Uncoupled spectral analysis with non-proportional damping

N. Blaise, T. Canor & V. Denoël

University of Liège (Belgium)

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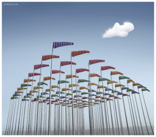
Analysis of structures under random excitations

Structures

are subjected to random excitations

Wind



Earthquake

...

and we have to solve the equation of motion

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{p}(t)$$

Mass; Damping; Stiffness; Nodal displacements; External forces

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Dynamic analysis of large structures

Modal basis

Normal modes of vibration — Modal damping matrix

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi} = \mathbf{I} \qquad \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi} = \mathbf{D} \qquad \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi} = \mathbf{\Omega}$$

Modal stiffness matrix (diagonal)

Rayleigh Damping

$$\mathbf{C} = \alpha\mathbf{K} + \beta\mathbf{M} \longrightarrow \mathbf{D} = \mathbf{D}_d$$

(diagonal)

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- Sources of non-proportionality
  - damping devices (TMD, TLCD), aerodynamic damping and...

D is not diagonal

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D is not diagonal

- Coupled system of equation of motion

Modal amplitudes

$$\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \boldsymbol{\Omega}\mathbf{q} = \mathbf{g}$$

Generalized forces

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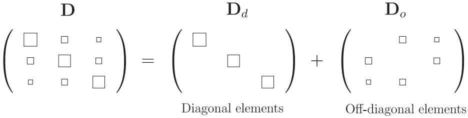
Dynamic analysis of large structures

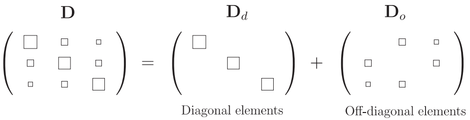
- Split damping matrix



[1] Rayleigh. (1877). The Theory of Sound.Vol. 1. New-York : Dover Publication

Dynamic analysis of large structures

- Split damping matrix



- Decoupling approximation[1]

$$\mathbf{H}_d = \left(-\mathbf{I}\omega^2 + j\omega\mathbf{D}_d + \mathbf{\Omega}\right)^{-1} \longrightarrow$$

Inversion of a diagonal matrix only
Decoupled system

[1] Rayleigh. (1877). The Theory of Sound.Vol. 1. New-York : Dover Publication

Dynamic analysis of large structures

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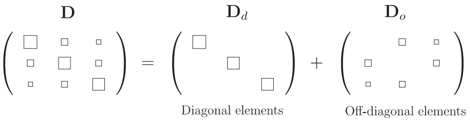

Decoupling approximation[1]

$$\mathbf{H}_d = (-\mathbf{I}\omega^2 + j\omega\mathbf{D}_d + \mathbf{\Omega})^{-1} \rightarrow$$

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Full matrix inversion

$$\mathbf{H} = (-\mathbf{I}\omega^2 + j\omega\mathbf{D} + \mathbf{\Omega})^{-1} \rightarrow$$

Full matrix inversion
Coupled system

$$\mathbf{H} = (\mathbf{I} + j\omega\mathbf{H}_d\mathbf{D}_o)^{-1}\,\mathbf{H}_d$$

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Asymptotic expansion method

Key-idea[1]

$$\mathbf{H} = \left(\mathbf{I} + j\omega \mathbf{H}_d \mathbf{D}_o\right)^{-1} \mathbf{H}_d$$

$$(\mathbf{I}+\mathbf{X})^{-1} \simeq \mathbf{I} - \mathbf{X} + \mathbf{X}^2 - \ldots = \mathbf{I} + \sum_{i=1}^{k} (-\mathbf{X})^i$$

<u>Condition:</u> $r(\mathbf{X}) = ||\boldsymbol{\lambda}||_\infty < 1$

Eigenvalues of $\mathbf{X}$

Approximation of H

Decoupling approximation (diagonal)

$$\mathbf{H}_k = \mathbf{H}_d + \underbrace{\sum_{i=1}^{k} (-j\omega)^i \left(\mathbf{H}_d \mathbf{D}_o\right)^i \mathbf{H}_d}$$

Corrections terms (non-diagonal)

No full matrix inversion

Inversion of a diagonal matrix only

↓

Approximate the coupled system

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[1] Denoël and Degée. (2009). Asymptotic expansion of slightly coupled modal dynamic transfer functions non-proportional damping. *Journal of Sound and Vibration* 328, 1-2, 1-8

Power spectral density matrices

- Exact solution

PSD matrix of generalized forces

$$\mathbf{S}^{(q)} = \mathbf{H}\mathbf{S}^{(g)}\mathbf{H}^*$$

PSD matrix of modal displacements

- Decoupling approximation

$$\mathbf{S}^{(q_d)} = \mathbf{H}_d\mathbf{S}^{(g)}\mathbf{H}_d^*$$

Formulation of the corrections terms

Proposed method[1]

$$\mathbf{S}^{(q_k)} = \mathbf{H}_k \mathbf{S}^{(g)} \mathbf{H}_k^*$$

Solution in the uncoupled system $\left(\mathbf{H}_d \mathbf{S}^{(g)} \mathbf{H}_d^*\right)$

$$\mathbf{S}^{(q_k)} = \mathbf{S}^{(q_d)} + \underbrace{\sum_{i=1}^{k} \mathbf{\Delta S}^{(q_i)}}_{\text{Corrections terms due to non-proportionality damping}}$$

[1] Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291

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Corrections terms

$(j\omega \mathbf{D}_o)$

$$\mathbf{\Delta S}^{(q_1)} = -\left(\mathbf{H}_d \mathbf{J}_o \mathbf{S}^{(q_d)} + \mathbf{S}^{(q_d)} \mathbf{J}_o^* \mathbf{H}_d^*\right)$$

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$$\mathbf{\Delta S}^{(q_i)} \overset{i \geq 1}{=} -\left(\mathbf{H}_d \mathbf{J}_o \mathbf{\Delta S}^{(q_{i-1})} + \mathbf{\Delta S}^{(q_{i-1})} \mathbf{J}_o^* \mathbf{H}_d^*\right) - \mathbf{H}_d \mathbf{J}_o \mathbf{\Delta S}^{(q_{i-2})} \mathbf{J}_o^* \mathbf{H}_d^*$$

[1] Canor, Blaise and Denoël. (2012). Efficient uncoupled stochastic analysis with non-proportional damping. *Journal of Sound and Vibration* 331, 24, 5283-5291

Importance of the corrections terms

- Uncorrelated generalized forces - Decoupled solution

$$\mathbf{S}^{(\mathbf{g})} = \mathbf{S}_d^{(\mathbf{g})}$$

$$\mathbf{S}_d^{(\mathbf{q}_d)} = \mathbf{H}_d \mathbf{S}_d^{(\mathbf{g})} \mathbf{H}_d^*$$

→ diagonal

Importance of the corrections terms

- Uncorrelated generalized forces - Decoupled solution

$$\mathbf{S}^{(\mathbf{g})} = \mathbf{S}_d^{(\mathbf{g})}$$

$$\underbrace{\mathbf{S}_d^{(\mathbf{q}_d)}}_{\text{diagonal}} = \mathbf{H}_d \mathbf{S}_d^{(\mathbf{g})} \mathbf{H}_d^*$$

- First two corrections terms

$$\Delta \mathbf{S}^{(\mathbf{q}_1)} = - \underbrace{\left(\mathbf{H}_d \mathbf{J}_o \mathbf{S}_d^{(\mathbf{q}_d)} + \mathbf{S}_d^{(\mathbf{q}_d)} \mathbf{J}_o^* \mathbf{H}_d^* \right)}_{\text{zero-diagonal matrix}}$$

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diagonal

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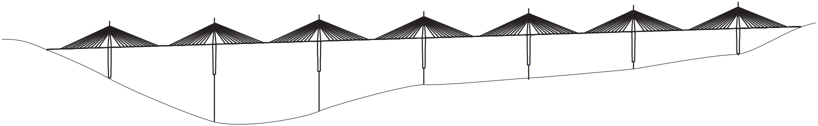
$$\Delta\mathbf{S}^{(\mathbf{q}_2)} = -\underbrace{\left(\mathbf{H}_d \mathbf{J}_o \Delta\mathbf{S}^{(\mathbf{q}_1)} + \Delta\mathbf{S}^{(\mathbf{q}_1)} \mathbf{J}_o^* \mathbf{H}_d^*\right)}_{\text{zero-diagonal matrix}} - \mathbf{H}_d \mathbf{J}_o \mathbf{S}_d^{(\mathbf{q}_d)} \mathbf{J}_o^* \mathbf{H}_d^*$$

full matrix

Viaduc of Millau

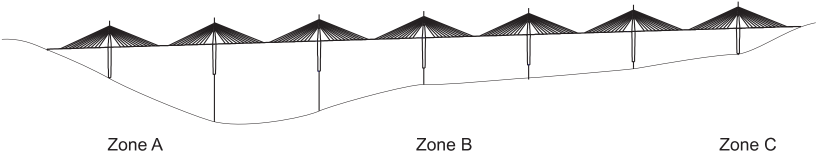
- Seven-span cable-stayed bridge (∼2.5 km long)



- Crosses the Tarn Valley about 350 m above the river
- Finite element model
  - 1425 nodes
  - 2439 beam elements with 12 DOFs

Model of wind

- Three zones



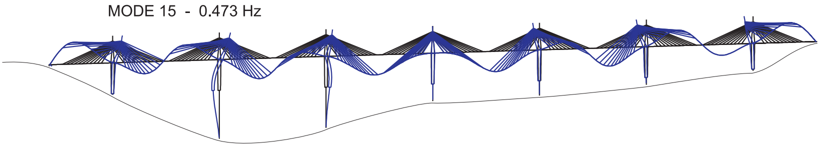
- Main characteristics from on-site measurements

Zones	U (m/s)	σ_u (m/s)	σ_v (m/s)	σ_w (m/s)
Zone A	38	6.5	6.5	4.5
Zone B	34	5.5	5.5	4.0
Zone C	36	5.5	5.5	4.0

- Considering aerodynamic damping $r(\mathbf{D}) = r(\mathbf{D}_s + \mathbf{D}_a) = 1.02$

Modal properties

- First 40 modes are kept for the modal analysis (< 1Hz)



- Structural modal damping matrix $\mathbf{D}_s \rightarrow \xi = 0.3\,\%$ in each mode

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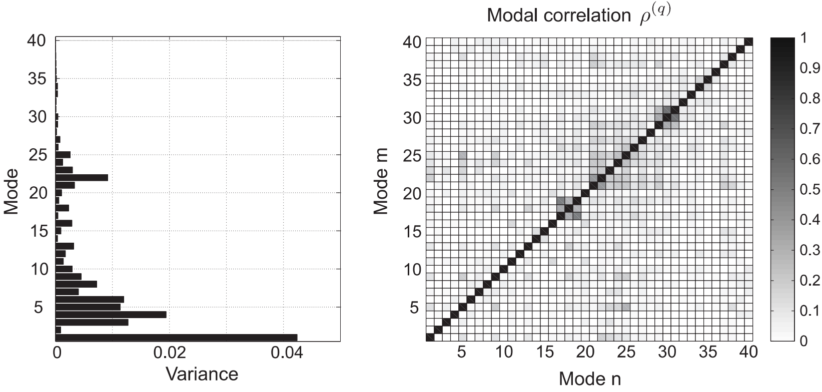
Exact solution

Variances and correlation of modal coordinates



Exact solution

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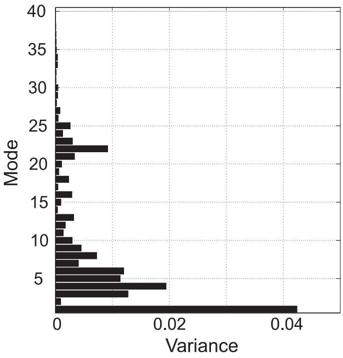
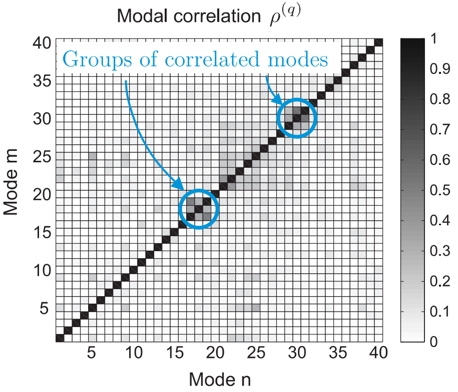
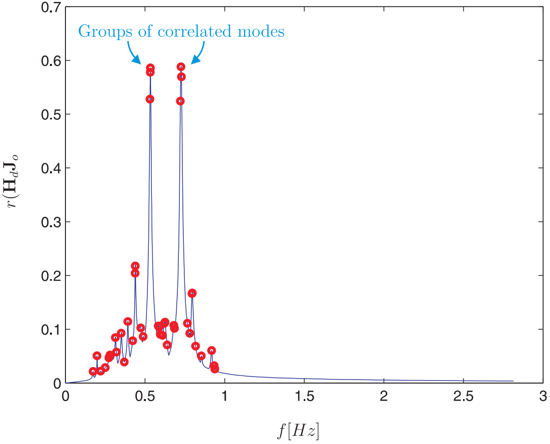




Asymptotic expansion method

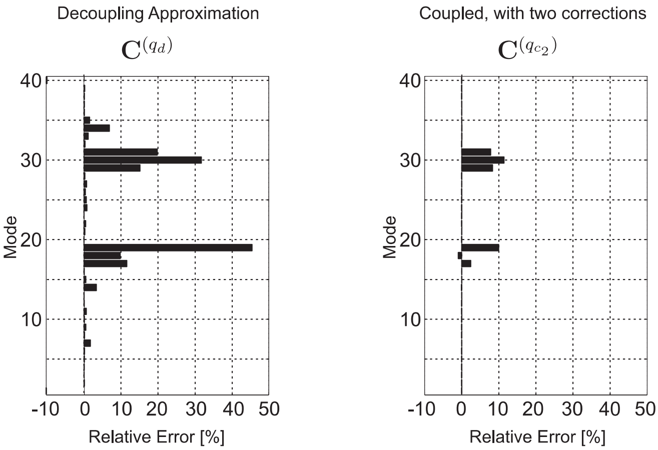
Condition: $r(\mathbf{H_d J_o}) = ||\boldsymbol{\lambda}||_\infty < 1$

Eigenvalues of $\mathbf{H}_d\mathbf{J}_0$



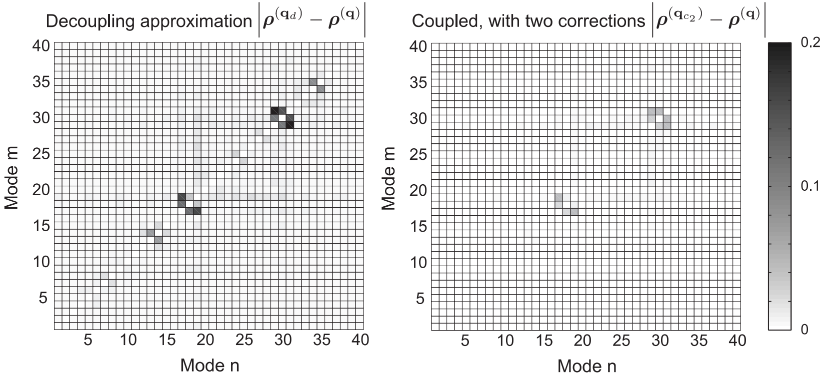
Asymptotic expansion method

- Variances



Asymptotic expansion method

- Correlation of modal coordinates



- Asymptotic expansion of the modal transfer matrix enables to approximate a coupled system with non-proportional damping based on the decoupled modal transfer matrix $\mathbf{H}_d$

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  - Second order approximation of $\mathbf{H}$ is sufficient

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- Studied case : Viaduc of Millau
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  - Second order approximation of **H** is sufficient
- Perspectives
  - Order of approximation function of the frequency
  - Background-resonant decomposition for the correction terms
  - Dynamic system with non-linear terms

The team...

Thomas Canor

Vincent Denoël

...thanks you for your kind attention

Questions ?

Read out more about us on : www.orbi.ulg.ac.be

Contact me at : N.Blaise@ulg.ac.be

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