COMPARATIVE STUDY OF ANALYTICAL FORMULAE FOR THE
FIRE RESISTANCE OF STEEL BEAM-COLUMNS

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ABSTRACT

This paper investigates the difference between the buckling formulae published in the
Eurocode 3 part 1.2 and the recommendations made in the final report of the Buckling Curves
in Case of Fire (BCCF) research project. This study compares the critical temperatures
obtained with both formulations to assess the impact on the fire endurance of steel columns
subjected to axial compression and bending. An extensive comparison of the ultimate
temperatures obtained with both formulations has been performed (382 profiles, buckling
about the strong and weak axis, 12 column lengths, 6 M/N ratios and uniform and triangular
bending moment distributions). Failure temperatures between 400°C and 860°C have been
considered. The formulations are also compared with Finite Elements (F.E.) calculations
performed for a S235 HEA 200 at 600°C. This analysis shows that for buckling about the
strong axis the BCCF method is better than the EC3 but for buckling about the weak axis the
EC3 predicts failure temperatures closer to the F.E. model than the BCCF formulation.
Finally, the ultimate temperatures predicted by the two formulations have also been compared
with experimental results from the database SCOFIDAT. This comparison shows that there is
no major difference between the two formulations for small and large bending moments. This
study concludes that the EC3 and BCCF formulations are generally equivalent and that either
formulation can be used.

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1. INTRODUCTION

In 1995, a new procedure for the calculation of steel beam-columns interaction curves in case of fire was proposed as a result of the Buckling Curves in Case of Fire (BCCF) research project funded by the European Convention for Coal and Steel (ECSC agreement: 7210 SA 316/515/618/931). The results were published by Talamona [11], Profil Arbed Recherches [12-14], the European Commission [15] and in international journals (Fransen et al. [16] and Talamona et al. [17]).

The proposed formulation was later accepted to become part of the Eurocode 3 part 1.2 to calculate the fire resistance of steel beam-columns. First they were adopted in the French National Annex, later in the ENV version of the EC3 and finally in the EN version [17].

It appeared recently that the equations published in the official version of EN 1993-1-2 [17] differ somehow from the equations proposed in the original work [11,12,14].

The motivation for this project is to clarify the doubt concerning the safety level of the Eurocode 3 created by the modification of the formulae originally proposed in 1995. The objectives are to identify precisely the differences between the two formulations and to examine their consequences on the fire endurance of steel columns subjected to axial compression and bending.

2. DESCRIPTION OF THE FORMULATIONS

2.1. BCCF Formulae

The original formulation proposed to calculate the fire resistance of steel beam-columns was published by the BCCF research partners in 1995 [11,12,14]. In accordance with this proposal, elements with cross-sections in classes 1 and 2 submitted to bending and axial compression must satisfy the following condition in case of fire:

\[
\frac{N_{x,k}}{X_{a,x}} + k_{x} \cdot M_{x,k} \cdot k_{y} \cdot M_{y,k} \leq 1
\]

The subscripts “y” and “z” refer to the strong axis (or y-axis) and to the weak axis (or z-axis) respectively (except for \( f \), which is the yield strength and \( k \) its reduction factor).

The interaction factors \( k_{x,y} \) and \( k_{x,z} \) should be determined by:

\[
\begin{align*}
    k_{x,y} & = 1 - \frac{\mu_{x,y} \cdot N_{x,k}}{X_{a,x} \cdot k_{y} \cdot f_{y}} & \text{but} \quad k_{x,y} \leq 3 \\
    k_{x,z} & = 1 - \frac{\mu_{x,z} \cdot N_{x,k}}{X_{a,z} \cdot k_{z} \cdot f_{z}} & \text{but} \quad k_{x,z} \leq 3
\end{align*}
\]

Where the coefficients \( \mu_{x,y} \) and \( \mu_{x,z} \) are given by:

\[
\begin{align*}
    \mu_{x,y} & = [2 \cdot \beta_{M,y} - 5] \cdot \frac{Z_{d,y}}{Z_{d,T}} + 0.44 \cdot \beta_{M,y} + 0.29 & \text{but} \quad \mu_{x,y} \leq 0.8 \quad \text{and} \quad Z_{d,T} \leq 1.1 \\
    \mu_{x,z} & = [2 \cdot \beta_{M,z} - 3] \cdot \frac{Z_{d,z}}{Z_{d,T}} + 0.71 \cdot \beta_{M,z} - 0.29 & \text{but} \quad \mu_{x,z} \leq 0.8
\end{align*}
\]

Using the equivalent uniform moment factors \( \beta_{M,y} \) and \( \beta_{M,z} \), defined as:

\[
\beta_{M,y} = 1.8 - 0.7 \cdot \psi_y
\]

\[
\beta_{M,z} = 1.8 - 0.7 \cdot \psi_z
\]
In equation (6), the subscript "i" is defined as "y" or "z" depending on the buckling axis considered. The non-dimensional slenderness estimated at elevated temperature is given by:

\[ \tilde{\lambda}_\theta = \lambda_\theta \frac{k_{20\theta}}{k_{E\theta}} \]  \hspace{1cm} (7)

2.2. Formulae from the EC3

The differences between the original formulae and the ones stated in EN 1993-1-2 lie in the equations used to determine the values of coefficients \( \mu_{\theta,y} \) and \( \mu_{\theta,z} \). In EN 1993-1-2 they are given by equations (8) and (9), which should be compared with equations (4) and (5) of the original proposal.

\[ \mu_{\theta,y} = (1.2 \cdot \beta_{\theta,y} - 1) \lambda_{\theta,y} + 0.44 \beta_{\theta,y} - 0.29 \] \hspace{1cm} \text{but } \mu_{\theta,y} \leq 0.8 \hspace{1cm} (8)

\[ \mu_{\theta,z} = (2 \cdot \beta_{\theta,z} - 5) \lambda_{\theta,z} + 0.44 \beta_{\theta,z} - 0.29 \] \hspace{1cm} \text{but } \mu_{\theta,z} \leq 0.8 \text{ and } \lambda_{\theta,z} \leq 1.1 \hspace{1cm} (9)

The equivalent uniform moment factors \( \beta_{\theta,y} \) and \( \beta_{\theta,z} \) remain unchanged.

2.3. Description of the Main Differences

Figures 1 and 2 show a comparison of the evolution of the ratios \( \mu_{\theta,y} \) and \( \mu_{\theta,z} \) as function of the relative slenderness calculated at elevated temperature (\( \lambda_\theta \)) and of the shape of the bending diagram (see Figure 3).

![Figure 1. \( \mu_{\theta,y} \) as function of \( \lambda_\theta \), y](image)

![Figure 2. \( \mu_{\theta,z} \) as function of \( \lambda_\theta \), z](image)

![Figure 3. Linear bending diagram](image)
Equation (4) of the original proposal has a limitation of the slenderness expressed at room temperature, \( \overline{A}_{20} \leq 1.1 \), which creates a variation of \( \mu_{i,a} \) as function of temperature (see equation (7)). The curves established at 700°C and 900°C represent the maximum and minimum values of the ratio \( \sqrt{F_{y,\theta}}/F_{y,\theta} \) (1.33 and 0.9428 respectively).

For bi-triangular bending moment distribution (\( \psi = -1 \)), the coefficients \( \mu_{i,a} \) and \( \mu_{i,b} \) are equal to the maximum value of 0.8 for both formulations. In this case, as the two formulations have the same \( \mu_{i,a} \) and \( \mu_{i,b} \) as input coefficients, the same ultimate temperature will be predicted. Thus this load case will be left out of this investigation.

The interaction curve given by equation (1) can be written in the following schematic way if the member is subjected to compression and bending only about one axis (y-axis or z-axis). If \( N' \) and \( M' \) are defined as:

\[
N' = \frac{N_{f,i,Ed}}{x_{\text{min},i} A_B k_{y,\theta} f_s Y_{M,i}}
\]

and

\[
M' = \frac{M_{i,Ed}}{k_{y,\theta} f_s Y_{M,i}}
\]

then equation (1), (2) and (3) become equal to the following system of equations:

\[
N' + M' = \mu_{i,a} \cdot N' \cdot M' = 1 \quad \text{if} \quad k_{s,i} = 1 - \mu_{i,a} \cdot N' \leq 3
\]

and

\[
N' + 3 \cdot M' = 1 \quad \text{if} \quad k_{s,i} = 1 - \mu_{i,a} \cdot N' > 3
\]

The other coefficients obtained from equations (4) to (9) remain unchanged.

In equation 11 and 12, the subscript “i” for \( M_{i,Ed} \), \( k_{y,i} \) and \( \mu_{i,a} \) correspond to the letter “y” and “z” depending on the bending axis considered. Figure 4 shows that the shape of the interaction curve is very sensitive to variations of the coefficient \( \mu_{i,a} \) when it varies from 0.8 to 0, which is the case for bi-triangular bending moment distribution and also short columns subjected to other bending moment distributions. As \( \mu_{i,a} \) decreases the M-N interaction curve becomes less and less sensitive to its variation. For example if \( \mu_{i,a} \) varies from -3 to -4, which are typical values of slender columns subjected to uniform bending moment distribution, the variation of the shape of the M-N curve is less than the previous variation from 0.8 to 0 (see figure 4). The linear descending branch near the \( N' \) axis comes from the limitation of \( k_{s,i} \) to a maximum value of 3 (from equation (2) and (3)) which means that equation (13) should be used instead of equation (12).

![Fig. 4. Influence of the coefficient \( \mu \) on the M-N interaction diagram](image-url)

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As the ultimate axial load capacity in pure compression and ultimate bending moment (M_u) in pure bending are independent from the coefficients \( \mu_{\mu,\rho} \) and \( \mu_{\rho,\rho} \), these values are not affected by the modifications introduced in the Eurocode 3 part 1.2. This implies that the starting and ending points of the M-N interaction curves are not affected by the modifications of the formulae and only the shape of the curve between pure compression and pure bending is modified.

### 3. COMPARISON OF THE FAILURE TEMPERATURES

#### 3.1. Comparison Between the Two Formulations

A comparison of the failure temperatures obtained with both formulations has been performed using the following assumptions:

- Profiles in class 1 and 2 in compression and bending have been considered (382 profiles).
- Steel S235 with reduction of the yield strength depending on the thickness of the flanges
  - 235 MPa for \( t_s \leq 16 \) mm
  - 225 MPa for \( t_s \leq 40 \) mm
  - 215 MPa for \( t_s \leq 100 \) mm
  - 195 MPa for \( t_s \leq 150 \) mm
- Twelve reduced slenderness have been considered (\( \bar{A}_{20} \) from 0.2 to 2.4 with an increment of 0.2)
- Six MN ratios (0.05, 0.1, 0.5, 1.0, 3.0 and 5.0 multiplied by the radius of gyration about the buckling axis considered)
- Buckling about the strong and weak axis
- Two bending moment distributions (uniform: \( \psi = 1 \) and triangular: \( \psi = 0 \))
- Failures temperatures between 400°C and 860°C

Note: axially loaded columns, columns in pure bending and bi-triangular (\( \psi = -1 \)) bending moment distribution have not been considered, as these load cases are not affected by the modification introduced in the EC3.

Figures 5, 6, 7 and 8 show a comparison of the failure temperatures obtained with the Eurocode 3 and the BCCF formulations for: profiles in class 1 and 2, non-dimensional slenderness from 0.2 to 2.4, uniform (\( \psi = 1 \)) and triangular (\( \psi = 0 \)) bending moment and buckling about the “\( y \)” and “\( z \)” axis. As can be seen there is no major difference between the temperatures predicted by both formulations. The biggest differences in ultimate temperatures are obtained for short columns submitted to a triangular bending moment and heavily loaded (failure temperatures under 600°C). These discrepancies are due to the fact that under these conditions the values of \( \mu_{\mu,\rho} \) and/or \( \mu_{\rho,\rho} \) are between 0.8 and -1 and it has been shown that in this range the M-N interaction curve is extremely sensitive to variations of \( \mu_{\mu,\rho} \) or \( \mu_{\rho,\rho} \). Noticeable differences also appear for slender columns (\( \bar{A}_{20} \geq 2.0 \)).

#### 3.2. Comparison with Finite Element Results

Figure 9 and 10 show the interaction curves for the case of a welded HEA 200 in S235, at 600 °C submitted to flexural buckling around the \( y \) axis and the \( z \) axis respectively. These figures show that for the major axis buckling the BCCF method is better than the Eurocode 3 but in the minor axis the Eurocode 3 seems to be better. Nevertheless both formulations provide similar results.
Fig. 5 - Buckling strong axis for $\psi = 1$ and $0.2 \leq \overline{\lambda}_20 \leq 2.4$
Fig. 6 - Buckling strong axis for \( \psi = 0 \) and \( 0.2 \leq \bar{A}_{20} \leq 2.4 \)
Fig. 7 - Buckling weak axis for $\psi = 1$ and $0.2 \leq \gamma_{20} \leq 2.4$

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Fig. 8 - Buckling weak axis for $\psi = 0$ and $0.2 \leq \frac{\lambda}{20} \leq 2.4$

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Fig. 9 - Interaction curves for beam column with bending around the major axis, \((N+M_x)\)

\[ L=1000\text{mm}; \quad \lambda_{x,\theta} = 0.159 \]
\[ L=3000\text{mm}; \quad \lambda_{x,\theta} = 0.477 \]
\[ L=5000\text{mm}; \quad \lambda_{x,\theta} = 0.795 \]

\[ L=7000\text{mm}; \quad \lambda_{x,\theta} = 1.112 \]
\[ L=10000\text{mm}; \quad \lambda_{x,\theta} = 1.589 \]
\[ L=14000\text{mm}; \quad \lambda_{x,\theta} = 2.225 \]

Fig. 10 - Interaction curves for beam column with bending around the minor axis, \((N+M_y)\)

\[ L=1000\text{mm}; \quad \lambda_{y,\theta} = 0.258 \]
\[ L=3000\text{mm}; \quad \lambda_{y,\theta} = 0.773 \]
\[ L=5000\text{mm}; \quad \lambda_{y,\theta} = 1.289 \]

\[ L=7000\text{mm}; \quad \lambda_{y,\theta} = 1.804 \]
\[ L=10000\text{mm}; \quad \lambda_{y,\theta} = 2.578 \]
3.3. Comparison with the Database SCOFIDAT

SCOFIDAT is a database that contains over 140 fire tests results performed on steel beam-columns under a wide range of loading, boundary conditions and fire exposures. The data was provided by seven labs. The experimental results obtained from axially loaded columns and tests with failure temperatures under 400°C were left out of this study, giving just above 60 experimental results that are considered. It has to be noted that all members were subjected to compression and uniform bending moment distribution.

![Comparison of the formulations with experimental results](image)

*Fig. 11. Comparison of the formulations with experimental results*

As can be seen figure 11, both formulations are in good agreement with experimental results. The slopes of the curves are both close to 1.0, the Y-intercept is small, and the regression coefficient above 0.8.

4. CONCLUSIONS

From this investigation, it can be concluded that the formulations from the Eurocode and BCCF research project are:
- Usually similar in terms of failure temperature
- When differences are observed the Eurocode 3 is usually conservative.
- Both formulations provide similar failure temperatures when the predicted temperatures are above 550°C
- Both formulations provide reasonable agreement with experimental results.

From the results presented in this paper it can be concluded that the new formulation is not likely to decrease the safety level of constructions. More investigation needs to be performed to assess the impact of these modifications on the competitiveness of steel structures.

5. NOMENCLATURE

\[ A \] area of the cross-section
\[ f_y \] yield strength at 20°C
\[ k_{y,\theta} \] reduction factor for the yield strength of steel at the steel temperature \( \theta \)
\[ k_{E,\theta} \] reduction factor for the elastic modulus of steel at the steel temperature \( \theta \)
\[ k_{\theta,\beta} \] interaction factor about the y-axis
\[
k_{z,y} \quad \text{interaction factor about the z-axis}
\]
\[
M_{y,zz} \quad \text{design moment resistance about the y-axis}
\]
\[
M_{z,zz} \quad \text{design moment resistance about the z-axis}
\]
\[
N_{a,zz} \quad \text{applied axial load}
\]
\[
W_{pl,y} \quad \text{plastic section modulus about the y-axis}
\]
\[
W_{pl,z} \quad \text{plastic section modulus about the z-axis}
\]
\[
\beta_{m,\psi} \quad \text{equivalent uniform moment factor}
\]
\[
\beta_{m,\chi} \quad \text{equivalent uniform moment factor}
\]
\[
\gamma_{m,\psi} \quad \text{partial factor for the relevant material property, for the fire situation}
\]
\[
\lambda_{20,y} \quad \text{non-dimensional slenderness at } 20^\circ\text{C}
\]
\[
\lambda_{20,z} \quad \text{non-dimensional slenderness at } 20^\circ\text{C}
\]
\[
\lambda_{\psi,y} \quad \text{non-dimensional slenderness at temperature } \theta
\]
\[
\lambda_{\psi,z} \quad \text{non-dimensional slenderness at temperature } \theta
\]
\[
\mu_{\psi} \quad \text{interaction coefficient about the y-axis}
\]
\[
\mu_{\chi} \quad \text{interaction coefficient about the z-axis}
\]
\[
\lambda_{\min,\psi} \quad \text{minimum value of } \lambda_{\psi}, \text{and } \lambda_{\chi}
\]
\[
\chi_{y,z} \quad \text{reduction factor for flexural buckling about the y-axis in the fire design situation}
\]
\[
\chi_{z,y} \quad \text{reduction factor for flexural buckling about the z-axis in the fire design situation}
\]
\[
\psi \quad \text{ratio between lowest and highest bending moment about the y-axis or z-axis}
\]

6. REFERENCES


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