the HR diagram
Challenges (modelling)

• explain **the range of modes excitation**
• and the presence of β Cephei and SPB stars in **low Z** environments
  
  • explain mode selection
Challenges (modelling)

→ Improve physics in models: rotation, mixing, convection, magnetic field...

• Good opacities are crucial!

• Correct initial chemical composition + its evolution (diffusion, radiative accelerations)

• Stellar modelling of fast rotators
Before Corot

• 16 Lacertae 2003
• HD129929 2003
• ν Eridani 2004
• θ Ophiuchi 2007
• 12 Lacertae 2009

Corot stars

• HD 180642 2011
• HD 51756 2011