POLYMORPHISM, AN EXTENSION OF BINARY OPENING

Application to contour filtering

MARC VAN DROOGENBROECK
Belgacom, New Developments
121, Boulevard E. Jacqmain
B-1210 Brussels, Belgium

Abstract. In this short paper we present a new operator called *polymorphism*, defined on the digital space $\mathbb{Z}^2$, which lies somewhere between the concept of binary opening and that of distance function. After the definition we give some properties that makes polymorphism attractive for both region description and filtering. For example, polymorphism can be proved translation invariant and increasing. In addition, the computation speed is about the same as for an opening, even if the image is a label image. Finally we detail an algorithm adapted to the filtering of a label image, such as could result from any segmentation, which maximizes the total polymorphism.

Key words: Contour filtering, Opening

1. The concept of polymorphism

To describe object shapes or sets, mathematical morphology uses basic templates called structuring elements which are translated throughout the plane in order to determine the similarity with the set under study. As a result of this comparison morphological operators are very sensitive to noise, especially for large structuring elements.

Although the binary opening tells how a structuring element fits into the set under study, it provides no information about the location of border points. On the contrary, the distance function expresses the distance of a point to the border of the set it belongs to, but it is not related to a particular structuring element. This suggests the definition of an intermediate operator that lies between the opening operator and the distance function.

1.1. Definition

In this paper, the sets X, B under study are subsets of $\mathbb{Z}^2$. The first concern when dealing with region representation is to know if a point x belongs to the opening of X by B, denoted $X \circ B$, or equivalently if $\exists B_p$ (=B translated by p) contained in X such that $x \in B_p$. This information is for example used to define morphological skeletons. The main drawback of morphological opening is that it only tells if a point belongs to $X \circ B$; no information is provided about the location of pixels inside $X \circ B$. The concept of polymorphism enhances the binary opening because it says how many translated structuring elements, included in X, contain x. The principle is illustrated in figure 1. We have drawn in gray two 3x3 squares included in X that contain x. The darker area indicates their intersection ; polymorphism values are larger inside this dark area.

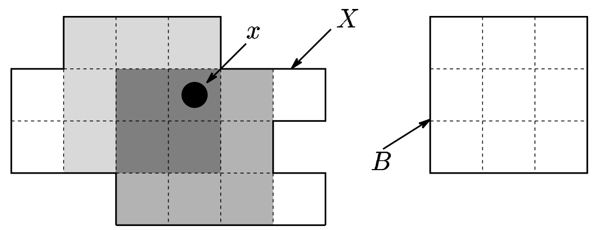


Fig. 1. Introduction to the polymorphism of a point. In this example, two distinct 3x3 squares included in X contain x.

Definition 1 *Let $X \subseteq \mathbb{Z}^2$ be a set and x an element of X. The* polymorphism *of an element x of X by the structuring element B, denoted $P_{X,B}(x)$, is the total number of distinct translated sets B_p that contain x:*

$$\begin{aligned} P_{X,B}(x) &= \mathcal{A}(\{p \in \mathbb{Z}^2 | x \in B_p \subseteq X\}) \\ &= \mathcal{A}(\{p \in X \ominus B | x \in B_p\}) \end{aligned} \quad (1)$$

where $\mathcal{A}$ is the set cardinality.

Definition 2 *The* polymorphism of a set *is the sum of the polymorphism of its elements. $P_{X,B}(x)$: $P_B(X) = \sum_{x \in X} P_{X,B}(x)$*

Let us note that the polymorphism of a set is equal to the product of the area of the eroded set $X \ominus B$ and the area of B (see [2] for a proof):

Theorem 3

$$P_B(X) = \mathcal{A}(X \ominus B)\mathcal{A}(B) \quad (2)$$

Figure 2 shows polymorphism values of set elements when B is the 3x3 square represented on the right. $P_B(X)$ equals 54. As can be seen low polymorphism values delimit the frontier and higher values appear at the centre of the set.

Polymorphism values of regions of a label image are represented in figure 3 as a gray-level image. The images are respectively the contours of the original image and polymorphism values computed with a 15x15 square. The contours are also superimposed on fig. 3(b).

1.2. Properties

It can be proved that polymorphism is an increasing and translation invariant operation. Moreover, $P_{B,X}(x)$ is bounded (all the proofs can be found in [2]):

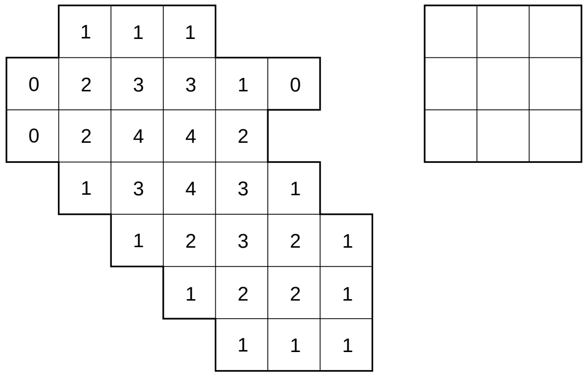
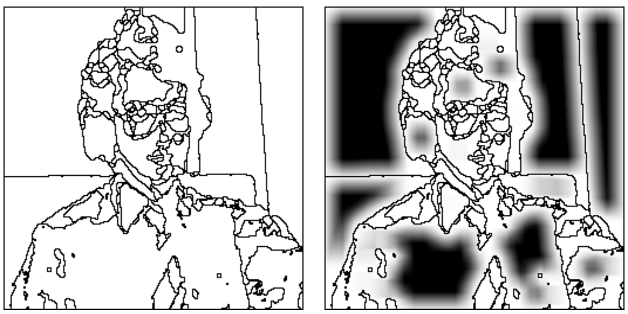


Fig. 2. Polymorphism values for a 3x3 structuring element.



(a) Contours of a label image. (b) Polymorphism values for a 15x15 square.

Fig. 3. Polymorphism values of all the regions of a label image. Darker grey tones correspond to higher values in (b).

Proposition 4

$$\text{If} \quad X \subseteq Y \quad \text{then} \quad P_B(X) \leq P_B(Y) \qquad (3)$$

$$P_B(X_p) = P_B(X) \quad \text{for all} \quad p \in \mathbb{Z}^2 \qquad (4)$$

$$\forall x \in \mathbb{Z}^2, \quad P_{X,B}(x) \leq \mathcal{A}(B) \qquad (5)$$

Link with the morphological opening. As was suggested in the introduction, polymorphism is closely related to the opening. The following property makes it apparent.

Proposition 5 *The polymorphism is an extension of the morphological opening. Indeed,*

$$P_B(x) \neq 0 \quad \text{if and only if} \quad x \in X \circ B \qquad (6)$$

1.3. The computation of polymorphism

There exists an efficient algorithm to perform polymorphism values [2]. The algorithm works in accordance with a decomposition in terms of an erosion and a kind of dilation; it is based on the following relationship: $\forall x \in X,\ P_{X,B}(x) = \mathcal{A}[(X \ominus B) \cap (\check{B})_x]$ where $\check{}$ denotes the transposition operation, and it implements a histogram analysis as developed by Huang *et al.* [1]. Experience shows that the computation time is similar to the computation time of an opening, even in the case of a label image.

2. Filtering process

Suppose we need to fill $X \circ B$ with the structuring element B. Points having polymorphism value 1 should be considered first, as they belong to a single structuring element. These points are also very sensitive to local shape modification. However, if we add points to X, $P_B(X)$ only increases. In the same order, it could happen that the minimal skeleton of $(X \cup y)$, where $y \in \mathbb{Z}^2$, is smaller than the minimal skeleton of X.

The proposed filtering scheme, valid for label images that consist of several regions X_k, tries to augment the total polymorphism value (i.e. $\sum_k P_B(X_k)$) by transferring points between regions. The process relies on B and is made of three steps:

(1) Initialization: compute the polymorphism value for each pixel;

(2) Assign all points having values above a given threshold to their original region;

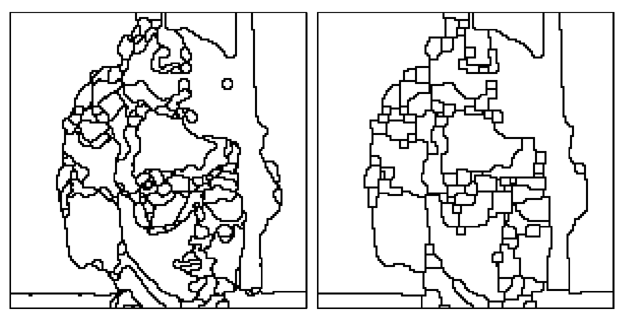
(3) Iterative stage :

  (3.1) Try to change points with low polymorphism values from a region to a neighboring region. The definitive change is the one that gives the highest increase of the total polymorphism;

  (3.2) Update the polymorphism values locally around the transferred point;

  (3.3) Return to (3.1) until stability achieved.

Let ζ_i be the filtering process implementation based on a square i pixels wide. Figure 4 shows the filtered contours obtained by means of $\zeta_4\zeta_3\zeta_2$.



(a) Original contours. (b) Contours filtered by $\zeta_4\zeta_3\zeta_2$.

Fig. 4. "Polymorphic" filter applied to the contours of a label image.

References

1. T. Huang, G. Yang, and G. Tang. A fast two-dimensional median filtering algorithm. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 27(1):13–18, February 1979.
2. M. Van Droogenbroeck. *Traitement d'images numériques au moyen d'algorithmes utilisant la morphologie mathématique et la notion d'objet : application au codage.* PhD thesis, Université catholique de Louvain, May 1994.