Proceedings of the 36th
Conference on Decision & Control
San Diego, California USA • December 1997

An Adaptive High-Gain Observer for Nonlinear Systems

Eric Bullinger Frank Allgöwer [1]
Institut für Automatik
ETH Zürich
CH 8092 Zürich, Switzerland

Abstract

In this paper we present a high-gain observer for a general class of nonlinear SISO systems for which the high-gain parameter is determined on-line in an adaptive fashion. The adaptation scheme is simple and universal in the sense that it is independent of the system the observer is designed for. We prove that the observer output error becomes smaller than a user specified bound for large times and that the adaptation converges. The assumptions required for the adaptive high-gain observer are the same as for the non-adaptive high-gain observer, namely that the system is uniformly observable for any $u(t)$.

keywords: nonlinear observers, adaptive, high-gain, robustness, bioreactor.

1 Introduction

For systems that are uniformly observable for any $u(t)$ (i.e. the states of the system can be determined from the output of the system and its derivatives, independently of the input)[3], a high-gain observer has been suggested in [9]. One of the advantages of this observer are its excellent robustness properties [9]. By choosing the observer gain k large enough (therefore the name "high-gain") the observer error can be made arbitrarily small. The difficulty in practical applications is, however, the determination of an appropriate value for the observer gain. For values too low, the desired bounds on the observer error cannot be achieved. For values unnecessarily high, the sensitivity to noise increases, thus limiting the practical use.

In this paper we propose an adaptation scheme for the observer gain of the high-gain observer in [9] such that its advantages are retained and that the observer gain is adjusted automatically until the observer output error becomes smaller than a desired target value.

The paper is organized as follows: in Section 2 we recall the main result on the high-gain observer in [9]. In Section 3 we present the adaptation scheme and prove the boundedness of the observer error and convergence of the adaptation scheme. The paper is concluded with a simple bioreactor example.

2 High-Gain Observer

The theory of high-gain observers as in [9] assumes that the system is given in observability normal form [11], also called the generalized controller canonical form [2]. In principle, every uniformly observable SISO-system with input u and output y can be transformed into this normal form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= \phi(\mathbf{x}, \mathbf{u}) \\ y &= x_1. \end{aligned} \tag{1}$$

with $\mathbf{x} = [x_1, \dots, x_n]^T$ and $\mathbf{u} = [u, \dot{u}, u^{(2)}, \dots, u^{(n)}]^T$.

A possibility to observe such systems (1) is the high-gain observer [9]. The structure of the high-gain observer is a simple chain of integrators, each "corrected" by the injection of the output error $(y - \hat{y})$ multiplied by a factor depending on the constant observer gain k:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + p_1 k (y - \hat{y}) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + p_2 k^2 (y - \hat{y}) \\ &\vdots \\ \dot{\hat{x}}_{n-1} &= \hat{x}_n + p_{n-1} k^{n-1} (y - \hat{y}) \\ \dot{\hat{x}}_n &= p_n k^n (y - \hat{y}) \\ \hat{y} &= \hat{x}_1. \end{aligned} \tag{2}$$

where $\hat{\mathbf{x}} = [\hat{x}_1, \dots, \hat{x}_n]^T$ denotes the estimate of the states $\mathbf{x}$ and $\hat{y}$ the estimate of the system's output (cf. Figure 1).

In contrast to the classical Luenberger observer [6], the high-gain observer does not consist of a replica of the system (1) plus correction terms as the nonlinearity $\phi(\mathbf{x}, \mathbf{u})$ is not modeled. The observer error will be denoted by $\mathbf{e}$ with $\mathbf{e}(t) = [e_1(t), \dots, e_n(t)]^T$ with $e_i(t) = x_i(t) - \hat{x}_i(t)$.

[1] correspondence address: {bullinger,allgower}@aut.ee.ethz.ch
phone: + 41 1 632 3557, fax: + 41 1 632 1211

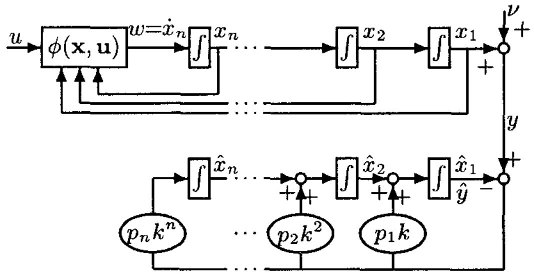


Figure 1: High-gain observer for a system in observability normal form.

The following theorem is proven in [9].

Theorem 1 (High-Gain Observer). *Assume that*

A1) the system (1) *exhibits no finite escape time and*

A2) the nonlinearity ϕ *in* (1) *is bounded,. i.e.* $\|\phi(\mathbf{x},\mathbf{u})\| \leq \mu \quad \forall \mathbf{x}, \mathbf{u}$.

If the coefficients $\{p_1, \dots, p_n\}$ *are such that* $s^n + \sum_{j=1}^{n} p_j s^{n-j}$ *is a Hurwitz polynomial with distinct roots, then for all* $d > 0$ *and all times* $\bar{t} > 0$ *there exists a finite observer gain* $\bar{k}$ *such that for all constant* $k \geq \bar{k}$ *the observer error satisfies:*

$$\|\mathbf{e}(t)\| \leq d \quad \forall t \geq \bar{t}.$$

This means that by an appropriate choice of the observer gain k the observer error $\mathbf{e}$ can be made arbitrarily small in an arbitrarily short time.

In the Laplace domain, the relationship between $w(s) = \mathcal{L}\{\dot{x}_n(t)\}$, an additive output noise $\nu(s)$ (see Fig. 1), and the observer output error $e_1(s) = y(s) - \hat{y}(s)$ is given by:

$$e_1(s) = \frac{1}{(s+k)^n} w(s) + \frac{s^n}{(s+k)^n} \nu(s), \tag{3}$$

where, for simplicity, $p_i = \binom{n}{i}$. From (3) it is obvious that the larger the observer gain k is chosen, the smaller the influence w and thus of the nonlinearity ϕ (and therefore of u) on the observer output error. For a large observer gain, the observer is thus very robust, provided the dimension of the state-space is known [9]. However, for large values of k the additive output noise is damped in the observer output error, and therefore undamped in the observer output.

3 Adaptive High-Gain Observer

To overcome the difficulty of having to choose the observer gain k, we propose to use a simple adaptation law to find the appropriate observer gain for all $t \geq t_0$:

$$\begin{aligned} \frac{d}{dt}s(t) &= \begin{cases} \gamma \cdot |y(t) - \hat{y}(t)|^2 & \text{for } |y(t) - \hat{y}(t)| > \lambda \\ 0 & \text{for } |y(t) - \hat{y}(t)| \leq \lambda \end{cases} \\ t_i &: \ s(t_i) = S_i \qquad i = 0, 1, 2, \dots \\ k(t) &= S_i \qquad \forall t \in [t_i, t_{i+1}) \end{aligned} \tag{4}$$

where $\lambda > 0$, $\gamma > 0$, $\beta > 0$, S_0 are pregiven and

$$S_{i+1} - S_i = \beta\, e^{i^2} \quad \forall i \geq 0. \tag{5}$$

The idea behind this adaptation law is that the observer gain k is piecewise constant and takes values S_i. The switch to the new S_i depends on the monotonically increasing parameter s. Whenever s reaches a new threshold S_i (the time when this occurs is denoted by t_i) the observer gain takes this value S_i. The S_i are predefined as a monotonically increasing sequence such that their growth rate is larger than e^i. One possibility to guarantee this is given in (5). For any other choice with growth rate larger than e^i, like

$$S_{i+1} - S_i = \beta e^{i \ln(i)},$$

Theorem 2 below also holds. Thus, k(t) is increased step-wise as long as $y - \hat{y}$ lies outside λ and cannot decrease.

We now prove that the high-gain observer (2) with a time-varying observer gain k as in adaptation law (4) guarantees convergence of the adaptation law and boundedness of the observer error.

Theorem 2. *Assume that system* (1) *satisfies assumptions A1) and A2). If the coefficients* $\{p_1, \dots, p_n\}$ *are such that* $s^n + \sum_{j=1}^{n} p_j s^{n-j}$ *is a Hurwitz polynomial then a high-gain observer* (2) *with the adaptation law* (4), (5) *for the observer parameter* k *achieves for any* $\lambda > 0, \gamma > 0, \beta > 0$ *and any* S_0*:*

a) $k(t) < k_\infty < \infty \quad \forall t$

b) the total length of time for which the observer output error is larger than λ *is finite, i.e.* $\exists t_{max} < \infty : \int_{\mathbf{T}} dt < t_{max}$, *where* $\mathbf{T} = \{t | \ \|y(t) - \hat{y}(t)\| \geq \lambda\}$.

When the observer output error has been larger than λ for t_{max}, the observer output error will afterwards always be smaller than λ, which implies that the adaptation has converged.

Proof of a). The error differential equations are:

$$\begin{aligned} \dot{e}_1 &= e_2 && - \ p_1 k\, e_1 \\ \dot{e}_2 &= e_3 && - \ p_2 k^2\, e_1 \\ &\vdots \\ \dot{e}_{n-1} &= e_n && - \ p_{n-1} k^{n-1}\, e_1 \\ \dot{e}_n &= \phi(\mathbf{x},\mathbf{u}) && - \ p_n k^n\, e_1. \end{aligned} \tag{6}$$

With a coordinate change as in singular perturbation theory:

$$\begin{aligned} \tau &= t \cdot k \\ \tilde{e}_j &= e_j/k^{j-1}, \quad j = 1, \dots, n, \end{aligned} \tag{7}$$

the transformed differential equations are:

$$\begin{aligned} \tfrac{d}{d\tau}\tilde{e}_1 &= \tilde{e}_2 & - \; p_1\,\tilde{e}_1 \\ \tfrac{d}{d\tau}\tilde{e}_2 &= \tilde{e}_3 & - \; p_2\,\tilde{e}_1 \\ &\vdots & \\ \tfrac{d}{d\tau}\tilde{e}_{n-1} &= \tilde{e}_n & - \; p_{n-1}\,\tilde{e}_1 \\ \tfrac{d}{d\tau}\tilde{e}_n &= \tfrac{1}{k^n}\phi(\mathbf{x},\mathbf{u}) & - \; p_n\,\tilde{e}_1. \end{aligned} \tag{8}$$

To simplify the notation we write (8) as:

$$\frac{d}{d\tau}\tilde{\mathbf{e}} = A\tilde{\mathbf{e}} + B\phi(\mathbf{x},\mathbf{u}) \tag{9}$$

with:

$$A = \begin{bmatrix} -p_1 & 1 & 0 & & \\ -p_2 & 0 & 1 & & \\ \vdots & & & \ddots & \\ -p_{n-1} & 0 & 0 & & 1 \\ -p_n & 0 & 0 & & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T; \qquad B = \frac{1}{k^n} \cdot b.$$

As $s^n + \sum_{j=1}^{n} p_j s^{n-j}$ is a Hurwitz polynomial, A is a Hurwitz matrix. Thus, there exists a symmetric, positive definite matrix P, s.t.

$$A^T P + PA = -I. \tag{10}$$

Therefore, a Lyapunov function candidate is:

$$V = \tilde{\mathbf{e}}^T P \tilde{\mathbf{e}}$$

with the properties:

$$\begin{gathered} V > 0 \quad \forall\, \tilde{e} \neq 0 \\ V = 0 \Longleftrightarrow \tilde{e} = 0 \\ \sigma_{min}^2(P)\|\tilde{\mathbf{e}}\|^2 \leq V \leq \sigma_{max}^2(P)\|\tilde{\mathbf{e}}\|^2, \end{gathered}$$

where $\sigma(\cdot)$ denotes a singular value.

First we define a finite constant $\bar{k}$:

$$\bar{k} = \sqrt[n]{\frac{8\,\|p\|\,\mu}{\lambda}}, \tag{11}$$

where $p = Pb$ denotes the last column of P.

The proof of a) is by contradiction. Assume that $s(t) \to \infty$. Then there exists a finite time denoted t_{i^*} s.t.:

$$k(t) > \bar{k} \;\; \forall t > t_{i^*}.$$

We now look at a single time interval $[t_i, t_{i+1})$, where k is constant by (4). The derivative of V along a trajectory of (9) is:

$$\begin{aligned} \frac{d}{d\tau}V &= \frac{d\tilde{\mathbf{e}}}{d\tau}^T P\tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T P \frac{d\tilde{\mathbf{e}}}{d\tau} \\ &= \left(A\tilde{\mathbf{e}} + B\phi(\mathbf{x},\mathbf{u})\right)^T P\,\tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T P \left(A\tilde{\mathbf{e}} + B\phi(\mathbf{x},\mathbf{u})\right). \end{aligned}$$

With (10) this is equal to:

$$\frac{d}{d\tau}V = -\|\tilde{\mathbf{e}}\|^2 + 2\tilde{\mathbf{e}}^T p\;\phi(\mathbf{x},\mathbf{u})\;\frac{1}{k^n}. \tag{12}$$

For $t \geq t_{i^*}$, i.e. $t \in [t_{i^*+i}, t_{i^*+i+1})$, $i \geq 0$, we can bound the derivative of V by:

$$\begin{aligned} \frac{d}{d\tau}V &\leq -\|\tilde{\mathbf{e}}\|^2 + 2\,\|\tilde{\mathbf{e}}\|\,\|p\|\,\mu\,\frac{1}{\bar{k}^n} \\ &\leq -\|\tilde{\mathbf{e}}\|^2 + \frac{1}{4}\|\tilde{\mathbf{e}}\|\,\lambda \\ &< -\frac{1}{2}\|\tilde{\mathbf{e}}\|^2, \quad \forall\, \|\tilde{\mathbf{e}}\| > \lambda/2. \end{aligned}$$

Thus, V is a "Lyapunov function" for $\|\tilde{\mathbf{e}}\| > \lambda/2$ and $\|\tilde{\mathbf{e}}(t)\|$ is decreasing exponentially on any time interval $[t_{i^*+i}, t_{i^*+i+1}), i \geq 0$ [10] as long as $\|\tilde{\mathbf{e}}\| > \lambda/2$:

$$\|\tilde{\mathbf{e}}(t)\| \leq \kappa(P) e^{-k(t_{i^*+i})\frac{t - t_{i^*+i}}{4\,\sigma_{max}(P)}} \|\tilde{\mathbf{e}}(t_{i^*+i})\|. \tag{13}$$

where $\kappa(P)$ is the condition number of P, i.e.

$$\kappa(P) = \frac{\sigma_{max}(P)}{\sigma_{min}(P)}.$$

We now look at the effect of switching ($i \geq 0$):

$$\begin{aligned} e_j(t_{i+1}) &= \lim_{t \to t_{i+1}} e_j(t) \\ &= k(t_i)^{j-1} \lim_{t \to t_{i+1}} \tilde{e}_j(t) \\ \tilde{e}_j(t_{i+1}) &= \left(\frac{1}{k(t_{i+1})}\right)^{j-1} e_j(t_{i+1}) \\ &= \xi^{j-1} \lim_{t \to t_{i+1}} \tilde{e}_j(t), \quad \xi = \frac{k(t_i)}{k(t_{i+1})}, \end{aligned}$$

and hence

$$\tilde{\mathbf{e}}(t_{i+1}) = \Xi \lim_{t \to t_i} \tilde{\mathbf{e}}(t),$$

with $\Xi = diag(1, \xi, \cdots, \xi^{n-1})$.

As $\xi \in (0, 1)$, for all $i \geq 0$, the switching does not increase $\|\tilde{\mathbf{e}}(t)\|$:

$$\begin{aligned} \|\tilde{\mathbf{e}}(t_{i+1})\| &= \|\Xi \cdot \lim_{t \to t_{i+1}} \tilde{\mathbf{e}}(t)\| \\ &\leq \underbrace{\sigma_{max}(\Xi)}_{=1} \cdot \lim_{t \to t_{i+1}} \|\tilde{\mathbf{e}}(t)\|. \end{aligned} \tag{14}$$

Combining (13) and (14), we see that the increase with growing i of $\|\tilde{\mathbf{e}}(t_{i^*+i})\|$ for any $i \geq 0$ can be bounded:

$$\|\tilde{\mathbf{e}}(t_{i^*+i+1})\| \;\; \leq \;\; \kappa(P)\;\|\tilde{\mathbf{e}}(t_{i^*+i})\|. \tag{15}$$

With (7), it follows that:

$$|y(t) - \hat{y}(t)| = |\tilde{e}_1(t)| \leq \|\tilde{\mathbf{e}}(t)\|. \tag{16}$$

For $i \geq 0$, applying (16) to (4) results in:

$$\begin{aligned} S_{i^*+i+1} - S_{i^*+i} &= \int_{t_{i^*+i}}^{t_{i^*+i+1}} \dot{s}(t)dt \\ &= \int_{t_{i^*+i}}^{t_{i^*+i+1}} \gamma \, |y(t) - \hat{y}(t)|^2 dt \\ &\leq \int_{t_{i^*+i}}^{t_{i^*+i+1}} \gamma \, ||\tilde{\mathbf{e}}(t)||^2 dt. \end{aligned}$$

Using (13) the right hand side can be bounded:

$$\begin{aligned} S_{i^*+i+1} - S_{i^*+i} &\leq \gamma \kappa(P)^2 ||\tilde{\mathbf{e}}(t_{i^*+i})||^2 \\ &\quad \cdot \int_{t_{i^*+i}}^{t_{i^*+i+1}} e^{-k(t_{i^*+i}) \frac{t - t_{i^*+i}}{2\,\sigma_{max}(P)}} dt \\ &\leq \kappa(P)^2 \frac{2\gamma\sigma_{max}(P)}{k(t_{i^*+i})} ||\tilde{\mathbf{e}}(t_{i^*+i})||^2. \end{aligned}$$

With (15) we finally get:

$$S_{i^*+i+1} - S_{i^*+i} \leq \kappa(P)^{2i+2} \frac{2\gamma\sigma_{max}(P)}{k(t_{i^*})} ||\tilde{\mathbf{e}}(t_{i^*})||^2. \quad (17)$$

By (5), the growth condition on S, there exists a finite index $\bar{i}$, such that the left side of (17) is greater than the right side. Thus, the assumption that $s(t) \to \infty$ does not hold and, therefore, the observer gain must be bounded:

$$\lim_{t\to\infty} k(t) = k(t_{i^*+\bar{i}}) < k(t_{i^*+\bar{i}+1}) < \infty.$$

Setting $k_\infty = k(t_{i^*+\bar{i}+1})$ completes the proof of a).

Proof of b). From (4) and (16) we get that:

$$\begin{aligned} k_\infty - k_0 &= \int_0^\infty \dot{s}(t)dt \geq \lambda\gamma \int_{\mathbf{T}} dt \\ \text{and } t_{max} &= \frac{k_\infty - k_0}{\lambda\gamma} < \infty. \end{aligned}$$

■

Theorem 3. *Assume that system (1) satisfies assumptions A1) and A2). If the coefficients $\{p_1, \ldots, p_n\}$ are such that $s^n + \sum_{j=1}^n p_j s^{n-j}$ is a Hurwitz polynomial and $k(t) \geq \hat{k}$ $\forall t \geq \hat{t}$ then the high-gain observer (2) achieves:*

$$||\mathbf{e}(t)|| \leq \begin{cases} \frac{2||p||\mu}{\hat{k}} & \hat{k} \geq 1 \\ \frac{2||p||\mu}{\hat{k}^n} & \hat{k} < 1 \end{cases} \quad \textit{for large } t,$$

where p is defined as in (11).

Proof of Theorem 3. We will show that for any fixed k the observer error $||\mathbf{e}||$ converges to a domain included in the k-dependent ball around the origin.

From (12) it can be seen that

$$\frac{d}{d\tau}V < 0 \quad \forall \, ||\tilde{\mathbf{e}}|| > \frac{2||p||\mu}{k^n}.$$

Therefore, $||\tilde{\mathbf{e}}||$ converges to a subdomain of $\tilde{\Omega} = \left\{ \mathbf{x} \,\middle|\, ||\mathbf{x}|| \leq \frac{2||p||\mu}{k} \right\}$.
As $\mathbf{e} = K\tilde{\mathbf{e}}$, with $K = diag(1, k, \ldots, k^{n-1})$, $||\mathbf{e}||$ converges to a domain contained in the region Ω:

$$\begin{aligned} \Omega &= \left\{ \mathbf{x} \,\middle|\, ||\mathbf{x}|| \leq ||K^{-1}|| \frac{2||p||\mu}{k} \right\} \\ &= \left\{ \mathbf{x} \,\middle|\, ||\mathbf{x}|| \leq \max\{1, k^{-(n-1)}\} \frac{2||p||\mu}{k} \right\}. \end{aligned}$$

Therefore, for large t,

$$||\mathbf{e}(t)|| \leq \left\{ \begin{array}{ll} \frac{2||p||\mu}{k} & k \geq 1 \\ \frac{2||p||\mu}{k^n} & k < 1 \end{array} \right\}.$$

■

Summarizing Theorems 2 and 3, the adaption of the proposed high-gain observer converges; for large times the observer output error is smaller than some user specified bound and the observer error is bounded.

Remark 1. *The idea of the proof of Theorem 2 a) can be used to generalize the results in [9] to the case of nondistinct roots.*

Remark 2. *From (13) holding on any interval $[t_i, t_{i+1})$, $i \geq i^*$, it does not follow that (13) holds for all $t \geq t_{i^*}$ because the switching can increase the error, i.e. it can happen that $||\mathbf{e}(t_i)|| < ||\mathbf{e}(t_{i+1})||$.*

Corollary. *If S_0 is chosen s.t.*

$$S_0 \geq \frac{2||p||\mu}{\lambda}$$

then the norm of the observer error $\mathbf{e}(t)$ is bounded by λ for large times, i.e.

$$||\mathbf{e}|| \leq \lambda \quad \textit{for large } t.$$

Proof of Corollary. The result follows immediately from Theorem 3 and is therefore omitted. ■

The proposed adaptive high-gain observer is easy to implement (as only the state-space dimension of the system has to be known) and retains the advantages of the non-adaptive high-gain observer. Robustness is improved by the adaptation law as it enables the user to start with a small observer gain that is increased only as needed. In a non-adaptive scheme the observer gain is usually chosen in a conservative way, which causes the high-gain observer to be less performant in the presence of output measurement noise than the adaptive high-gain observer.

4 Example

To demonstrate the adaptive high-gain observer, the proposed method is applied to a bioreactor as given in [1].

The bioreactor is represented by the following simple model derived from material balances:

$$\begin{aligned} \dot{m} &= \tfrac{a_1 m s}{a_2 m + s} - u\, m \\ \dot{s} &= -\tfrac{a_3 a_1 m s}{a_2 m + s} - u\, s + u\, a_4 \\ y &= m. \end{aligned} \tag{18}$$

m and s denote the concentration of the microorganism and the substrate respectively, u is the substrate inflow rate which is considered as input. All state variables are strictly positive and the parameters are $a_1 = a_2 = a_3 = 1$, $a_4 = 0.1$, $m(0) = 0.075$, $s(0) = 0.03$.

The system (18) can easily be transformed into the observability normal form: the new state variables x_1, x_2 being defined by:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \Phi(m, s, u, \dot{u}) = \begin{bmatrix} m \\ \dot{m} \end{bmatrix}. \tag{19}$$

In [4] it has been shown that assumptions A1) and A2) are satisfied for $u \in (0, a_1)$.

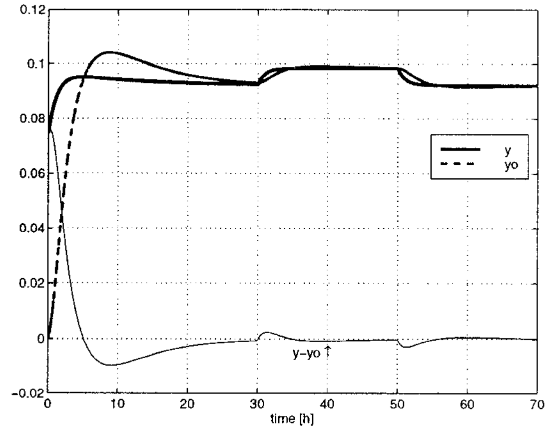
For the observer, the following values are used: $\lambda = 0.02, \gamma = 100$, $p_1 = 1$, $p_2 = 0.2$, $S_0 = 0.1$, $S_i = S_0\, e^{\alpha i^2}$, $\alpha = 0.0001$, which also satisfies growth condition (5). The following substrate input flow profile is used for the simulations:

$$u(t) = \begin{cases} 0.08\, h^{-1} & t \in [0, 30)h \\ 0.02\, h^{-1} & t \in [30, 50)h \\ 0.08\, h^{-1} & t > 50h \end{cases}.$$

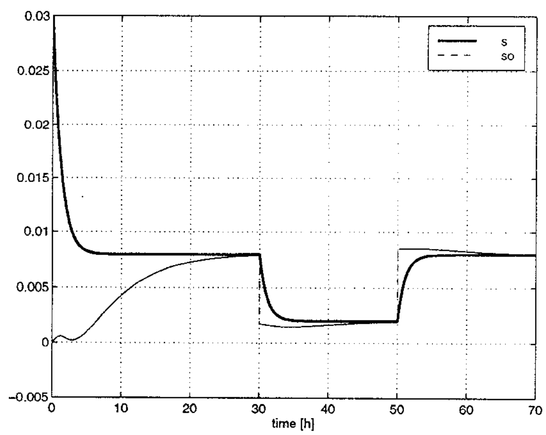
y is the plant output without, yn with noise. yo is the observer output. so is the estimated value for s, calculated via $\Phi^{-1}(x_{o_1}, x_{o_2}, u, \dot{u})$, where x_{o_1}, x_{o_2} are the states of the observer.

The first simulation is shown in Figure 2. It can be seen that without noise the state-estimates come close to the true value of the states. After the transient phase the error is smaller than the target error $\lambda = 0.01$. In Figure 2.a the output of the bioreactor (the concentration of the microorganism) and its estimate as well as the input are shown. In Figure 2.b the concentration of the substrate and its calculated estimate are shown. Figure 2.c shows that k increases rapidly (because of the large value γ) and then stays constant at a value that is not "high" as the name of the observer implies.

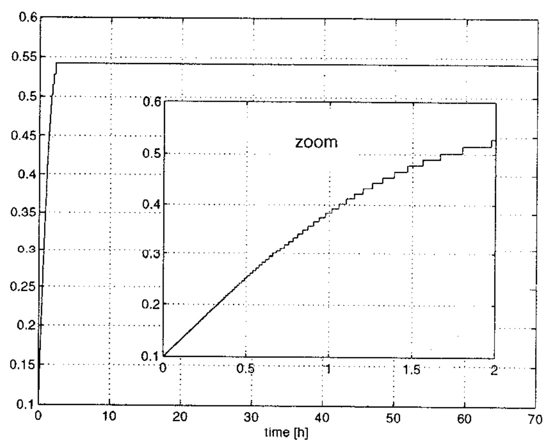
For the second simulation (Figure 3) band-limited white noise with a rather high power spectral density of 0.2510^{-6} and a sampling time of 0.01 h is used. Here, the observer behaves similarly to the case without noise (Figure 2). Especially note that the observer gain in the presence of a low noise level (Figure 3.c) is not much higher than in the case without noise.



(a) "True" (y) and estimated (yo) concentration of microorganism and observer output error $y - y_o$.

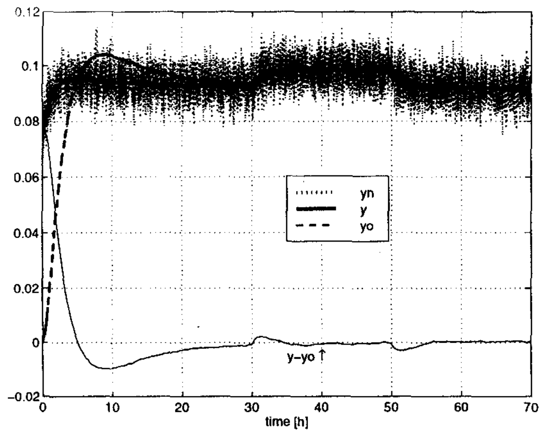


(b) Estimated (so) and "true" (s) concentration of substrate.

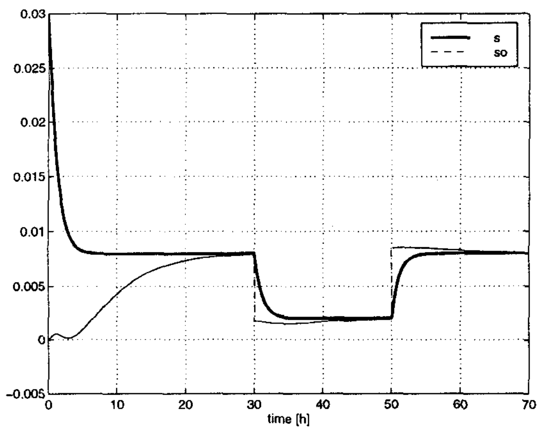


(c) Observer gain k.

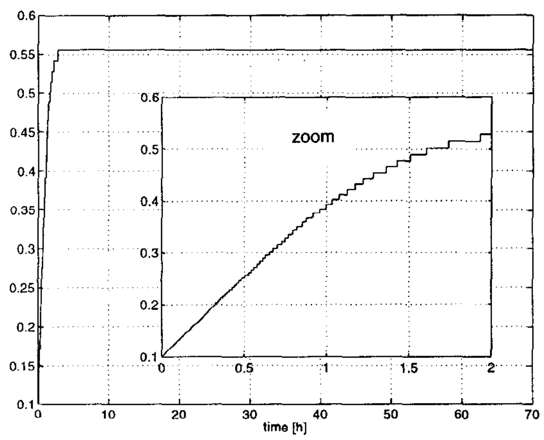
Figure 2: State-estimation of the bioreactor without noise.



(a) "True" (y), "true" with additive noise (yn), and estimated (yo) concentration of microorganism and observer output error $y - y_o$.



(b) Estimated (so) and "true" (s) concentration of substrate.



(c) Observer gain k.

Figure 3: State-estimation of the bioreactor with noise.

Conclusions

In this paper we introduced an adaptive extension to the high-gain observer originally proposed in [8]. The adaptation of the observer gain is done via a very simple adaptation law that is universal in the sense that it is independent of the system to be observed. Thus, this adaptation fits in nicely into the philosophy of the high-gain observer. We have proved that with the adaptive high-gain observer the observer output error becomes smaller than an arbitrary user-specified bound for large times and that the adaptation converges. The assumptions needed are the same as required for the non-adaptive high-gain observer.

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