The Airline Container Loading Problem with Pickup and Delivery

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26th European Conference on Operational Research
EURO-INFORMS Joint International Meeting
Rome 1-4 July, 2013

Outline

Outline

Context of the Research

⇒ **Problem Statement**:

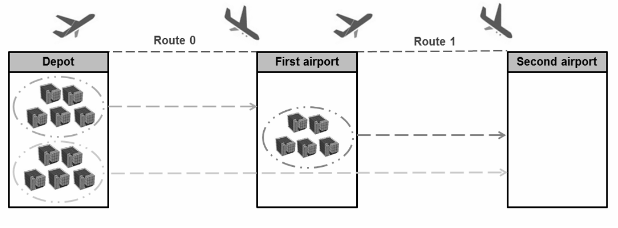
"How to **optimally load** a **set of containers/pallets** (ULDs) into a **cargo aircraft** that has to serve **multiple destinations** under some safety, structural, economical, environmental and manoeuvrability **constraints?**"

- Transport of goods by air
- Airlines were among first industries to have used OR methods
- Still numerous challenges
  - Volatility and increasing trend in the oil prices
  - Increasing pressure for greater focus on environmental concerns
  - More attention to spendings
- Load planning has possibilities for costs cutting because it is still a manual task

Outline

Description of the Problem

- A cargo aircraft has to deliver goods at two consecutive airports[1]



- The optimal location for all ULDs into the aircraft in order to minimize:

  ⇒ The fuel consumption during the entire trip
  ⇒ The loading time at the intermediate destination

[1] Generalization could be easily done to more than two destinations

Summary of the model

Minimize $\sum_{\forall k \in \mathbb{K}}$ (deviation most aft CG) and # ULDs to unload

subject to:

Each ULD is loaded
Each ULD fits in a position
A position accepts only one ULD
Some positions are overlapping: not simultaneously used
Longitudinal stability: The CG is within certified limits
Lateral balance
Maximum weight per position
Combined load limits
Cumulative load limits
Regulations for hazardous goods
Two parts of larger ULDs in adjacent positions
P & D

⇒ "Assignment Problem / Combinatorial Problem"
⇒ Integer Linear Problem

Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)

Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)

Main references for the basic problem (CG)

1. Limbourg, S., Schyns, M., and Laporte, G. (2011). Automatic Aircraft Cargo Load Planning. *Journal of the Operational Research Society*
2. Souffriau, W., Demeester, P. and Vanden Berghe, G. and De Causmaecker,P. (2008). The Aircraft Weight and Balance Problem. *Proceedings of ORBEL 22*, Brussels, pp. 44–45.
3. Mongeau, M. and Bès, C. (2003). Optimization of Aircraft Container Loading, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 39, pp. 140–150.

Outline

Summary of the model

$$\text{Min } \alpha \times \sum_{\forall k \in \mathbb{K}} \epsilon_k + \beta \times \sum_{\forall j \in \mathbb{P}} n_j$$

Subject to:

OF: Fuel consumption

$$CG_k - OCG_k - \epsilon_k \leq 0 \quad \forall k \in \mathbb{K}$$
$$CG_k - OCG_k + \epsilon_k \geq 0 \quad \forall k \in \mathbb{K}$$

OF: Loading time

$$\sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in B_j} x_{i'j'1} - n_j \times |B_j| - (1 - x_{ij1}) \times |B_j| \leq 0 \quad \forall j \in \mathbb{P},\ \forall i \in \mathbb{U}_3$$

Longitudinal stability

$$minCG_k \leq c_k \leq maxCG_k \quad \forall k \in \mathbb{K}$$

Lateral stability

$$-\bar{D} \leq \sum_{i(\mathbb{U}_1 \cup \mathbb{U}_3)} w_i \left(\sum_{j \in \mathbb{P}_R} x_{ij1} - \sum_{j \in \mathbb{P}_L} x_{ij1}\right) \leq \bar{D}$$
$$-\bar{D} \leq \sum_{i(\mathbb{U}_2 \cup \mathbb{U}_3)} w_i \left(\sum_{j \in \mathbb{P}_R} x_{ij2} - \sum_{j \in \mathbb{P}_L} x_{ij2}\right) \leq \bar{D}$$

Allowable positions

$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} x_{ij1} \leq 1 \quad \forall j \in \mathbb{P}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} x_{ij2} \leq 1 \quad \forall j \in \mathbb{P}$$
$$x_{ijk} = 0 \quad \forall i \in \mathbb{U},\ \forall j \in \mathbb{P},\ \forall k \in \mathbb{R} \mid U_i \text{ does not fit in } P_j$$
$$x_{ij1} + x_{i'j'1} \leq 1 \quad \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{E}_j$$
$$x_{ij2} + x_{i'j'2} \leq 1 \quad \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{E}_j$$

Full load

$$\sum_{j \in \mathbb{P}} x_{ij1} = 1 \quad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3)$$
$$\sum_{j \in \mathbb{P}} x_{ij2} = 1 \quad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3)$$

Weight restrictions

$$w_i \times x_{ij1} \leq \bar{W}_j \quad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$w_i \times x_{ij2} \leq \bar{W}_j \quad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap O_a^d \neq \emptyset} x_{ij1} o_{ija}^d \leq \bar{O}_a^d \quad \forall d \in \mathbb{D},\ \forall a \in \mathbb{O}^d$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap O_a^d \neq \emptyset} x_{ij2} o_{ija}^d \leq \bar{O}_a^d \quad \forall d \in \mathbb{D},\ \forall a \in \mathbb{O}^d$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} F_c \neq \emptyset} \sum_{l=1}^{a} x_{ij1} f_{ijl} \leq \bar{F}_a \quad \forall a \in \mathbb{F}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} F_c \neq \emptyset} \sum_{l=1}^{a} x_{ij2} f_{ijl} \leq \bar{F}_a \quad \forall a \in \mathbb{F}$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} T_c \neq \emptyset} \sum_{l=1}^{a} x_{ij1} t_{ijl} \leq \bar{T}_a \quad \forall a \in \mathbb{F}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} T_c \neq \emptyset} \sum_{l=1}^{a} x_{ij2} t_{ijl} \leq \bar{T}_a \quad \forall a \in \mathbb{T}$$

P & D

$$x_{ij1} = 0 \quad \forall i \notin (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$x_{ij2} = 0 \quad \forall i \notin (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$x_{ij1} - x_{ij2} \leq n_j \quad \forall i \in \mathbb{U}_3,\ \forall j \in \mathbb{P}$$
$$x_{ij2} - x_{ij1} \leq n_j \quad \forall i \in \mathbb{U}_3,\ \forall j \in \mathbb{P}$$
$$\sum_{\forall i' \in \mathbb{U}_2} \sum_{\forall j' \in \mathbb{B}_j} x_{i'j'2} + (x_{ij1} - n_j)|B_j| \leq |B_j| \quad \forall j \in \mathbb{P}, \forall i \in \mathbb{U}_3$$
$$x_{ij'1} - n_j' + n_j \leq 1 \quad \forall i \in \mathbb{U}_3,\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{P} \quad | \ j' \text{ is before } j$$

Hazardous goods

$$x_{ij1} + x_{i'j'1} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'};\ \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \text{and } \forall j, j' \in \mathbb{P}$$
$$x_{ij2} + x_{i'j'2} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'};\ \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \text{and } \forall j, j' \in \mathbb{P}$$

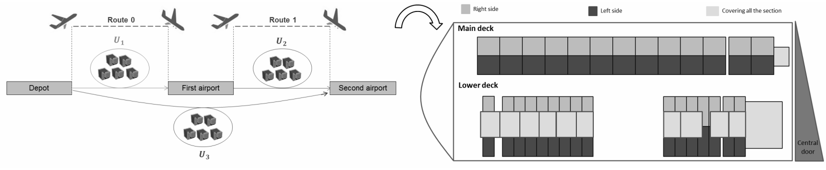
Larger ULDs

$$x_{ij1} \leq \sum_{\forall j' \in \mathbb{A}_j} x_{i'j'1} \quad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall i' \in \mathbb{L}_i,\ \forall j \in \mathbb{P}$$
$$x_{ij2} \leq \sum_{\forall j' \in \mathbb{A}_j} x_{i'j'2} \quad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall i' \in \mathbb{L}_i,\ \forall j \in \mathbb{P}$$

Outline

Main Parameters and Variables

- **Set of routes** $\mathbb{K}$
- **Set of ULDs** $\mathbb{U}$
  - According to their origin and destination: three subsets of ULDs: $\mathbb{U}_1$, $\mathbb{U}_2$, $\mathbb{U}_3$
- **Set of positions** $\mathbb{P}$
- There is **only one central door** situated at the extremity of the aircraft



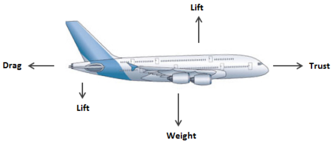
Binary Variables

$$x_{ijk} = \begin{cases} 1 & \text{if ULD } i \text{ is in position } j \text{ during the route } k \\ 0 & \text{otherwise} \end{cases}$$

Outline

Objective function: Most Aft CG

In terms of fuel consumption, the optimal location for the CG is the most aft within certified limits



In mathematical terms, it gives:

$$\textbf{Min} \sum_{\forall k \in \mathbb{K}} \epsilon_k$$

Subject to:

$$\left.\begin{array}{ll} CG_k - OCG_k - \epsilon_k & \leq 0 \\ CG_k - OCG_k + \epsilon_k & \geq 0 \end{array}\right\} \forall k \in \mathbb{K}$$

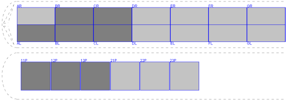
where :

-CG_k is the CG obtained after assignment of ULDs in the aircraft during the route k

-OCG_k is the optimal CG, i.e. most aft CG on the route k

Objective function: minimize # ULDs to Unload

The loading time is function of the # of ULDs to be unloaded
We want to prevent the unnecessary unloads at the intermediate destination



In mathematical terms, it gives:

$$\textbf{Min} \sum_{\forall j \in \mathbb{P}} n_j$$

Subject to:

$$\sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in \mathbb{B}_j} x_{i'j'1} - n_j \times |B_j| - (1 - x_{ij1}) \times |B_j| \leq 0 \quad \forall j \in \mathbb{P}, \ \forall i \in \mathbb{U}_3$$

where :
-n_j is a binary variable
-B_j is the set of all position situated behind j relative to the door

Multi-objective Optimization

In definitive, both objectives have to be considered together:

$$\textbf{Min} \quad E(\alpha) \times \sum_{\forall k \in \mathbb{K}} \epsilon_k + E(\beta) \times \sum_{\forall j \in \mathbb{P}} n_j$$

Subject to:

$$\sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in \mathbb{B}_j} x_{i'j'1} - n_j \times |B_j| - (1 - x_{ij1}) \times |B_j| \leq 0 \quad \forall j \in \mathbb{P}, \ \forall i \in \mathbb{U}_3$$

$$\left.\begin{array}{ll} CG_k - OCG_k - \epsilon_k & \leq 0 \\ CG_k - OCG_k + \epsilon_k & \geq 0 \end{array}\right\} \forall k \in \mathbb{K}$$

where :
-α is the additional cost (fuel + emissions) for a deviation of one inch from the most aft CG

-β is the cost associated with the loading time of one ULD (in terms of wages, fees to the airport for the usage of the runway...)

Outline

Standard Constraints

Stability

Longitudinal stability: CG within certified limits
Lateral balance of the aircraft

Full Load

Each ULD is loaded

Allowable positions

Each ULD fits in a position
A position accepts only one ULD
Overlapping positions: not simultaneously used

Weights restrictions

Maximum weight per position
Combined load limits
Cumulative load limits

Specific Constraints

Routes constraints

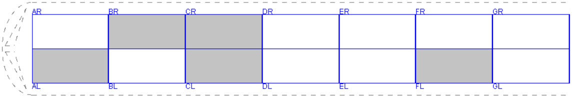
$$x_{ij1} = 0 \qquad \forall i \notin (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$x_{ij2} = 0 \qquad \forall i \notin (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$

Same position for ULDs not unloaded

$$x_{ij1} - x_{ij2} \leq n_j \qquad \forall i \in \mathbb{U}_3,\ \forall j \in \mathbb{P}$$
$$x_{ij2} - x_{ij1} \leq n_j \qquad \forall i \in \mathbb{U}_3,\ \forall j \in \mathbb{P}$$

Allowable positions for ULDs loaded at the intermediate destination

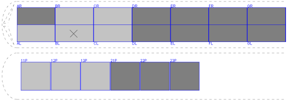
$$\sum_{\forall i' \in \mathbb{U}_2} \sum_{\forall j' \in \mathbb{B}_j} x_{i'j'2} + (x_{ij1} - n_j)|B_j| \leq |B_j| \qquad \forall j \in \mathbb{P}, \forall i \in \mathbb{U}_3$$

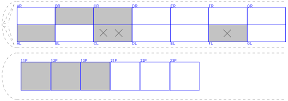


Specific Constraints

Changing the position of an ULD with minimal unloading

$$x_{ij'1} - n'_j + n_j \leq 1 \qquad \forall i \in \mathbb{U}_3,\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{P} \mid j\text{' is before } j$$





Hazardous goods

$$x_{ij1} + x_{i'j'1} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'};\ \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \text{and } \forall j, j' \in \mathbb{P}$$

$$x_{ij2} + x_{i'j'2} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'};\ \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \text{and } \forall j, j' \in \mathbb{P}$$

ULDs of larger dimensions

$$x_{ij1} \leq \sum_{\forall j' \in \mathbb{A}_j} x_{i'j'1} \qquad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall i' \in \mathbb{L}_i,\ \forall j \in \mathbb{P}$$

$$x_{ij2} \leq \sum_{\forall j' \in \mathbb{A}_j} x_{i'j'2} \qquad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall i' \in \mathbb{L}_i,\ \forall j \in \mathbb{P}$$

Outline

The case of a Boeing 747[2]

Data

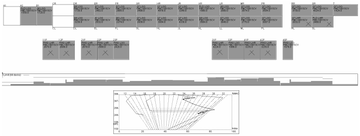
- Parameters alpha and beta equal to 1
- 38 ULDs distributed as follows:

Origin	Destination	Number of ULDs
FRA	LGG	28
FRA	LHR	10
LGG	LHR	/

- 67 standard positions, plus 10 larger ones

Results

- √ All constraints satisfied
- √ Deviations CG's: 0 and 0.0028
- √ # ULDs unloaded: 5
- √ Computation time: 18'21"



[2]Model implemented in Java using IBM ILOG CPLEX 12 and tested on real data with personal computer (Windows 7, Intel Core i5-2450M, 2.50GHz, 8.00 GB of RAM)

Additional results

- Small instances (15 ULDS) and Larger instances (35 ULDs)
- Same set of data
- Variations of the origin and destination (randomly)

Results (15 tests for each)

	Small instances	Large instances
Status of solution	Optimal	Optimal
Longest computation time	17'	nearly 8 hours

General trends

- The computation time $\nearrow$ when more ULDs present on several routes
- But it also depends on the number of position accepted by each ULD

Outline

Conclusion and outlooks

- The goal of this paper was the development of a mixed integer linear programming model for loading optimally a set of unit load devices into a cargo aircraft that visits successively two airports
- This sequence of destinations had never been considered in the literature before us.
- We can summary results of our research as follows:

Objective	Implies	Results
Min fuel consumption	CG's close to the aft limit	$\surd$
Min loading time	Minimizing the unloads	$\surd$

- Better results in terms of both fuel consumption and loading time.
- Computation time $\nearrow$ when the distribution of ULDs becomes more balanced

Conclusion and outlooks

<u>To do list</u>

- √ Mathematical formulation of the model
- √ Implementation of the model
- √ Tests on real instances
- ☐ Consideration of lateral doors
- ☐ Complexity of the model
- ☐ Development of Heuristics ?

Contact me

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Thank you for your attention !