Laboratoire de Techniques Aéronautiques et Spatiales (ASMA)
Milieux Continus & Thermomécanique

Simulation of crashworthiness problems with improved implicit time integration methods for non-linear dynamics.

L. Noels, L. Stainier, J.-P. Ponthot

University of Liège, LTAS-MCT, Belgium

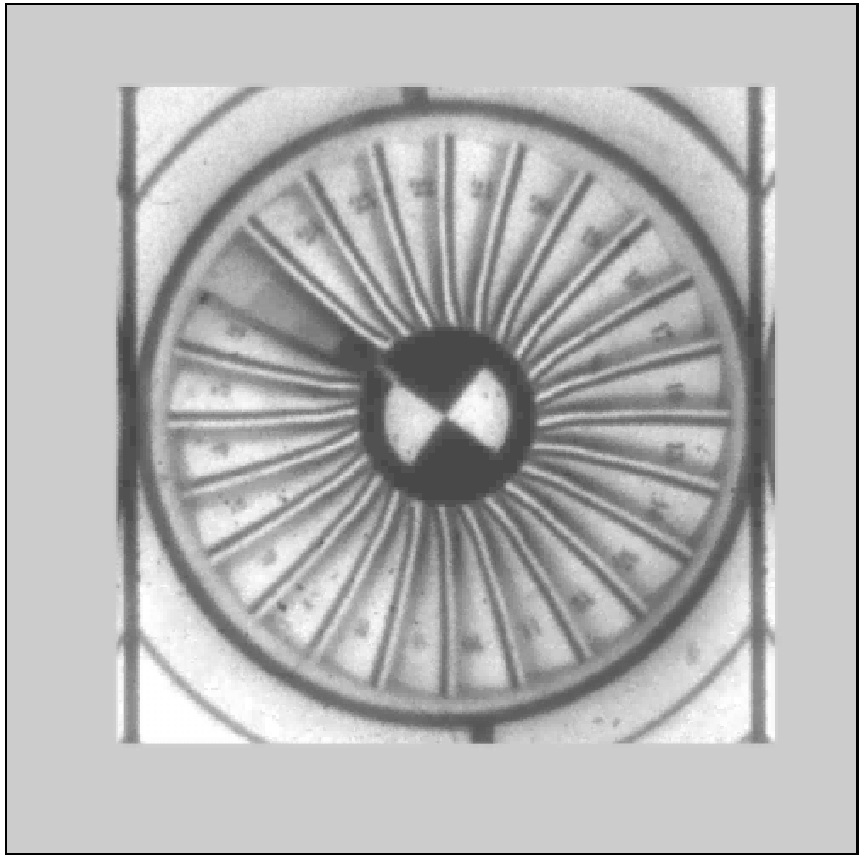
Tel: +32-(0)4-366-91-26
Fax: +32-(0)4-366-91-41
e-mail: mct@ulg.ac.be

Chemin des chevreuils 1
B-4000 Liège - Belgium
www.ulg.ac.be/ltas-mct/

Introduction
Industrial problems

- Industrial context:
  - Structures must be able to resist to crash situations
  - Numerical simulations is a key to design structures
  - Efficient time integration in the non-linear range is needed

- Goal:
  - Numerical simulation of blade off and wind-milling in a turboengine
  - Example from SNECMA



Scope of the presentation

1. Scientific motivations
2. Consistent scheme in the non-linear range
3. Combined implicit/explicit algorithm
4. Complex numerical examples
5. Conclusions & perspectives

1. Scientific motivations
Dynamics simulations

- Scientific context:
  - Solids mechanics
  - Large displacements
  - Large deformations
  - Non-linear mechanics

- Spatial discretization into finite elements:
  - Balance equation $$M\,\ddot{\vec{x}} + \vec{F}^{\,\text{int}} = \vec{F}^{\,\text{ext}}$$
  - Internal forces formulation $$\vec{F}^{\,\text{int}} = \int_{V_0} \Sigma \mathbf{f}^{T} \vec{D} J dV_0$$

Σ: Cauchy stress; **F**: deformation gradient; $\mathbf{f} = \mathbf{F}^{-1}$**;**

$\vec{D} = \partial\varphi / \partial\vec{x}_0$: derivative of the shape function; J : Jacobian

1. Scientific motivations
Dynamics simulations

- Temporal integration of the balance equation
- 2 methods:
  - Explicit method

$$\left.\begin{array}{l}\vec{x}_n\\ \dot{\vec{x}}_n\\ \ddot{\vec{x}}_n\end{array}\right\}\xrightarrow{\text{approximation}}\ddot{\vec{x}}_{n+1}=\mathbf{M}^{-1}\left(\vec{F}_n^{\,ext}-\vec{F}_n^{\,int}\right)\xrightarrow{\text{deduction}}\vec{x}_{n+1},\dot{\vec{x}}_{n+1}$$

    - Non iterative
    - Limited needs in memory
    - Conditionally stable (small time step)

    } Very fast dynamics

  - Implicit method

$$\left.\begin{array}{l}\vec{x}_n\\ \dot{\vec{x}}_n\\ \ddot{\vec{x}}_n\end{array}\right\}\xrightarrow{\text{extrapolation}}\left\{\begin{array}{l}\vec{x}_{n+1}\\ \dot{\vec{x}}_{n+1}\\ \ddot{\vec{x}}_{n+1}\end{array}\right\}\xleftrightarrow{\text{iterations}}\left\{\begin{array}{l}\mathbf{M}\ddot{\vec{x}}+\vec{F}^{\,int}=\vec{F}^{\,ext}\\ \vec{x}_{n+1}=f(\vec{x}_n,\dot{\vec{x}}_n,\dot{\vec{x}}_{n+1},\ddot{\vec{x}}_n,\ddot{\vec{x}}_{n+1})\\ \dot{\vec{x}}_{n+1}=f(\dot{\vec{x}}_n,\vec{x}_n,\vec{x}_{n+1},\ddot{\vec{x}}_n,\ddot{\vec{x}}_{n+1})\end{array}\right.$$

    - Iterative
    - More needs in memory
    - Unconditionally stable (large time step)

    } Slower dynamics

1. Scientific motivations
Implicit algorithm: our opinion

- If wave propagation effects are negligible

  → Implicit schemes are more suitable

  – Sheet metal forming (springback, superplastic forming, …)
  – Crashworthiness simulations (car crash, blade loss, shock absorber, …)

- Nowadays, people choose explicit scheme mainly because of difficulties linked to implicit scheme:
  – Lack of smoothness (contact, elasto-plasticity, …)

    → convergence can be difficult

  – Lack of available methods (commercial codes)

- Little room for improvement in explicit methods
- Complex problems can take advantage of combining explicit and implicit algorithms

1. Scientific motivations
Conservation laws

- Conservation of linear momentum (Newton's law)
  - Continuous dynamics $\frac{\partial \mathbf{M}\dot{\vec{x}}}{\partial t} = \vec{F}^{\text{ext}}$
  - Time discretization $\sum_{nodes} \mathbf{M}\dot{\vec{x}}_{n+1} - \mathbf{M}\dot{\vec{x}}_n = \Delta t \sum_{nodes} \vec{F}^{\text{ext}}_{n+1/2}$ & $\sum_{nodes} \vec{F}^{\text{int}}_{n+1/2} = 0$
- Conservation of angular momentum
  - Continuous dynamics $\frac{\partial \vec{x} \wedge \mathbf{M}\dot{\vec{x}}}{\partial t} = \vec{x} \wedge \vec{F}^{\text{ext}}$
  - Time discretization $\sum_{nodes} \sum_{nodes} \vec{x}_{n+1} \wedge \mathbf{M}\dot{\vec{x}}_{n+1} - \vec{x}_n \wedge \mathbf{M}\dot{\vec{x}}_n = \Delta t \sum_{nodes} \vec{x}_{n+1/2} \wedge \vec{F}^{\text{ext}}_{n+1/2}$

    & $\sum_{nodes} \vec{x}_{n+1/2} \wedge \vec{F}^{\text{int}}_{n+1/2} = 0$
- Conservation of energy

  W$^{\text{int}}$: internal energy;
  W$^{\text{ext}}$: external energy;
  D^{int}: dissipation (plasticity …)

  - Continuous dynamics $\frac{\partial}{\partial t} K + \frac{\partial}{\partial t} W^{\text{int}} = \frac{\partial}{\partial t} W^{\text{ext}} - \dot{D}^{\text{int}}$
  - Time discretization $W^{\text{int}}_{n+1} - W^{\text{int}}_n + \Delta D^{\text{int}} = \sum_{nodes} \vec{F}^{\text{int}}_{n+1/2} \bullet [\vec{x}_{n+1} - \vec{x}_n]$ &

$$\sum_{nodes} \frac{1}{2}\mathbf{M}\dot{\vec{x}}_{n+1} \bullet \dot{\vec{x}}_{n+1} - \frac{1}{2}\mathbf{M}\dot{\vec{x}}_n \bullet \dot{\vec{x}}_n + W^{\text{int}}_{n+1} - W^{\text{int}}_n + \Delta D^{\text{int}} = \sum_{nodes} \vec{F}^{\text{ext}}_{n+1/2} \bullet [\vec{x}_{n+1} - \vec{x}_n]$$

1. Scientific motivations
Explicit algorithms

- Central difference (no numerical dissipation)

$$\dot{\vec{x}}_{n+1/2} = \dot{\vec{x}}_{n-1/2} + \Delta t \ \ddot{\vec{x}}_n$$

$$\vec{x}_{n+1} = \vec{x}_n + \Delta t \ \dot{\vec{x}}_{n+1/2}$$

$$\ddot{\vec{x}}_{n+1} = M^{-1}\left[\vec{F}^{\,ext}_{n+1} - \vec{F}^{\,int}_{n+1}\right]$$

- Hulbert & Chung (numerical dissipation) [CMAME, 1996]

$$[1-\alpha_M]\,\ddot{\vec{x}}_{n+1} = M^{-1}\left[\vec{F}^{\,ext}_{n} - \vec{F}^{\,int}_{n}\right] - \alpha_M \ \ddot{\vec{x}}_n$$

$$\vec{x}_{n+1} = \vec{x}_n + \Delta t \ \dot{\vec{x}}_n + \Delta t^2\left[\frac{1}{2} - \beta\right]\ddot{\vec{x}}_n + \Delta t^2 \beta \ \ddot{\vec{x}}_{n+1}$$

$$\dot{\vec{x}}_{n+1} = \dot{\vec{x}}_n + \Delta t[1-\gamma]\,\ddot{\vec{x}}_n + \Delta t \gamma \ \ddot{\vec{x}}_{n+1}$$

- Small time steps $\longrightarrow$ conservation conditions are approximated

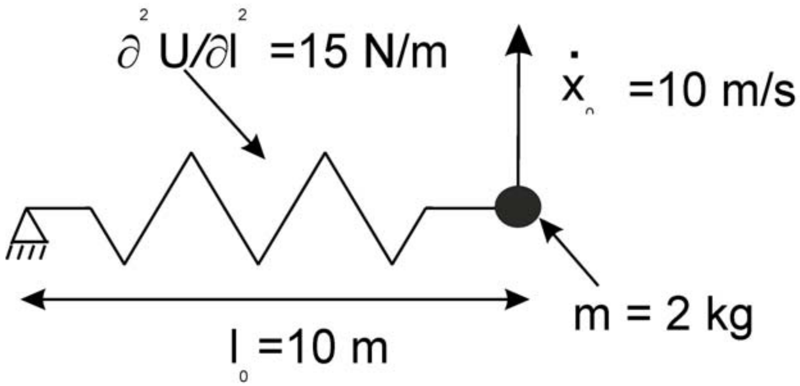
- Numerical oscillations may cause spurious plasticity

1. Scientific motivations
Implicit algorithms

- α-generalized family (Chung & Hulbert [JAM, 1993])
  - Newmark relations:
$$\begin{cases} \ddot{\vec{x}}_{n+1} = \dfrac{1}{\beta\,\Delta t^2}\left[\vec{x}_{n+1} - \vec{x}_n - \Delta t\,\dot{\vec{x}}_n - \left[\dfrac{1}{2} - \beta\right]\Delta t^2\,\ddot{\vec{x}}_n\right] \\ \dot{\vec{x}}_{n+1} = \dfrac{\gamma}{\beta\,\Delta t}\left[\vec{x}_{n+1} - \vec{x}_n + \left[\dfrac{\beta}{\gamma} - 1\right]\Delta t\,\dot{\vec{x}}_n + \left[\dfrac{\beta}{\gamma} - \dfrac{1}{2}\right]\Delta t^2\,\ddot{\vec{x}}_n\right] \end{cases}$$
  - Balance equation:
$$\frac{1-\alpha_M}{1-\alpha_F} M\,\ddot{\vec{x}}_{n+1} + \frac{\alpha_M}{1-\alpha_F} M\,\ddot{\vec{x}}_n + \left[\vec{F}^{\text{int}}_{n+1} - \vec{F}^{\text{ext}}_{n+1}\right] + \frac{\alpha_F}{1-\alpha_F}\left[\vec{F}^{\text{int}}_n - \vec{F}^{\text{ext}}_n\right] = 0$$
  - $\alpha_M = 0$ and $\alpha_F = 0$ (no numerical dissipation)
    - Linear range: consistency (i.e. physical results) demonstrated
    - Non-linear range with small time steps: consistency verified
    - Non-linear range with large time steps: total energy conserved but without consistency (e.g. plastic dissipation greater than the total energy, work of the normal contact forces > 0, …)
  - $\alpha_M \neq 0$ and/or $\alpha_F \neq 0$ (numerical dissipation)
    - Numerical dissipation is proved to be positive only in the linear range

1. Scientific motivations
Numerical example: mass/spring-system

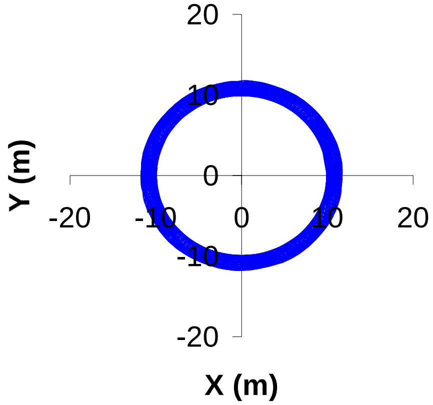
- Example: Mass/spring system (2D) with an initial velocity perpendicular to the spring (Armero & Romero [CMAME, 1999])



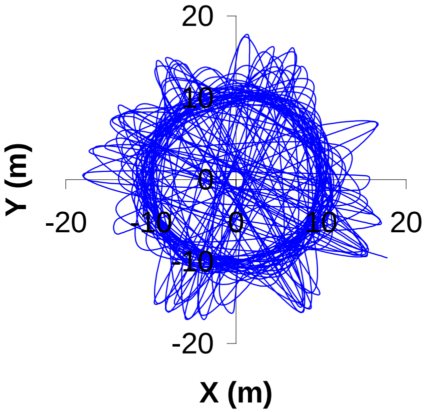
Explicit method: Δt_{crit} ~ 0.72s;

1 revolution ~ 4s

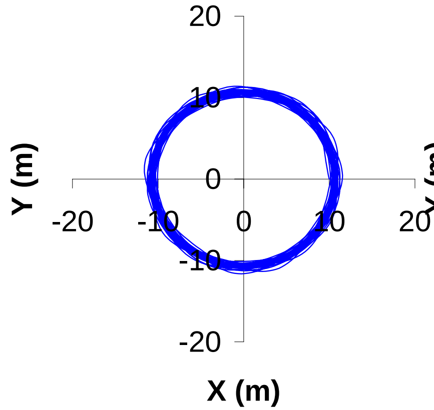
  - Newmark implicit scheme (no numerical damping)
  - Chung-Hulbert implicit scheme (numerical damping)



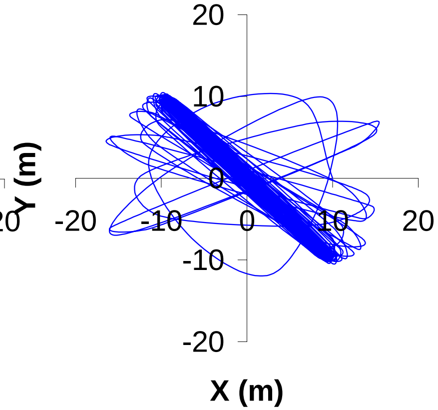
Δt=1s



Δt=1.5s



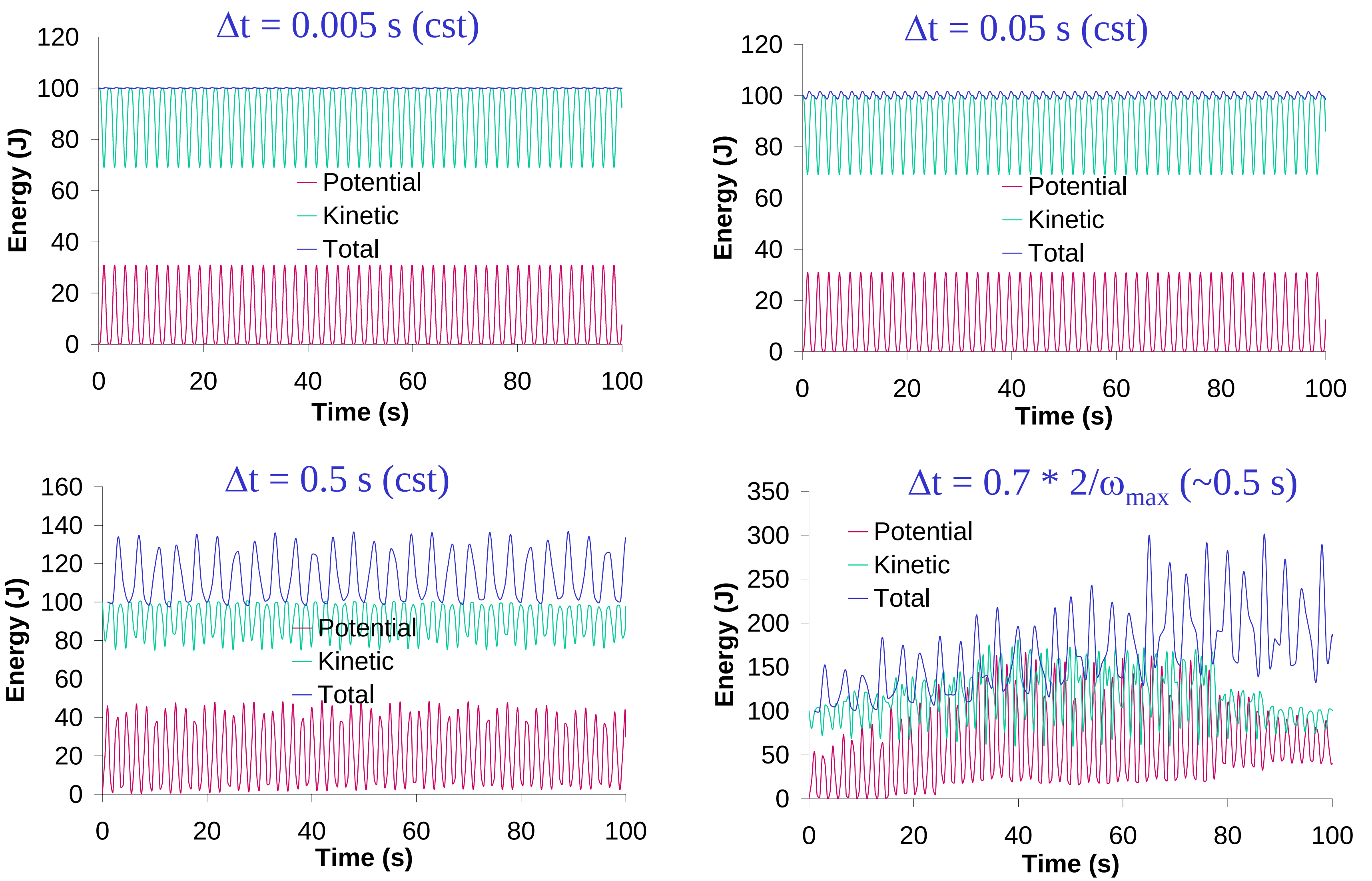
Δt=1s



Δt=1.5s

- Same mass/spring system with a central difference scheme

Δt_{crit} ~ 0.72s computed from the maximal pulsation of the system (2 degrees of freedom)



2. Consistent scheme in the non linear range Principle

- Consistent implicit algorithms in the non-linear range:
  - The Energy Momentum Conserving Algorithm or EMCA (Simo et al. [ZAMP 92], Gonzalez & Simo [CMAME 96]):
    - Conservation of the linear momentum
    - Conservation of the angular momentum
    - Conservation of the energy (no numerical dissipation)
  - The Energy Dissipative Momentum Conserving algorithm or EDMC (Armero & Romero [CMAME, 2001]):
    - Conservation of the linear momentum
    - Conservation of the angular momentum
    - Numerical dissipation of the energy is proved to be positive

2. Consistent scheme in the non linear range Principle

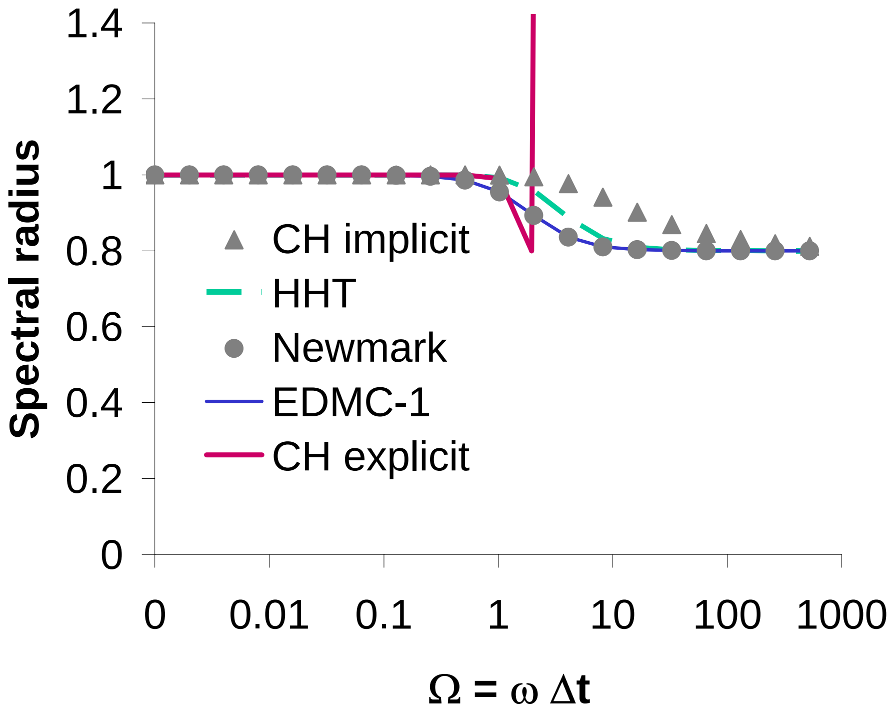
- Based on the mid-point scheme (Simo et al. [ZAMP, 1992])
  - Relations displacements /velocities/accelerations
    $$\begin{cases} \dfrac{\ddot{\vec{x}}_{n+1} + \ddot{\vec{x}}_n}{2} = \dfrac{\dot{\vec{x}}_{n+1} - \dot{\vec{x}}_n}{\Delta t} \\ \dfrac{\dot{\vec{x}}_{n+1} + \dot{\vec{x}}_n}{2} \left(+ \dot{\vec{x}}^{\text{diss}}_{n+1} \right) = \dfrac{\vec{x}_{n+1} - \vec{x}_n}{\Delta t} \end{cases}$$
  - Balance equation
    $$M \frac{\ddot{\vec{x}}_{n+1} + \ddot{\vec{x}}_n}{2} = \vec{F}^{\text{ext}}_{n+1/2} - \vec{F}^{\text{int}}_{n+1/2} \left(- \vec{F}^{\text{diss}}_{n+1/2} \right)$$
  - EMCA:
    - With $\vec{F}^{\text{int}}_{n+1/2} \neq \int_{V_0} \Sigma^{n+1/2} \mathbf{f}_0^{n+1/2^T} \vec{D} J_0^{n+1/2} dV_0$ and $\vec{F}^{\text{ext}}_{n+1/2}$ designed to verify conserving equations
    - No dissipation forces and no dissipation velocities
  - EDMC:
    - Same internal and external forces as in the EMCA
    - With $\vec{F}^{\text{diss}}_{n+1/2}$ and $\dot{\vec{x}}^{\text{diss}}_{n+1}$ designed to achieve positive numerical dissipation without spectral bifurcation

2. Consistent scheme in the non linear range
Dissipation property

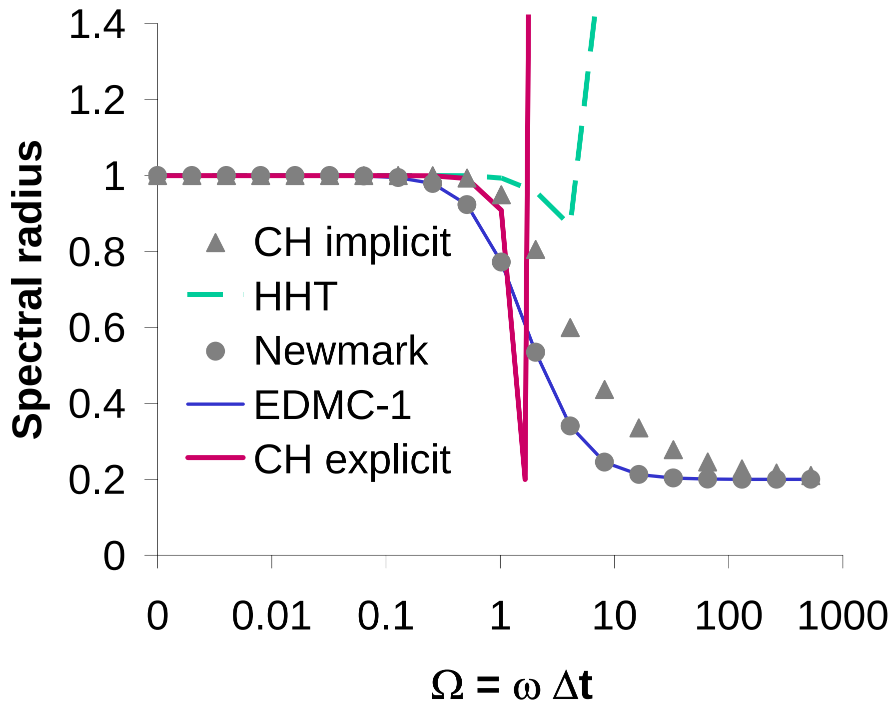
- Comparison of the spectral radius
  - Integration of a linear oscillator: $\vec{x}_{n+1} = \exp\left(\frac{\ln \rho^2}{\Delta t} t_{n+1}\right)\cos\left(\omega \quad t_{n+1}\right)$

ρ: spectral radius; ω: pulsation

Low numerical dissipation

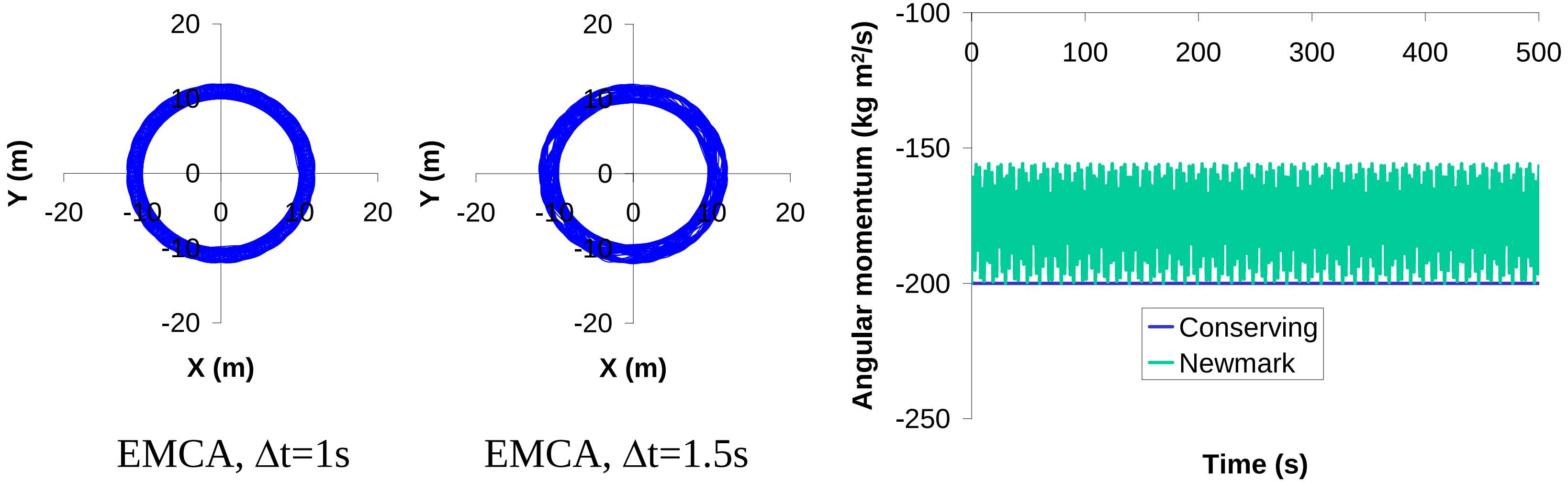


High numerical dissipation



- Forces of the spring for any potential V
  - Without numerical dissipation (EMCA) (Gonzalez & Simo [CMAME, 1996])

$$\vec{F}^{\,int}_{n+1/2} = \frac{V(l_{n+1}) - V(l_n)}{l_{n+1}^2 - l_n^2}\left[\vec{x}_{n+1} + \vec{x}_n\right]$$



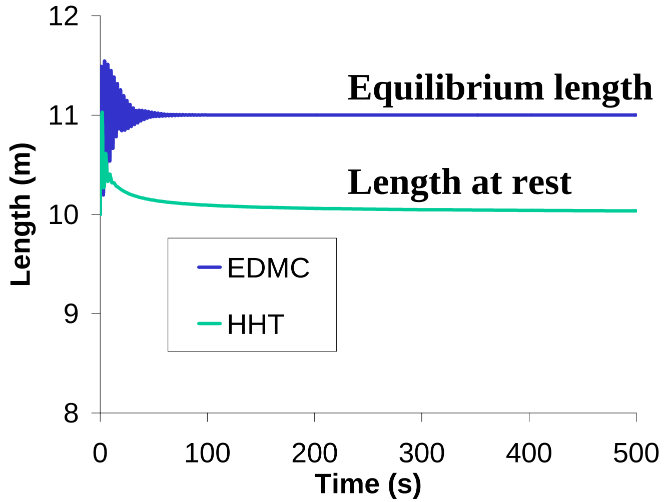
EMCA, Δt=1s   EMCA, Δt=1.5s

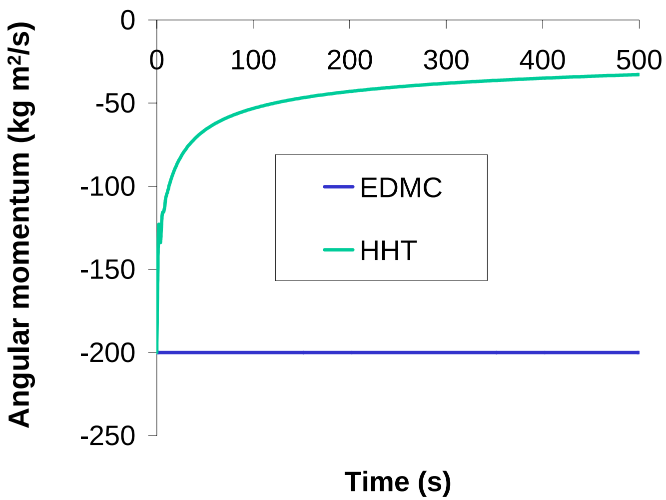
  - The consistency of the EMCA solution does not depend on Δt
  - The Newmark solution does-not conserve the angular momentum

2. Consistent scheme in the non linear range
The mass/spring system

– With numerical dissipation (EDMC 1st order) with dissipation parameter 0<χ<1 (Armero&Romero [CMAME, 2001]), here χ = 0.111

$$\begin{cases} \dot{\vec{x}}^{\,diss}_{n+1} = \chi \dfrac{\left\|\dot{\vec{x}}_{n+1}\right\| - \left\|\dot{\vec{x}}_{n}\right\|}{\left\|\dot{\vec{x}}_{n+1}\right\| + \left\|\dot{\vec{x}}_{n}\right\|} \dfrac{\left[\vec{x}_{n+1} + \vec{x}_{n}\right]}{2} \\ \vec{F}^{\,diss}_{n+1/2} = \chi \dfrac{V''\left(\dfrac{l_{n+1}+l_n}{2}\right)}{l_{n+1}+l_n} \left[l_{n+1}-l_n\right] \dfrac{\left[\vec{x}_{n+1} + \vec{x}_{n}\right]}{2} \end{cases}$$





– Only EDMC solution preserves the driving motion:

- The length tends towards the equilibrium length
- Conservation of the angular momentum is achieved

Formulations in the literature: hyperelasticity

- Hyperelastic material (stress derived from a potential *V*):
  - Saint Venant-Kirchhoff hyperelastic model (Simo et al. [ZAMP, 1992])
  - General formulation for hyperelasticity (Gonzalez [CMAME, 2000]):

$$\vec{F}^{\text{int}}_{n+1/2} = \int_{V_0} \frac{\mathbf{F}_0^n + \mathbf{F}_0^{n+1}}{2} \left[\frac{\partial V}{\partial \mathbf{GL}} \left(\frac{\mathbf{GL}_0^n + \mathbf{GL}_0^{n+1}}{2} \right) + \underbrace{O\left(\left\| \mathbf{GL}_0^{n+1} - \mathbf{GL}_0^n \right\|^2 \right)}_{f\left(V^{n+1}, V^n\right)} \right] \vec{D} dV_0$$

F: deformation gradient

GL: Green-Lagrange strain

V: potential

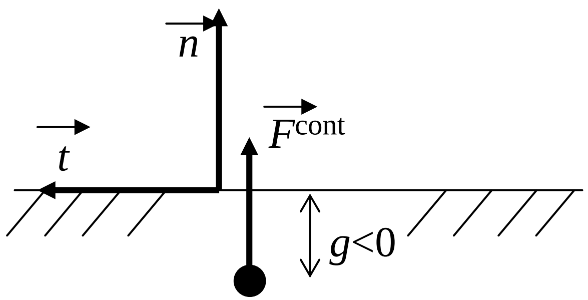
$\vec{D} = \partial\varphi / \partial\vec{x}_0$

φ: shape functions

  - Classical formulation: $\vec{F}^{\text{int}} = \int_{V_0} \mathbf{F} \frac{\partial V}{\partial \mathbf{GL}} \vec{D} dV_0$
  - Hyperelasticity with elasto-plastic behavior: energy dissipation of the algorithm corresponds to the internal dissipation of the material (Meng & Laursen [CMAME, 2001])

Formulations in the literature: contact

- Description of the contact interaction:



$\vec{n}$: normal

$\vec{t}$: tangent

g: gap

$\vec{F}^{cont}$: force

- Computation of the classical contact force:
  - Penalty method $\vec{F}^{cont} = -k_N g \; \vec{n}$
  - Augmented Lagrangian method $\vec{F}^{cont} = \Lambda^{(k)}\vec{n} - k_N g \; \vec{n}$
  - Lagrangian method $\vec{F}^{cont} = \Lambda \vec{n}$

k_N: penalty

Λ: Lagrangian

2. Consistent scheme in the non linear range
Formulations in the literature: contact

- Penalty contact formulation (normal force proportional to the penetration "gap") (Armero & Petöcz [CMAME, 1998-1999]):
  - Computation of a dynamic gap for slave node $\vec{x}$ projected on master surface $\vec{y}(u)$

$$g_{n+1}^{\mathrm{d}} = g_n^{\mathrm{d}} + \vec{n}_{n+1/2} \bullet \left[\vec{x}_{n+1} - \vec{x}_n - \vec{y}_{n+1}(u_{n+1/2}) + \vec{y}_n(u_{n+1/2}) \right]$$

  - Normal forces derived from a potential V

$$\vec{F}_{n+1/2}^{\mathrm{cont}} = \frac{V\left(g_{n+1}^{\mathrm{d}}\right) - V\left(g_n^{\mathrm{d}}\right)}{g_{n+1}^{\mathrm{d}} - g_n^{\mathrm{d}}} \vec{n}_{n+1/2}$$

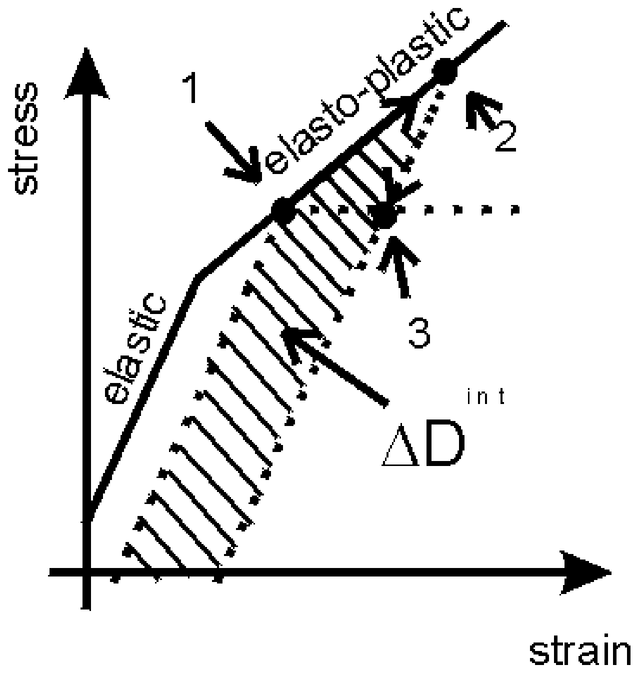
- Augmented Lagrangian and Lagrangian consistent contact formulation (Chawla & Laursen [IJNME, 1997-1998]):
  - Computation of a gap rate

$$\Delta t \;\; g_{n+1/2}^{\mathrm{r}} = \vec{n}_{n+1/2} \bullet \left[\vec{x}_{n+1} - \vec{x}_n - \vec{y}_{n+1}(u_{n+1/2}) + \vec{y}_n(u_{n+1/2}) \right]$$

2. Consistent scheme in the non linear range
Developments for a hypoelastic model

- The EMCA or EDMC for hypoelastic constitutive model:
  - Valid for hypoelastic formulation of (visco) plasticity
  - Energy dissipation from the internal forces corresponds to the plastic dissipation

- Hypoelastic model:
  - stress obtained incrementally from a hardening law
  - no possible definition of an internal potential!
  - Idea: the internal forces are established to be consistent on a loading/unloading cycle



2. Consistent scheme in the non linear range
Developments for a hypoelastic model

- Incremental strain tensor:

  E: natural strain tensor; **F**: deformation gradient

$$\mathrm{E}_n^{n+1} = \frac{1}{2}\ln\left(\mathbf{F}_n^{n+1^{\mathrm{T}}}\mathbf{F}_n^{n+1}\right)$$

- Elastic incremental stress:

  Σ: Cauchy stress; *H*: Hooke stress-strain tensor

$$\Delta\Sigma_n^{n+1} = H : \mathrm{E}_n^{n+1}$$

- Plastic stress corrections:

$$\mathrm{s}^{\mathrm{c}}{}_n^{n+1} = f(\sigma^{vm}, \varepsilon^{p})$$

  (radial return mapping: Wilkins [MCP, 1964], Maenchen & Sack [MCP, 1964], Ponthot [IJP, 2002])

  s$^{\mathbf{c}}$: plastic corrections; **σ**$^{\mathbf{vm}}$: yield stress;

  ε^{p}: equivalent plastic strain

- Final Cauchy stresses:

$$\Sigma^{n+1} = \mathrm{R}_n^{n+1}\left[\Sigma^{n} + \Delta\Sigma_n^{n+1} - \mathrm{s}^{\mathrm{c}}\right]\mathrm{R}_n^{n+1^{\mathrm{T}}}$$

  (final rotation scheme: Nagtegaal & Veldpaus [NAFP, 1984], Ponthot [IJP, 2002])

  R: rotation tensor;

- Classical forces formulation:

$$\vec{F}^{\,\mathrm{int}}_{n+1} = \int_{V_0} \Sigma^{n+1}\mathbf{f}_0^{n+1^{T}}\vec{D}\,J\,dV_0$$

  f = F$^{\mathbf{-1}}$**;** *D* derivative of the shape function;

  J : Jacobian

2. Consistent scheme in the non linear range
Developments for a hypoelastic model

- EMCA (<u>without</u> numerical dissipation):
  - Balance equation $M \frac{\ddot{\vec{x}}_{n+1}+\ddot{\vec{x}}_n}{2} = \vec{F}^{ext}_{n+1/2} - \vec{F}^{int}_{n+1/2}$
  - New internal forces formulation:

$$\vec{F}^{int}_{n+1/2} = \frac{1}{4}\int_{V_0} \left[\mathbf{I}+\mathbf{F}_n^{n+1}\right]\left[\Sigma^n + \mathbf{C}^*\right] \mathbf{f}_0^{n^T} \vec{D} J^n + \left[\mathbf{I}+\mathbf{f}_n^{n+1}\right]\left[\Sigma^{n+1}+\mathbf{C}^{**}\right] \mathbf{f}_0^{n+1^T} \vec{D} J^{n+1} dV_0$$

F: deformation gradient; **f**: inverse of **F;** *D* derivative of the shape function; *J* : Jacobian = det **F;**
Σ: Cauchy stress

  - Correction terms C* and C**: (<u>second order</u> correction in the plastic strain increment)

$$\begin{cases} \mathbf{C}^* = \dfrac{\Delta D^{int}/J_0^n - \mathbf{GL}_n^{n+1^{pl}} : \Sigma^n}{\mathbf{GL}_n^{n+1} : \mathbf{GL}_n^{n+1}} \mathbf{GL}_n^{n+1} \\[2ex] \mathbf{C}^{**} = \dfrac{\Delta D^{int}/J_0^{n+1} - \mathbf{A}_n^{n+1^{pl}} : \Sigma^{n+1}}{\mathbf{A}_n^{n+1} : \mathbf{A}_n^{n+1}} \mathbf{A}_n^{n+1} \end{cases}$$

ΔD^int: internal dissipation due to the plasticity; **A**: Almansi incremental strain tensor (**A** = $\mathbf{A}^{pl}$ + $\mathbf{A}^{el}$);
GL: Green-Lagrange incremental strain (**GL** = $\mathbf{GL}^{pl}$ + $\mathbf{GL}^{el}$)

  - Verification of the conservation laws

$$\begin{cases} \sum_{nodes} \vec{F}^{int}_{n+1/2} = 0 \quad \& \quad \sum_{nodes} \vec{x}_{n+1/2} \wedge \vec{F}^{int}_{n+1/2} = 0 \\[2ex] \Delta D^{int} = \int_{cycle} \sum_{nodes} \vec{F}^{int}_{n+1/2} \bullet [\vec{x}_{n+1} - \vec{x}_n] \end{cases}$$

2. Consistent scheme in the non linear range
Developments for a hypoelastic model

- EDMC (1st order accurate with numerical dissipation):
  - Balance equation $$M\frac{\ddot{\vec{x}}_{n+1}+\ddot{\vec{x}}_{n}}{2}=\vec{F}^{\text{ext}}_{n+1/2}-\vec{F}^{\text{int}}_{n+1/2}-\vec{F}^{\text{diss}}_{n+1/2}$$
  - New dissipation forces formulation:

$$\vec{F}^{\text{diss}}_{n+1/2}=\frac{1}{4}\int_{V_0}\left[\mathbf{I}+\mathbf{F}_n^{n+1}\right]\mathbf{D}^{*}\mathbf{f}_0^{n\,T}\vec{D}+\left[\mathbf{I}+\mathbf{f}_n^{n+1}\right]\mathbf{D}^{**}\mathbf{f}_0^{n+1\,T}\vec{D}\,dV_0$$

  - Dissipating terms D* and D**:

$$\begin{cases}\mathbf{D}^{*}=\dfrac{\chi/2\;\mathbf{E}^{\text{el}\,n+1}_{n}:H:\mathbf{E}^{\text{el}\,n+1}_{n}\,J_0^{n}}{\mathbf{GL}_n^{n+1}:\mathbf{GL}_n^{n+1}}\mathbf{GL}_n^{n+1}\\[2ex]\mathbf{D}^{**}=\dfrac{\chi/2\;/\mathbf{E}^{\text{el}\,n+1}_{n}:H:\mathbf{E}^{\text{el}\,n+1}_{n}\,J_0^{n+1}}{\mathbf{A}_n^{n+1}:\mathbf{A}_n^{n+1}}\mathbf{A}_n^{n+1}\end{cases}$$

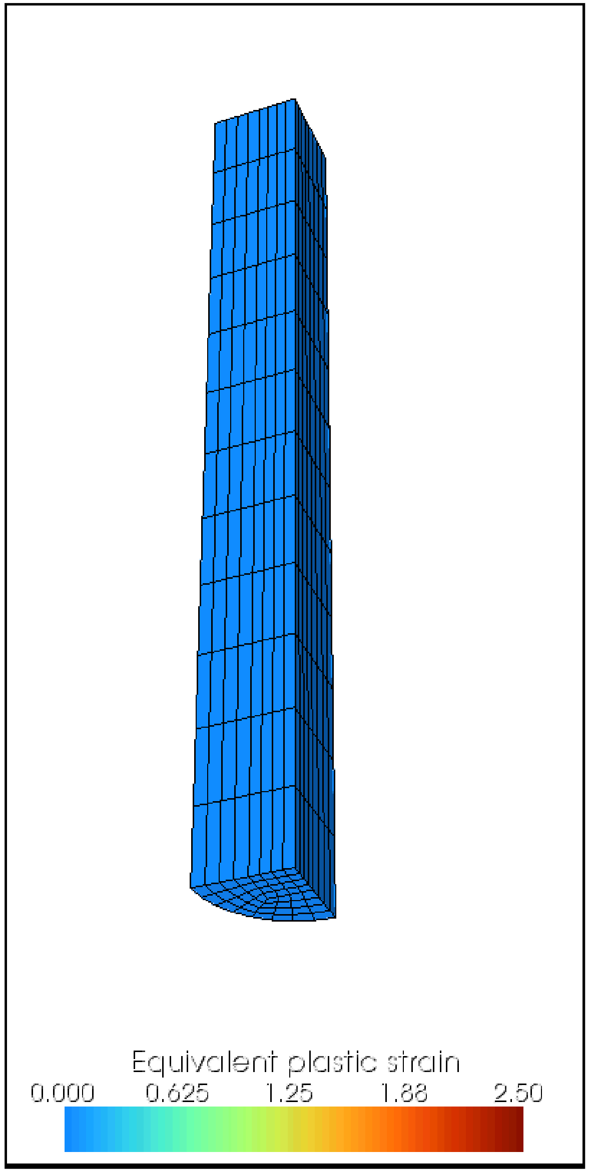
  - Verification of the conservation laws

$$\begin{cases}\sum_{nodes}\vec{F}^{\text{diss}}_{n+1/2}=0 \quad \& \quad \sum_{nodes}\vec{x}_{n+1/2}\wedge\vec{F}^{\text{diss}}_{n+1/2}=0\\[2ex]\Delta D^{\text{num}}=\sum_{nodes}\vec{F}^{\text{diss}}_{n+1/2}\bullet[\vec{x}_{n+1}-\vec{x}_{n}]+\mathbf{M}\dot{\vec{x}}^{\text{diss}}_{n+1/2}\bullet[\dot{\vec{x}}_{n+1}-\dot{\vec{x}}_{n}]\end{cases}$$

ΔD^{num}: numerical dissipation

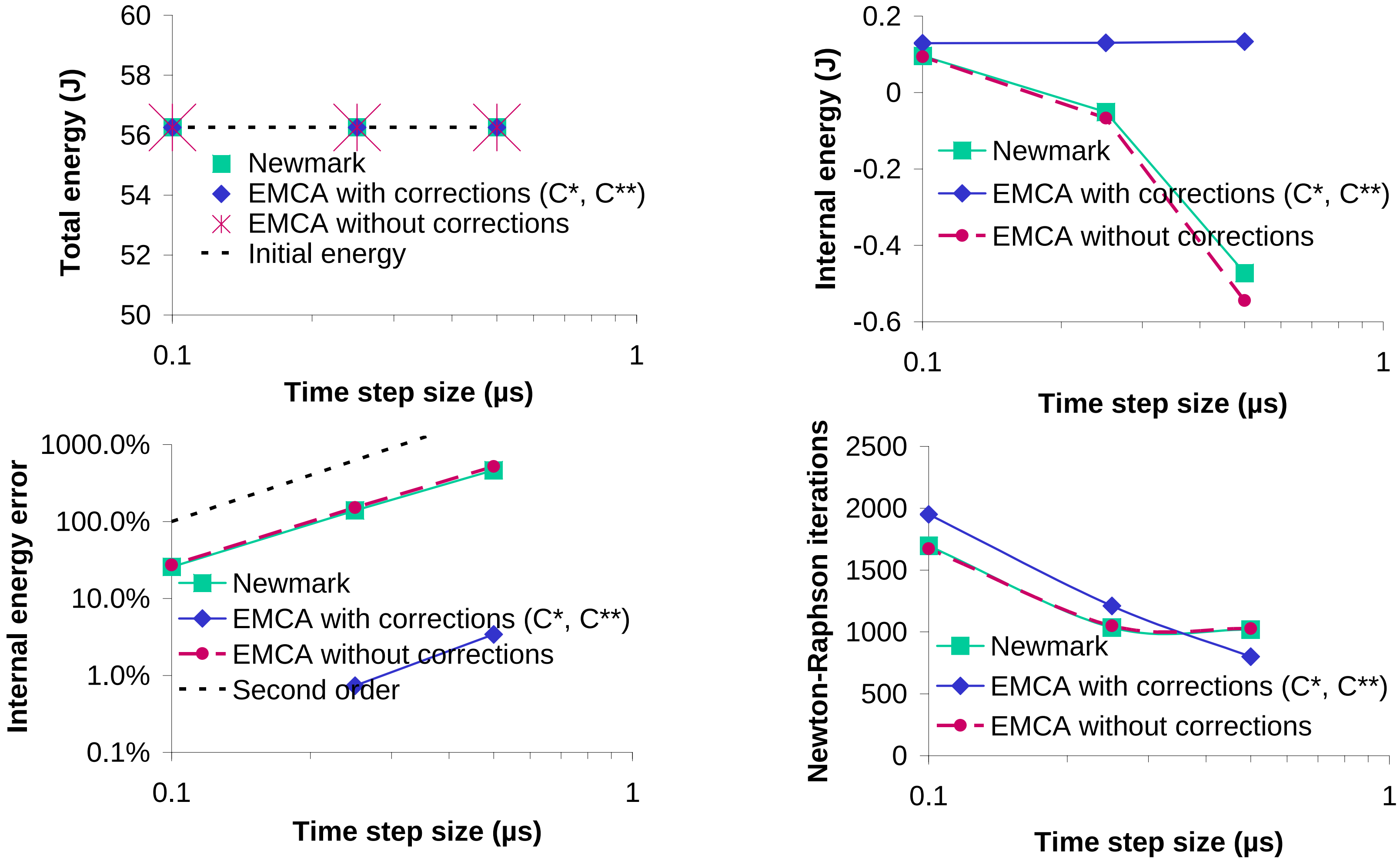
2. Consistent scheme in the non linear range
Numerical example: Taylor bar

- Impact of a cylinder :
  - Hypoelastic model
  - Elasto-plastic hardening law
  - Simulation during 80 µs



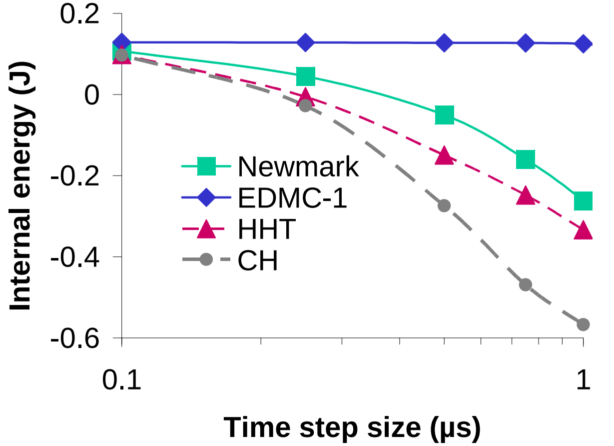
2. Consistent scheme in the non linear range
Numerical example: Taylor bar

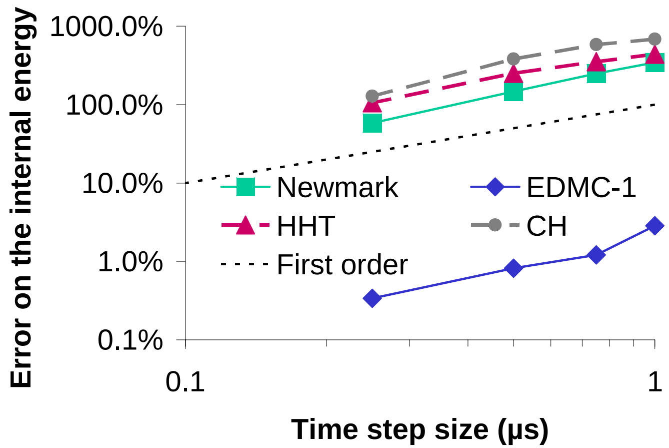
- Simulation without numerical dissipation: final results



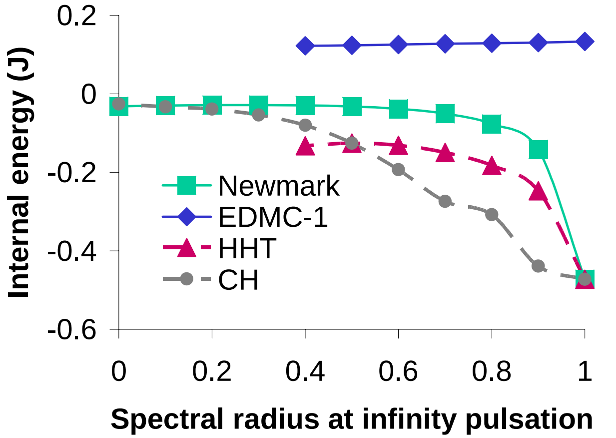
2. Consistent scheme in the non linear range
Numerical example: Taylor bar

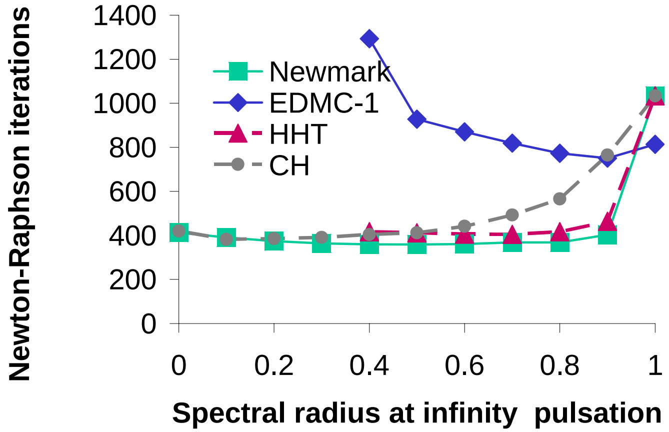
- Simulations with numerical dissipation: final results
  - Constant spectral radius at infinity pulsation = 0.7





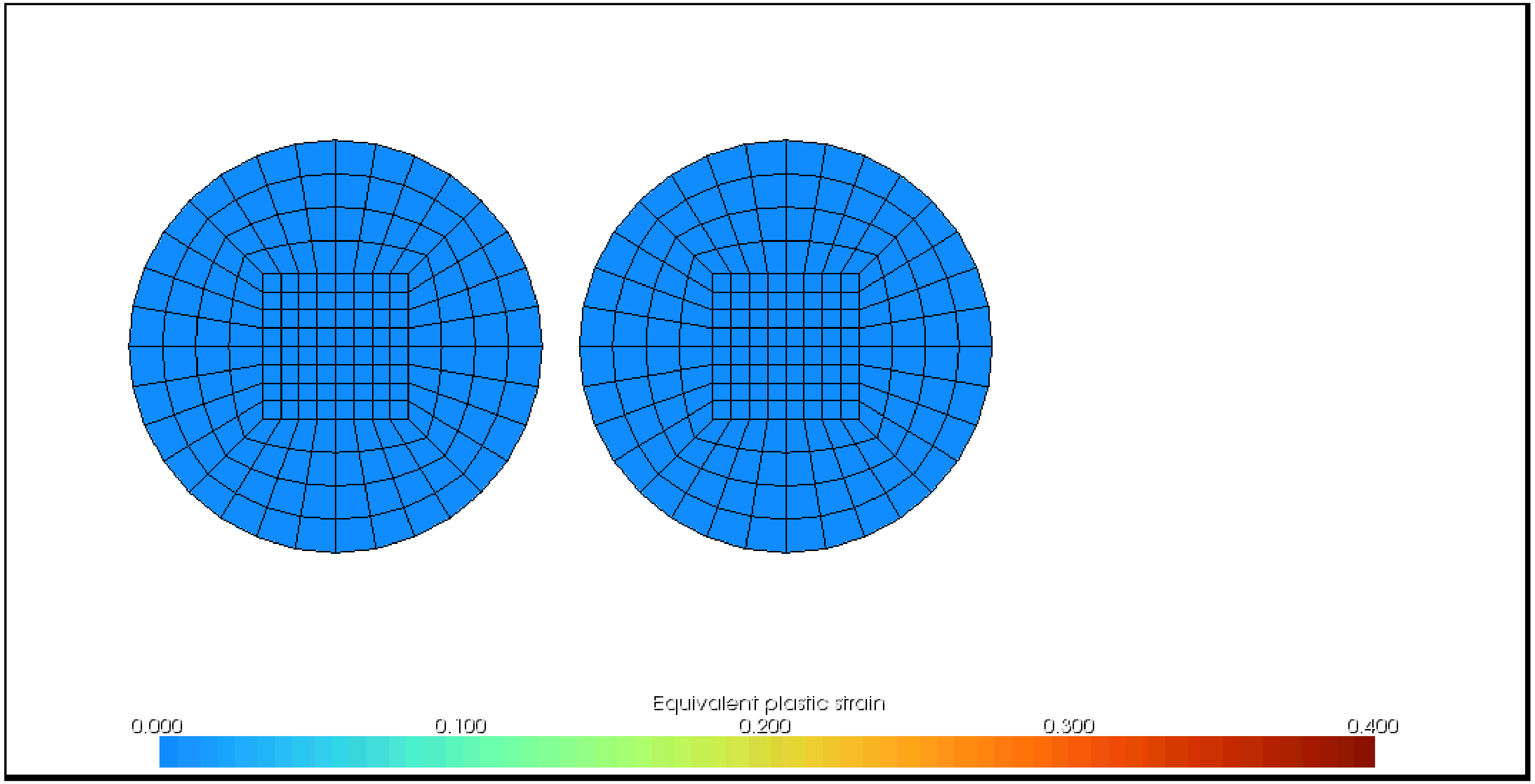
  - Constant time step size = 0.5 μs





Numerical example: impact of two 2D-cylinders

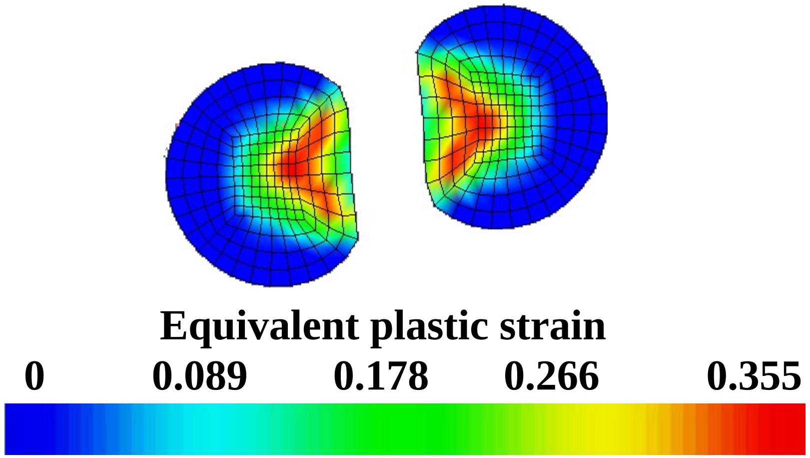
- Impact of 2 cylinders (Meng&Laursen) :
  - Left one has a initial velocity (initial kinetic energy 14J)
  - Elasto perfectly plastic hypoelastic material
  - Simulation during 4s



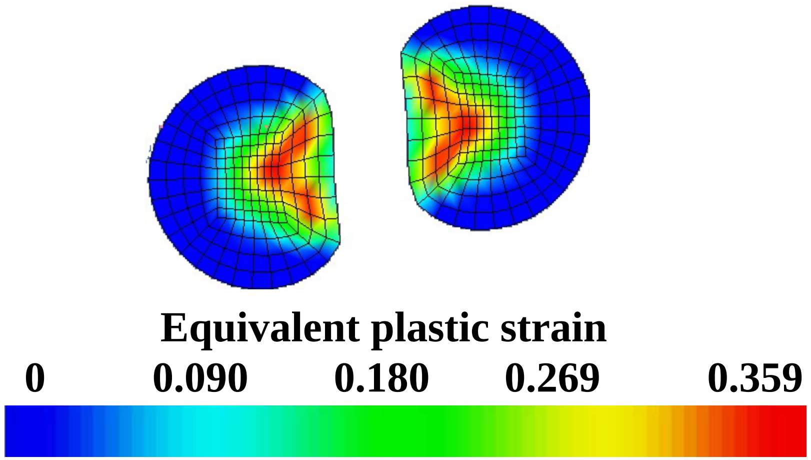
Numerical example: impact of two 2D-cylinders

- Results comparison at the end of the simulation

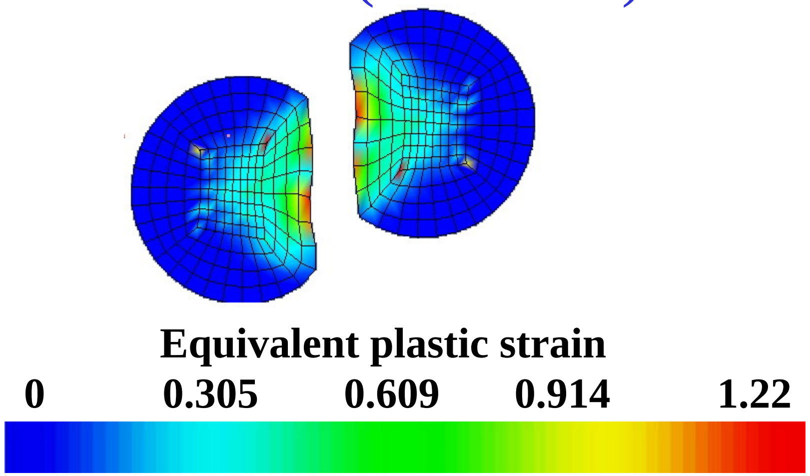
Newmark(Δt=1.875 ms)



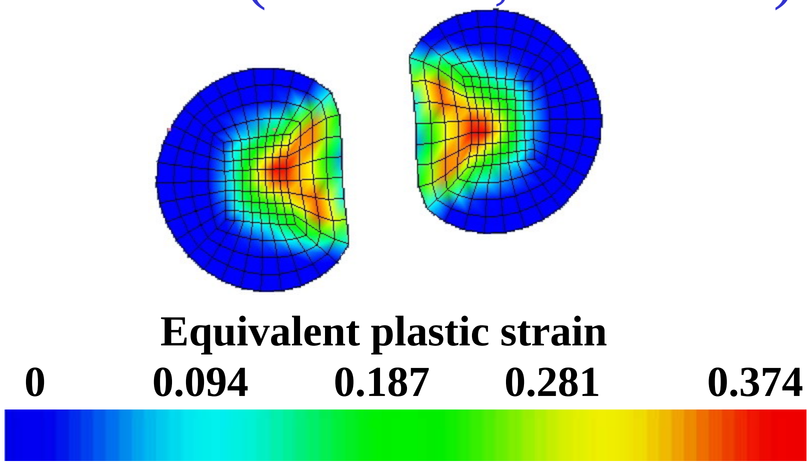
EMCA (with cor., Δt=1.875 ms)



Newmark(Δt=15 ms)



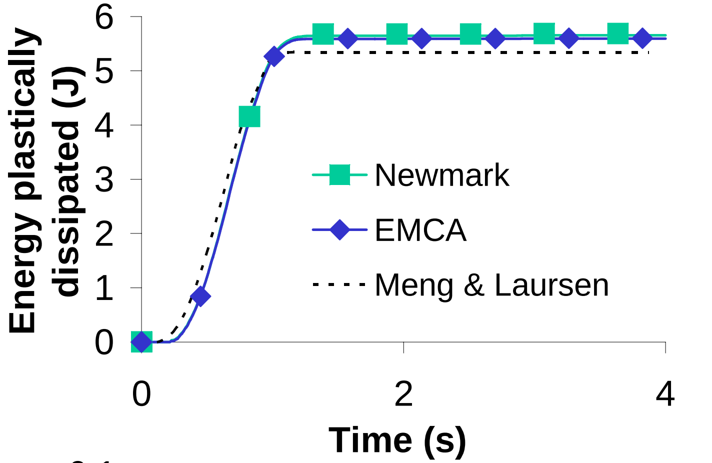
EMCA (with cor., Δt=15 ms)

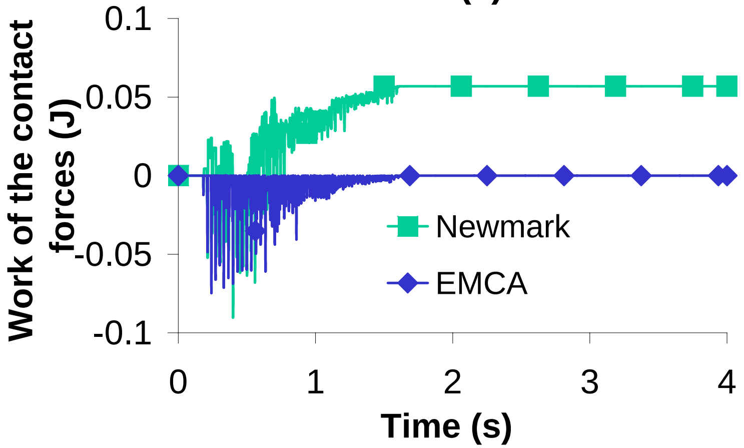


2. Consistent scheme in the non linear range
Numerical example: impact of two 2D-cylinders

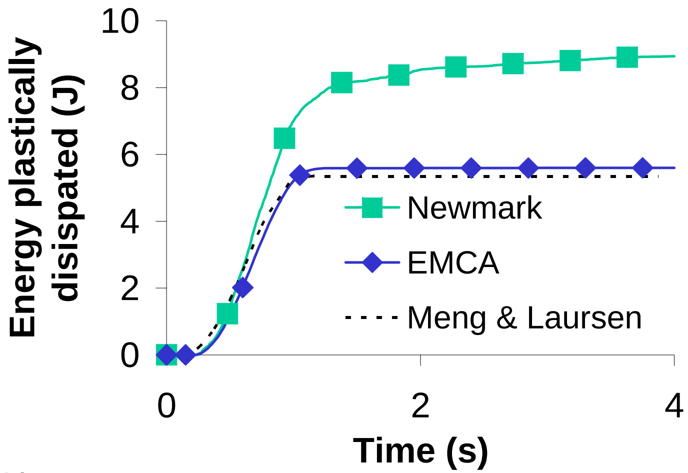
- Results evolution comparison

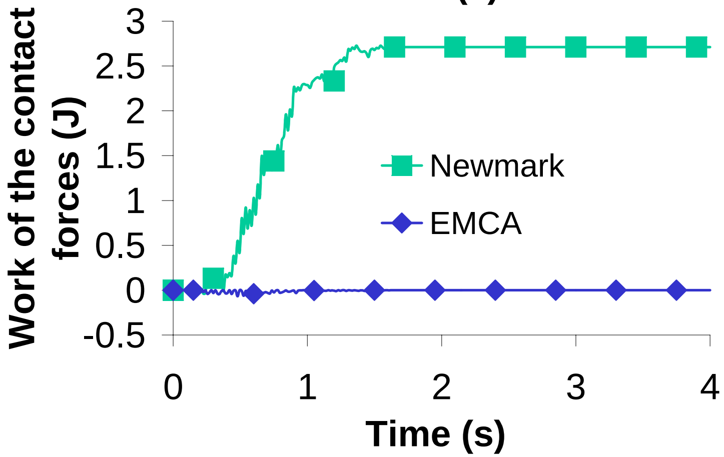
Δt=1.875 ms





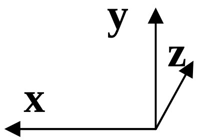
Δt=15 ms

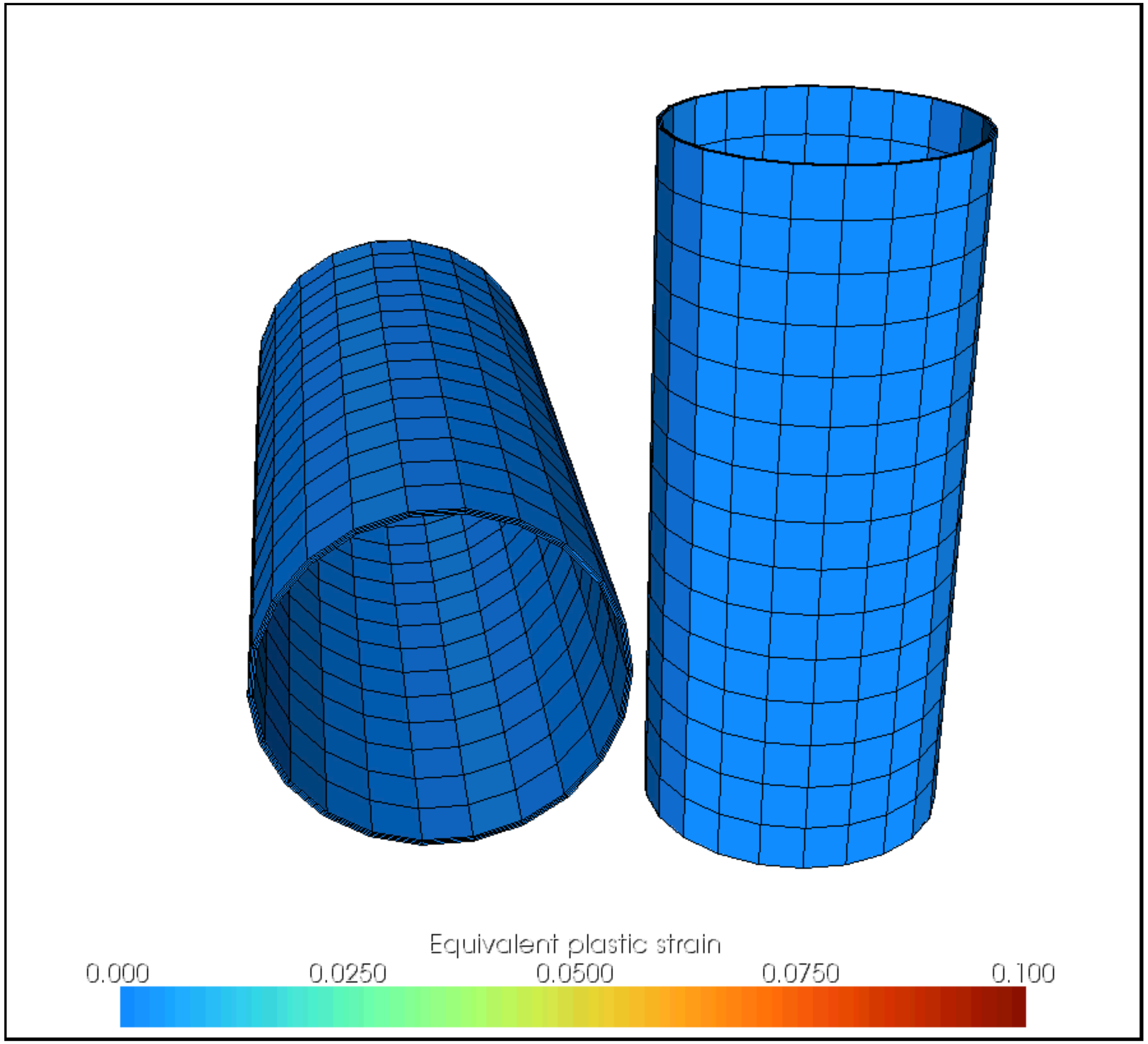




2. Consistent scheme in the non linear range
Numerical example: impact of two 3D-cylinders

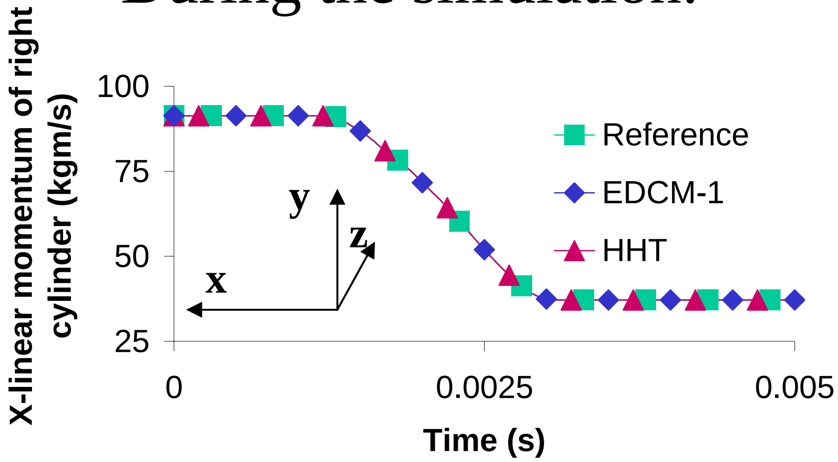
- Impact of 2 hollow 3D-cylinders:
  - Right one has a initial velocity ($\dot{\vec{x}}_{0X} = 10\dot{\vec{x}}_{0Y}$)
  - Elasto-plastic hypoelastic material (aluminum)
  - Simulation during 5ms
  - Use of numerical dissipation
  - Frictional contact

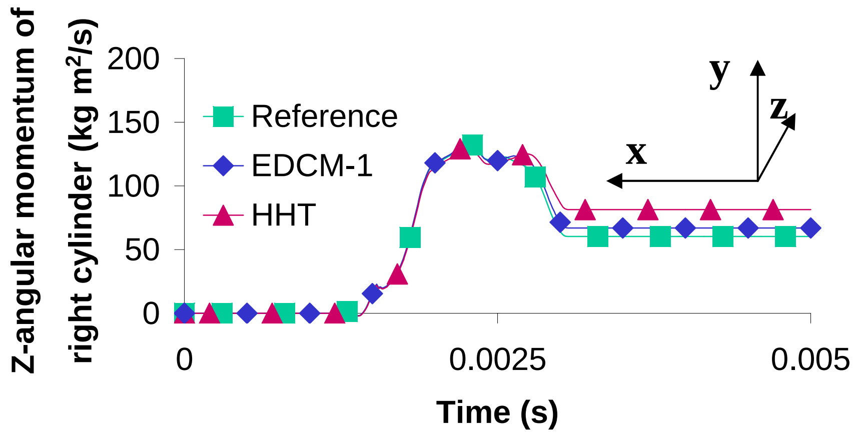




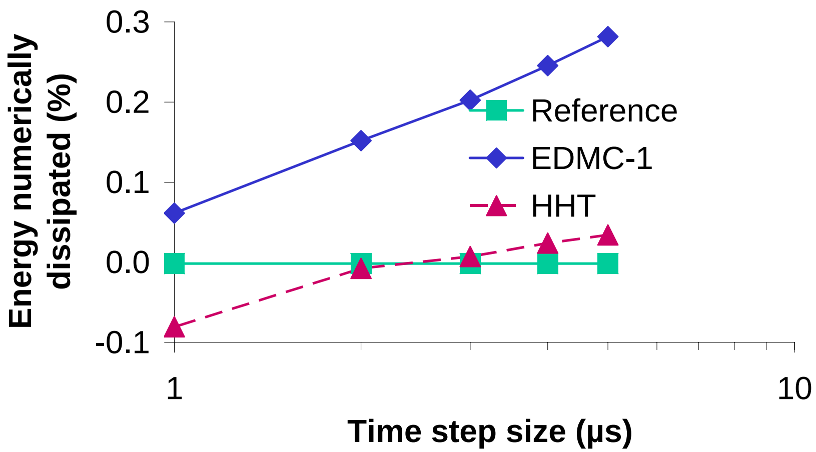
2. Consistent scheme in the non linear range
Numerical example: impact of two 3D-cylinders

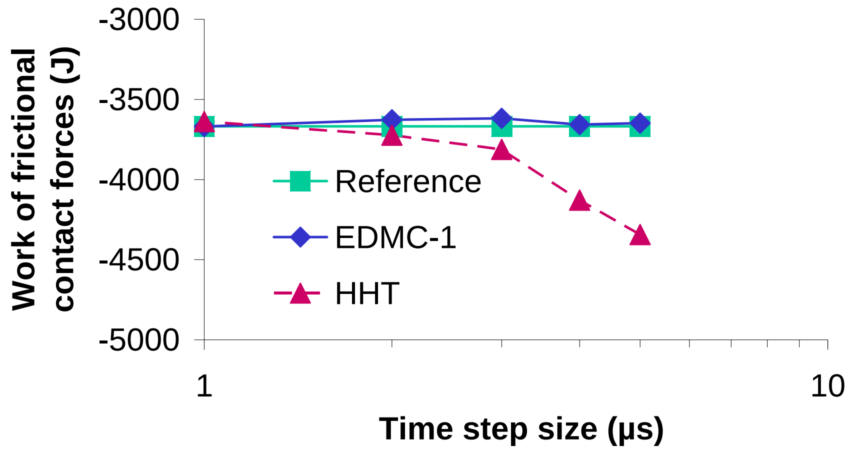
- Results comparison with a reference (EMCA; Δt=0.5μs):
  - During the simulation:





  - At the end of the simulation:





2. Consistent scheme in the non linear range
Extension to the use of incremental potential

- Hyperelastic material with use of the variational formulation of visco-plastic updates [Ortiz & Stainier, CMAME 1999]

$$\mathbf{S}^{n+1} = 2\frac{\partial \Delta D_{\text{eff}}}{\partial \mathbf{C}^{n+1}}\left(\mathbf{C}_0^{n+1}, \mathbf{C}_0^{n}\right)$$

C: right Cauchy-Green strain

S: second Piola-Kirchhoff stress

ΔD_{eff}: incremental potential

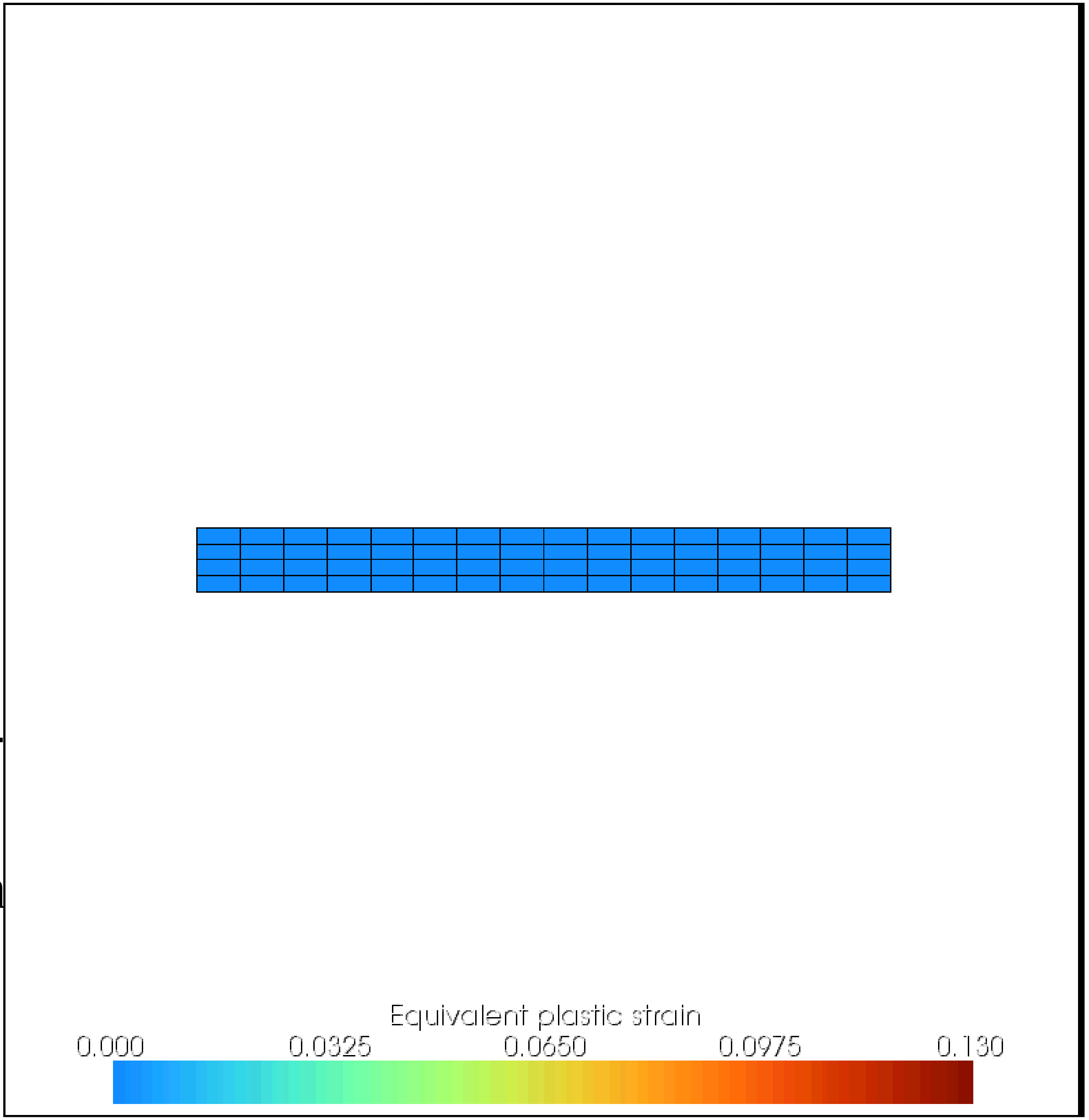
- Use of a similar form than the Gonzalez formulation [CMAME 2000] for an elastic model

$$\vec{F}^{\text{int}}_{n+1/2} = \int_{V_0} \frac{\mathbf{F}_0^{n} + \mathbf{F}_0^{n+1}}{2}\left[2\frac{\partial \Delta D_{\text{eff}}}{\partial \mathbf{C}}\left(\frac{\mathbf{C}_0^{n}+\mathbf{C}_0^{n+1}}{2}, \mathbf{C}_0^{n}\right) + \overbrace{f\left(\Delta D_{\text{eff}}\left(\mathbf{C}_0^{n+1}, \mathbf{C}_0^{n}\right), \frac{\partial \Delta D_{\text{eff}}}{\partial \mathbf{C}}\left(\frac{\mathbf{C}_0^{n}+\mathbf{C}_0^{n+1}}{2}, \mathbf{C}_0^{n}\right)\right)}^{O\left(\left\|\mathbf{C}_0^{n+1}-\mathbf{C}_0^{n}\right\|^2\right)}\right]\vec{D}\,dV_0$$

F: deformation gradient; φ: shape functions; $\vec{D} = \partial\varphi / \partial\vec{x}_0$

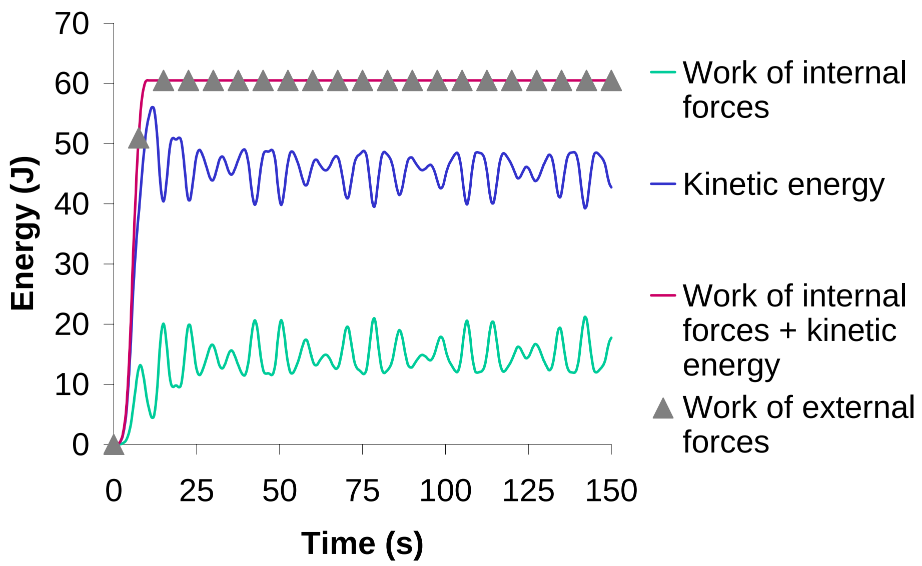
2. Consistent scheme in the non linear range
Simulation of a tumbling beam

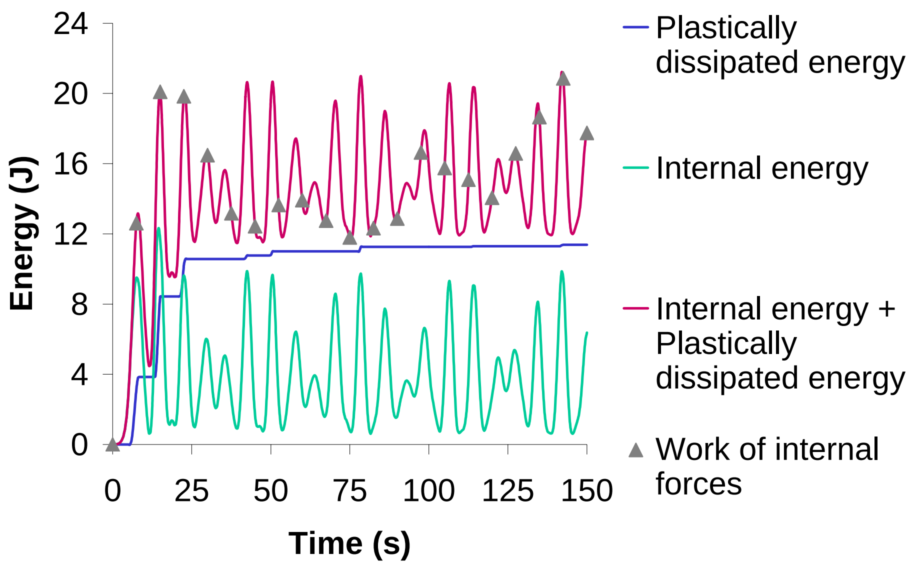
- Tumbling beam:
  - Initial symmetrical loads ($t < 10s$)
  - Elasto perfectly plastic hyperelastic material
  - Simulation during 150s
  - Use of the variational formulation of elasto-plastic updates
  - Conserving algorithm



2. Consistent scheme in the non linear range
Simulation of a tumbling beam

- Time evolution of the results:





3. Combined implicit/explicit algorithm
Automatic shift

- Shift from an implicit algorithm to an explicit one:
  - Evaluation of the ratio r*

$$r^* = \frac{CPU \quad 1 \quad \text{implicit} \quad \text{step}}{CPU \quad 1 \quad \text{explicit} \quad \text{step}}$$

  - Explicit time step size depends on the mesh

    Ωb: stability limit;

    ω max: maximal eigen pulsation

$$\Delta t_{\text{expl}} = \frac{\Omega_b}{\omega_{\max}}$$

  - Implicit time step size depends on the integration error (Géradin)

    e_{int}: integration error;

    Tol: user tolerance

$$\begin{cases} e_{\text{int}} = \Delta t^3 \dddot{x} \approx \dfrac{\Delta t_{\text{impl}}^2 \sum_{i=nodes} \Delta \left|\ddot{\vec{x}}\right|_i}{e_{\text{ref}}} \\ \dfrac{\Delta t_{\text{impl}}^{\text{new}}}{\Delta t_{\text{impl}}^{\text{old}}} = \left[\dfrac{2 e_{\text{int}}}{Tol}\right]^{1/2.5} \end{cases}$$

  - Shift criterion

    μ : user security

$$\Delta t_{\text{impl}} < \frac{r^* \Delta t_{\text{expl}}}{\mu}$$

3. Combined implicit/explicit algorithm
Automatic shift

- Shift from an explicit algorithm to an implicit one:
  - Evaluation of the ratio r*

$$r^* = \frac{CPU \quad 1 \quad \text{implicit} \quad \text{step}}{CPU \quad 1 \quad \text{explicit} \quad \text{step}}$$

  - Explicit time step size depends on the mesh

    Ωb: stability limit;

    $\omega\, max$: maximal eigen pulsation

$$\Delta t_{\text{expl}} = \frac{\Omega_b}{\omega_{\max}}$$

  - Implicit time step size interpolated form a acceleration difference

    Tol: user tolerance

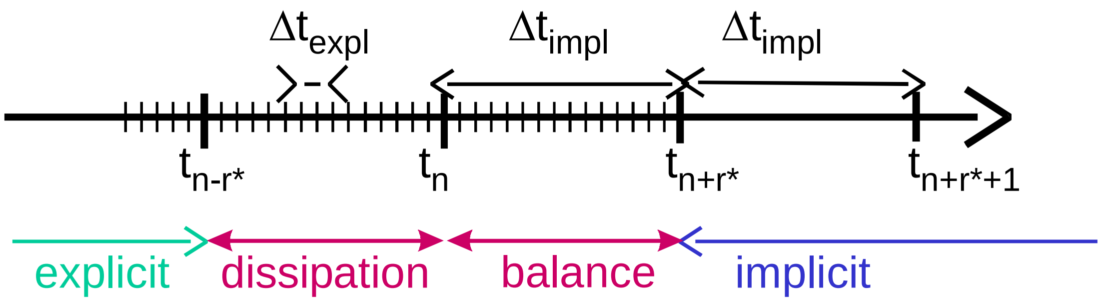
$$\Delta t_{\text{impl}} = \left(\frac{Tol \;\; e_{\text{ref}} \sqrt{\Delta t_{\text{expl}}}}{2 \sum_{i=nodes} \Delta \left| \ddot{\vec{x}} \right|_i} \right)^{2/5}$$

  - Shift criterion

$$\Delta t_{\text{impl}} > \mu r^* \Delta t_{\text{expl}}$$

    μ : user security

- Stabilization of the explicit solution:



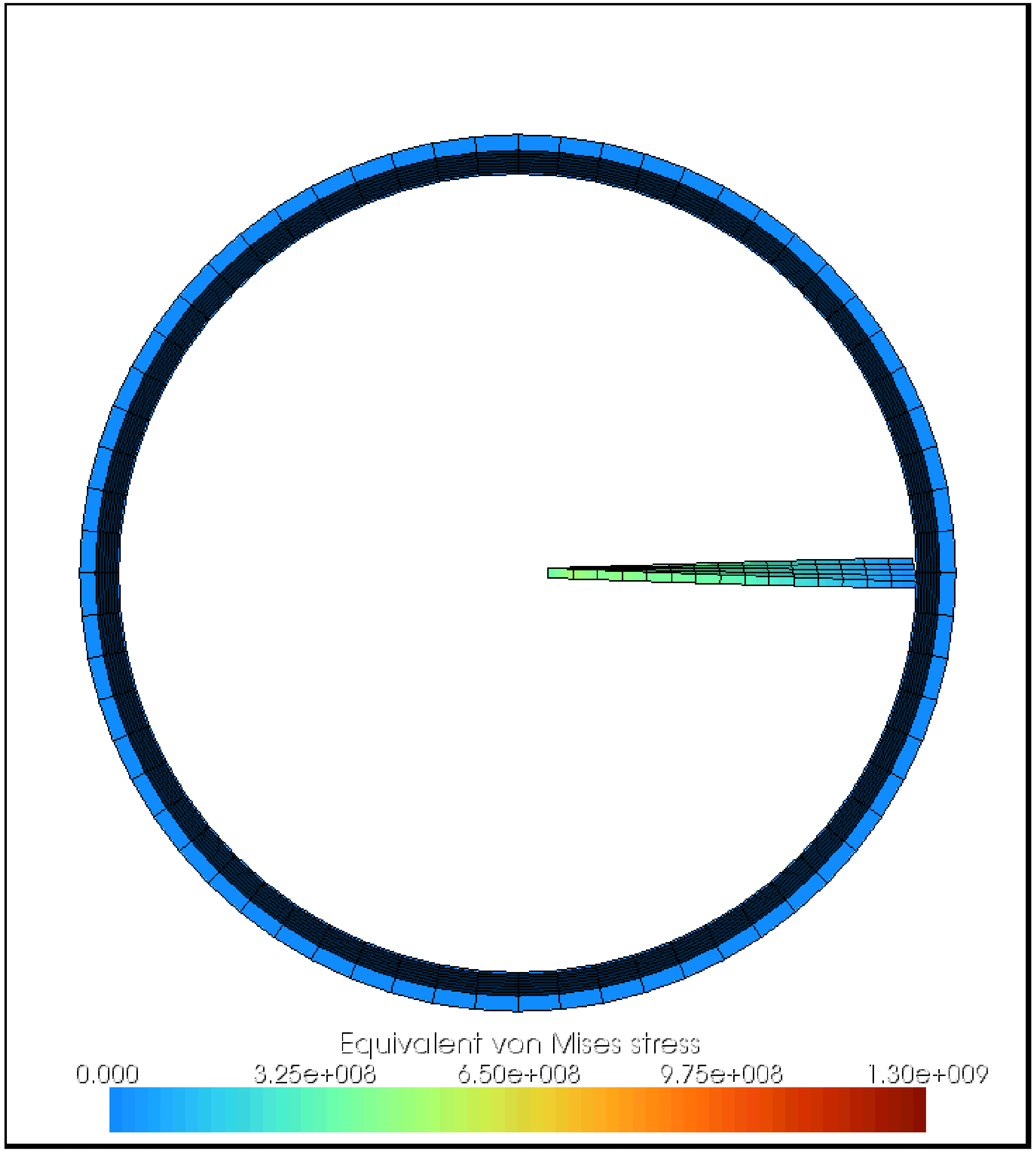
  – Dissipation of the numerical modes: spectral radius at bifurcation equal to zero.

  – Consistent balance of the r* last explicit steps:

$$M\,\frac{\ddot{\vec{x}}_{n+r^*}+\ddot{\vec{x}}_{n}}{2}=\vec{F}^{\,\mathrm{ext}}_{n+r^*/2}-\vec{F}^{\,\mathrm{int}}_{n+r^*/2}\left(-\vec{F}^{\,\mathrm{diss}}_{n+r^*/2}\right)$$

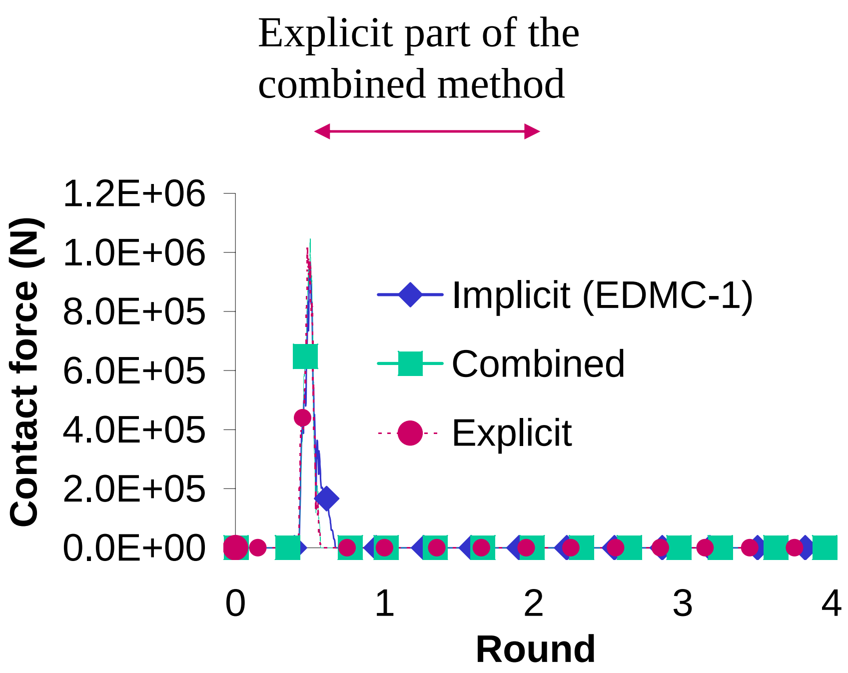
3. Combined implicit/explicit algorithm
Numerical example: blade casing interaction

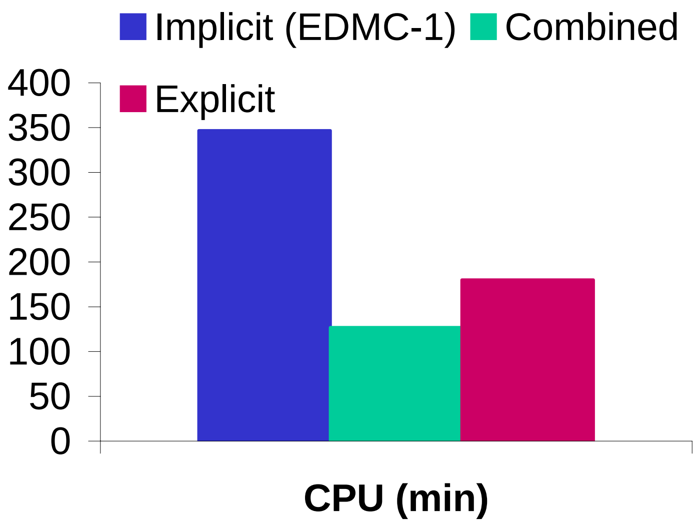
- Blade/casing interaction :
  - Rotation velocity 3333rpm
  - Rotation center is moved during the first half revolution
  - EDMC-1 algorithm
  - Four revolutions simulation



3. Combined implicit/explicit algorithm
Numerical example: blade casing interaction

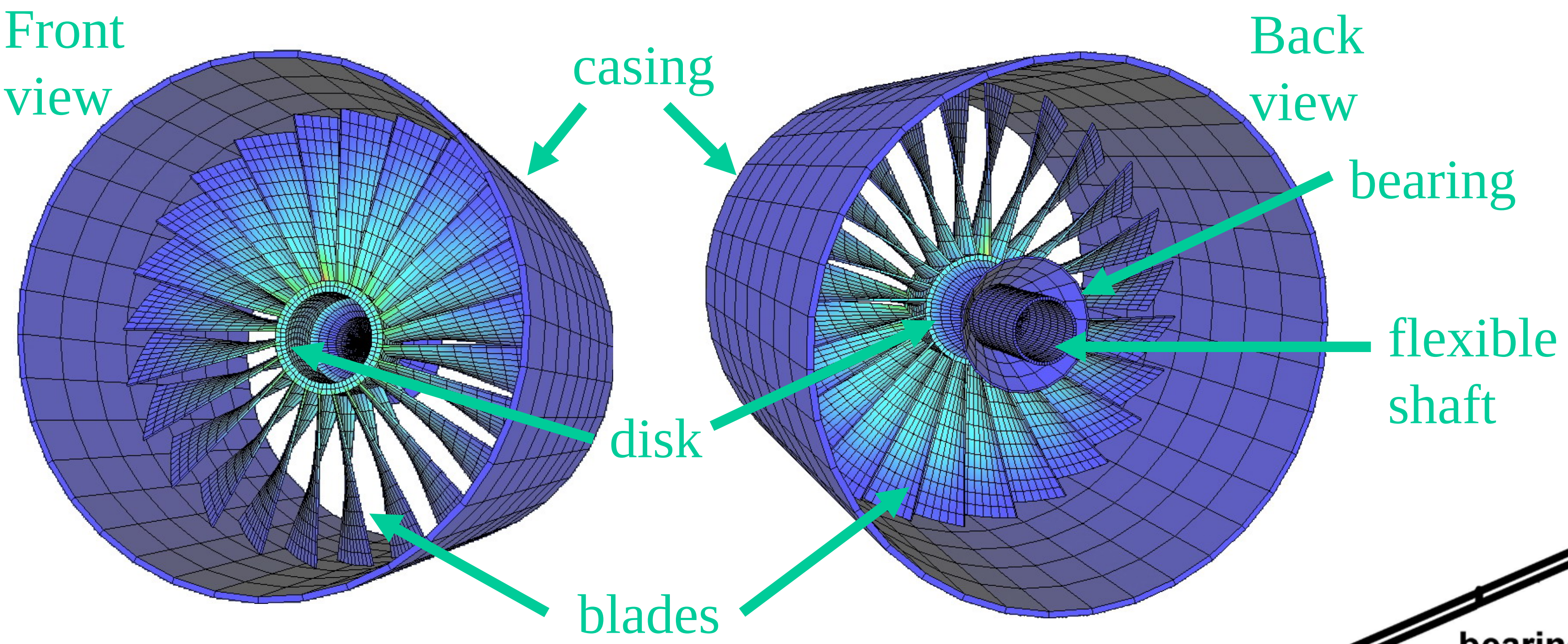
- Final results comparison:

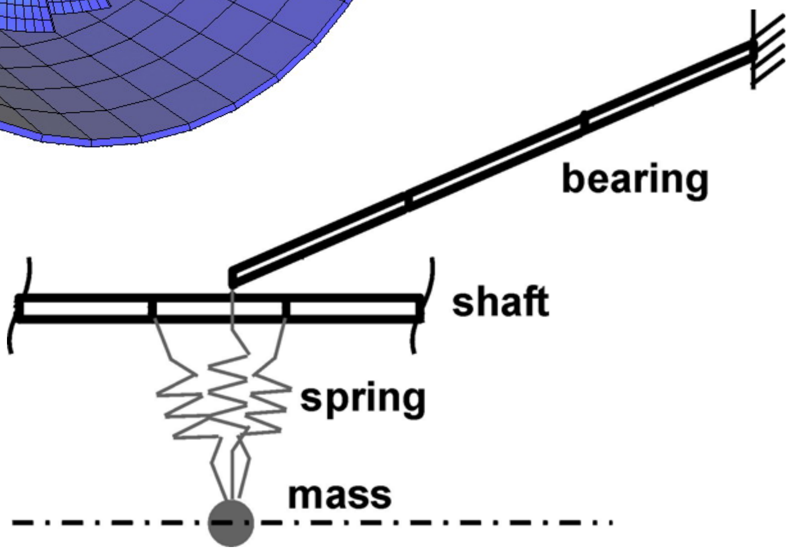




4. Complex numerical examples
Blade off simulation

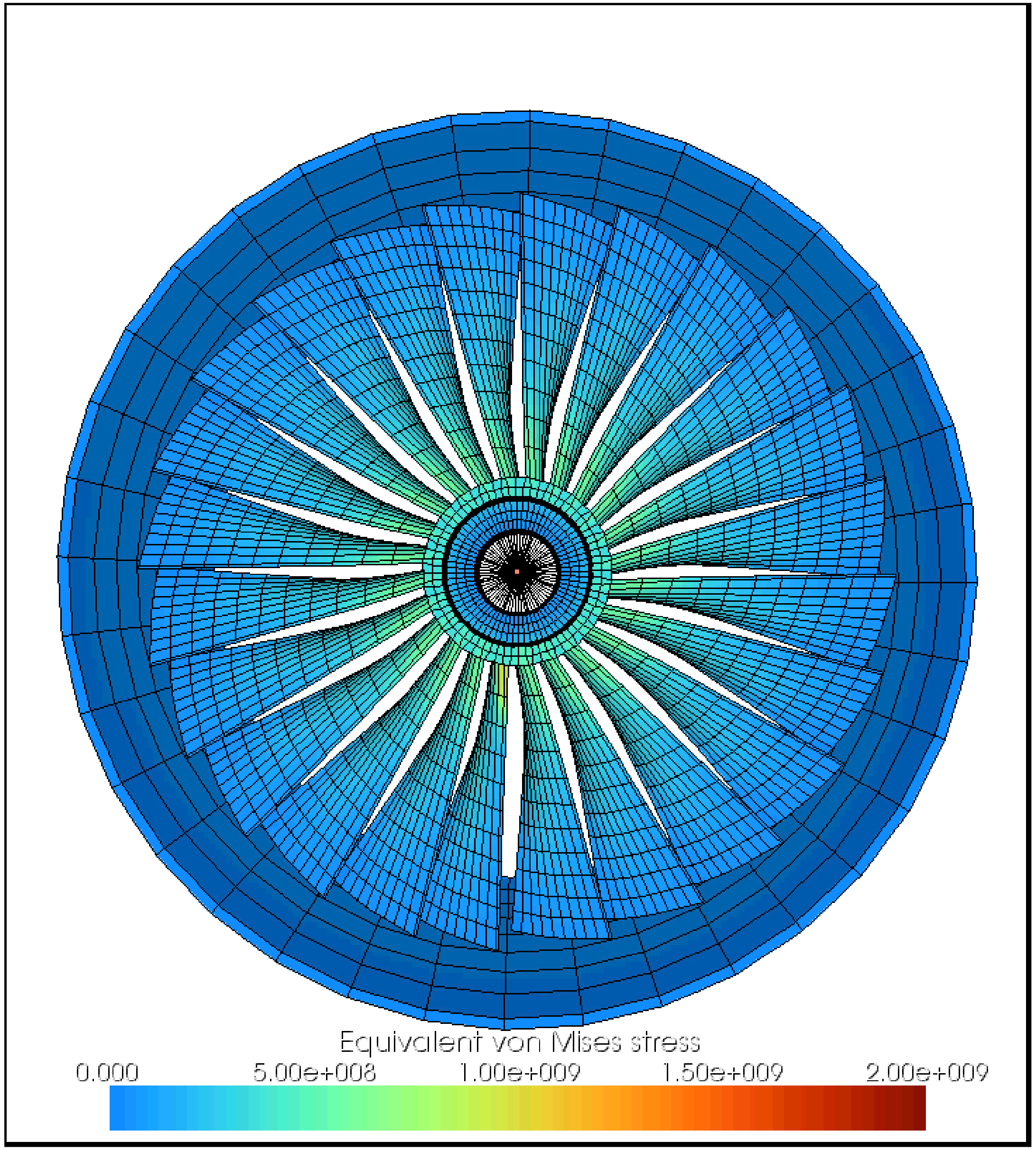
- Numerical simulation of a blade loss in an aero engine





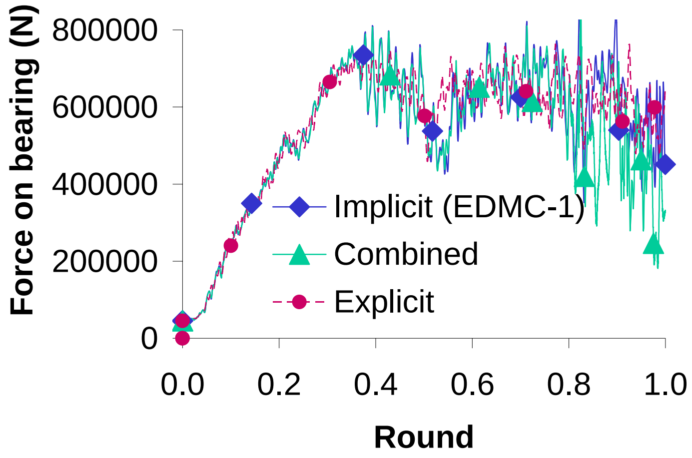
4. Complex numerical examples
Blade off simulation

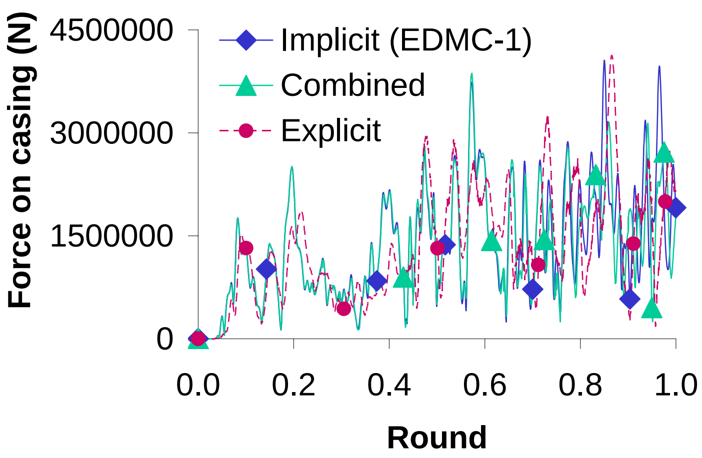
- Blade off :
  - Rotation velocity 5000rpm
  - EDMC algorithm
  - 29000 dof's
  - One revolution simulation
  - 9000 time steps
  - 50000 iterations (only 9000 with stiffness matrix updating)



4. Complex numerical examples
Blade off simulation

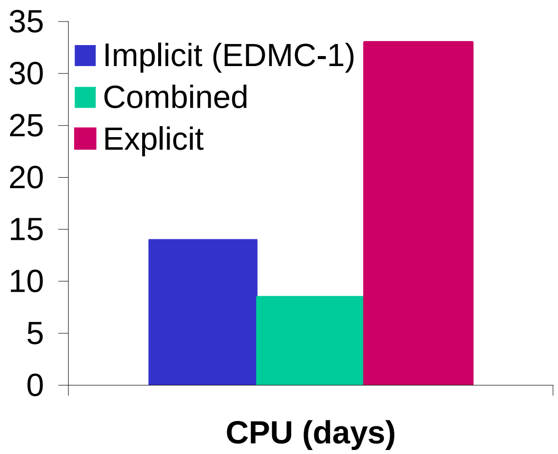
- Final results comparison:



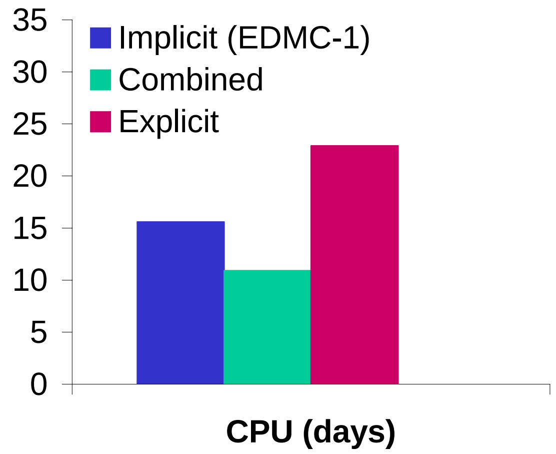


- CPU time comparison before and after code optimization:

Before optimization

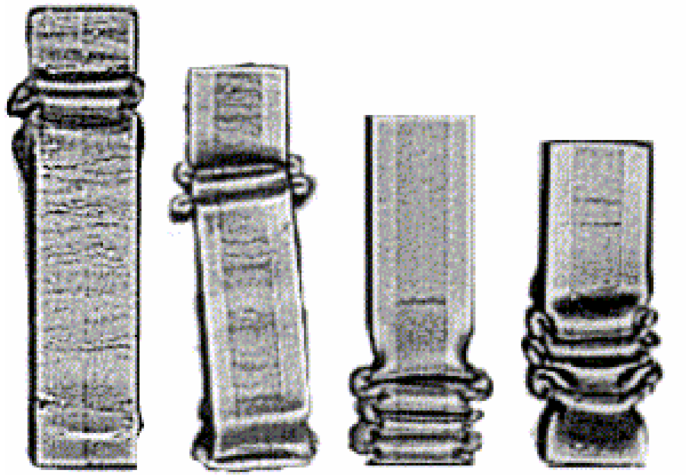


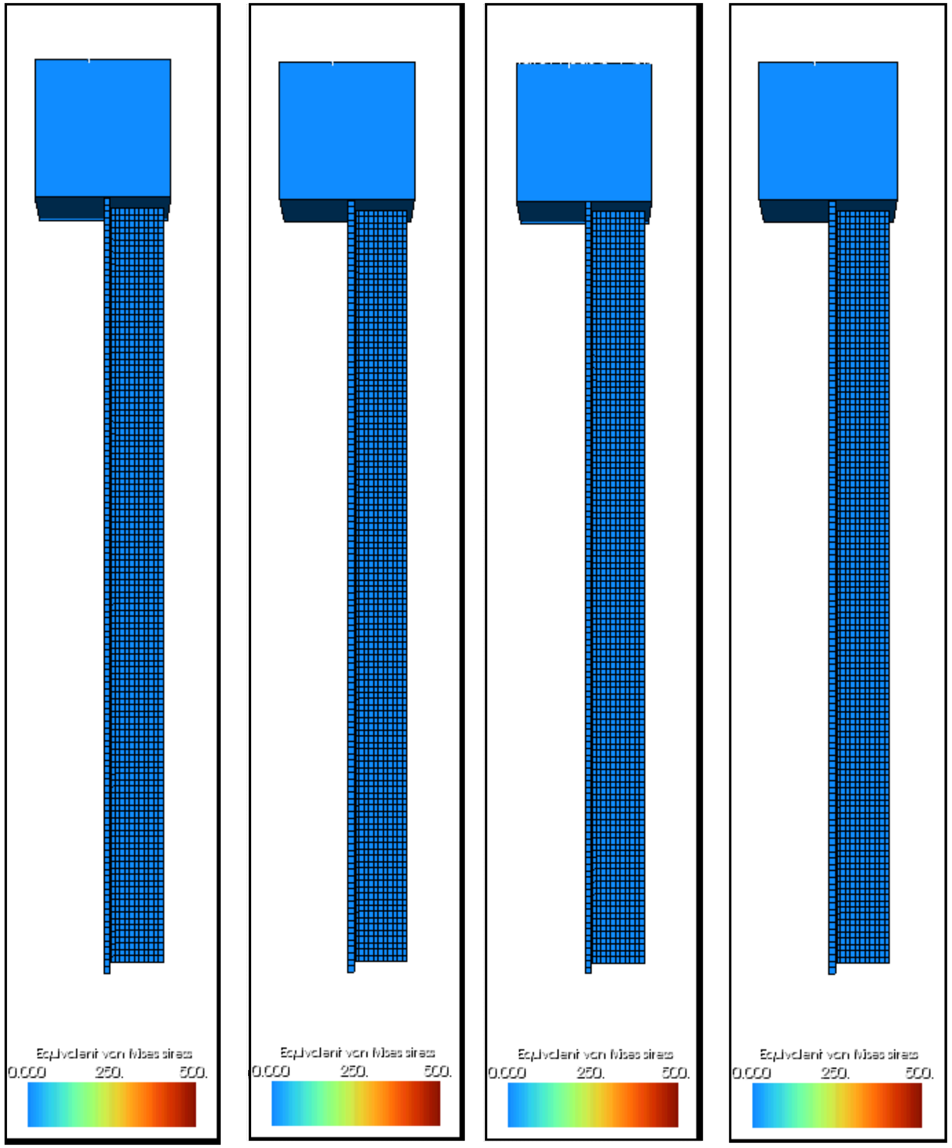
After optimization



4. Complex numerical examples
Dynamic buckling of square aluminum tubes

- Absorption of 600J with different impact velocities :
  - EDMC algorithm
  - 16000 dof's / 2640 elements
  - Initial asymmetry
  - Comparison with the experimental results of Yang, Jones and Karagiozova [IJIE, 2004]

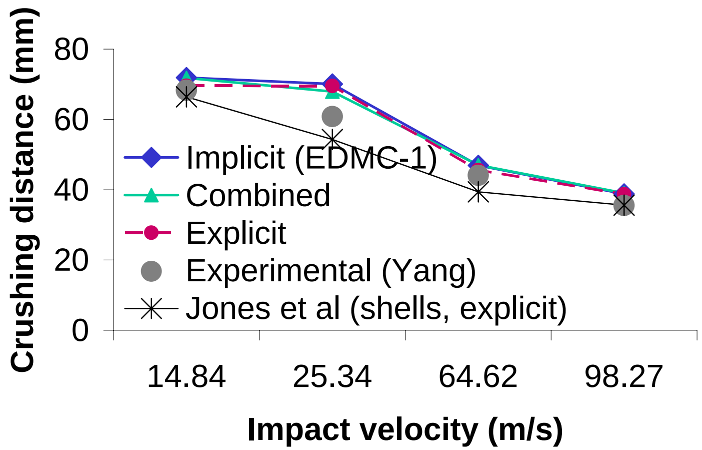


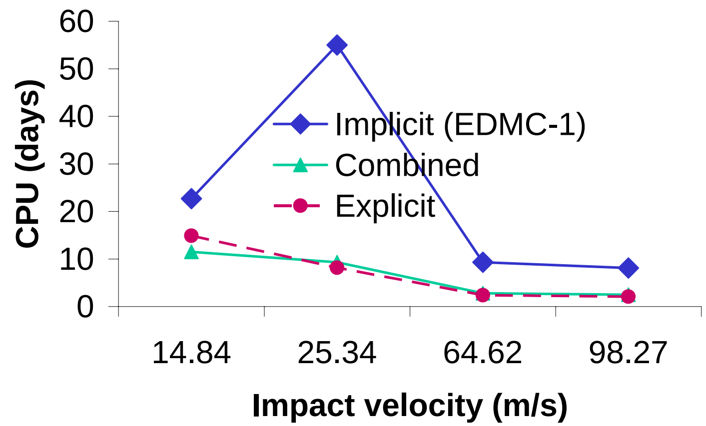


Impact velocity : 98.27 m/s 64.62 m/s 25.34 m/s 14.84 m/s

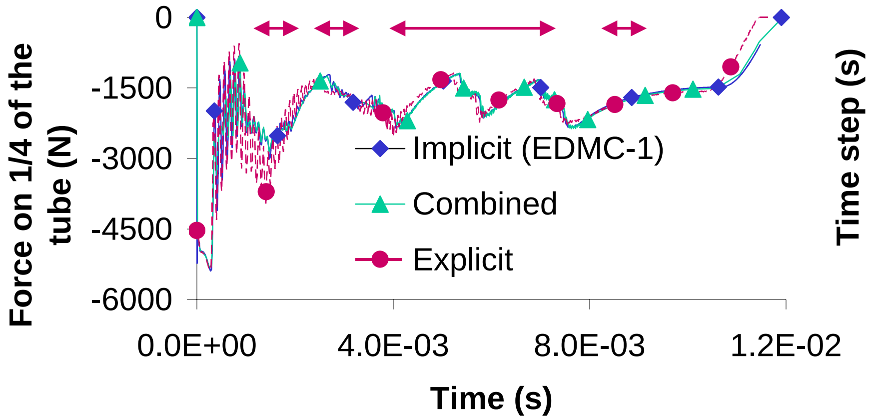
Dynamic buckling of square aluminum tubes

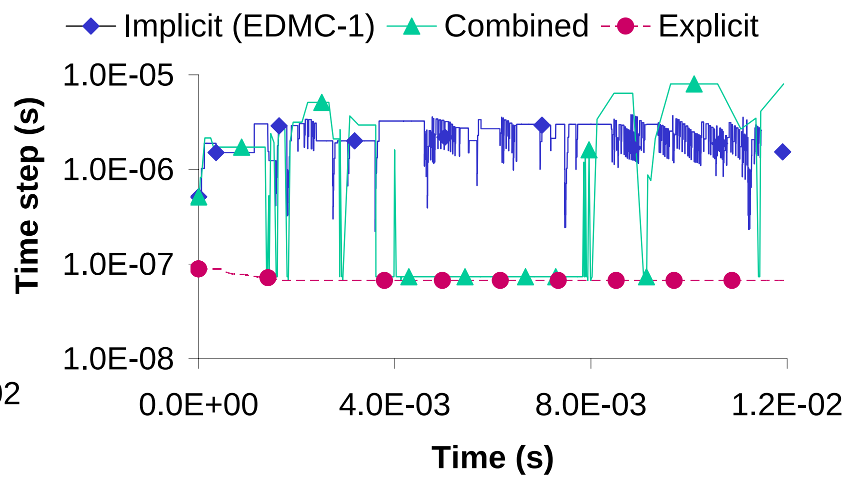
- Final results comparison:





- Time evolution for the 14.84 m/s impact velocity:





5. Conclusions & perspectives
Advantages of new developments

- Advantages of the consistent scheme:
  – Conservation laws and physical consistency are verified for each time step size in the non-linear range
  – Conservation of angular momentum even if numerical dissipation is introduced

- Advantages of the implicit/explicit combined scheme:
  – Reduction of the CPU cost
  – Automatic algorithms
  – No lack of accuracy
  – Remains available after code optimizations

5. Conclusions & perspectives
Drawbacks of new developments

- Drawbacks of the consistent scheme:
  - Mathematical developments needed for each element, material…
  - More complex to implement
- Drawback of the implicit/explicit combined scheme:
  - Implicit and explicit elements must have the same formulation

5. Conclusions & perspectives
Future works

- Development of a second order accurate EDMC scheme
- Extension to a hyper-elastic model based on an incremental potential (in progress)
- Development of a thermo-mechanical consistent scheme
- Modelization of wind-milling in a turbo-engine