Equivalent Static Wind Loads for structures with non-proportional damping

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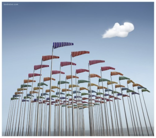
Analysis of structures under random excitations

Structures

are subjected to random excitations

Wind



Earthquake

...

and we have to solve the equation of motion

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{p}(t)$$

Mass; Damping; Stiffness; Nodal displacements; External forces

Analysis of structures in a modal basis

Modal basis

$$\mathbf{x} = \mathbf{\Phi}\mathbf{q}$$

- $\mathbf{\Phi}$: Normal modes of vibration
- $\mathbf{q}$: Modal amplitude

Equation of motion

$$\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{\Omega}\mathbf{q} = \mathbf{g}$$

- $\mathbf{D}$: Modal damping matrix $(\mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi})$
- $\mathbf{\Omega}$: Modal stiffness matrix (diagonal)
- $\mathbf{g}$: Generalized forces

Modal damping matrix

- Assumption of proportionality

$$\mathbf{C} = \alpha\mathbf{K} + \beta\mathbf{M} \longrightarrow \mathbf{D} = \mathbf{D}_d \quad \text{(diagonal)}$$

- Sources of non-proportionality : damping devices (Tuned Mass Damper, Tuned Liquid Column Damper), aerodynamic damping...

Analysis of structures in a modal basis

Split damping matrix

$$\mathbf{D} = \mathbf{D}_d + \mathbf{D}_o$$

Diagonal elements ($\mathbf{D}_d$)

Off-diagonal elements ($\mathbf{D}_o$)

Decoupling approximation

$$\mathbf{H}_d = \frac{1}{-\mathbf{I}\omega^2 + i\omega\mathbf{D}_d + \mathbf{\Omega}}$$

→ Inversion of a diagonal matrix only

Full matrix inversion

$$\mathbf{H} = \left(-\mathbf{I}\omega^2 + j\omega\mathbf{D} + \mathbf{\Omega}\right)^{-1}$$

→ Full matrix inversion

$$\mathbf{H} = \left(\mathbf{H}_d^{-1} + j\omega\mathbf{D}_o\right)^{-1}$$

$$\mathbf{H} = \left(\mathbf{I} + j\omega\mathbf{H}_d^{-1}\mathbf{D}_o\right)^{-1}\mathbf{H}_d$$

Asymptotic expansion method

- Modal transfer matrix[1]

$$\mathbf{H} = \left(\mathbf{I} + j\omega \mathbf{H}_d^{-1} \mathbf{D}_o\right)^{-1} \mathbf{H}_d$$

$$\frac{\mathbf{I}}{\mathbf{I}+\mathbf{X}} \simeq \mathbf{I} - \mathbf{X} + \mathbf{X}^2 - \ldots = \mathbf{I} + \sum_{i=1}^{k} (-\mathbf{X})^i$$

[1] Denoël and Degée. (2009). Asymptotic expansion of slightly coupled modal dynamic transfer functions non-proportional damping. *Journal of Sound and Vibration* 328, 1-2, 1-8

[2] Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291

Asymptotic expansion method

- Modal transfer matrix[1]

Decoupling approximation (diagonal)

$$\mathbf{H}_k = \mathbf{H}_d + \underbrace{\sum_{i=1}^{k} (-j\omega)^i (\mathbf{H}_d \mathbf{D}_o)^i \mathbf{H}_d}_{\text{Corrections terms (non-diagonal)}}$$

No full matrix inversion

Inversion of a diagonal matrix only

↓

Reduction of computation time

[1] Denoël and Degée. (2009). Asymptotic expansion of slightly coupled modal dynamic transfer functions non-proportional damping. *Journal of Sound and Vibration* 328, 1-2, 1-8

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Asymptotic expansion method

Modal transfer matrix[1]

$$\mathbf{H}_k = \underbrace{\mathbf{H}_d}_{\text{Decoupling approximation (diagonal)}} + \underbrace{\sum_{i=1}^{k} (-j\omega)^i (\mathbf{H}_d \mathbf{D}_o)^i \mathbf{H}_d}_{\text{Corrections terms (non-diagonal)}}$$

No full matrix inversion

Inversion of a diagonal matrix only

↓

Reduction of computation time

- Index of diagonality : $\max_{\omega} \rho(j\omega \mathbf{H}_d \mathbf{D}_o) = o(1)$

Stochastic modal analysis[2]

$$\mathbf{S}^{(q_k)} = \mathbf{H}_k \mathbf{S}^{(g)} \mathbf{H}_k^*$$

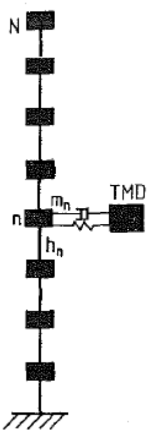
↓

$$\mathbf{S}^{(q_k)} = \underbrace{\mathbf{S}^{(q_d)}}_{\text{Solution in the uncoupled system}} + \underbrace{\sum_{i=1}^{k} \Delta \mathbf{S}^{(q_i)}}_{\text{Corrections terms due to non-proportionality damping}}$$

[1] Denoël and Degée. (2009). Asymptotic expansion of slightly coupled modal dynamic transfer functions non-proportional damping. *Journal of Sound and Vibration* 328, 1-2, 1-8

[2] Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291

370 m TV transmission tower



- 9-lumped-mass cantiliver beam model
- Random excitation : **wind**
- Tuned Mass Damper connected at the 4^{th}mass
  - Mass : 494 tons
  - Stiffness : 1,061 kN/m
  - Damping ratio : 17%
  - Tuned to the fundamental frequency : 0.2Hz
  - Index of diagonality $\rho(\mathbf{D}) = 1.87$
  - Condition : $\max_{\omega} \rho(i\omega \mathbf{H}_d \mathbf{D}_o) = 0.61 < 1$
- Structural data from[1]

[1] Xu, Samali, and Kwok. (2009). Control of along-wind response of structures by mass and liquid dampers. *Journal of Engineering Mechanics* 118, 1, 20-39

Model of wind

- Mean wind profile → power law
- A one-dimensional Gaussian velocity turbulence field
  - Spectrum of longitudinal turbulence
  - Linearized expression of the applied forces
  - Spanwise coherence function → decreasing exponential
- Aerodynamic data from [1]

[1] Xu, Samali, and Kwok. (2009). Control of along-wind response of structures by mass and liquid dampers. *Journal of Engineering Mechanics* 118, 1, 20-39

Modal properties

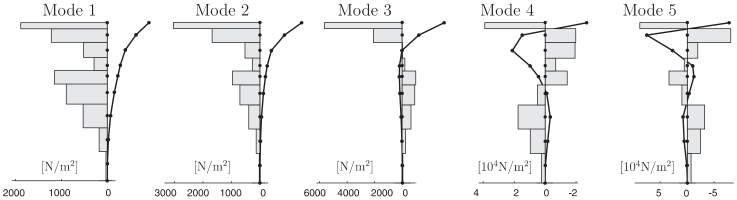
Inertial forces per unit surface

$$\Psi_m = \mathbf{K}\Phi_m$$

Ψ_m: mth inertial force

Φ_m: mth modal shape

First five modes



f_1=0.204Hz	f_2=0.258Hz	f_3=0.352Hz	f_4=0.925Hz	f_5=1.39Hz
ξ_1=6.7%	ξ_2=10.6%	ξ_3=1.9%	ξ_4=0.3%	ξ_5=0.2%

Structural design

Structural responses

$$\mathbf{r} = \mathbf{O}\mathbf{x} \rightarrow \mathbf{r} = \mathbf{\Upsilon}\mathbf{q}$$

Matrix of influence coefficients ($\mathbf{O}$)

Matrix of modal structural responses ($\mathbf{\Upsilon}$)

Envelope values (min and max) of the structural responses

- Maximum value of the jth structural response

$$r_j^{k,max} = g_j^{r_k} \sigma_j^{r_k}$$

peak factor ($g_j^{r_k}$)

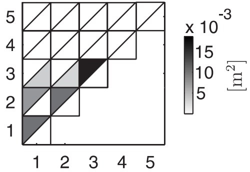
standard deviation ($\sigma_j^{r_k}$)

- Covariance matrix of the structural responses

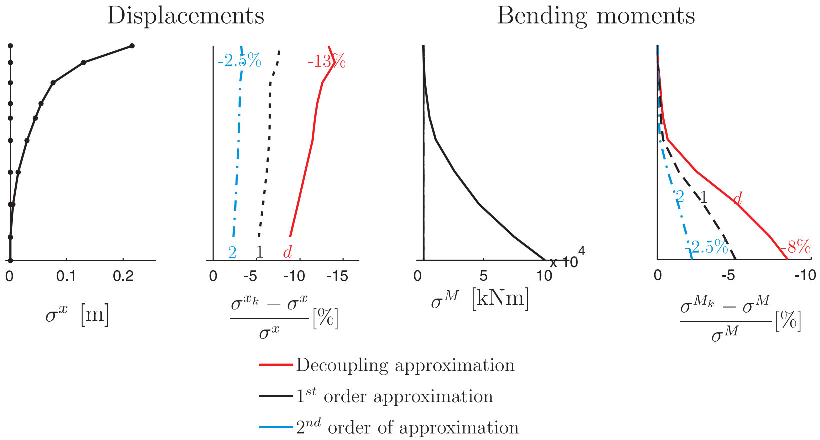
$$\mathbf{C}^{(r_k)} = \mathbf{\Upsilon}\mathbf{C}^{(q_k)}\mathbf{\Upsilon}^T$$

$$\mathbf{C}^{(q_k)} = \int_{-\infty}^{+\infty} \mathbf{S}^{(q_k)}\, d\omega = \mathbf{C}^{(q_d)} + \sum_{i=1}^{k} \mathbf{\Delta C}^{(q_i)}$$

Covariance matrix

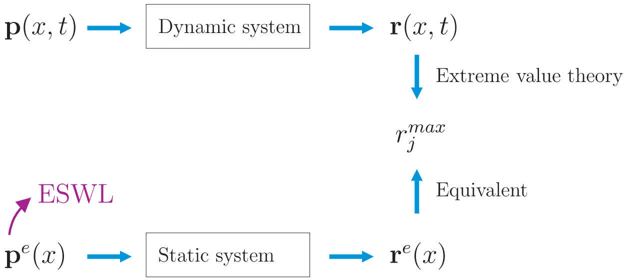
		Max. relative errors		
	Exact solution (k=∞)	Decoupled (k=0)	kth approximation of **H** (k=1)	(k=2)
$C^{0},(q)$		30%	21%	6%

Standard deviations



Definition

- Equivalent static wind loads (ESWL)



- Chen & Kareem [1] propose to construct ESWLs as weighted combinations of inertial forces $\boldsymbol{\psi}_m$

$$\mathbf{p}_j^e = g_j \sum_{m=1}^{M} W_{jm} \boldsymbol{\psi}_m$$

[1] Chen, and Kareem. (2009). Equivalent static wind loads for buffeting response of bridges by mass and liquid dampers. *Journal of Structural Engineering-Asce* 127, 12, 1467-1475

Asymptotic expansion method

- Weighted combinations of the inertial forces

$$\mathbf{p}_j^{e,k} = g_j^k \sum_m^M W_{jm}^k \boldsymbol{\psi}_m$$

kth approximation of the weighting coefficients

$$W_{jm}^k = \alpha_j^k W_{jm}^d + \sum_{i=1}^k \Delta W_{jm}^i$$

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kth approximation of the weighting coefficients

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- Definition of the ESWL

kth approximation of the ESWL

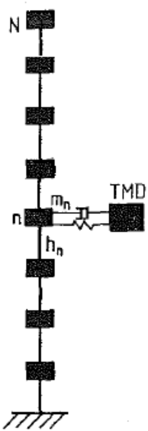
$$\mathbf{p}_j^{e,k} = \alpha_j^k \mathbf{p}_j^{e,d} + \Delta \mathbf{p}_j^{e,k}$$

correction resulting from the non-proportionality of damping

ESWL for the uncoupled system

scaled coefficients

Five structural responses



M_0 : the bending moment at the base

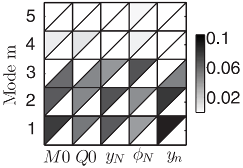
Q_0 : the shear force at the base

y_n : horizontal displacement at the fourth level

y_N : the horizontal displacement at the top

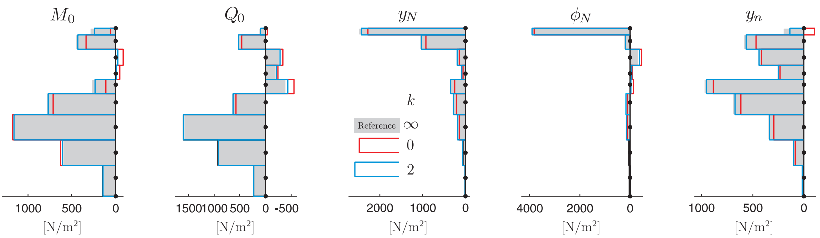
ϕ_N : the rotation at the top

Weighting coefficients

		Max. relative errors		
	Exact solution (k=∞)	Decoupled (k=0)	k^{th} approximation of $\mathbf{H}$ (k=1)	(k=2)
W		37% ($W_2 - Q_0$)	19% ($W_1 - y_n$)	7% ($W_2 - M_0$)

ESWL

Structure with TMD



ESWL

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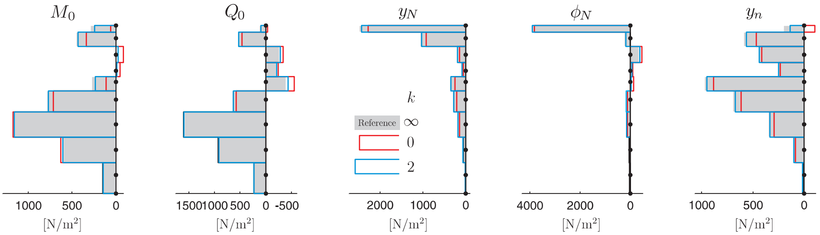
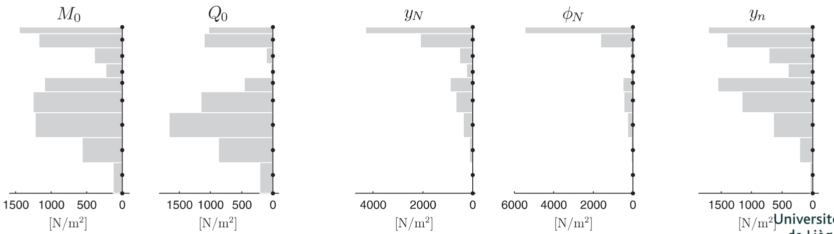


Structure **without** TMD



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- Perspectives
  - Order of approximation function of the frequency
  - Background-resonant decomposition for the correction terms
  - Dynamic system with non-linear terms

Thank you for your attention.

Questions ?

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Co-workers

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