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IDENTIFICATION OF MECHANICAL SYSTEMS WITH LOCAL NONLINEARITIES THROUGH DISCRETE-TIME VOLTERRA SERIES AND KAUTZ FUNCTIONS

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ABSTRACT

Mathematical modeling of mechanical structures is an important research area in structural dynamics. The goal is to obtain a model that accurately predicts the dynamics of the system. However, the nonlinear effects caused by large displacements and boundary conditions like gaps, backlash, joints, as well as large displacements are not as well understood as the linear counterpart. This paper identifies a non-parametric discrete-time Volterra model of a benchmark nonlinear structure consisting of a cantilever beam connected to a thin beam at its free end. The time-domain data of the modal test are used to identify the Volterra kernels. To facilitate the identification process, the kernels are expanded with orthogonal Kautz functions to decrease the number of parameters to be identified. The nonlinear parameters are also estimated by the updating of a finite element model with local nonlinearity involving the optimization of residue of the numerical and experimental kernels. The capability of the representation of the nonlinear phenomena is investigated through numerical simulations. The paper concludes by indicating the advantages and drawbacks of the Volterra series for modeling the behavior of nonlinear structures with suggestions to overcome the disadvantages found during the tests.

1. INTRODUCTION

Mathematical modeling of mechanical structures is an important research topic in structural dynamics. One of the goals of this area is to obtain a reliable model that can accurately predict the dynamic behavior of the system [1]. Model updating strategies for linear structures were extensively investigated in literature [2]. These procedures are generally based on the maximization of the correlation between the mathematical model and experimental vibration data measured on the structure. The objective functions usually depend upon modal parameters of the structure as for example, mode shapes, natural frequencies and frequency response functions. However, these concepts are based in the superposition principle, so that they are not valid in the case of nonlinear systems [3]. In this context, Volterra series are an attractive technique since they are a generalization of the linear model based on the impulse response function [4]. The Volterra series represent the output of a nonlinear system through a multidimensional convolution between the input signal and the Volterra kernels [5].

In the area of structural dynamics, most of the studies applying Volterra series consider their continuous time formulation of Volterra series, while the use of the discrete-time formulation is often disregarded. Cafferty and Tomlinson [6] identified analytic expressions for the Volterra kernels of automotive dampers in the frequency domain, termed high order frequency response functions (HOFRF), via the harmonic probing method. An experimental test is applied in the structure to identify the main diagonals of the nonlinear kernels and to calculate the nonlinear parameters of the damper. Silva (2005) [7] studied the identification of nonlinear aeroelastic systems using, among other techniques, Volterra models. Input and output signals of the aileron and the vibration measured in a active aeroelastic wing under test in a wind tunnel were used to identify the first three Volterra kernels of this system. da Silva *et al.* [8] applied the Volterra model in the problem of identification of a cantilever beam with a local nonlinearity in the free end of the beam. The authors expanded the Volterra model with orthogonal Kautz functions, which substantially decreased the number of terms required to represent the kernels. Following the same idea, da Silva [9] utilized the Volterra model to create a metric for nonlinear model updating, and treated a frame structure with local nonlinearity. The poles of the Kautz basis functions were selected by analyzing the predominant dynamics in the response of the nonlinear system. The results showed the possibility of application of the Volterra model in systems with polynomial and bi-linear nonlinearities.

This paper is in line with the works done by da Silva *et al.* [8] and da Silva [9], and addresses the identification of a Volterra series-based model of a benchmark nonlinear structure of the University of Liège consisting of a cantilever beam exhibiting geometrically nonlinear behavior [10]. Starting from the identified Volterra kernels, the nonlinear parameters of a finite element model of the beam are also estimated through model updating.

The paper is summarized as follows. Section 2 gives a background on discrete Volterra series and the orthonormal expansion of this model. Section 3 describes the benchmark structure used in this study. Section 4 shows the methodology used in the updating of the finite element model with local nonlinearity. Section 5 and 6 shows the identification of the Volterra kernels of the system and the updating of the nonlinear parameters of the model respectively. Finally, in the section 7 the main advantages and drawbacks of the proposed methodology are exposed and further steps of this research are outlined.

2. VOLTERRA SERIES

The Volterra series is a direct generalization of the concept of impulse response function of linear systems [5]. In the discrete-time formulation, the Volterra series express the response of the system $x(k)$ to an input $u(k)$ as:

$$x(k) = x_{lin}(k) + x_{quad}(k) + x_{cub}(k) + ... \tag{1}$$

where $x_{lin}(k)$ is the linear contribution of the response given by the convolution of the input $u(k)$ with the linear impulse response function, and $x_{quad}(k) + x_{cub}(k) + ...$ are the nonlinear contributions of the output $x(k)$. The application of the discrete Volterra series gives the following equation:

$$x(k) = \sum_{m=1}^{+\infty} \sum_{n_1=-\infty}^{+\infty} \dots \sum_{n_m=-\infty}^{+\infty} h_m(n_1, n_2, \dots, n_m) \prod_{i=1}^{m} u(k - n_i) \tag{2}$$

where m is the number of non-linear terms generally truncated in a low-order values M and $h_m(n_1, n_2, \dots, n_m)$ is the $m-th$ order Volterra kernel. Despite of this truncation, the most common nonlinearities can be well represented by this model, except Coulomb damping, hysteresis, backlash and others [8, 9]. Considering the first three terms ($x_{lin}(k)$, $x_{quad}(k)$ and $x_{cub}(k)$) it is possible to obtain:

$$\begin{aligned} x(k) = &\sum_{n_1=0}^{N_1} h_1(n_1)\, u(k-n_1) + \sum_{n_1=0}^{N_2}\sum_{n_2=0}^{N_2} h_2(n_1,n_2)\, u(k-n_1)\, u(k-n_2) + \\ &+ \sum_{n_1=0}^{N_3}\sum_{n_2=0}^{N_3}\sum_{n_3=0}^{N_3} h_3(n_1,n_2,n_3)\, u(k-n_1)\, u(k-n_2)\, u(k-n_3) \end{aligned} \tag{3}$$

where $h_1(n_1)$ is the first Volterra kernel (linear impulse response function), $h_2(n_1, n_2)$ is the second Volterra kernel and $h_3(n_1, n_2, n_3)$ is the third Volterra kernel. Unfortunately, the number of parameters to be estimated in Eq. (3) can be large, limiting its use in practical applications. One way to overcome this drawback is to expand the Volterra kernels using orthogonal basis functions. In the case of oscillating underdamped systems, as is the main interest of structural dynamics, Kautz functions can be applied [12]. The expansion of the Volterra kernels can be represented by:

$$h_1(n_1) = \sum_{i_1}^{M_1} \alpha(i_1)\, \psi_{i_1}(n_1) \tag{4}$$

$$h_2(n_1, n_2) = \sum_{i_1=1}^{M_2}\sum_{i_2=1}^{i_1} \alpha(i_1, i_2) \psi_{i_1}(n_1)\, \psi_{i_2}(n_2) \tag{5}$$

$$h_3(n_1, n_2, n_3) = \sum_{i_1=1}^{M_3}\sum_{i_2=i_1}^{M_3}\sum_{i_3=i_2}^{M_3} \alpha(i_1, i_2, i_3)\, \psi_{i_1}(n_1)\, \psi_{i_2}(n_2)\, \psi_{i_3}(n_3) \tag{6}$$

where M_1, M_2 and M_3 are the number of orthogonal functions used to describe the first and second kernels, respectively, $\alpha(i_1)$, $\alpha(i_1, i_2)$ and $\alpha(i_1, i_2, i_3)$ denotes the orthogonal Volterra kernels, and ψ_{i_1}, ψ_{i_2} and ψ_{i_3} are the orthogonal functions. Substituting Eqs. (4) and (5) in the Eq. (3), it is possible to obtain:

$$x(k) = \sum_{i_1=1}^{M_1} \alpha(i_1)\, l_{i_1}(k) + \sum_{i_1=1}^{M_2}\sum_{i_2=1}^{i_1} \alpha(i_1, i_2)\, l_{i_1}(k)\, l_{i_2}(k) + \sum_{i_1=1}^{M_3}\sum_{i_2=i_1}^{M_3}\sum_{i_3=i_2}^{M_3} \alpha(i_1, i_2, i_3)\, l_{i_1}(k)\, l_{i_2}(k)\, l_{i_3}(k) \tag{7}$$

where $l_{i_1}(k)$, $l_{i_2}(k)$ and $l_{i_3}(k)$ are the convolution between the input and the impulse response of the orthogonal functions. As the Volterra systems are linear in the parameters, it is possible to write the vector of output signal $\mathbf{X}$ as a multiplication between the regression matrix $\mathbf{\Lambda}$ with the input signal filtered by the orthogonal functions and the parameter vector θ with the orthogonal kernels:

$$\mathbf{X} = \mathbf{\Lambda}\theta \tag{8}$$

To estimate the Volterra kernels in the vector θ, it is possible to apply the classical least squares method and obtain this vector as [11]:

$$\theta = \mathbf{\Lambda}^{-1}\mathbf{X} = \left(\mathbf{\Lambda}^T\mathbf{\Lambda}\right)^{-1}\mathbf{\Lambda}^T\mathbf{X} \tag{9}$$

However, such an orthogonal expansion of the Volterra kernels requires a suitable function that represents the linear and nonlinear dynamics of the system. The aforementioned Kautz orthogonal functions turn out to be appropriate and write [12]:

$$\Psi_{2n}(z) = \frac{\sqrt{(1-c^2)(1-b^2)}\,z}{z^2 + b(c-1)z - c}\left[\frac{-cz^2 + b(c-1)z + 1}{z^2 + b(c-1)z - c}\right]^{n-1} \tag{10}$$

$$\Psi_{2n-1}(z) = \frac{\sqrt{1-c^2}\,z(z-b)}{z^2 + b(c-1)z - c}\left[\frac{-cz^2 + b(c-1)z + 1}{z^2 + b(c-1)z - c}\right]^{n-1} \tag{11}$$

where:

$$b = \frac{\left(\beta + \bar{\beta}\right)}{\left(1 + \beta\bar{\beta}\right)} \qquad c = -\beta\bar{\beta} \tag{12}$$

where β and $\bar{\beta}$ are the Kautz pole and the conjugate pole respectively. The Kautz poles in the continuous-time domain can be related to the dynamics of the structure through:

$$s_n = -\zeta_n\omega_n + j\omega_n\sqrt{1 - {\zeta_n}^2} \tag{13}$$

where ζ_n is the damping ratio of the pole, and ω_n is $n-th$ the frequency. For the application of the poles in the model, it is necessary to make a transformation of the poles to the discrete domain:

$$z_n = e^{s_n/F_s} \tag{14}$$

where F_s is the sampling frequency. Starting from the knowledge of the Kautz poles, an optimization algorithm can be used to minimize the prediction error of the Volterra model.

3. BENCHMARK STRUCTURE

The structure used as a benchmark in this study was proposed in the École Centrale de Lyon [13] and reproduced in the University of Liège in the framework of COST Action F3 [10]. This system is composed by a main cantilevered beam connected to a thin beam in the free end. The main geometrical and mechanical properties of the structure are listed in Table 1.

	Length [mm]	Width [mm]	Thickness [mm]	Material
Main beam	700	14	14	Steel
Thin beam	40	14	0.5	Steel

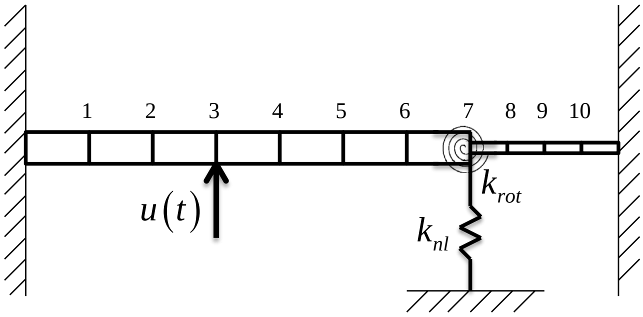
Table 1. Geometrical and mechanical properties of the benchmark structure.

Seven accelerometers regularly spanned the main beam, and an additional displacement sensor was positioned at its end (see the schematic diagram in Fig. 1). The excitation force was a band-limited (0 - 500 Hz) white-noise sequence sampled at 2560 Hz and applied in a horizontal plane by a shaker located 30 cm away from the clamping (position 3 in Fig. 1). Seven excitation levels were considered from 0.56 Nrms up to 27.73 Nrms, and 163840 samples were recorded for each level.

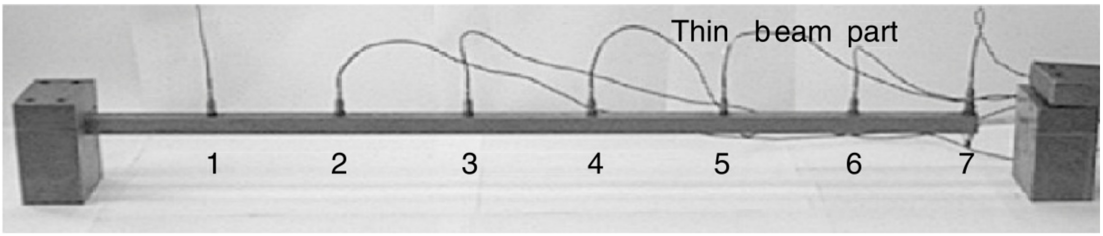
The structure was modeled with the finite element method (FEM), the main beam was described 7 Euler-Bernoulli beam elements and the thin beam with 4 beam elements with two degrees of freedom per node. The junction between the two beams was modeled as a linear rotational stiffness (k_{rot}). The geometrical nonlinearity is described using a non-integer exponent-type model of the form [14]:

$$f_{nl}(x) = k_{nl}|x|^{\eta} sign(x) \tag{15}$$

where f_{nl} is the nonlinear internal force, x the displacement, k_{nl} the nonlinear stiffness coefficient and η the exponent of the nonlinearity.



(a) Schematic diagram of the structure.



(b) Photo of the experimental setup [10].

Figure 1. ULg benchmark structure.

4. MODEL UPDATING METHODOLOGY

The updating of the finite element model based on the experimentally measured input and output signals is a two-step procedure. Firstly, a low-level data set (0.56 Nrms) and the displacement of the node 7 of the structure are used to update the Young's modulus (E) and the density (ρ) of the material, the rotational stiffness of the junction (k_{rot}), and the proportional damping coefficients of the mass matrix (β_M) and stiffness matrix (β_K). The linear objective function is formed as the norm of the difference between the magnitudes of the experimental (H_{exp}) and the analytic (H_{FEM}) frequency response functions calculated with the matrices of the equations of motion:

$$J(\mathbf{p}_{lin}) = \left\| H_{exp} - H_{FEM} \right\| \tag{16}$$

where $\mathbf{p}_{lin}$ is the vector with the parameters chosen to be updated:

$$\mathbf{p}_{lin} = [E, \rho, k_{rot}, \beta_M, \beta_K] \tag{17}$$

In the second step, a nonlinear high-level data set (5.5 Nrms) is exploited. The objective function now relies on the difference between the orthogonal Volterra kernels extracted experimentally and the orthogonal kernels computed with the finite element model. In this work, a model with 3 Volterra kernels is considered and the objective function is calculated as the difference between the two nonlinear kernels:

$$J(\mathbf{p}_{nl}) = \sum_j \sum_i \left[\alpha_{exp}(i,j) - \alpha_{FEM}(i,j)\right]^2 + \sum_k \sum_j \sum_i \left[\alpha_{exp}(i,j,k) - \alpha_{FEM}(i,j,k)\right]^2 \tag{18}$$

where $\mathbf{p}_{nl}$ is the vector with the nonlinear parameters to be identified:

$$\mathbf{p}_{nl} = [k_{nl}, \eta] \tag{19}$$

A flowchart summarizing the proposed methodology is shown in Fig. (2).

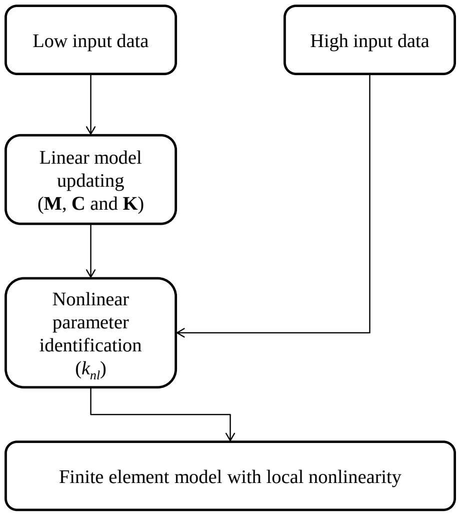
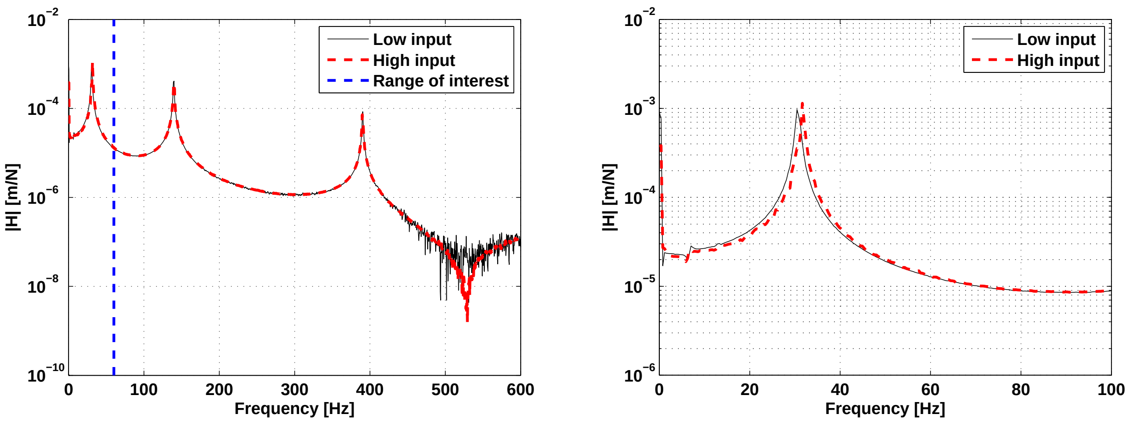


Figure 2. Flowchart of the model updating methodology.

5. VOLTERRA MODEL IDENTIFICATION

The Volterra model considered herein encompasses single input, single output systems, and thus cannot represent the benchmark structure as a multi-degree-of-freedom system. For this reason, experimental data was pre-processed with a digital low-pass filter with a cut-off frequency of 60 Hz to focus on the first mode of the structure for which the nonlinear effects are the most significant. After filtering, measured signals was then down-sampled to 128 Hz to limit the frequency band of interest. Fig. 3 shows the FRFs computed at low (0.56 Nrms) and high excitation level (5.5 Nrms) together with the frequency interval considered for the Volterra model identification. The change in the first natural frequency due to the nonlinearity is also visible in this figure.



(a) The first three modes of the system. (b) First mode shape.

Figure 3. Experimental FRFs of the benchmark structure.

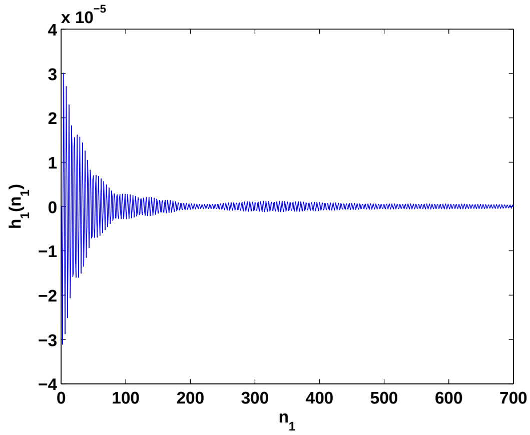
As previously mentioned in the Section 2 of this paper, to identify the orthogonal Volterra kernels, it is necessary to define the parameters of the Kautz function poles, *i.e* the damping ratio ζ_n and the frequency ω_n. Fortunately, these values are usually close to the values that describe the linear dynamics of the system [8]. As this work addresses a model with the first three Volterra kernels, three pairs of parameters must be chosen. The first kernel is represented with the estimated frequency of the first peak of the FRF computed at low level (ω_1)

and with a damping ratio (ζ_1) estimated using the half-power bandwidth method [15]. As for the nonlinear parameters of the Kautz poles needed to identify the second and third kernels, they are computed via an optimization routine, namely the sequential quadratic programming (SQP) [16], minimizing the prediction error of the model. The resulting parameters of the Kautz poles are listed in Table 2.

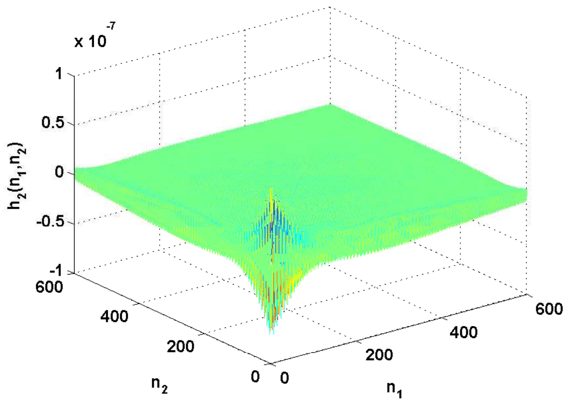
	ω_1 [Hz]	ζ_1	ω_2 [Hz]	ζ_2	ω_3 [Hz]	ζ_3
Value	30.69	0.0041	48.04	0.0026	31.43	0.0191

Table 2. Information for the identification of the Kautz poles.

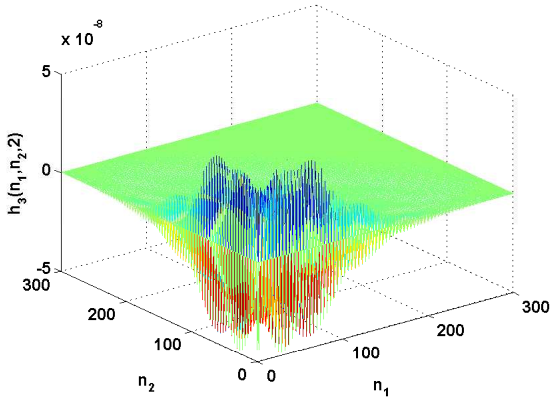
Starting from the definition of the Kautz filters, it is then possible to solve a least-squares problem to calculate the Volterra kernels. The Volterra kernels in the physical basis are shown in Fig. (4). It is possible to observe the first kernel as a classical impulse response of a linear system and the second kernel as a bi-dimensional impulse response while the third kernel must be cut to able a partial graphical representation, since it has four dimensions.



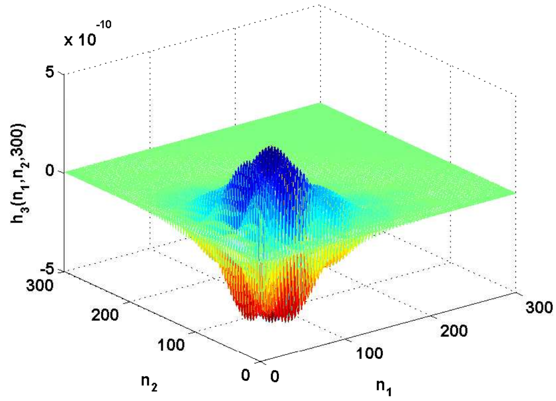
(a) First Volterra kernel.



(b) Second Volterra kernel.



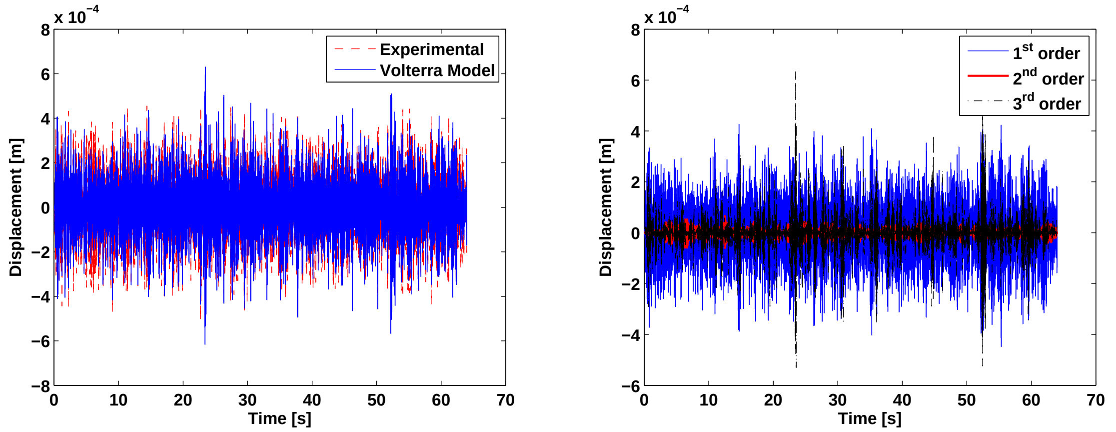
(c) Third Volterra kernel (cut in 2).



(d) Third Volterra kernel (cut in 300).

Figure 4. Volterra kernels in the physical basis.

Fig. 5 (a) depicts a comparison between the original and the estimated displacement signals while Fig. 5 (b) represents the components of the response of the system. The third-order component is the most significant contribution in the response, indicating that the nonlinearity in the system has a dominant cubic component.

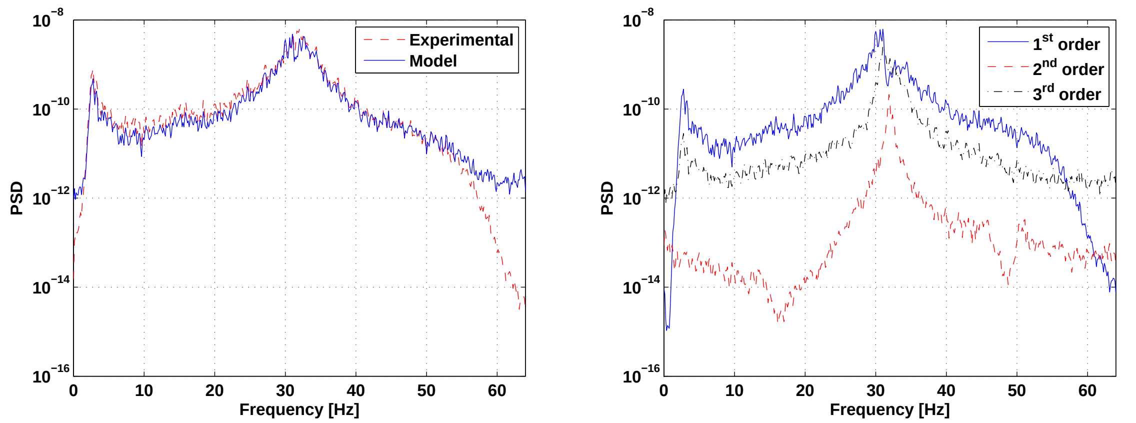


(a) Comparison between the original and estimated signals.

(b) Components of the response in time domain.

Figure 5. Output time signals generated by the estimated Volterra model.

Fig. 6 (a) shows a comparison between the power spectral densities of the estimated and original signals (computed using a hanning window with 4 averages and 50 % of overlap). Fig. 6 (b) confirms that the third-order component is the most significant nonlinear contribution in the system response.



(a) Comparison between the orignal and estimated PSDs.

(b) PSDs of the components of the response.

Figure 6. PSDs of the output signals generated by the estimated Volterra model.

6. FINITE ELEMENT MODEL UPDATING

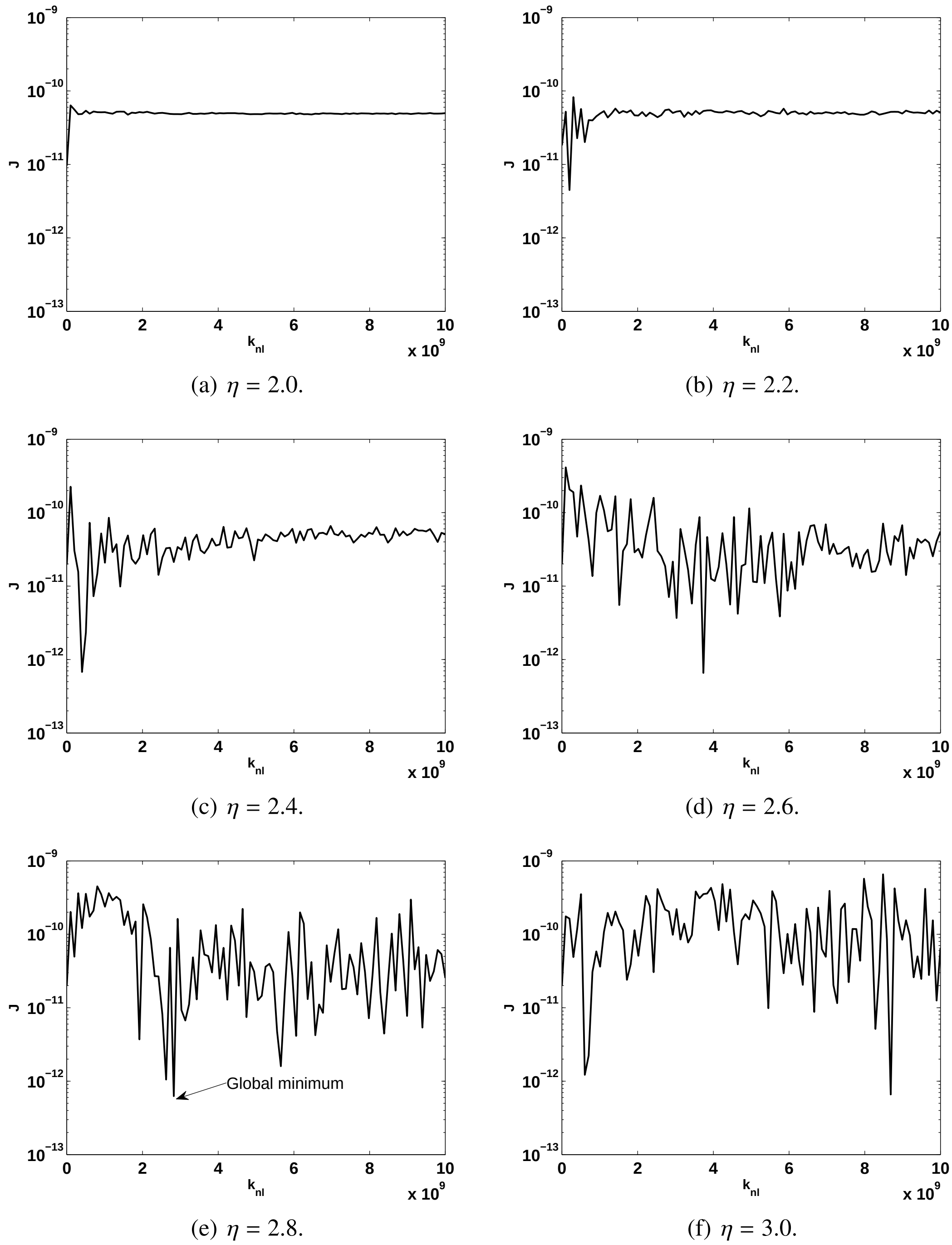
As already explained in Section 5, the finite element model updating procedure comprised two main steps. In the first one, the linear parameters of the structure was updated using low-level data and yield the global mass, damping and stiffness matrices. To this end, a hybrid optimization algorithm was used combining a genetic algorithm [17] and SQP optimization [16]. One however notes that the SQP algorithm does not contribute significantly to the minimization of the objective function. The search ranges and the values obtained by the optimization process are given in Table 3.

After defined the motion equation matrices, the second step of the model updating was performed with the nonlinear parameters identification. For this step, to get a better insight

	E [GPa]	ρ [kg/m^3]	k_{rot} [N/rad]	β_M	β_K
Search range	[105;315]	[3930;11790]	[0.1;10^4]	[10^{-7};10^{-3}]	[10^{-7};10^{-3}]
Obtained value	263.4	11736	6.56	8.02×10^{-6}	5.4×10^{-6}

Table 3. Search range and values obtained in the linear model updating.

in the nonlinear objective function based in the Volterra model, the objective function was mapped for values of $k_{nl} = [10^4; 10^{10}][N/m^\eta]$ and $\eta = [2.0; 2.2; 2.4; 2.6; 2.8; 3.0]$. Fig. (7) shows the objective function for each of the values of the exponent η.



(a) η = 2.0. (b) η = 2.2.

(c) η = 2.4. (d) η = 2.6.

(e) η = 2.8. (f) η = 3.0.

Figure 7. Objective function of the nonlinear parameters of the benchmark structure.

The nonlinear objective function shows a very complex behavior with many local minimum points. This feature can difficult the solution of the nonlinear parameter identification problem with classical gradient-based optimization algorithms. The optimal values of the parameters are $k_{nl} = 2.83 \times 10^9$ $[N/m^\eta]$ and $\eta = 2.8$, respectively, as indicated in Fig. 7 (e). These values are in good agreement with the results obtained in previous studies using the conditioned reverse path (CRP) method [10] and the proper orthogonal decomposition (POD) [14], as summarized in Table 4.

	CRP	POD	Volterra
k_{nl} $[N/m^\eta]$	2.08×10^9	1.65×10^9	2.83×10^9
η	2.8	2.8	2.8

Table 4: Comparison between the nonlinear parameter identification results of different studies.

It is interesting to point out that the estimated exponent $\eta = 2.8$ confirms the values calculated using other methods and is close to 3 (cubic nonlinearity). This result also reaffirms that the third-order response of the Volterra model was found as the most significant nonlinear component in the output of the system (see Fig. 5 and Fig. 6).

Fig. (8) presents a comparison between the experimental and modeled responses of the structure in the time domain. Fig. 9 compares the experimental high-level FRF and the FRF computed with the FEM-based model (hanning window with 10 averages and 50% of overlap). The FRFs estimated with the low- and high-amplitude input signals using the updated model are eventually superimposed in Fig. (10), considering the identified geometrical nonlinearity. The updated model accurately reproduces the increase in the first natural frequency with the increase of the input signal, experimentally observed in Fig. 3.

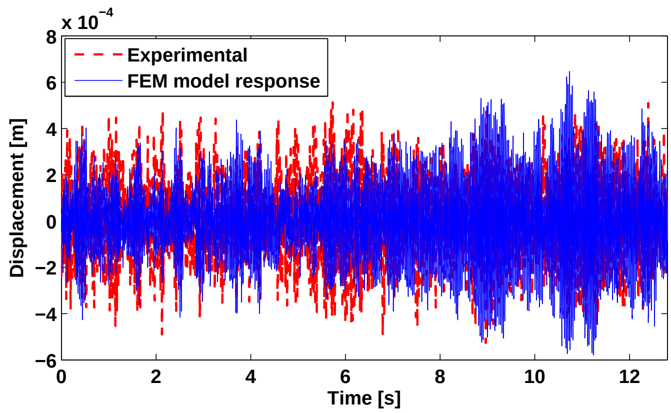


Figure 8: Comparison between the experimental response and estimated by the nonlinear updated model.

7. FINAL REMARKS

This work proposed the application of Volterra series expanded in orthogonal Kautz functions to model the nonlinear multiple convolution between the input and output signals of the benchmark structure of ULg. The orthogonal Volterra kernels were identified using experimental data from the system. A nonlinear parameter identification procedure was done to identify the parameters of the restoring force of the local nonlinearity of the system with a objective function based on the difference between the experimental nonlinear kernels and the nonlinear kernels extracted with the response of the FEM model.

The results showed a good agreement with previous works reported in [10] and in [14] and the FRF of the FEM model showed to reproduce the same kind of effect in the first natural

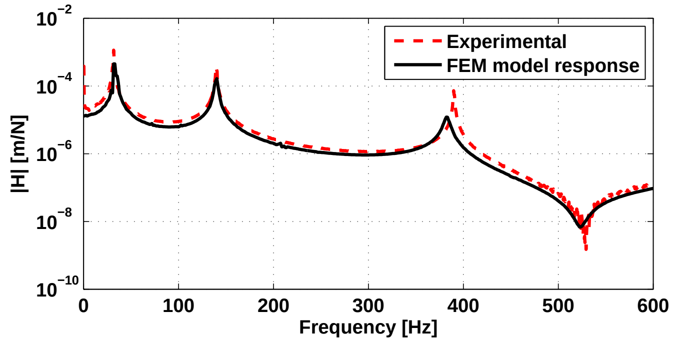
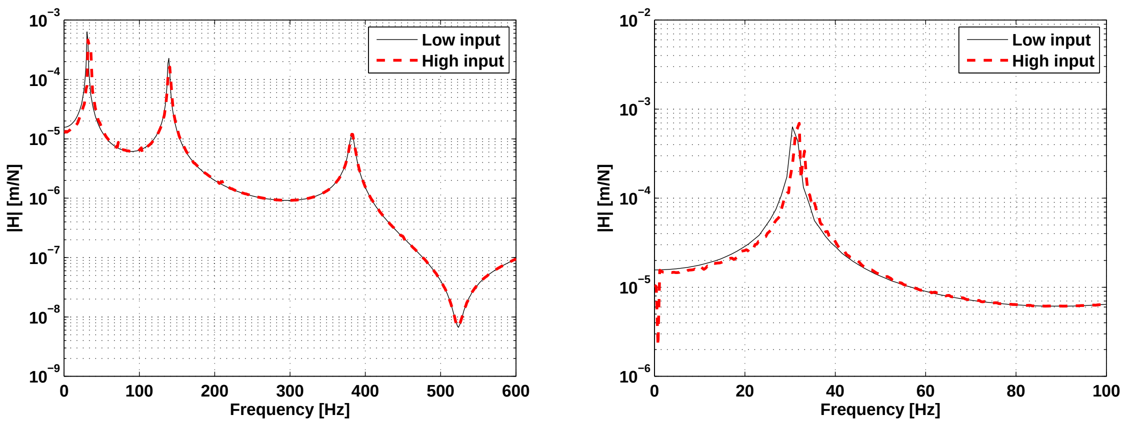


Figure 9: Comparison between the FRFs estimated with the experimental data and estimated by the nonlinear updated model.



(a) The first three modes of the system.

(b) First mode shape.

Figure 10. FRF estimated with the response of the FEM model of the benchmark structure.

frequency as observed in the FRF calculated with the experimental data. However, the main drawback of the proposed methodology is to consider only one mode shape of the structure and in a way that the technique can not represent the nonlinearities in multiple modes of the structure. Other limitation of the proposed nonlinear objective function was the complexity of the shape of this function that can difficult the solution of the problem of nonlinear parameter identification.

The future steps of this research include the application of the Volterra series to the problem of damage detection in nonlinear structures with a damage index based on the prediction error of the Volterra model and in the deviation of the kernels of the model. Another important issue to be addressed is the main drawbacks presented in these final remarks.

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