Modeling of Damage to Crack Transition using a Coupled Discontinuous Galerkin / Cohesive Extrinsic Law Framework

L. Wu , G. Becker and L. Noels

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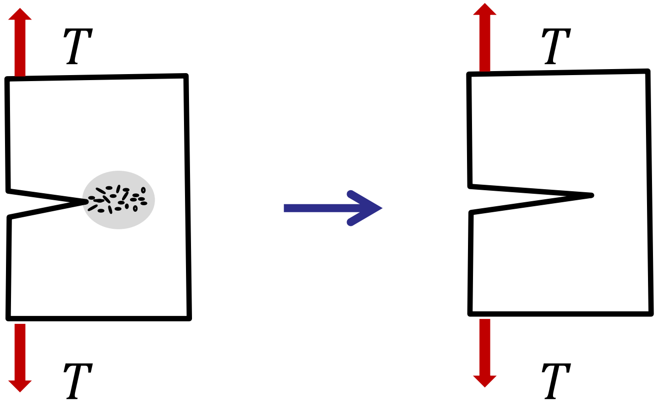
Introduction

- Material fracture process
  - Damage accumulation
  - crack



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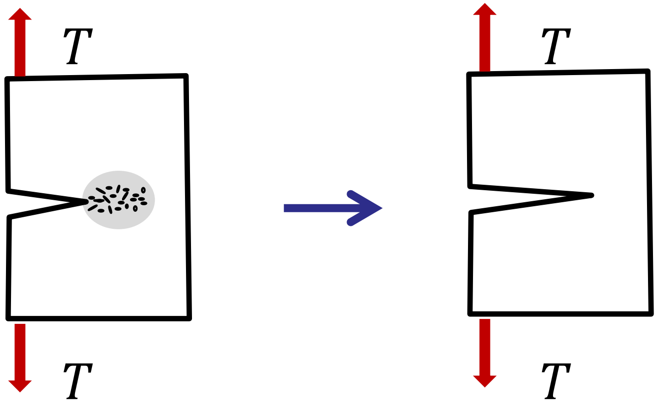
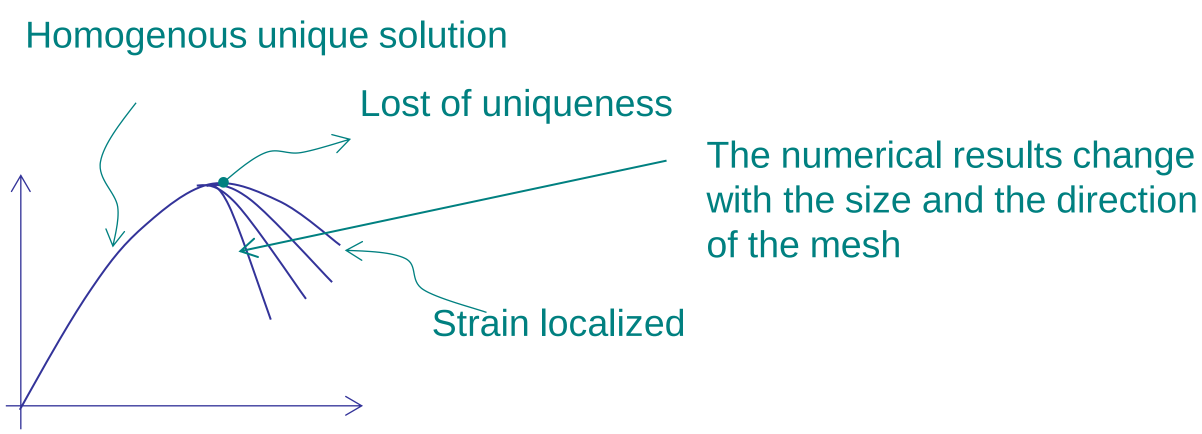


- Finite element solutions for strain softening problems suffer from:
  - The loss the uniqueness and strain localization
  - Mesh dependence



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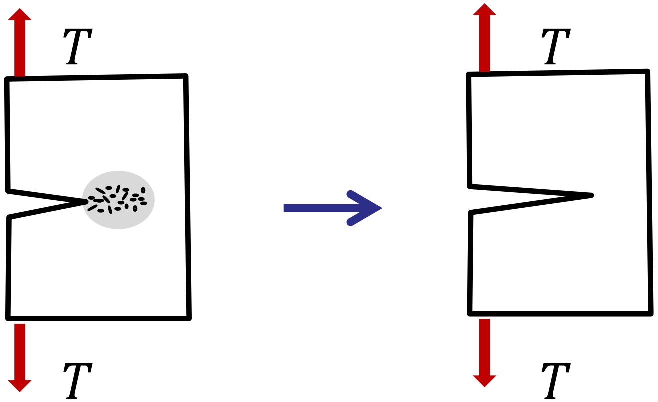

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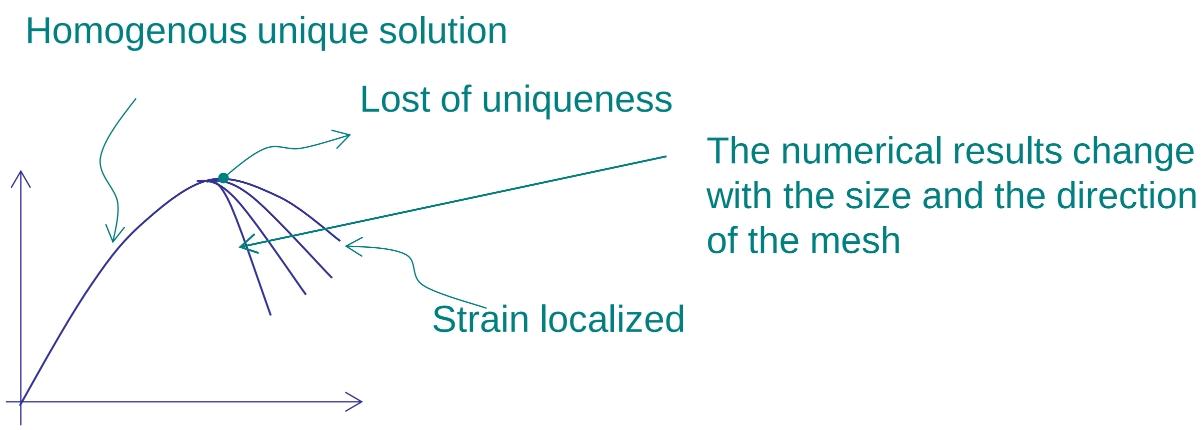

- Discontinuous Galerkin / Cohesive Extrinsic Law Framework

Non-local damage model to cohesive zone model

- Implicit gradient enhanced damage model [Peerlings et al. 96, Geers et al. 97, …]
  - A state variable is replaced by a non-local value reflecting the interaction between neighboring material points

$$\tilde{a}(\boldsymbol{x}) = \frac{1}{V_C}\int_{V_C} a(\boldsymbol{y})w(\boldsymbol{y};\boldsymbol{x})\mathrm{d}V$$

  - Use Green functions as weight $w(y; x)$ ⇨ Implicit gradient enhanced model

$$\tilde{a} - c\nabla^2\tilde{a} = a \quad \text{with} \quad \frac{\partial\tilde{a}}{\partial n} = n_i\frac{\partial\tilde{a}}{\partial x_i} = 0$$

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  - General form for anisotropic cases

$$\tilde{a} - \nabla\cdot\left(\mathbf{c}_g\cdot\nabla\tilde{a}\right) = a \qquad \mathbf{n}\cdot\left(\mathbf{c}_g\cdot\nabla\tilde{a}\right) = 0$$

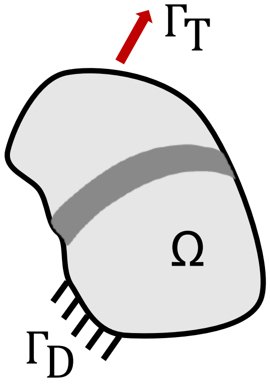
  - Damage evolution $D(\tilde{e};\, t)$

$$\tilde{e} - \nabla\cdot\left(\mathbf{c}_g\cdot\nabla\tilde{e}\right) = e \qquad e = \sqrt{\sum_{i=1,2,3}(\varepsilon_i^+)^2}$$

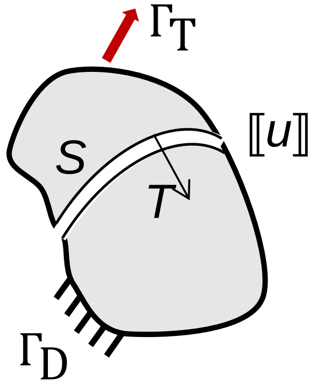
Non-local damage model to cohesive zone model

- Energy equivalence of damage model and cohesive zone model
  - Cohesive law can be constructed from damage model [Planas et al. 1993, Cazes et al. 2009…]

Non-local damage



Cohesive zone



$$\sigma = (1-D)\mathbf{C}^{\mathbf{e}} : \boldsymbol{\varepsilon}$$

$$Y = -\frac{1}{2}\boldsymbol{\varepsilon} : \mathbf{C}^{\mathbf{e}} : \boldsymbol{\varepsilon}$$

Free energy

$$\frac{1}{2}\int_{\Omega}(1-D)\boldsymbol{\varepsilon} : \mathbf{C}^{\mathbf{e}} : \boldsymbol{\varepsilon}\mathrm{d}V$$

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Before damage localization

$D = D'$ (diffuse damage)

$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}'$

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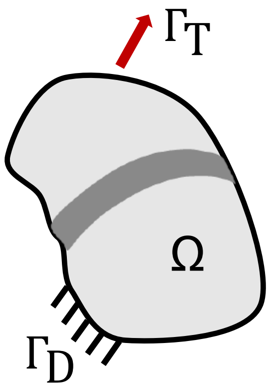
Non-local damage



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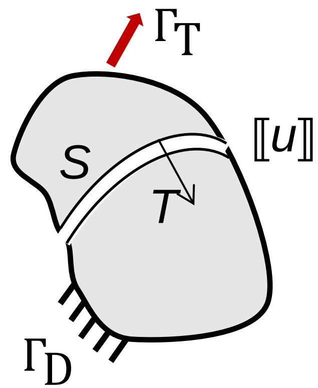

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$$\frac{1}{2}\int_{\Omega}(1-D')\boldsymbol{\varepsilon}' : \mathbf{C}^{\mathbf{e}} : \boldsymbol{\varepsilon}'\,\mathrm{d}V + \frac{1}{2}\int_{S} T[\![u]\!]\,\mathrm{d}A$$

Out of damage localization zone

$D = D'$ (diffuse damage)

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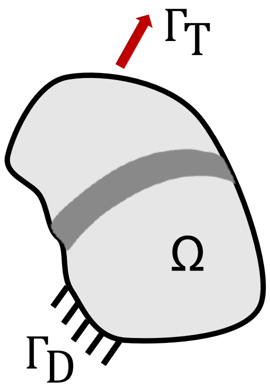
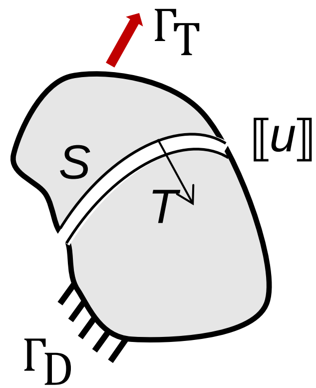
Non-local damage | **Cohesive zone**





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Increment of dissipated energy

$$\mathrm{d}\Phi_{\mathrm{loc}} = \int_{\Omega}(-Y\mathrm{d}D)\mathrm{d}V \;=\; \mathrm{d}\Phi_{\mathrm{S}} = \frac{1}{2}\int_{S}(T\mathrm{d}[\![u]\!] - [\![u]\!]\,\mathrm{d}T)\mathrm{d}A$$

Dissipated energy

$$\int_{\Omega}\left(\int_{D'}^{1} -Y\mathrm{d}D\right)\mathrm{d}V \;=\; \int_{S}\int_{0}^{\infty} T\,[\![u]\!]\,\mathrm{d}A$$

Out of damage localization zone

$D = D'$ (diffuse damage)

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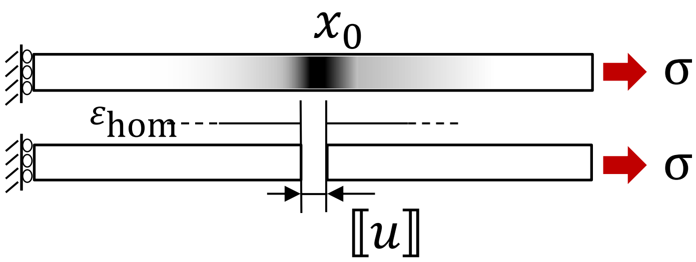
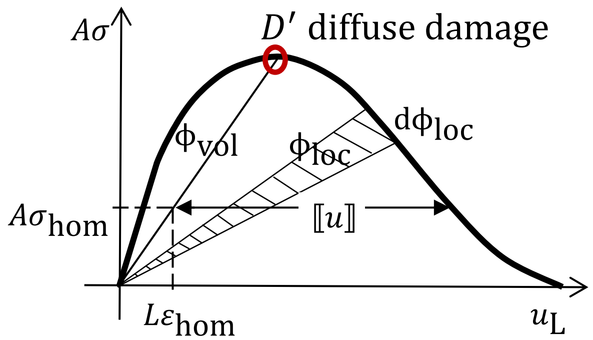
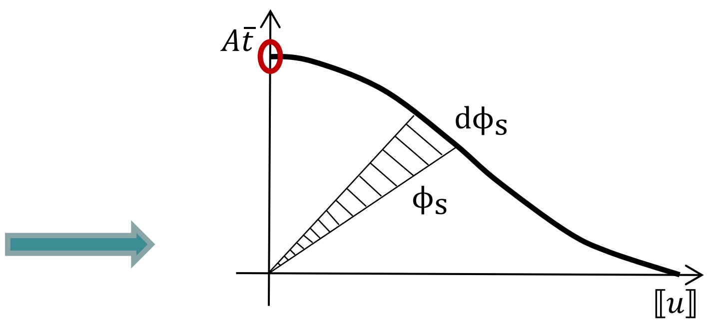
- 1D case analysis
  - *A* – cross section of the bar
  - σ – tensile stress
  - u_L – the displacement at right end of the bar







Transition at *D'*

$$[\![u]\!] = \int_0^L (\varepsilon - \varepsilon_{hom}) \mathrm{d}L$$

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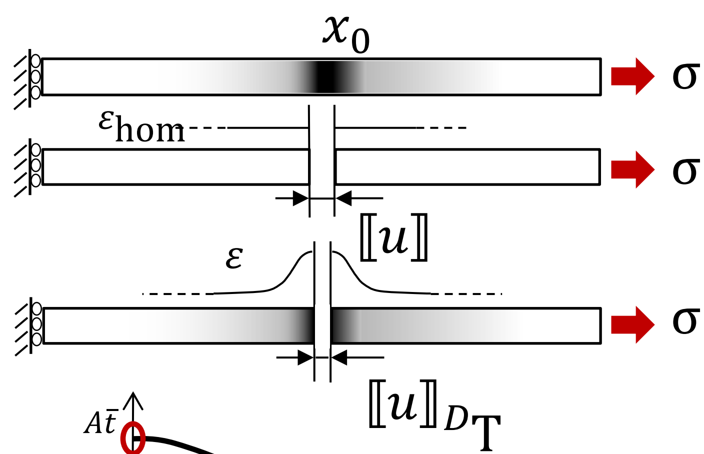

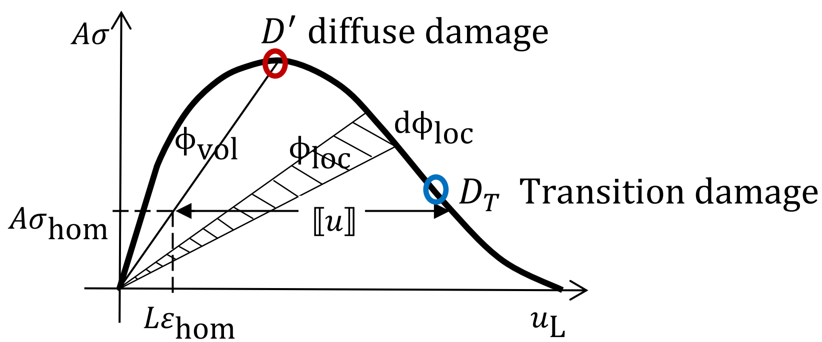

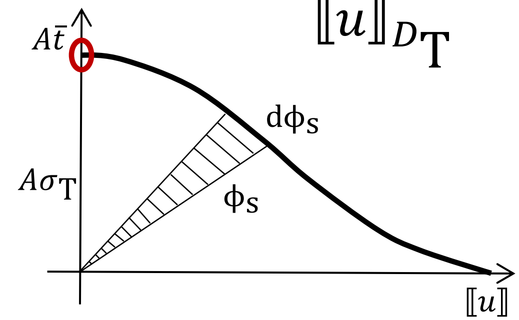

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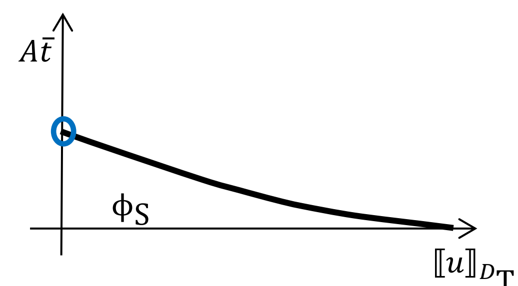
Transition at D_T

$$[\![u]\!]_{D_T} \neq [\![u]\!]$$



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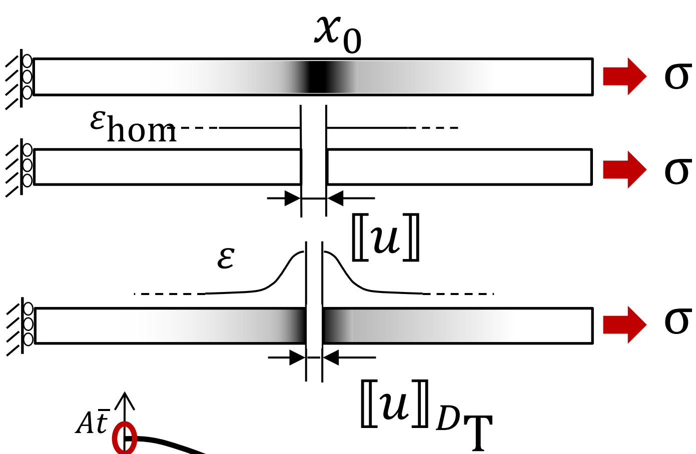

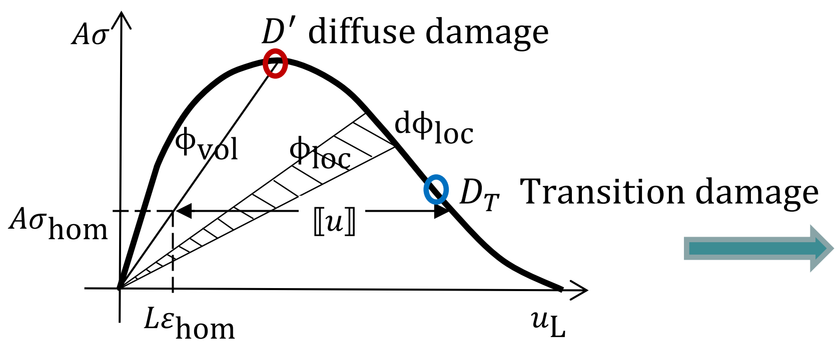

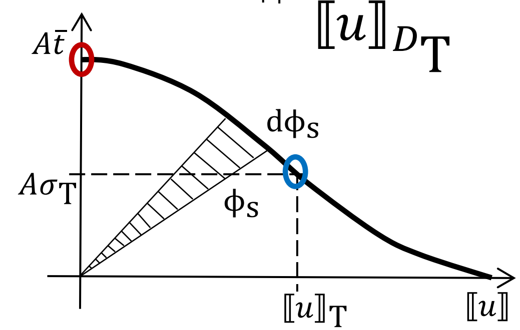

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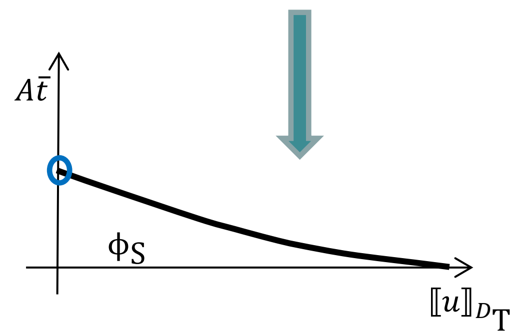
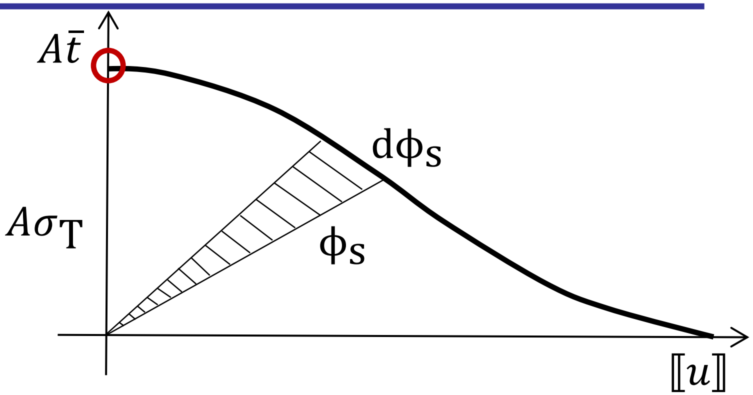


Key issue: ϕ_S !!!

Non-local damage model to cohesive zone model

- 1D case analysis
  - Numerical solution



Non-local damage model to cohesive zone model

- 1D case analysis
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  - Approximation at high damage

When damage is rather high:
[Dufour et al. 2008, …]

$$\tilde{\varepsilon}_C \approx \tilde{\varepsilon}_D(\varepsilon_{\text{hom}}, [\![u]\!])$$

Stress equivalence:

$$(1-D')E\varepsilon_{\text{hom}} = (1-D(\tilde{\varepsilon}_C))E\varepsilon$$

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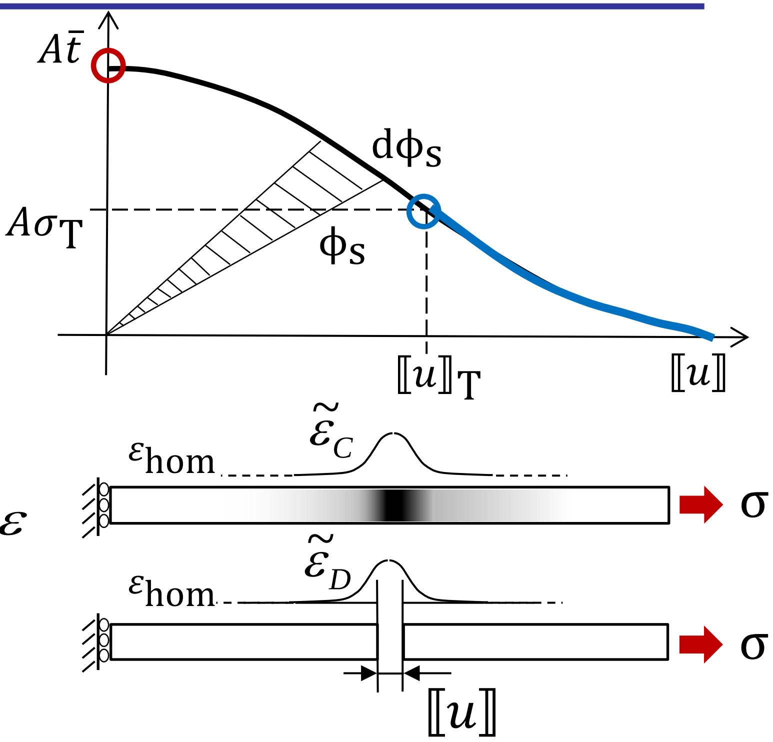

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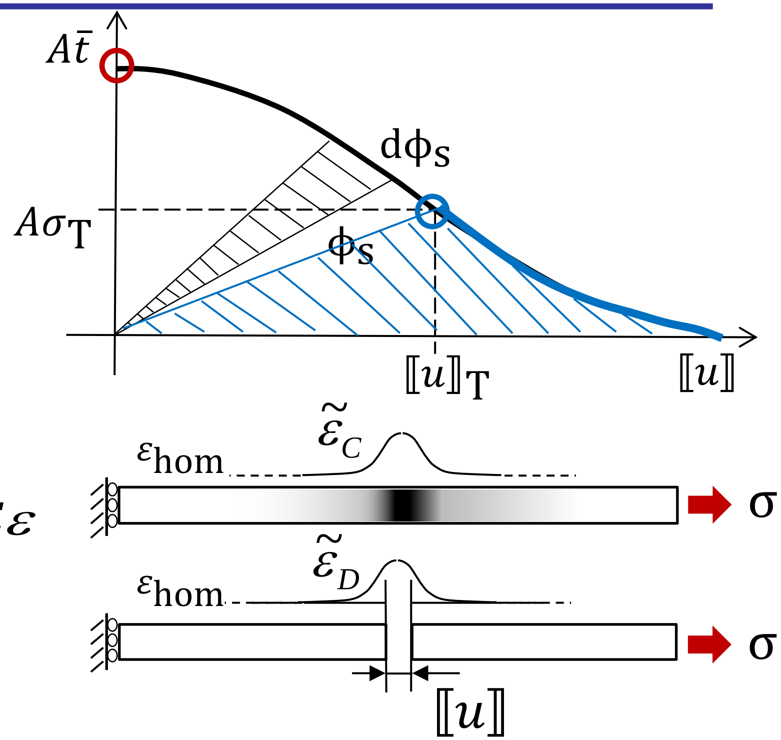

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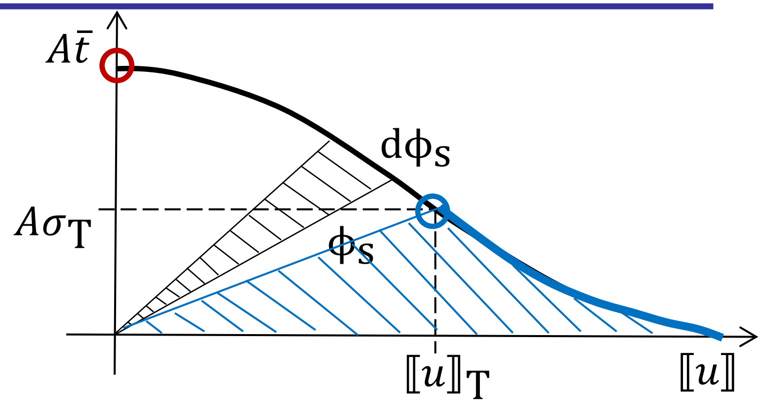

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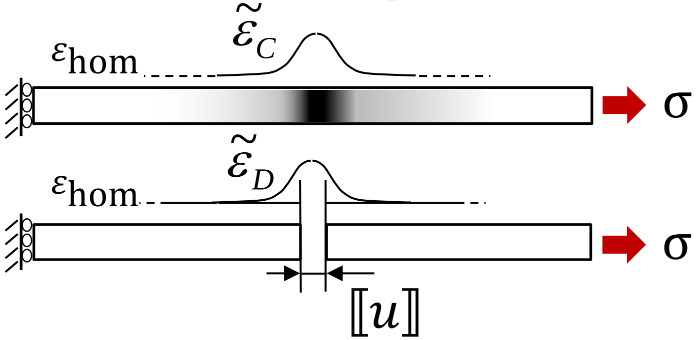
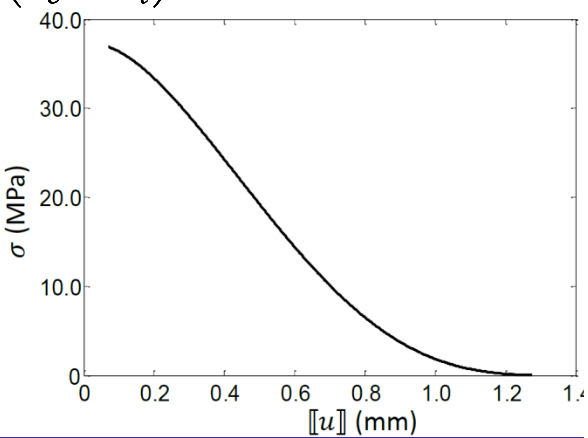


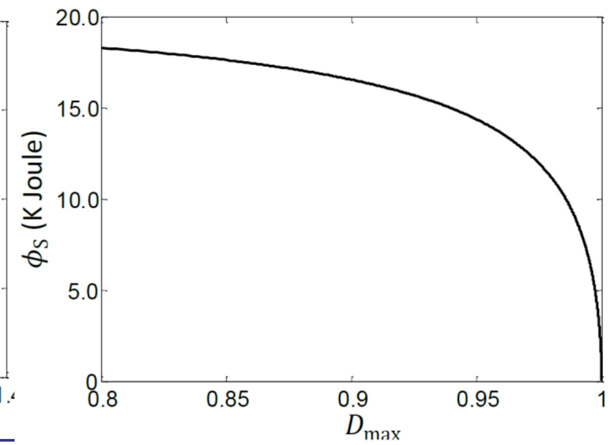
Example: [Geers et al. 1999, …]

$$D = 1 - \left(\frac{\kappa_i}{\kappa}\right)^{\beta}\left(\frac{\kappa_c - \kappa}{\kappa_c - \kappa_i}\right)^{\alpha} \qquad \kappa\{x\} = \max[\tilde{\varepsilon}(x,\tau)|\tau \le t]$$

E	3.2[GPa]
ν	0.28
κ_i	0.011
κ_c	0.5
α	5.0
β	0.75

$$\boldsymbol{c}_g = \text{diag}\{2.0\}\ mm^2$$





- Problems with cohesive elements
  - Intrinsic Cohesive Law (ICL)
    - Cohesive elements inserted from the beginning
    - Drawbacks:
      - Efficient if a priori knowledge of the crack path
      - Mesh dependency [Xu & Needelman, 1994]
      - Initial slope modifies the effective elastic modulus
      - This slope should tend to infinity [Klein et al. 2001]:
        - » Alteration of a wave propagation
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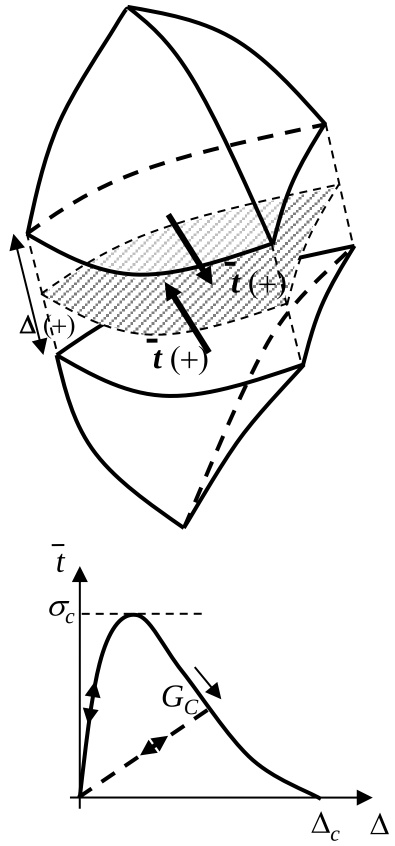

Discontinuous Galerkin / Cohesive Extrinsic Law Framework

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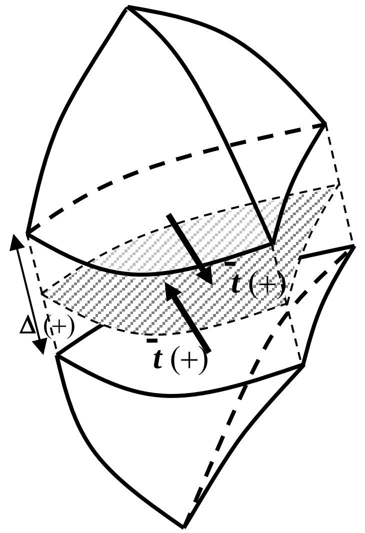
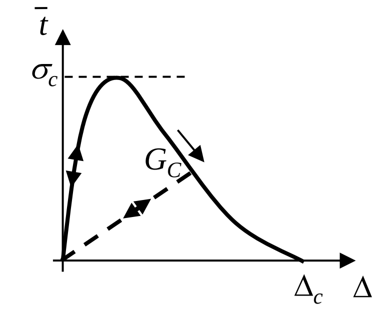
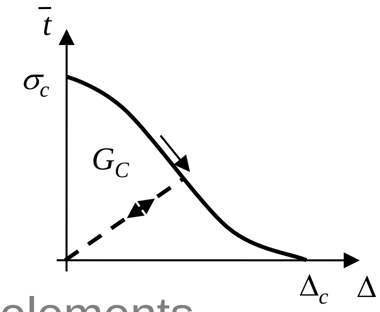
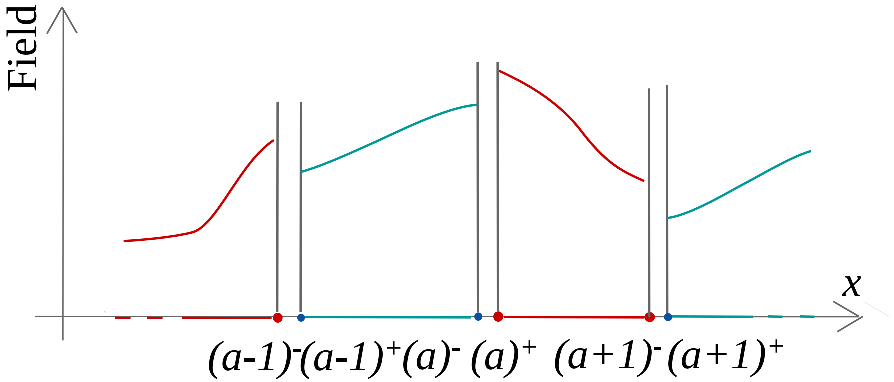
  - Extrinsic Cohesive Law (ECL)
    - Cohesive elements inserted on the fly when failure criterion is verified [Ortiz & Pandolfi 1999]
    - Drawback
      - Complex implementation in 3D (parallelization)
- Solution
  - Use discontinuous Galerkin methods embedding interface elements







- Discontinuous Galerkin (DG) methods
  - Finite-element discretization
  - Same **discontinuous** polynomial approximations for the
    - **Test** functions $\boldsymbol{u}_h$ and
    - **Trial** functions $\delta\boldsymbol{u}$



  - Definition of operators on the interface trace:
    - **Jump** operator: $[\![\bullet]\!] = \bullet^+ - \bullet^-$
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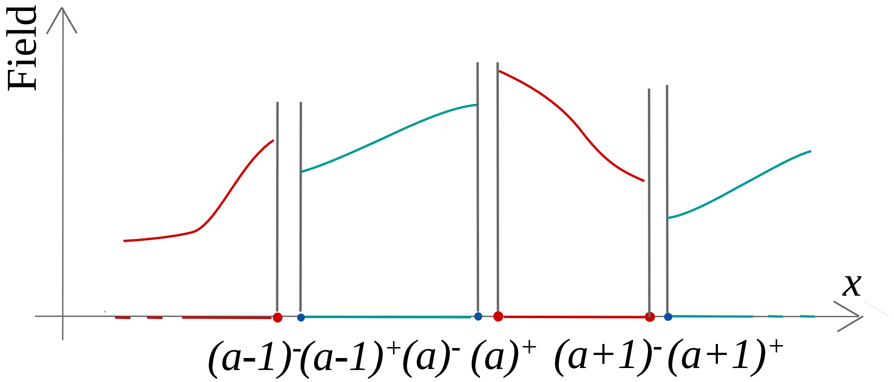

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    - **Jump** operator: $[\![\bullet]\!] = \bullet^+ - \bullet^-$
    - **Mean** operator: $\langle \bullet \rangle = \frac{\bullet^+ + \bullet^-}{2}$
  - Continuity is weakly enforced, such that the method
    - Is consistent
    - Is stable
    - Has the optimal convergence rate

Discontinuous Galerkin / Cohesive Extrinsic Law Framework

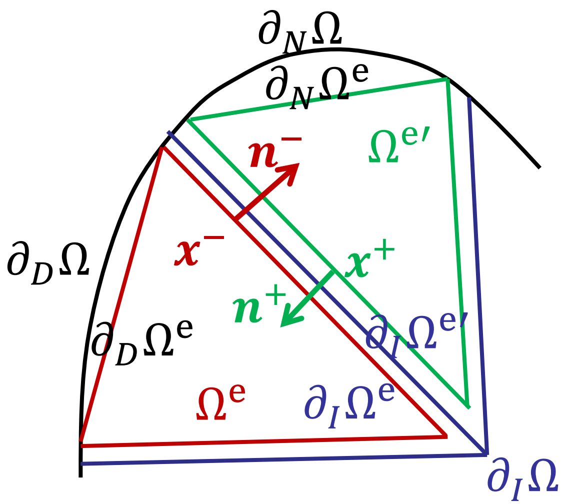
- Governing equations

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma}^T = \boldsymbol{0} \\ \tilde{e} - \nabla \cdot \left(\mathbf{c}_{\mathrm{g}} \cdot \nabla \tilde{e} \right) = e \end{cases}$$

Boundary conditions

$$\begin{cases} \boldsymbol{\sigma} \cdot \boldsymbol{n} = \bar{\boldsymbol{t}} \\ \boldsymbol{n} \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{e} \right) = 0 \end{cases}$$

- Weak formulation obtained by integration by parts **on each element** Ω^e



$$\sum_{\mathrm{e}} \int_{\Omega^{\mathrm{e}}} \boldsymbol{\nabla} \cdot \sigma^T(\boldsymbol{u}_h) \cdot \delta \boldsymbol{u} \, \mathrm{d}\Omega = 0$$

$$\sum_{\mathrm{e}} \int_{\Omega^{\mathrm{e}}} -\boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla} \delta \boldsymbol{u} \, \mathrm{d}\Omega + \sum_{e} \int_{\partial \Omega^{\mathrm{e}}} \delta \boldsymbol{u} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} \mathrm{d}\partial\Omega = 0$$

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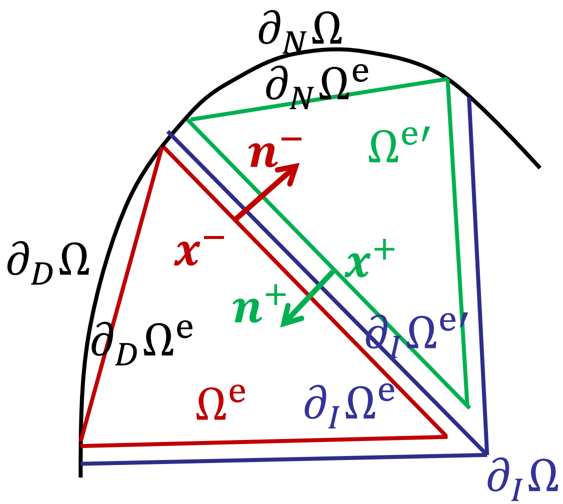

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$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla} \delta \boldsymbol{u} \, \mathrm{d}\Omega + \int_{\partial_I \Omega} [\![\delta \boldsymbol{u} \cdot \boldsymbol{\sigma}]\!] \cdot \boldsymbol{n}^- \mathrm{d}\partial\Omega = \int_{\partial_N \Omega} \bar{\boldsymbol{t}} \cdot \delta \boldsymbol{u} \mathrm{d}\partial\Omega$$

New interface termes

Discontinuous Galerkin / Cohesive Extrinsic Law Framework

- Governing equations

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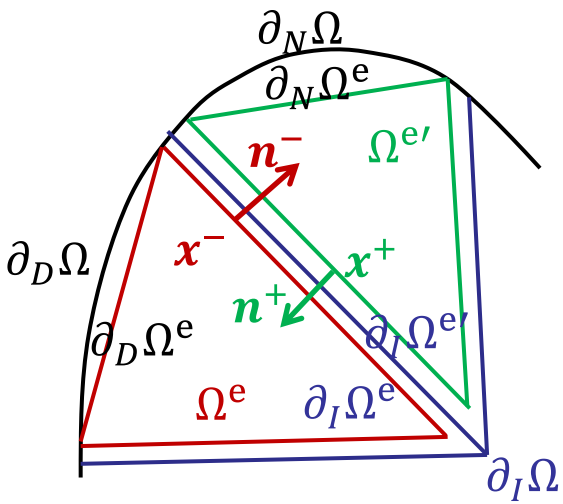
$$\begin{cases} \boldsymbol{\sigma} \cdot \boldsymbol{n} = \bar{\boldsymbol{t}} \\ \boldsymbol{n} \cdot \left(\boldsymbol{c}_g \cdot \nabla \tilde{e} \right) = 0 \end{cases}$$

- Weak formulation obtained by integration by parts **on each element** Ω^e



$$\sum_e \int_{\Omega^e} \boldsymbol{\nabla} \cdot \sigma^T(\boldsymbol{u}_h) \cdot \delta \boldsymbol{u} \, \mathrm{d}\Omega = 0$$

$$\sum_e \int_{\Omega^e} -\boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla} \delta \boldsymbol{u} \, \mathrm{d}\Omega + \sum_e \int_{\partial \Omega^e} \delta \boldsymbol{u} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} \mathrm{d}\partial\Omega = 0$$

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla} \delta \boldsymbol{u} \, \mathrm{d}\Omega + \int_{\partial_I \Omega} [\![\delta \boldsymbol{u} \cdot \boldsymbol{\sigma}]\!] \cdot \boldsymbol{n}^- \mathrm{d}\partial\Omega = \int_{\partial_N \Omega} \bar{\boldsymbol{t}} \cdot \delta \boldsymbol{u} \mathrm{d}\partial\Omega$$

New interface termes

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla} \delta \boldsymbol{u} \, \mathrm{d}\Omega + \int_{\partial_I \Omega} [\![\delta \boldsymbol{u}]\!] \cdot \langle \boldsymbol{\sigma} \rangle \cdot \boldsymbol{n}^- \mathrm{d}\partial\Omega +$$

$$+ \int_{\partial_I \Omega} [\![\delta \boldsymbol{u}]\!] \otimes \boldsymbol{n}^- : \langle \frac{\beta_s}{h_s} \mathrm{C}^{\mathrm{e}} \rangle : [\![\boldsymbol{u}]\!] \otimes \boldsymbol{n}^- \mathrm{d}\partial\Omega + \int_{\partial_I \Omega} [\![\boldsymbol{u}]\!] \cdot \langle \mathrm{C}^{\mathrm{e}} : \boldsymbol{\nabla} \delta \boldsymbol{u} \rangle \cdot \boldsymbol{n}^- \mathrm{d}\partial\Omega = \int_{\partial_N \Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{n} \mathrm{d}\partial\Omega$$

Stabilization

Enforcement of the compatibility

Discontinuous Galerkin / Cohesive Extrinsic Law Framework

- Combining with cohesive law

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla}\delta\boldsymbol{u}\, \mathrm{d}\Omega + \int_{\partial_I\Omega} \alpha \bar{\boldsymbol{t}}^{-}(\llbracket \boldsymbol{u} \rrbracket) \cdot \llbracket \delta\boldsymbol{u} \rrbracket \mathrm{d}\partial\Omega$$

$$+ \int_{\partial_I\Omega} (1-\alpha) \llbracket \delta\boldsymbol{u} \rrbracket \cdot \langle \boldsymbol{\sigma} \rangle \cdot \boldsymbol{n}^{-} \mathrm{d}\partial\Omega + \int_{\partial_I\Omega} (1-\alpha) \llbracket \delta\boldsymbol{u} \rrbracket \otimes \boldsymbol{n}^{-} : \langle \frac{\beta_s}{h_s} \mathrm{C}^{\mathbf{e}} \rangle : \llbracket \boldsymbol{u} \rrbracket \otimes \boldsymbol{n}^{-} \mathrm{d}\partial\Omega$$

$$+ \int_{\partial_I\Omega} (1-\alpha) \llbracket \boldsymbol{u} \rrbracket \cdot \langle \mathrm{C}^{\mathbf{e}} : \boldsymbol{\nabla}\delta\boldsymbol{u} \rangle \cdot \boldsymbol{n}^{-} \mathrm{d}\partial\Omega = \int_{\partial_N\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{n} \mathrm{d}\partial\Omega$$

- Transition from damage to crack
  - Critical damage D_T

Discontinuous Galerkin / Cohesive Extrinsic Law Framework

- Combining with cohesive law

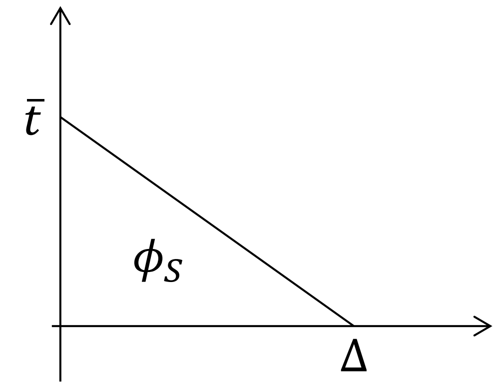
$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}_h) : \boldsymbol{\nabla}\delta\boldsymbol{u} \, \mathrm{d}\Omega + \int_{\partial_I\Omega} \alpha \bar{\boldsymbol{t}}^{-}([\![\boldsymbol{u}]\!]) \cdot [\![\delta\boldsymbol{u}]\!] \mathrm{d}\partial\Omega$$

$$+ \int_{\partial_I\Omega} (1-\alpha)[\![\delta\boldsymbol{u}]\!] \cdot \langle\boldsymbol{\sigma}\rangle \cdot \boldsymbol{n}^{-} \mathrm{d}\partial\Omega + \int_{\partial_I\Omega} (1-\alpha)[\![\delta\boldsymbol{u}]\!] \otimes \boldsymbol{n}^{-} : \langle \frac{\beta_s}{h_s} \mathrm{C}^{\mathbf{e}} \rangle : [\![\boldsymbol{u}]\!] \otimes \boldsymbol{n}^{-} \mathrm{d}\partial\Omega$$

$$+ \int_{\partial_I\Omega} (1-\alpha)[\![\boldsymbol{u}]\!] \cdot \langle \mathrm{C}^{\mathbf{e}} : \boldsymbol{\nabla}\delta\boldsymbol{u} \rangle \cdot \boldsymbol{n}^{-} \mathrm{d}\partial\Omega = \int_{\partial_N\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{n} \mathrm{d}\partial\Omega$$

- Transition from damage to crack
  - Critical damage D_T
  - Effective stress

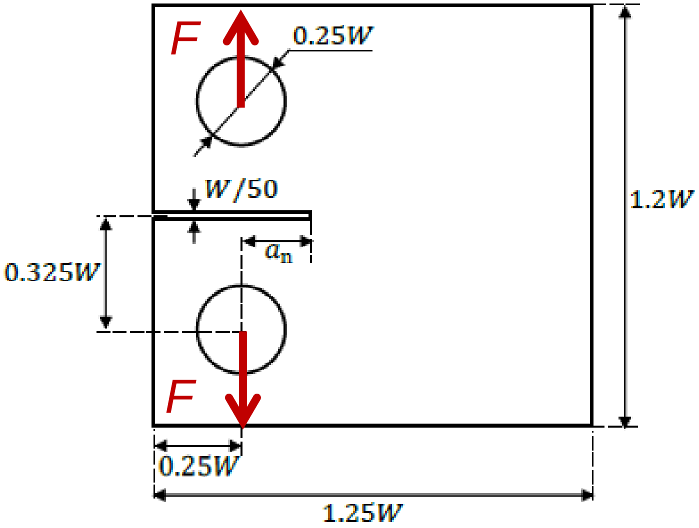
$$\sigma_c = \frac{\sigma}{(1-D_T)}$$



- TSL Characterized by
  - Strength σ_c &
  - Critical energy release rate ϕ_S

- Compact tension specimen [Geers et al. 1999, …]

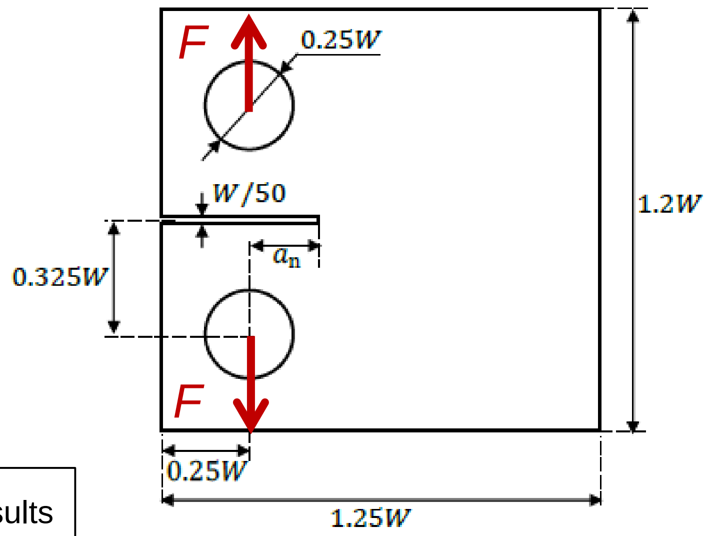
W=50 mm
an=10 mm
Thickness: 3.8 mm



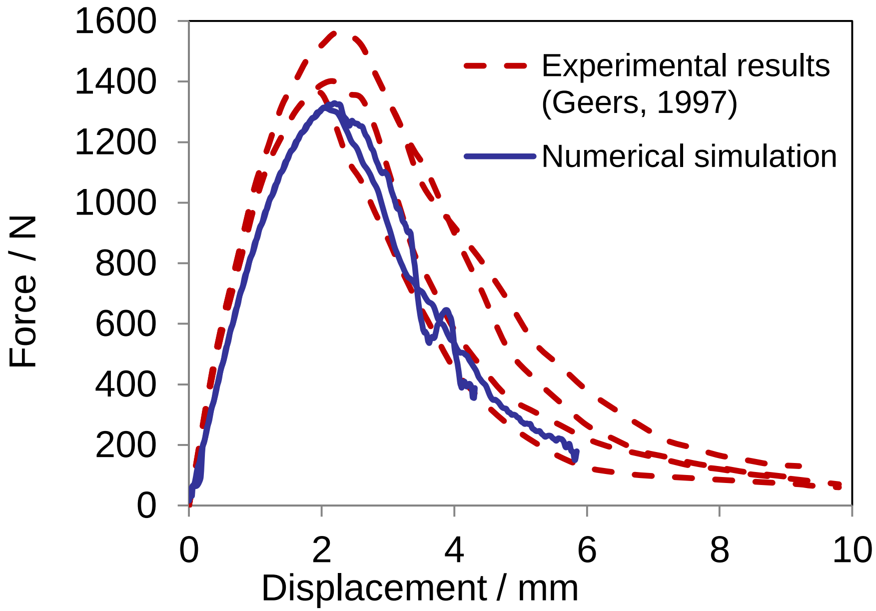
- 3D model
- Results

- Compact tension specimen [Geers et al. 1999, …]

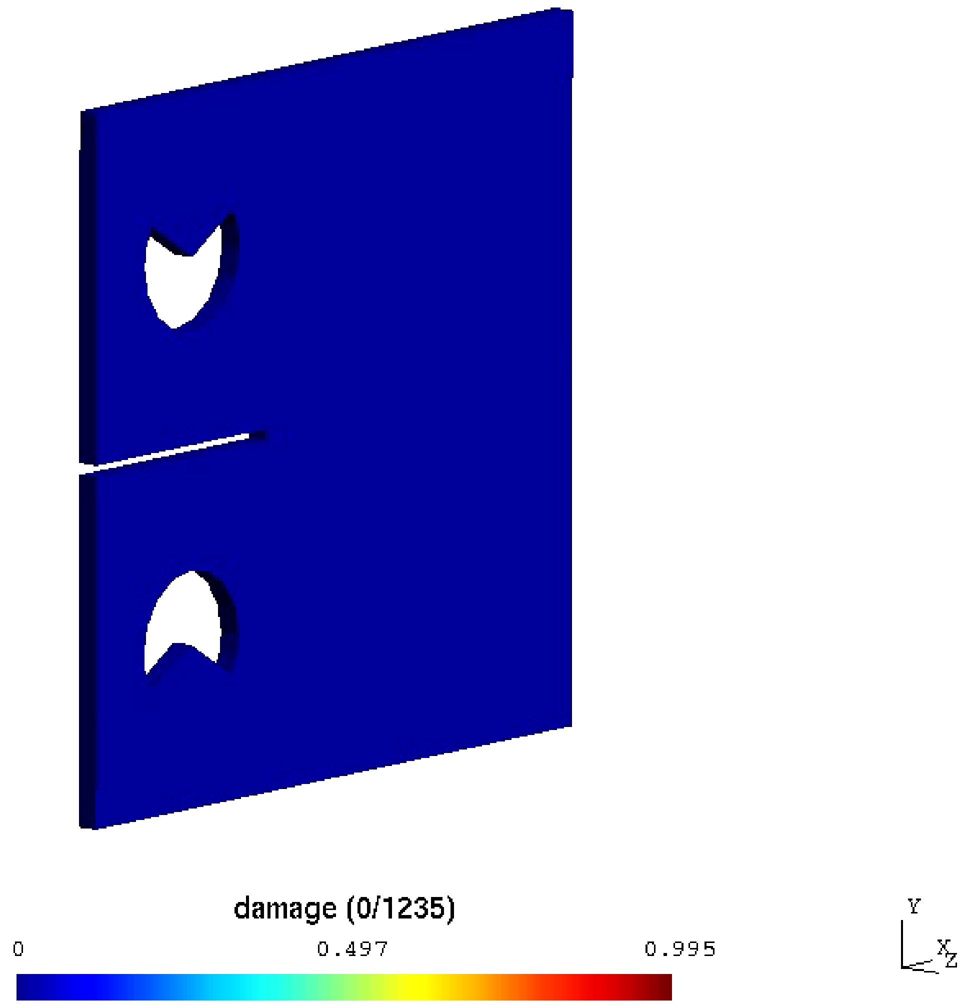
W=50 mm
an=10 mm
Thickness: 3.8 mm



  - 3D model
  - Results



- Compact tension specimen
  - Results



Conclusions

- Implicit gradient enhanced damage
  - Easy implementation
  - Extra degree of freedom on nodes
- Damage to crack
  - Cohesive law needs to be constructed
    - High damage (approximation)
    - Low damage (numerical solution)
  - Transition criterion from the information of damage and stress
- DG method
  - Computationally efficient // method
  - Consistent
  - Extrinsic cohesive law