A one-field formulation of elasto-plastic shells with fracture applications

BECKER Gauthier & NOELS Ludovic

Computational & Multiscale Mechanics of Materials, ULg
Chemin des Chevreuils 1, B4000 Liège, Belgium
Gauthier.Becker@ulg.ac.be
L.Noels@ulg.ac.be

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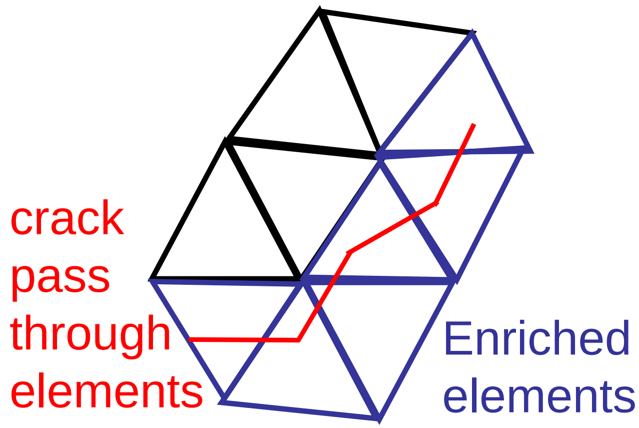
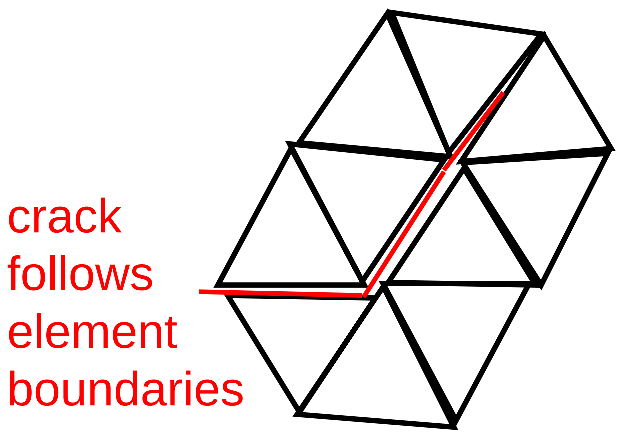
- The fracture process is modeled by cohesive elements to study
  - Dynamic crack propagation
  - Fragmentation

  - XFEM



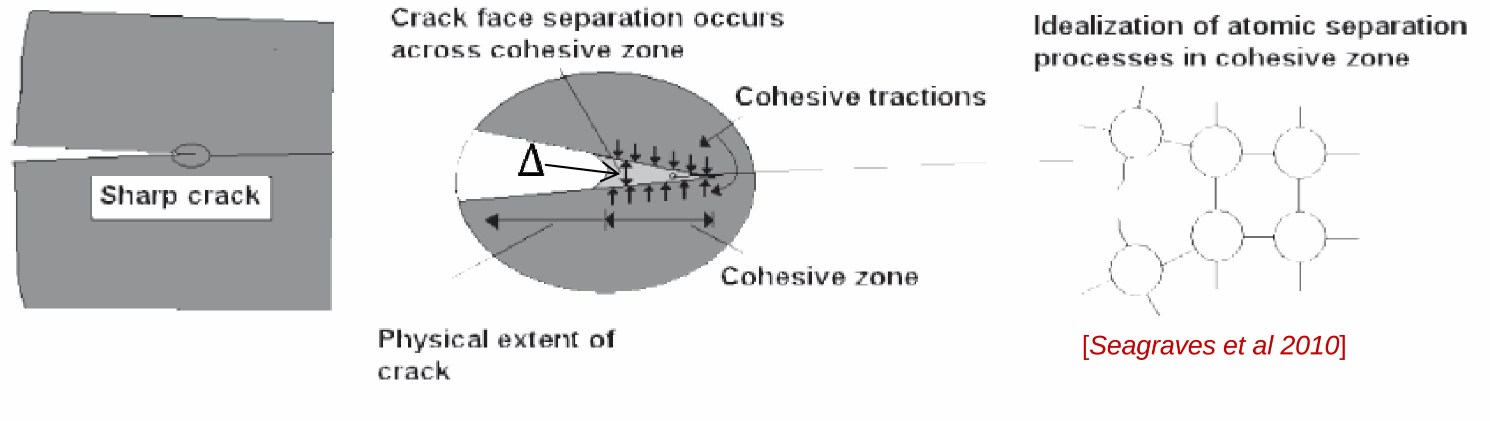
Commonly used for crack propagation

  - Interface elements

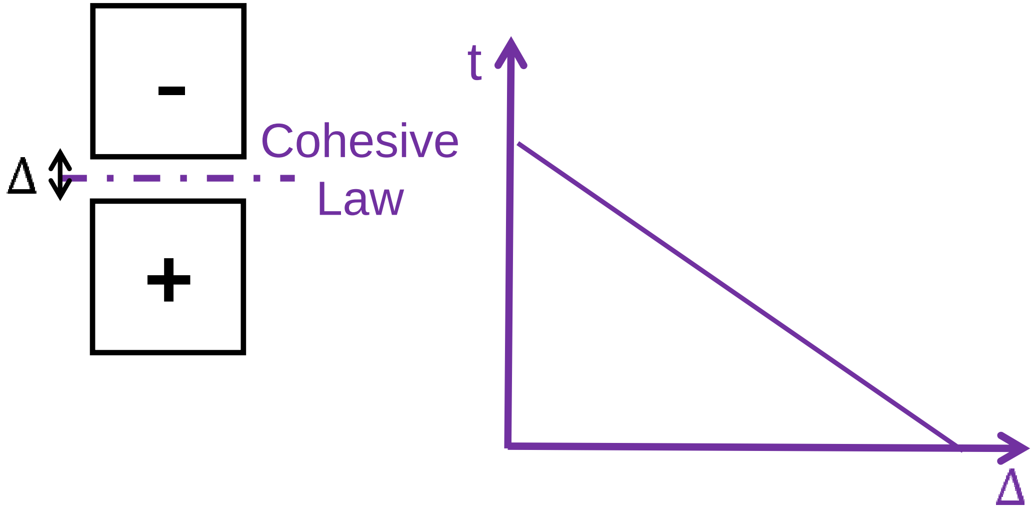


Dynamic phenomena (crack propagation due to impact, fragmentation)

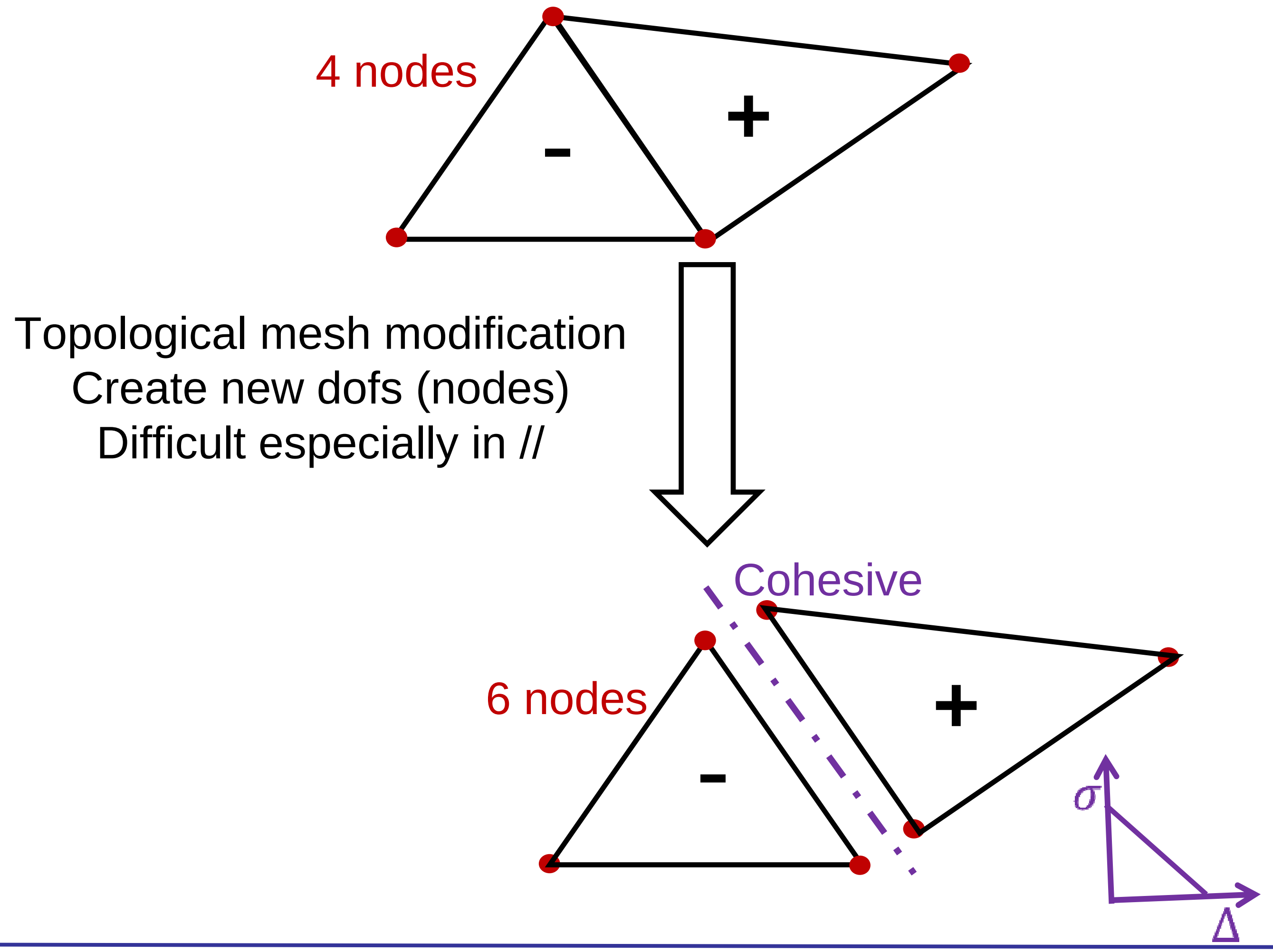
- Cohesive zone model is very appealing to model crack initiations in a numerical model
  - Model the separation of crack lips in brittle materials



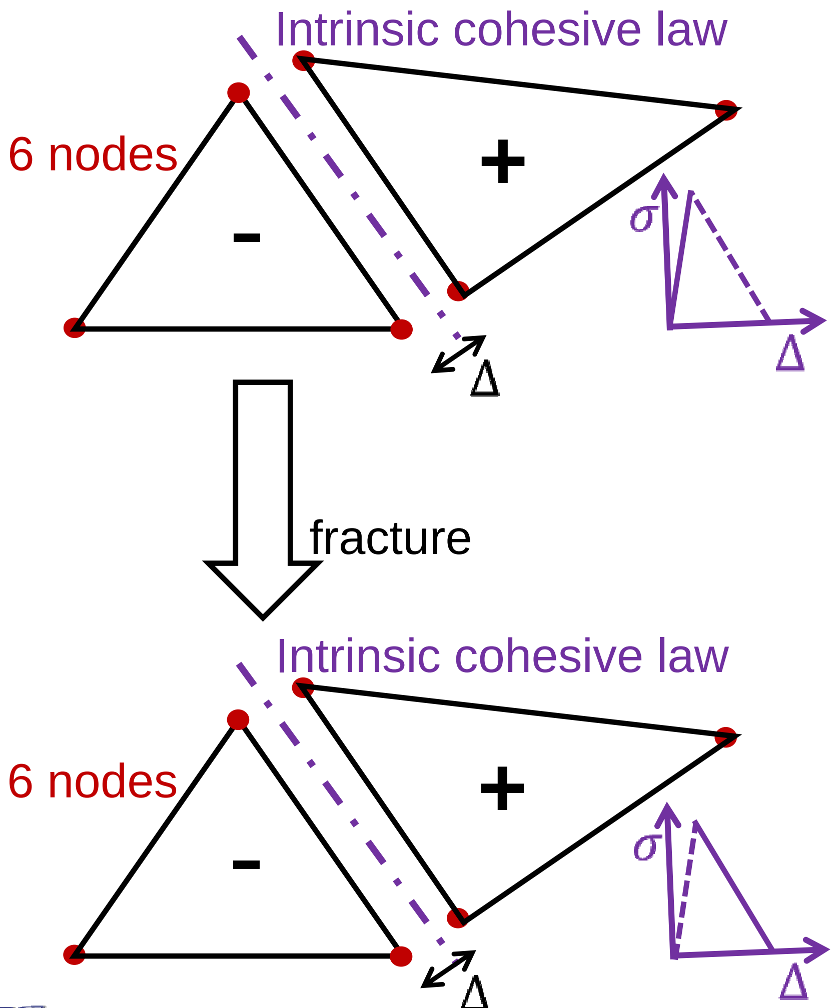
[*Seagraves et al 2010*]



- The insertion of cohesive elements during the simulation is difficult to implement as it requires topological mesh modifications
  - Extrinsic cohesive approach



- A recourse to an intrinsic cohesive law is generally done with FEM
  - Intrinsic cohesive approach



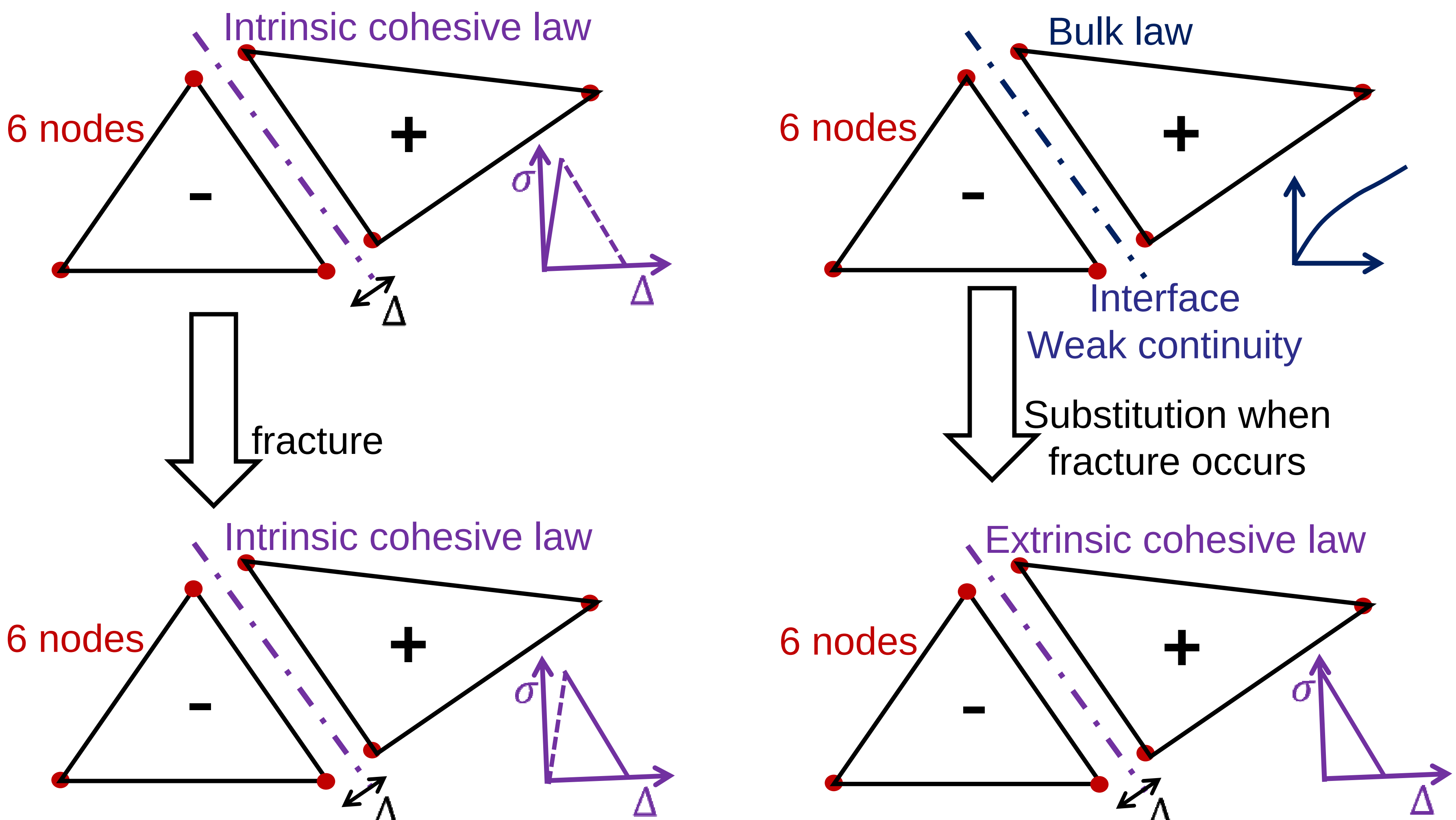
- An intrinsic cohesive law leads to numerical problems [*Seagraves et al 2010*]

  - Spurious stress wave propagation

  - Mesh dependency

  - Crack propagation rate too high

- Use of extrinsic cohesive law is easier when coupled with DG
  - FEM (continuous Galerkin)
  - Discontinuous Galerkin



Plan

- Develop a discontinuous Galerkin method for shells
  - One-field formulation
- Discontinuous Galerkin / Extrinsic Cohesive law framework
  - Develop a suitable cohesive law for thin bodies
- Applications
  - Fragmentation, crack propagation under blast loading

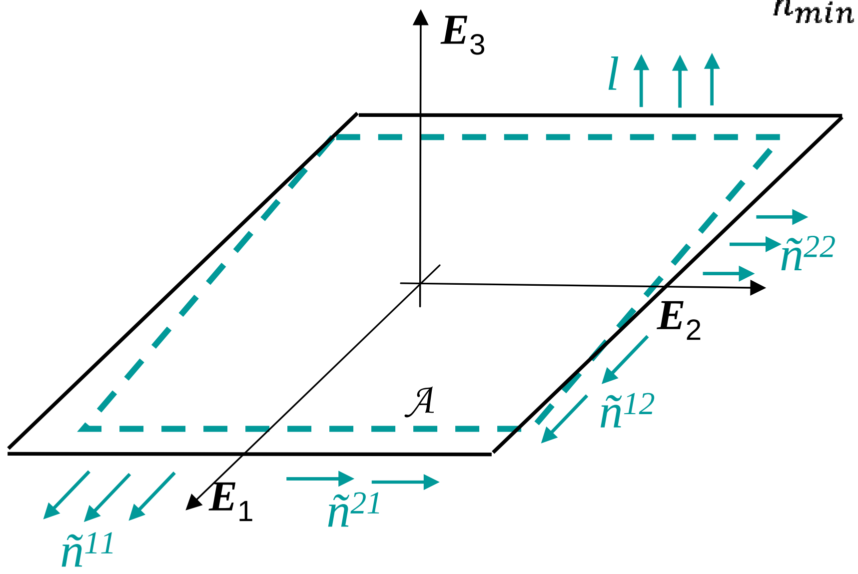
- The stress tensor σ is integrated on the thickness in the convected basis
  - Reduced stresses

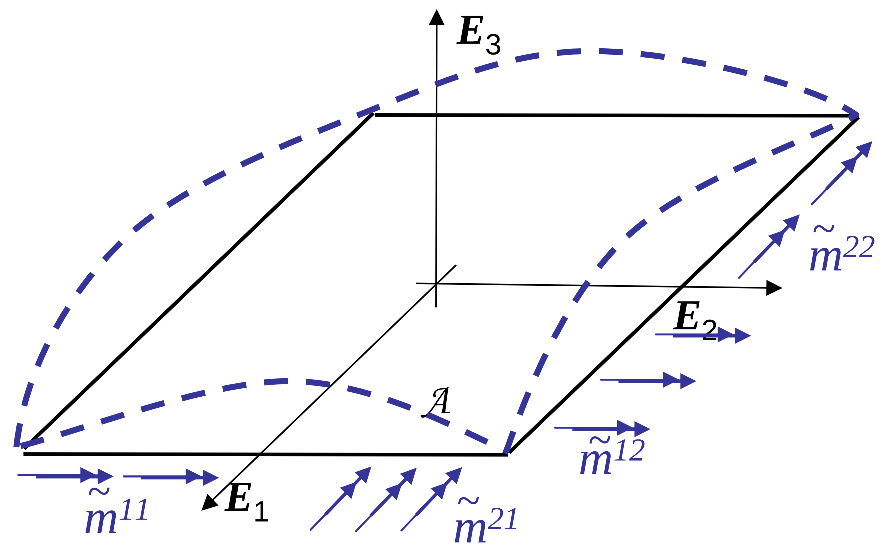
coupling

$$\boldsymbol{n}^\alpha = \frac{1}{\bar{j}}\int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma}\cdot\boldsymbol{g}^\alpha d\xi^3 = \left(\tilde{n}^{\alpha\beta} + \lambda_\mu^\beta \tilde{m}^{\alpha\mu}\right)\boldsymbol{\varphi}_{,\beta}$$

$$\tilde{\boldsymbol{m}}^\alpha = \frac{1}{\bar{j}}\int_{h_{min}}^{h_{max}} j\xi^3\boldsymbol{\sigma}\cdot\boldsymbol{g}^\alpha d\xi^3$$

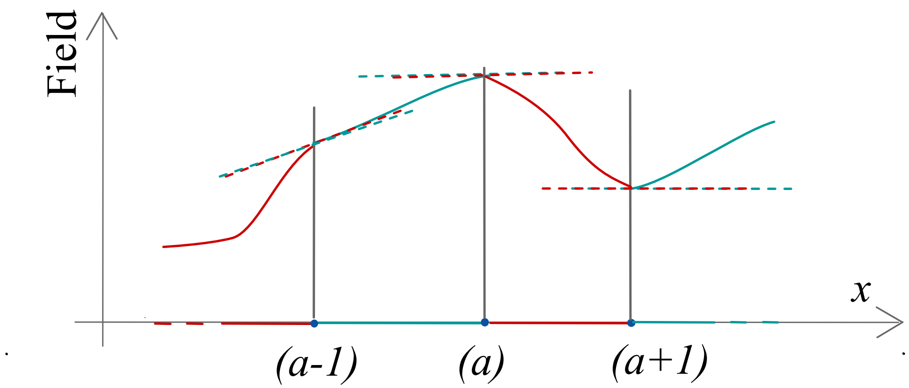
$$\boldsymbol{l} = \frac{1}{\bar{j}}\int_{h_{min}}^{h_{max}} j\boldsymbol{\sigma}\cdot\boldsymbol{g}^3 d\xi^3 \approx 0$$





Full-DG formulation of Kirchhoff-Love shells

- FEM (Continuous Galerkin)



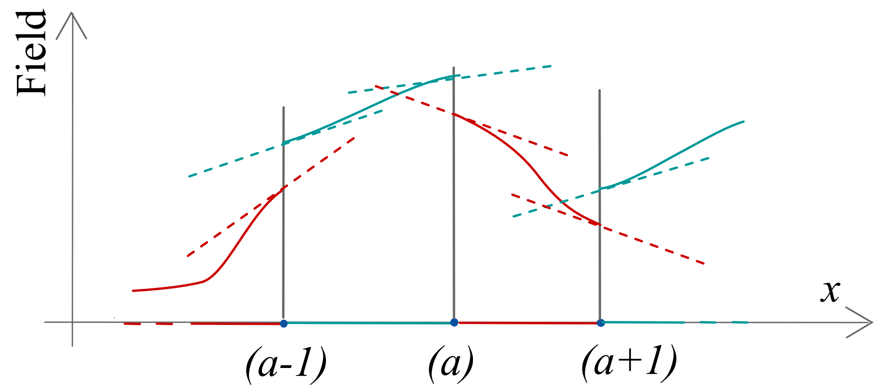
- Integration by parts of

$$\sum_e \left\{ \int_{A_e} \left[\left(\bar{j}\boldsymbol{n}^\alpha\right)_{,\alpha} \cdot \delta\boldsymbol{\varphi} + \left(\bar{j}\tilde{\boldsymbol{m}}^\alpha\right)_{,\alpha} \cdot \lambda_h \delta\boldsymbol{t} - \bar{j}\boldsymbol{l} \cdot \lambda_h \delta\boldsymbol{t} \right] dA = 0 \right.$$

$$\Downarrow$$

$$\sum_e \int_{A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi}_{,\alpha} + \bar{j}\tilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta\boldsymbol{t}_{,\alpha} - \left(\bar{j}\boldsymbol{l}\right)_{,\alpha} \cdot \int_\alpha \lambda_h \delta\boldsymbol{t}\, d\alpha' \right] dA = 0$$

- Discontinuous Galerkin



- Integration by parts on each element leading to

$$\sum_e \int_{A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi}_{,\alpha} + \bar{j}\tilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta\boldsymbol{t}_{,\alpha} - \left(\bar{j}\boldsymbol{l}\right)_{,\alpha} \cdot \int_\alpha \lambda_h \delta\boldsymbol{t}\, d\alpha' \right] dA$$

$$\left. + \int_{\partial A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi} \nu_\alpha^- + \bar{j}\tilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta\boldsymbol{t} \nu_\alpha^- + \bar{j}\boldsymbol{l} \cdot \int_\alpha \lambda_h \delta\boldsymbol{t}\, d\alpha' \; \nu_\alpha^- \right] dA \right\} = 0$$

Additional interface terms

- The equation of the full-DG formulation [Becker et al cmame2011, Becker et al ijnme2012]

$$\sum_e \int_{A_e} \left[\bar{j}\boldsymbol{n}^\alpha \cdot \delta\boldsymbol{\varphi}_{,\alpha} + \bar{j}\tilde{\boldsymbol{m}}^\alpha \cdot \lambda_h \delta\boldsymbol{t}_{,\alpha}\right] dA +$$

FEM (CG) equation

$$\sum_s \int_s \left[\langle \bar{j}\boldsymbol{n}^\alpha \rangle \cdot [\![\delta\boldsymbol{\varphi}]\!] + [\![\boldsymbol{\varphi}]\!] \cdot \langle \delta(\bar{j}\boldsymbol{n}^\alpha) \rangle + [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_\delta^- \left\langle \frac{\beta_2 \mathcal{H}_n^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu_\alpha^- d\partial A_e +$$

$$\sum_s \int_s \left[\langle \bar{j}\tilde{\boldsymbol{m}}^\alpha \rangle \cdot [\![\lambda_h \delta\boldsymbol{t}]\!] + [\![\boldsymbol{t}]\!] \cdot \langle (\bar{j}\lambda_h \tilde{\boldsymbol{m}}^\alpha) \rangle + [\![\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\gamma} \nu_\delta^- \left\langle \frac{\beta_1 \mathcal{H}_m^{\alpha\beta\gamma\delta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{t}]\!] \cdot \boldsymbol{\varphi}_{,\beta} \right] \nu_\alpha^- d\partial A_e +$$

$$\sum_s \int_s [\![\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \nu_\beta^- \left\langle \frac{\beta_3 \mathcal{H}_s^{\alpha\beta} \bar{j}_0}{h^s} \right\rangle [\![\delta\boldsymbol{\varphi}]\!] \cdot \boldsymbol{t} \quad \nu_\alpha^- d\partial A_e = 0$$

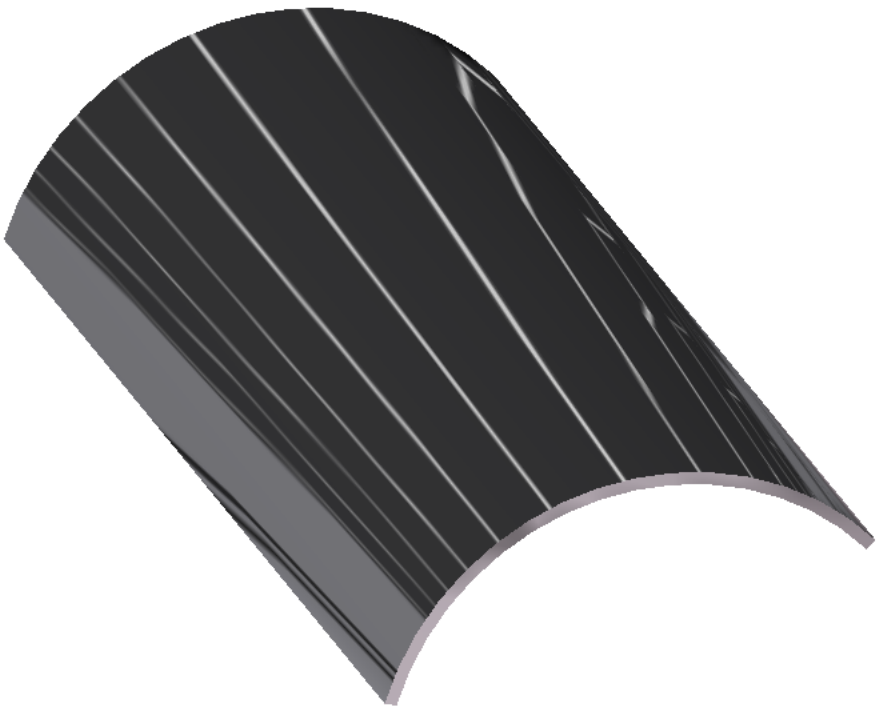
Consistency terms

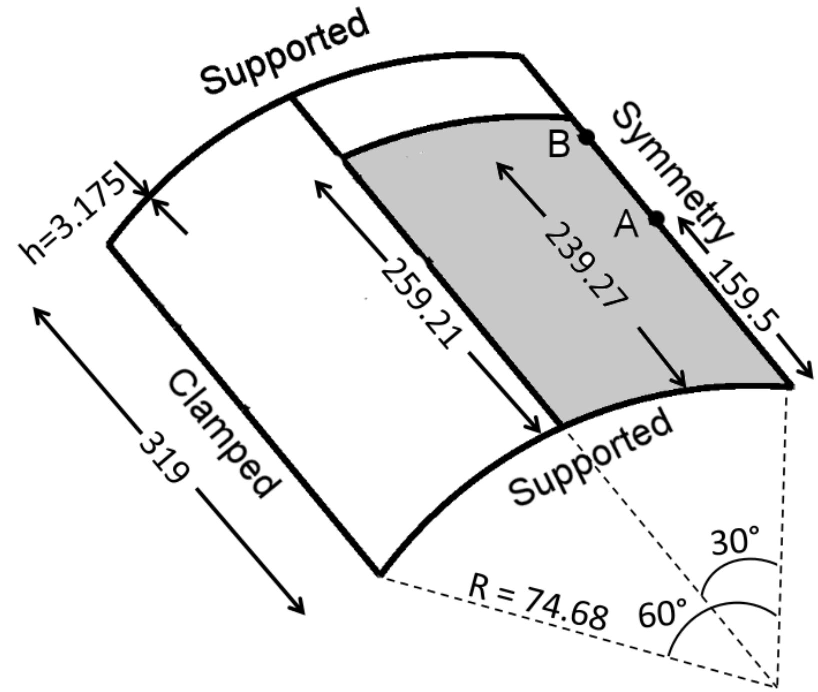
Symmetrization terms

Stabilization terms

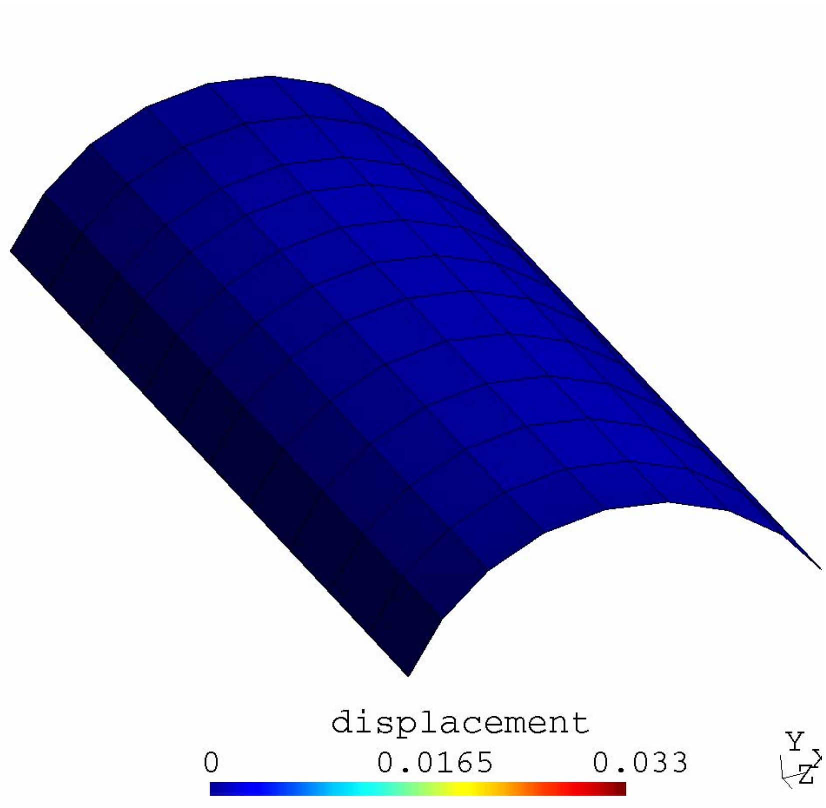
  - Application of the DG method gives 2 Bulk, 2 consistency, 2 symmetrization and 3 stabilization terms

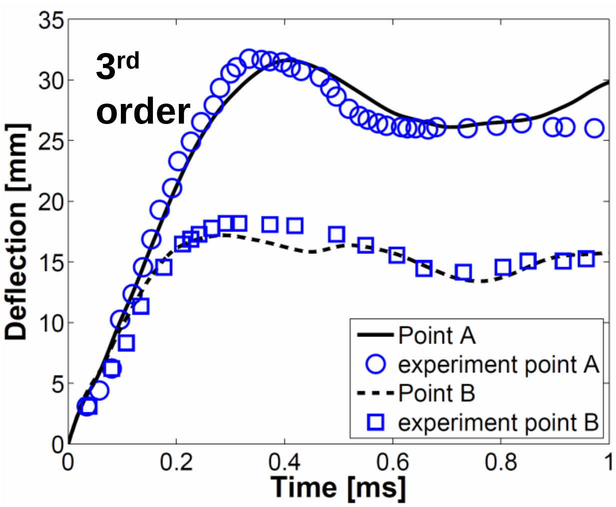
- A benchmark to prove the ability of the full-DG formulation to model continuous mechanics
  - J_2-linear hardening (elasto-plastic large deformations)
  - Panel loaded dynamically (explicit Hulbert-Chung scheme)





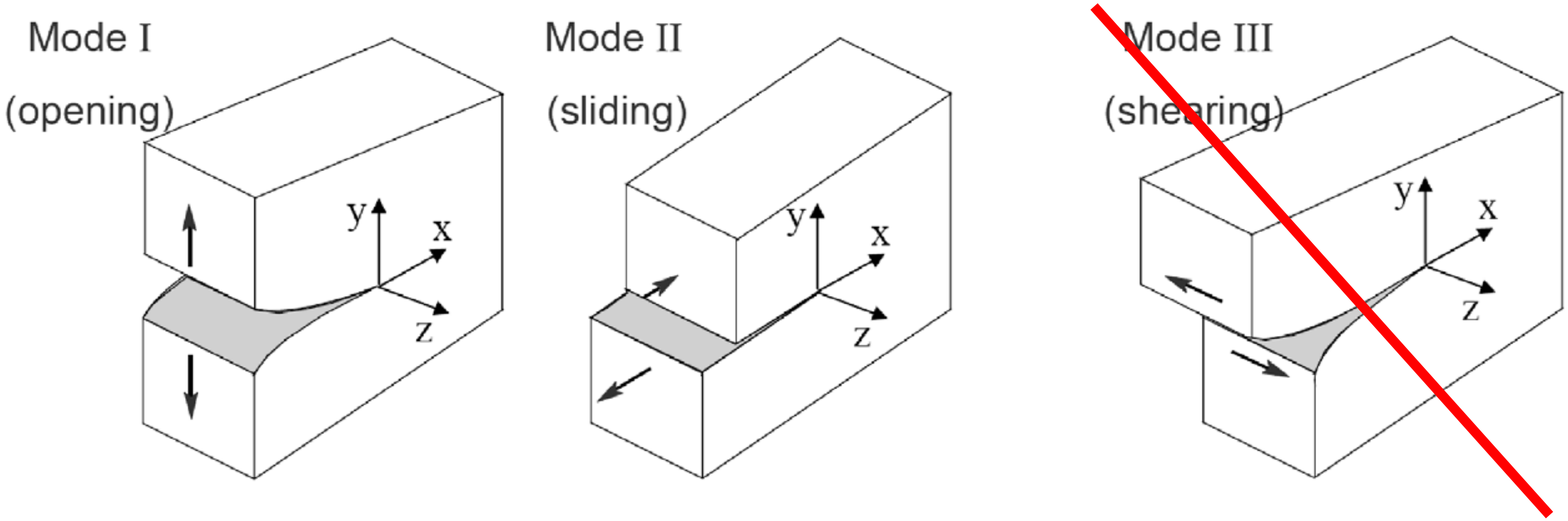
- A benchmark to prove the ability of the full-DG formulation to model continuous mechanics
  - J_2-linear hardening (elasto-plastic large deformations) panel loaded dynamically (explicit Hulbert-Chung scheme)





  - The results match experimental data

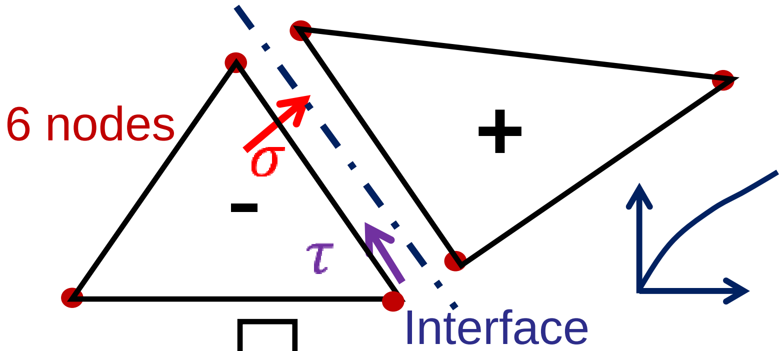
- Only modes I and II can be modeled by Kirchhoff-Love theory
  - Kirchhoff-Love → out-of-plane shearing is neglected



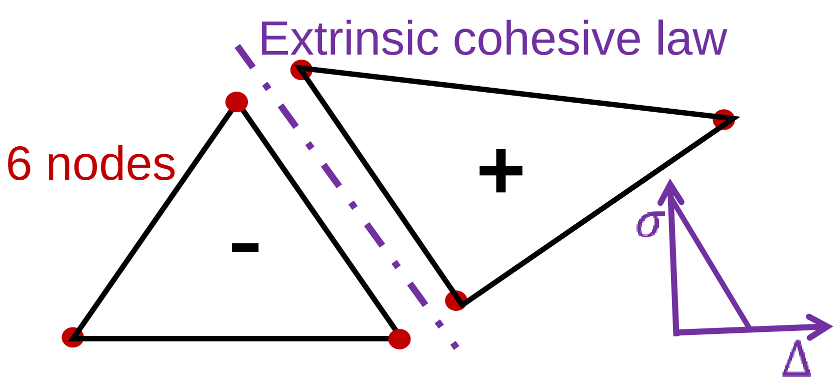
  - Model restricted to problems with negligible 3D effects at the crack tip

- Fracture criterion based on an effective stress

  – Camacho & Ortiz Fracture criterion [*Camacho et al ijss1996*]

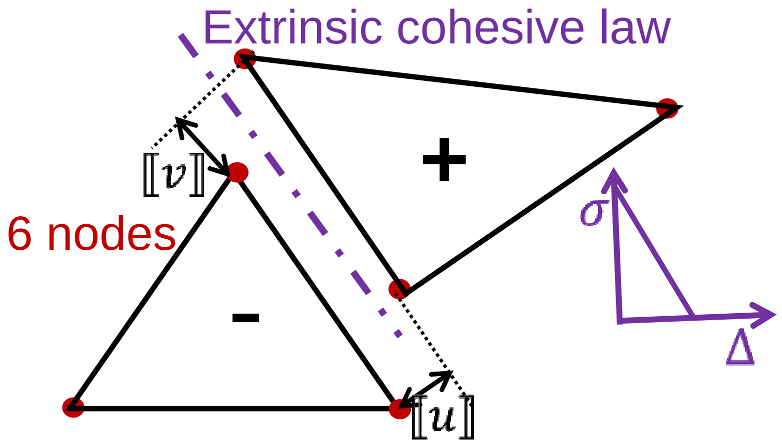


$$\sigma_{eff} > \sigma_c \quad \text{with} \quad \sigma_{eff} = \begin{cases} \sqrt{\sigma^2 + \beta^{-2}\tau^2} \text{ if } \sigma \geq 0 & \text{Traction} \\ \frac{1}{\beta} \ll |\tau| - \mu_c|\sigma| \text{ if } \sigma < 0 & \text{Compression} \end{cases}$$



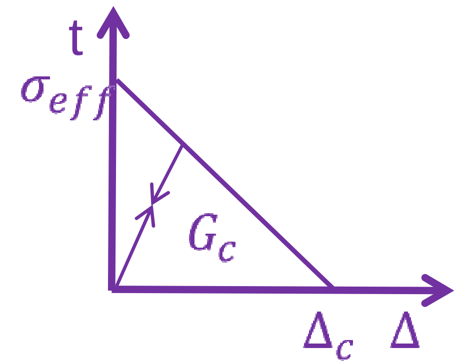
σ_c, β and μ_c are material parameters

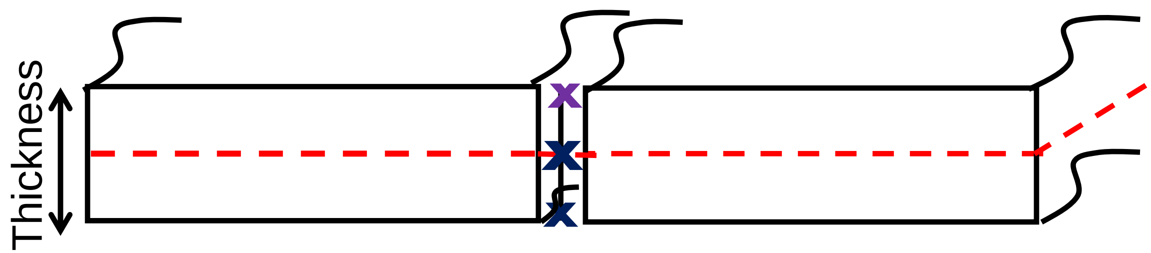
- The cohesive law is formulated in terms of an effective opening
  - Camacho & Ortiz Fracture criterion [*Camacho et al ijss1996*]



$$\Delta = \sqrt{[\![u]\!] + \beta^2 [\![v]\!]}$$

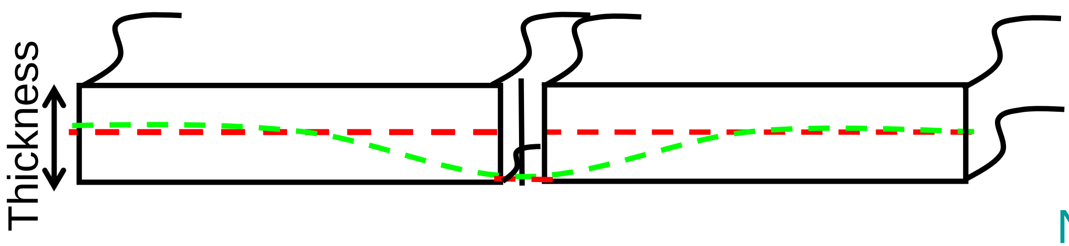
- Through-the-thickness crack propagation with shell elements?
  - No elements on thickness
  - Integrate the 3D TSL on the thickness [Cirak et al cmame2005]

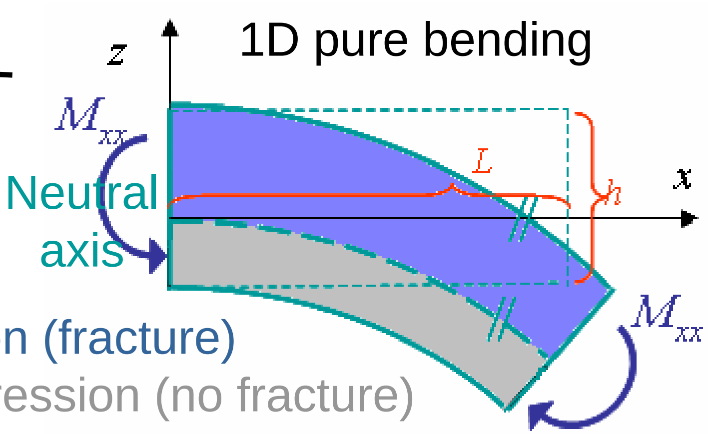




Fracture criterion is met
→ cohesive law
Unreached fracture
→ bulk law

  - The position of the neutral axis has to be recomputed to propagate the crack





Discontinuity
Continuity (Computation ?)

Tension (fracture)
Compression (no fracture)

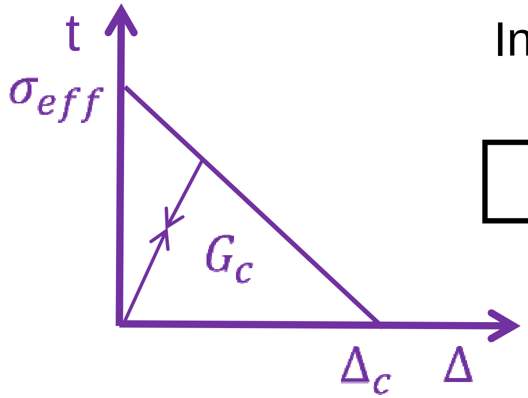
- The cohesive law can be formulated in terms of reduced stresses
  - Same as shell equations

Bulk law
Stress tensor $\boldsymbol{\sigma}$

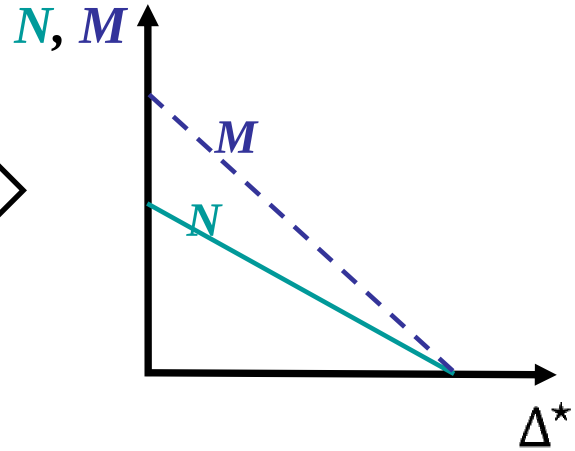
Integration on thickness

$$\boldsymbol{n}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j \boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^3$$

$$\tilde{\boldsymbol{m}}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{min}}^{h_{max}} j \xi^3 \boldsymbol{\sigma} \cdot \boldsymbol{g}^{\alpha} d\xi^3$$



Integration on thickness

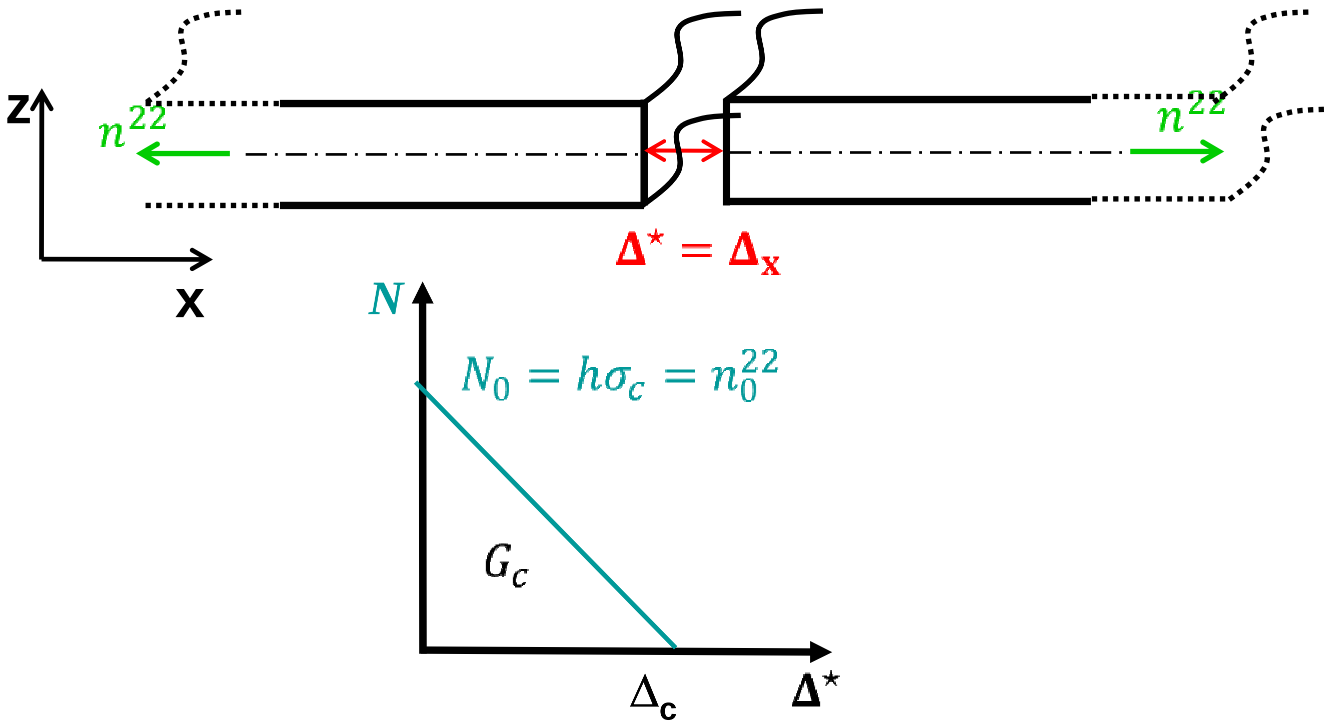


  - Similar concept suggested by Zavattieri [Zavattieri jam2006]

- Define Δ^* and $N(\Delta^*)$, $M(\Delta^*)$ to dissipate an energy equal to hG_C during the fracture process [Becker et al ijnme2012, Becker et al ijf2012]

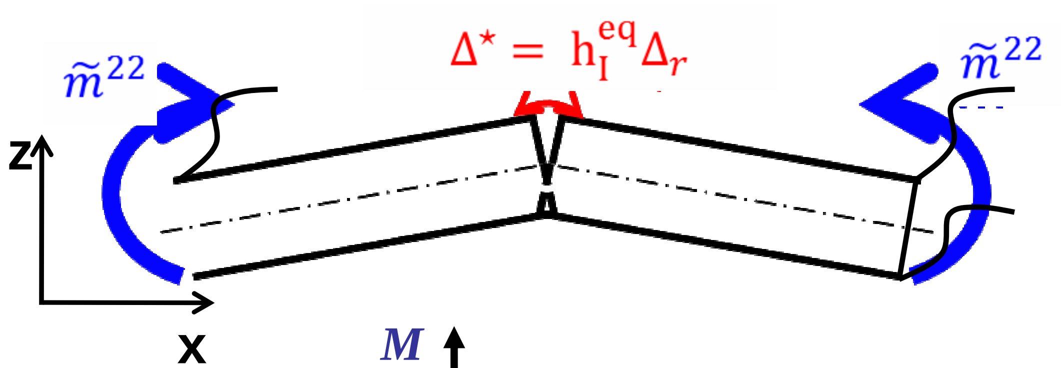
$$G_N + G_M = hG_c$$

- The law $N(\Delta^*)$ is defined to release an energy hG_C in pure tension
  - Pure mode I

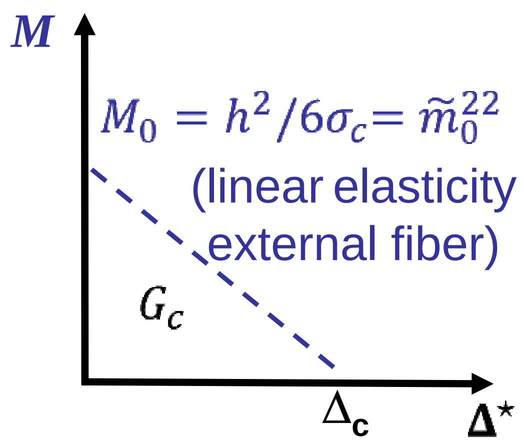


$$\int_0^{\Delta_c} N(\Delta_x)d\Delta_x = \frac{N_0\Delta_c}{2} = \frac{2h\sigma_c G_c}{2\sigma_c} = hG_c$$

- The law $M(\Delta^*)$ is defined to release an energy hG_C in pure bending
  - Pure mode I



$$\frac{\tilde{m}_0^{22}}{h_I^{eq}} = h\sigma_c$$

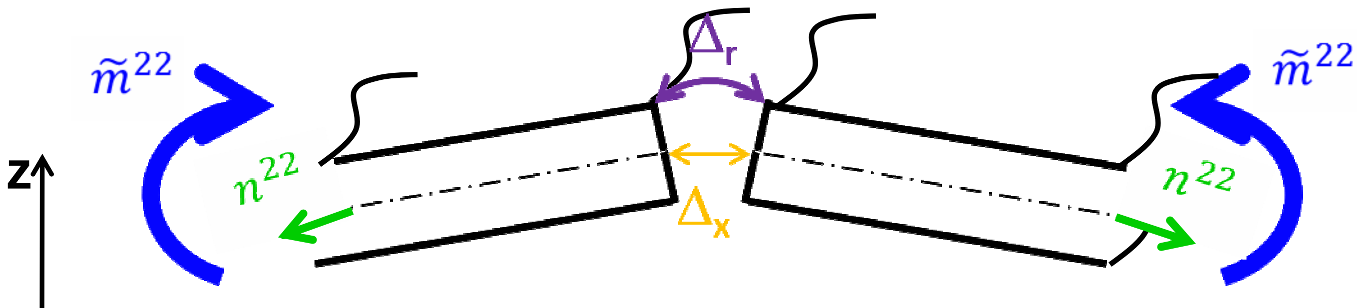


$$h_I^{eq} = \frac{h}{6}$$

$$\int_0^{\Delta_{rc}} M(\Delta^\star)d\Delta_r = \int_0^{\Delta_c} \pm\frac{6}{h}M_0\left(1-\frac{\Delta^\star}{\Delta_c}\right)d\Delta^\star = \frac{6}{h}\frac{h^2\sigma_c}{6}\frac{\Delta_c}{2} = hG_c$$

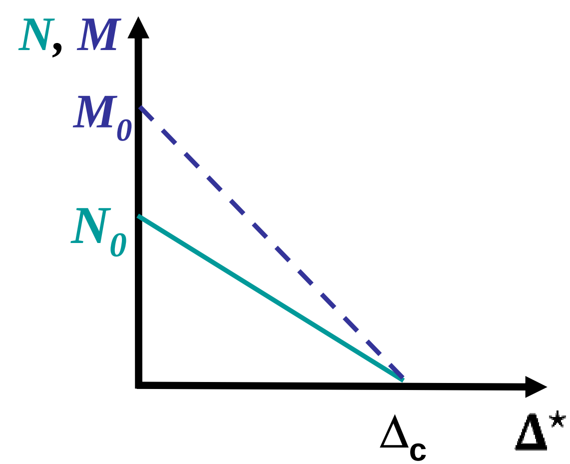
- Using the superposition principle the energy released for any couple N, M is equal to hG_c [Becker et al ijnme2011]

  - Pure mode I



$$\Delta_I^\star = (1-\eta_I)\Delta_x + \eta_I\, h_I^{eq}\, \Delta_r$$

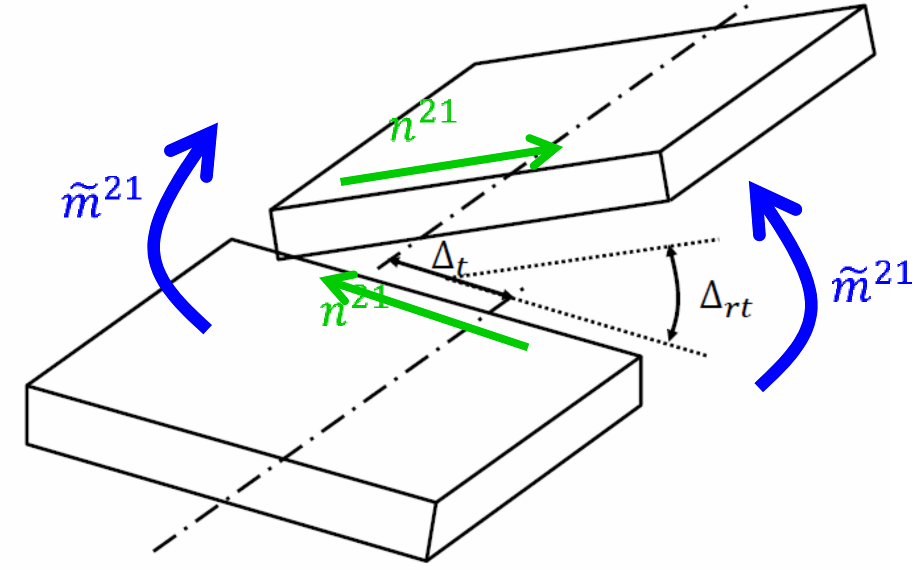
$$h_I^{eq} = \frac{M_0}{h\sigma_c - N_0}$$

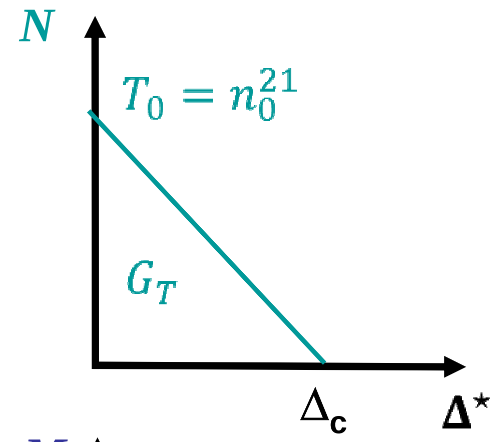


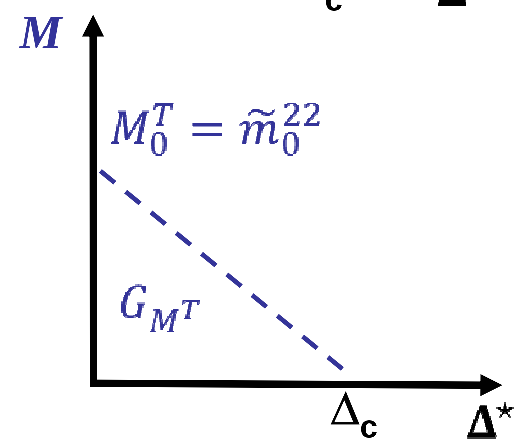
  - Coupling parameter

$$\eta_I = \frac{\left|1/h_I^{eq} M_0\right|}{N_0 + \left|1/h_I^{eq} M_0\right|} = \frac{h\sigma_c - N_0}{h\sigma_c}$$

- The cohesive model for mode I can be extended to mode II







$$G_T + G_{M^T} = h\beta G_c$$

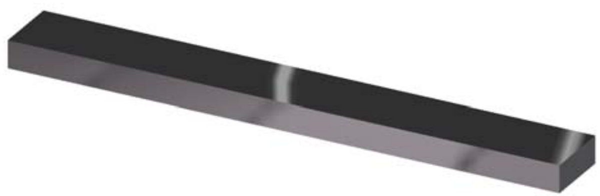
$$\Delta_{II}^\star = (1-\eta_{II})\Delta_t + \eta_{II} h_{II}^{eq} \Delta_{rt}$$

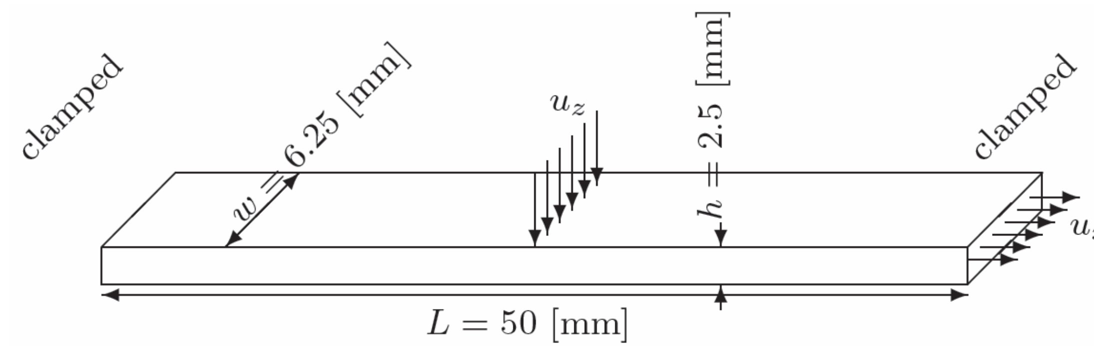
$$h_{II}^{eq} = \frac{M_0^T}{h\beta\sigma_c - T_0}$$

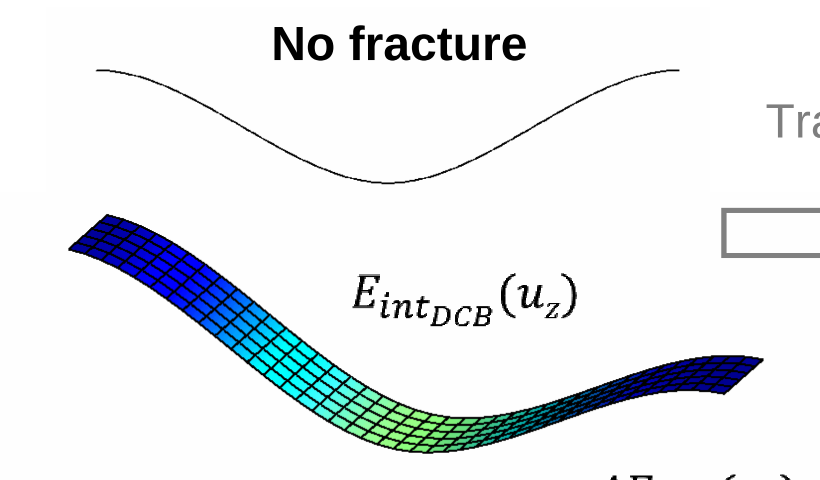
- Coupling parameter

$$\eta_{II} = \frac{\left|1/h_{II}^{eq} M_0^T\right|}{T_0 + \left|1/h_{II}^{eq} M_0^T\right|} = \frac{h\beta\sigma_c - T_0}{h\beta\sigma_c}$$

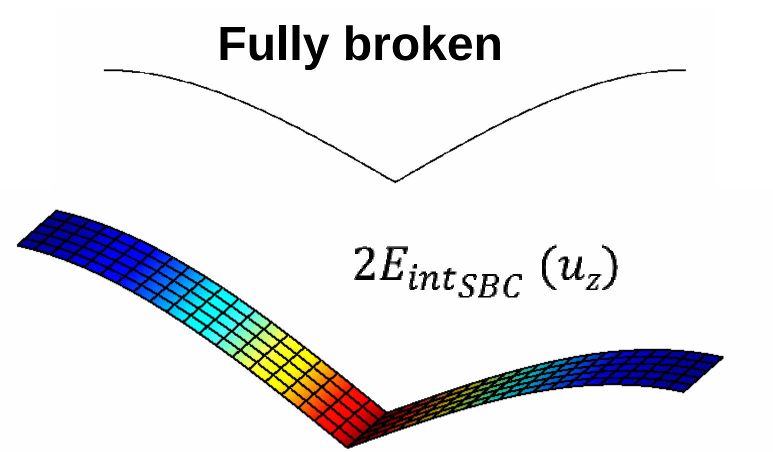
- The transition between uncracked to fully cracked body depends on ΔE_{int}
  - Double clamped elastic beam loaded in a quasi-static way







Transition ?

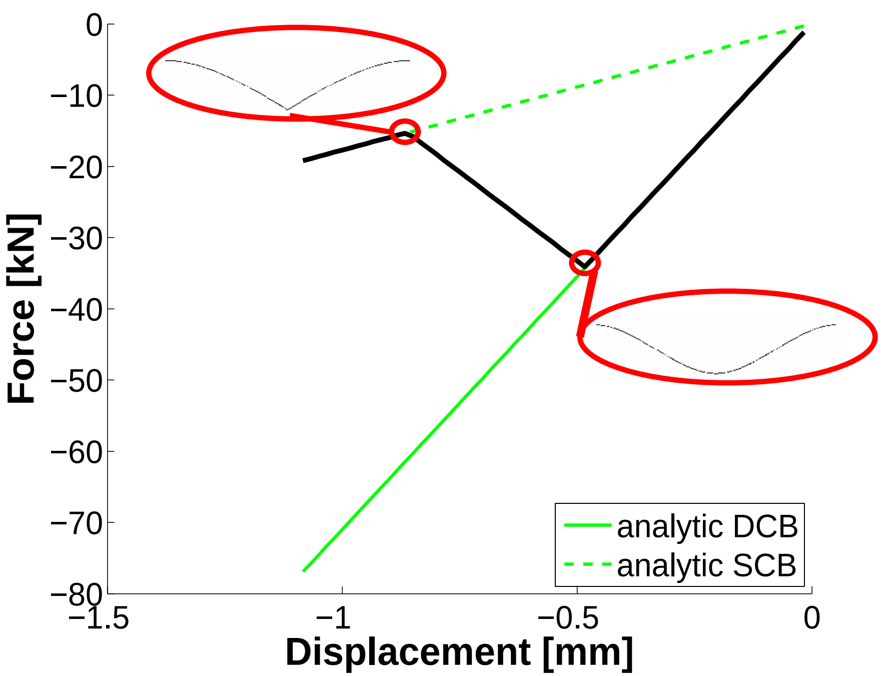


$$\Delta E_{int}(u_z) = E_{int_{DCB}}(u_z) - 2E_{int_{SBC}}(u_z)$$

- The framework can model stable/unstable crack propagation
  - Geometry effect (no pre-strain)

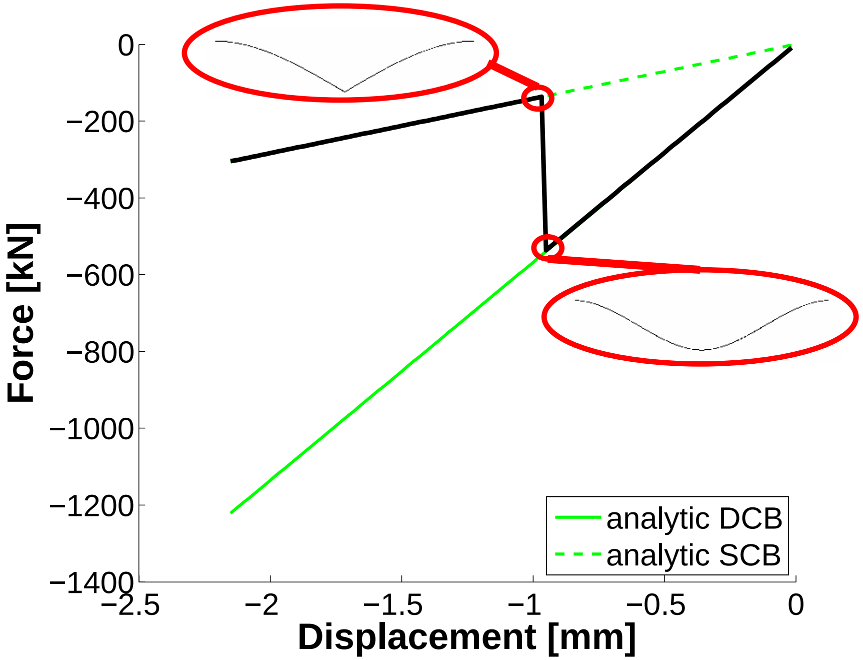
Stable transition

$\Delta E_{int}\left(u_z\right) < hGc$

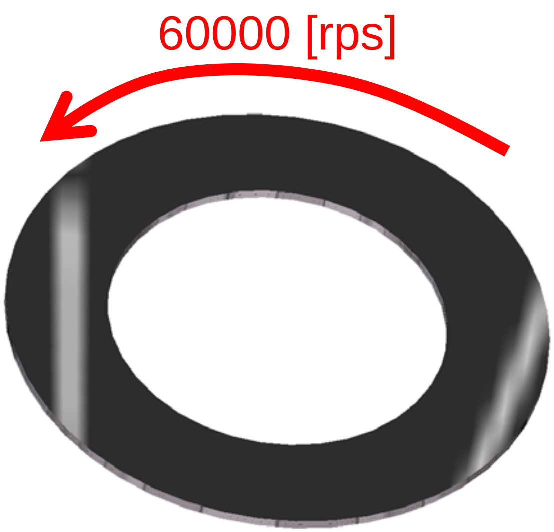


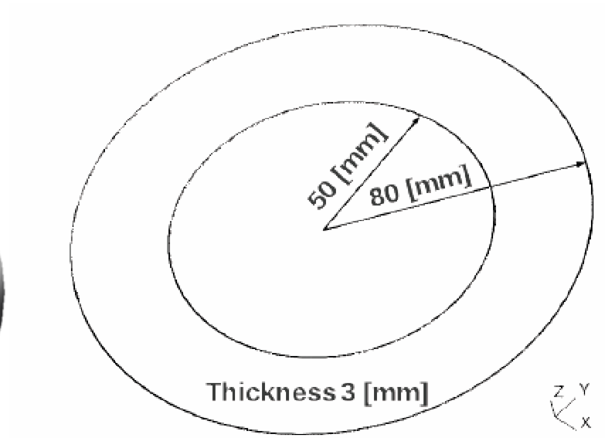
Unstable transition

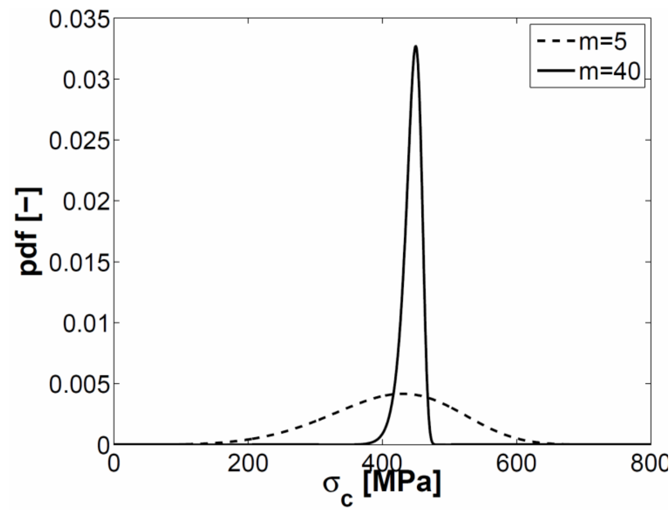
$\Delta E_{int}\left(u_z\right) > hGc$



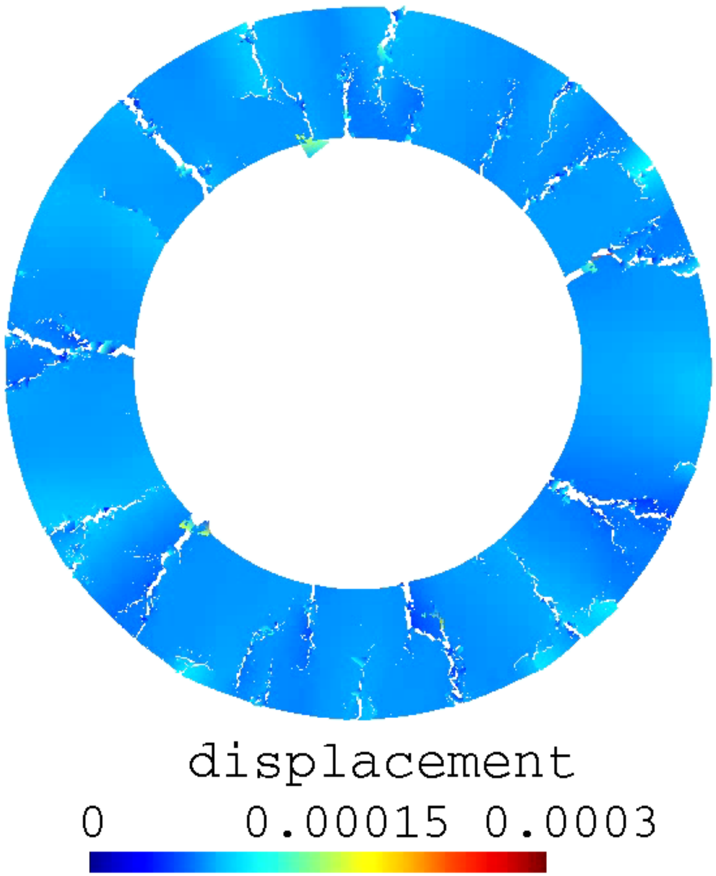
- A benchmark to investigate the fragmentation
  - Elastic plate ring loaded by a centrifugal force

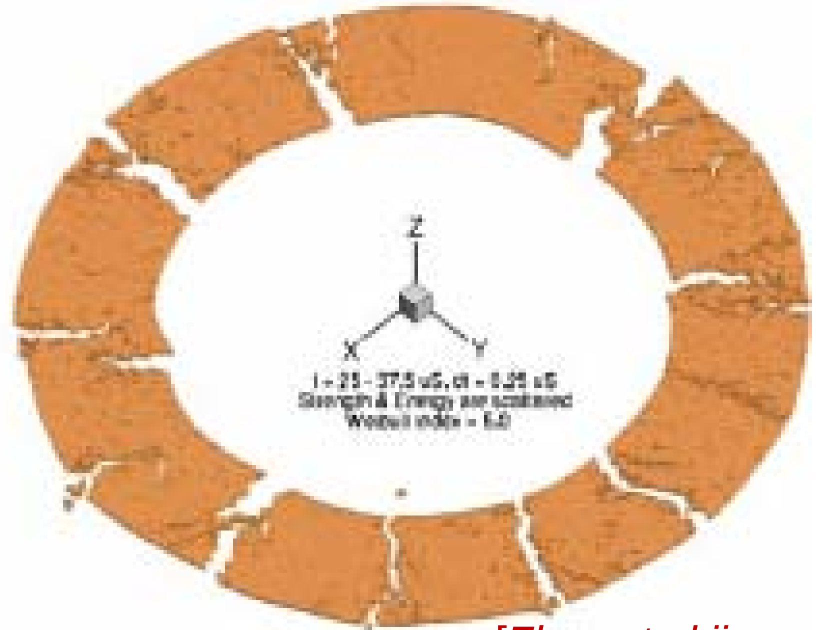






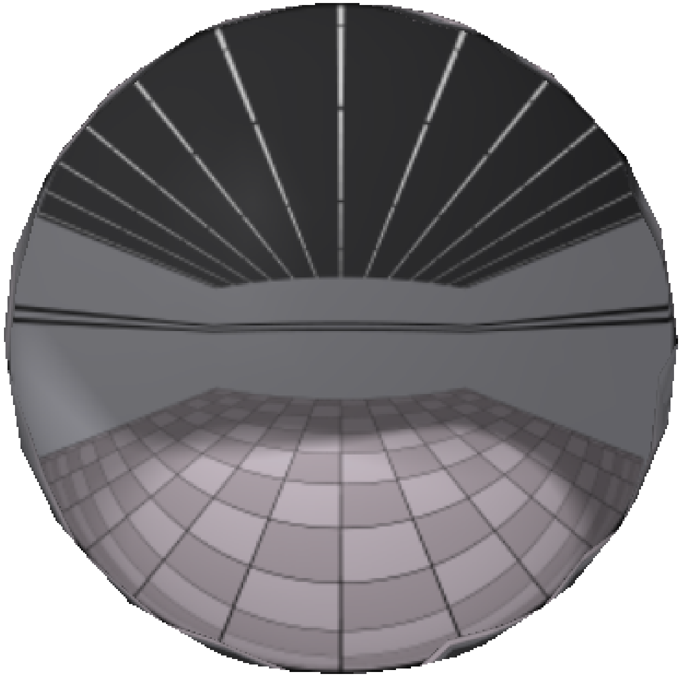
- Fragmentation is studied by the full-DG/ECL framework
  - Results are compared with the literature [*Zhou et al ijnme2004*]

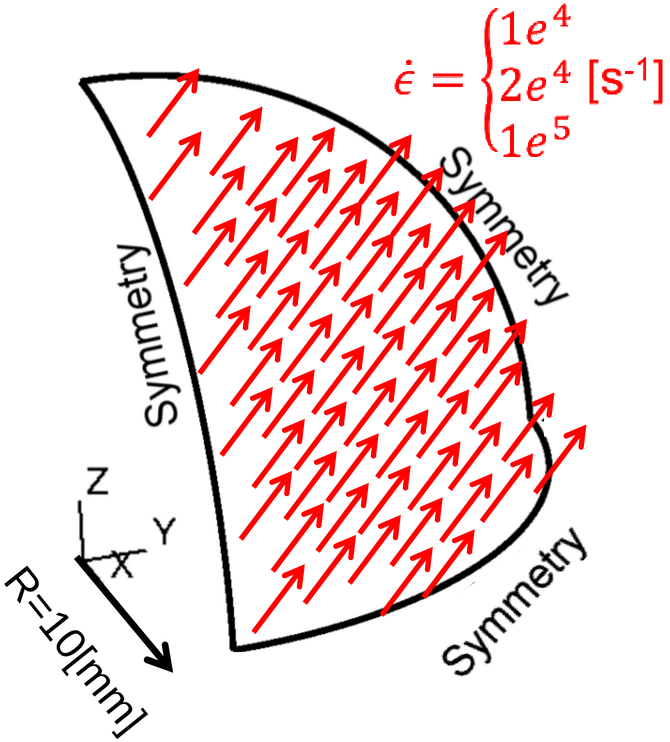




[*Zhou et al ijnme2004*]

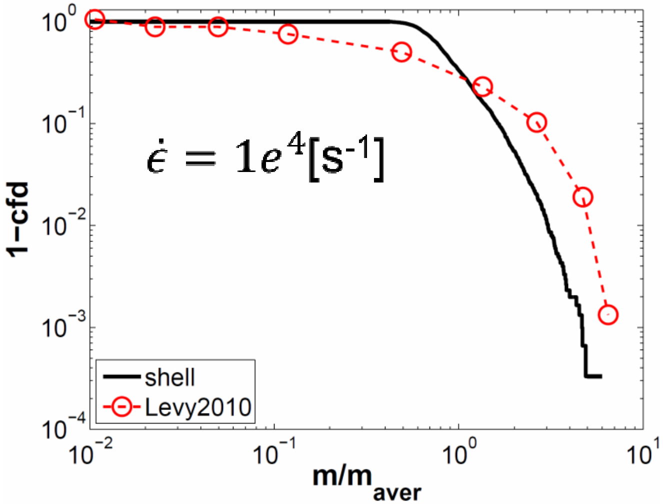
- Application to the dynamic fragmentation of a sphere
  - Elastic sphere under radial uniform expansion

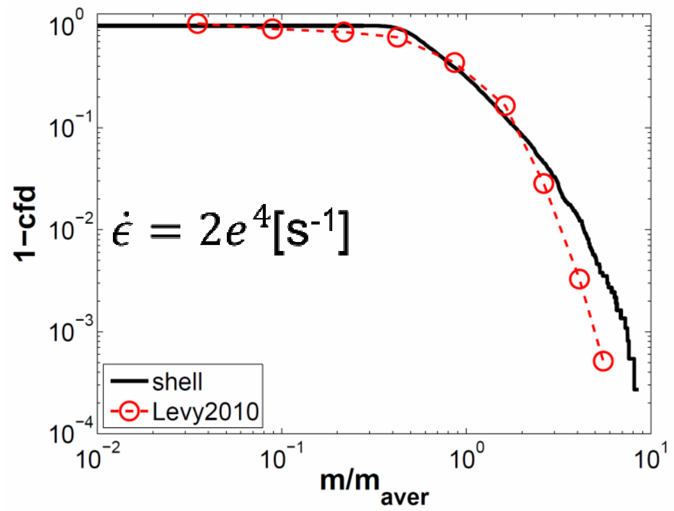


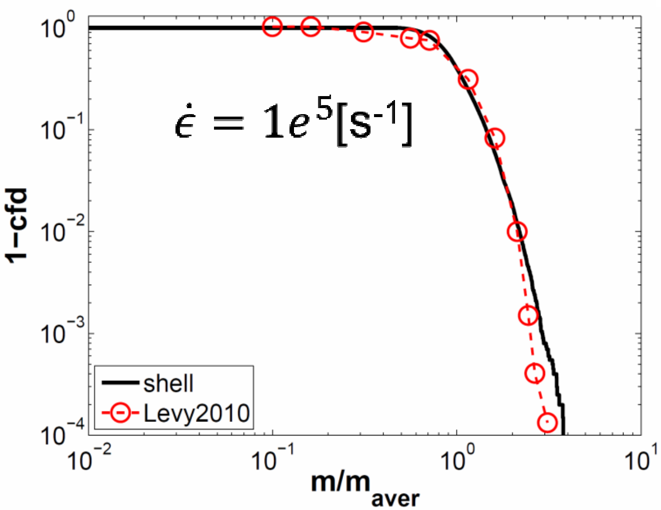


Applications of the DG/ECL framework

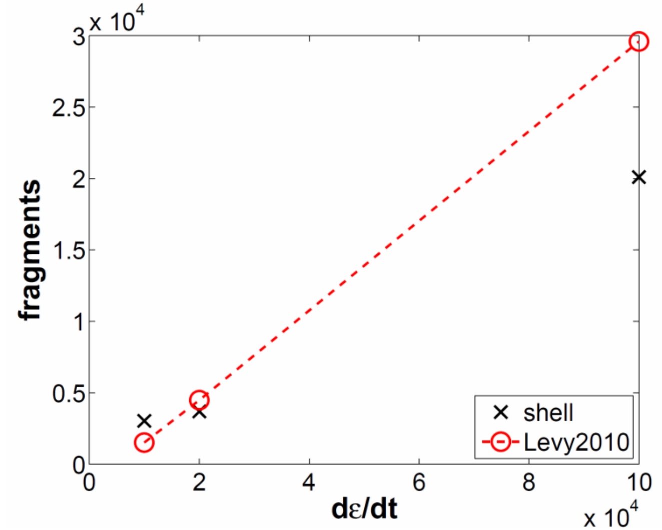
- The distribution of fragments and the number of fragments are in agreement with the literature [Levy EPFL2010]
  - Levy uses 3D elements





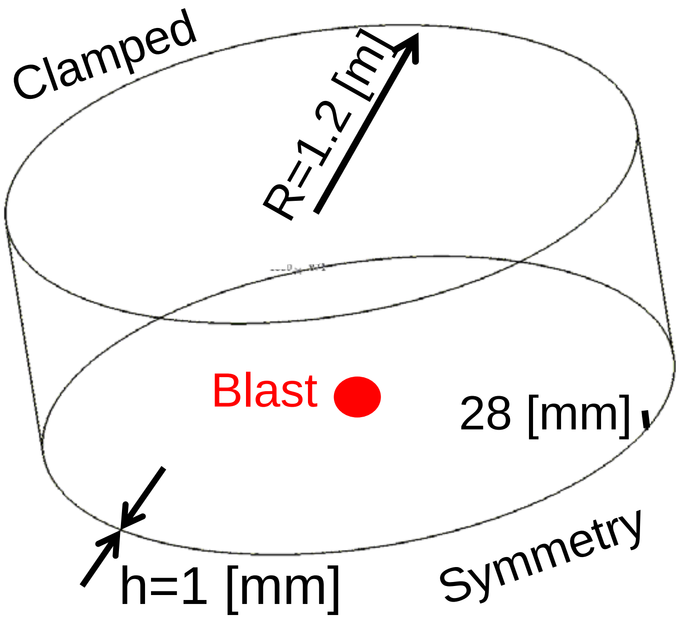


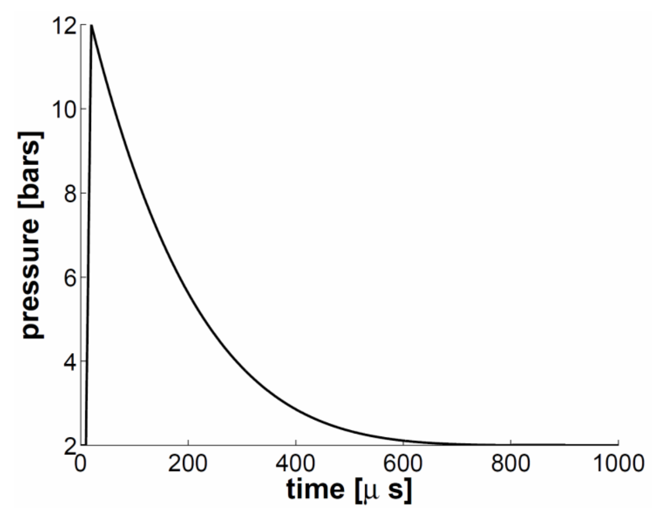
$\dot{\epsilon} = 1e^4[s^{-1}]$
2 588 265 Dofs
±48h on 32 cpus
(±17h for $\dot{\epsilon} = 1e^5[s^{-1}]$)



Applications of the DG/ECL framework

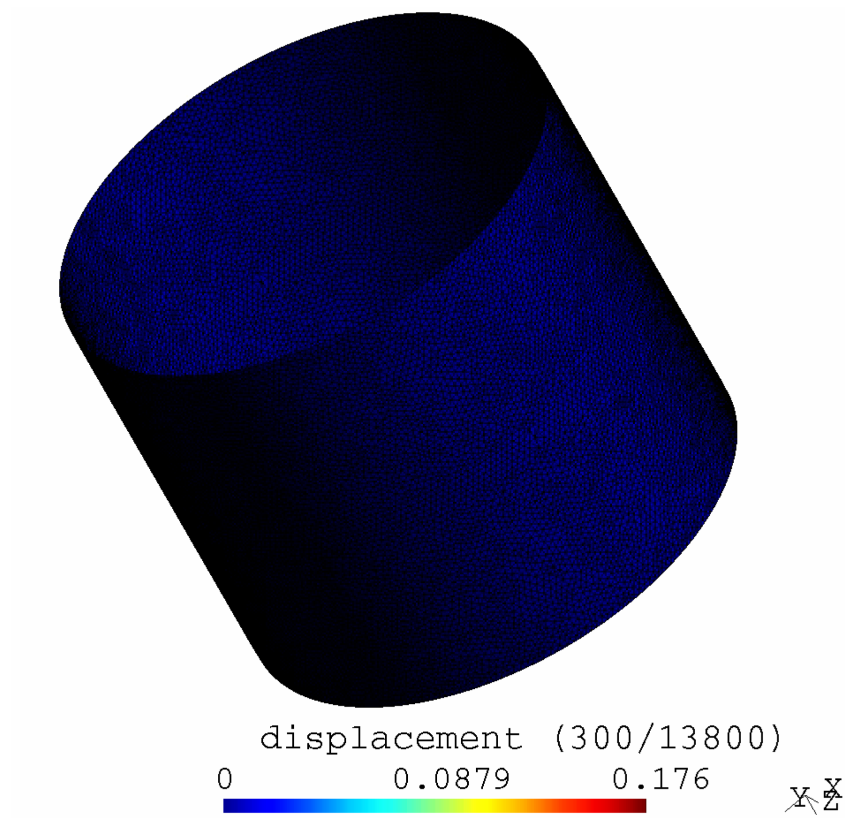
- Blast of an axially notched elasto-plastic cylinder (large deformations)



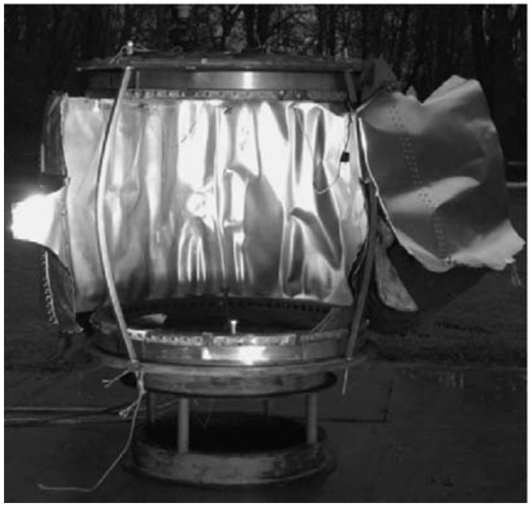


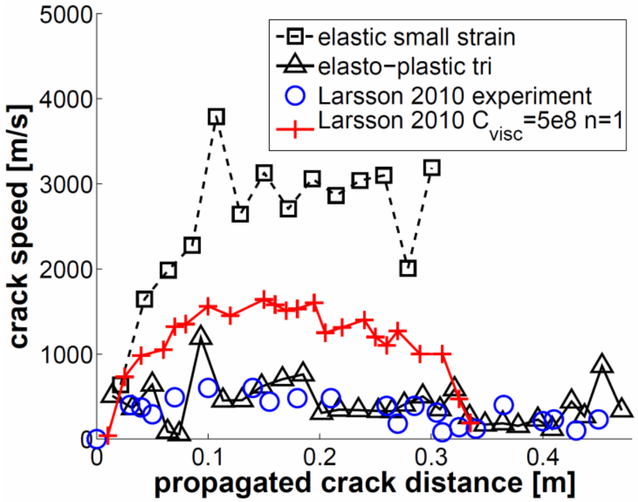
- Accounting for plasticity to capture the crack speed

  – Compare with the literature [Larson et al ijnme2011]

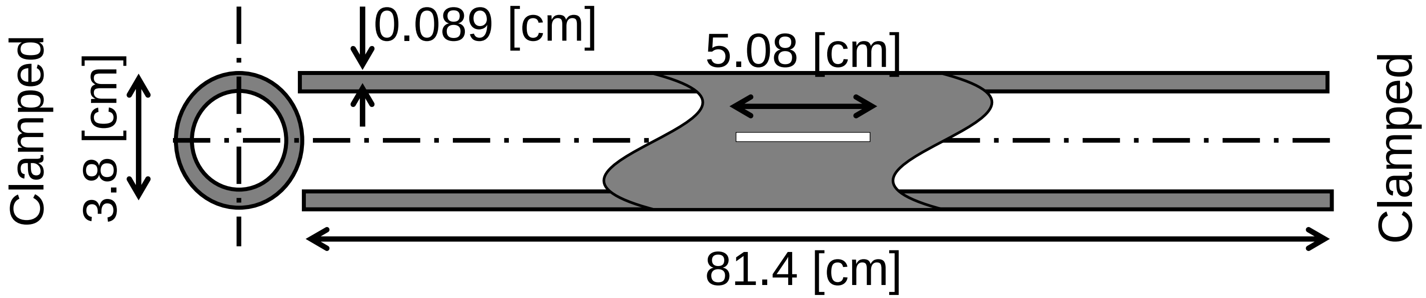


556 080 Dofs
±72h on 16 cpus



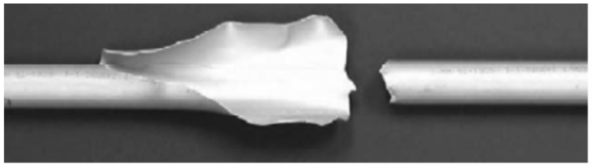


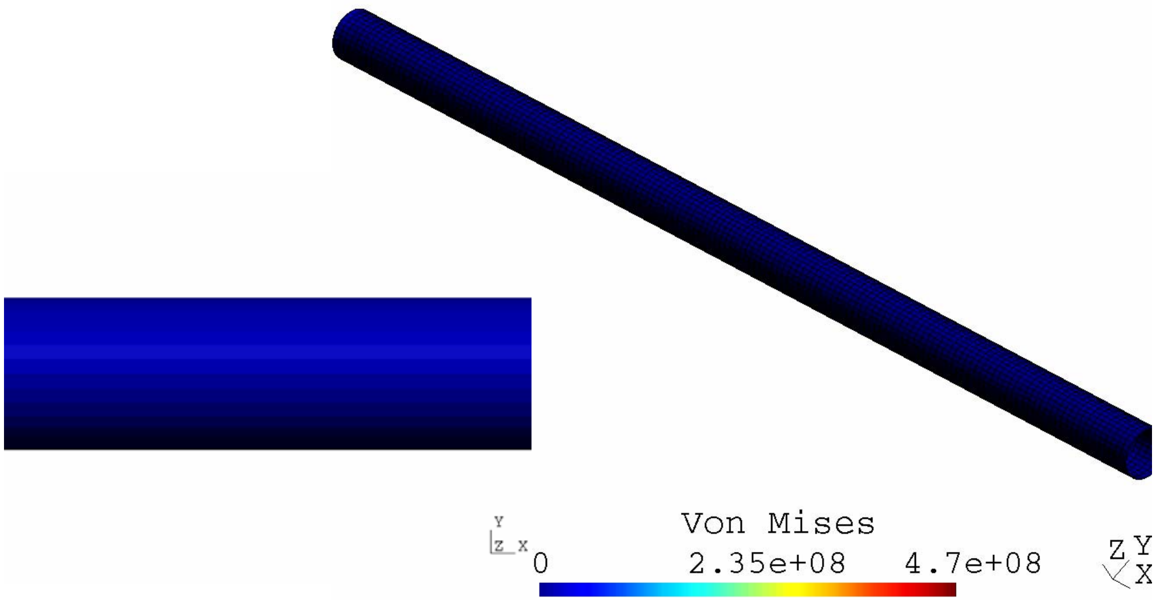
- Pressure wave passes through an axially notched elasto-plastic pipe (large deformations)



Applications of the DG/ECL framework

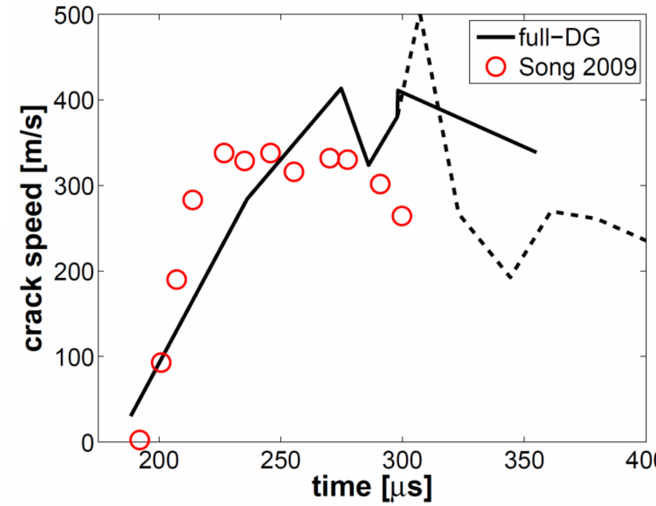
- Crack path and speed are well captured by the framework
  - Compare with the literature [Song et al jam2009]

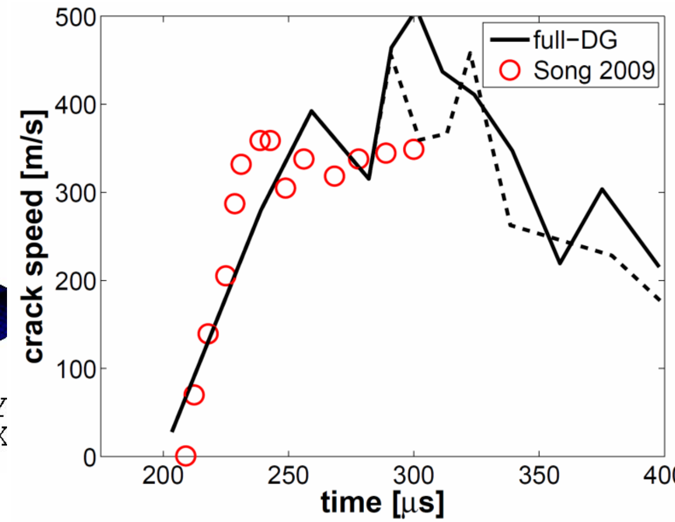




224 256 Dofs

±21 h on 12 cpus





Conclusions

- Full-DG / ECL framework allows accounting for fracture in dynamic simulations of thin bodies
  - One-field formulation
  - Crack propagation as well as fragmentation
  - Recourse to an elasto-plastic model is mandatory to capture crack speed
  - Affordable computational time for large models (using // implementation)

- Model the damage to crack transition by coupling a damage law with the full-DG/ECL framework
  - Replace the criterion based on an effective stress by a criterion based on the damage
  - Define the shape of the cohesive law

Thank you for your attention