Structural optimization of flexible components under dynamic loading within a multibody system approach:

A comparative evaluation of optimization methods based on a 2-dof robot application.

E. Tromme, O. Brüls, G. Virlez, P. Duysinx
Aerospace and Mechanical Engineering Department
University of Liège
Belgium
Introduction – Optimization of a connecting rod

- A component based approach
  - Experience - Empirical load case
  - Dynamic factor amplification for safety
  - Not optimal

- Multibody system based approach

Geometrical modeling

Multibody system dynamics

Optimization process
MBS: Several parameterizations

- Inertial Frame
  - No distinction
  - Rigid motion + small deformation
  - Absolute coordinates (FE)
  - Rigid + Elast. Coord.

Inertia forces are easily computed in an inertial reference frame. Internal forces are easily computed in a body-attached frame.
Equation of FEM-MBS dynamics

- Motion of the flexible body (FEM) is represented by absolute nodal coordinates $\mathbf{q}$ (Geradin & Cardona, 2001)

- Dynamic equations of multibody system
  \[
  \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{ext} - \mathbf{g}^{int} - \mathbf{g}^{gyr}
  \]

- Subject to kinematic constraints of the motion
  \[
  \Phi(\mathbf{q}, t) = 0
  \]

- The solution is based on a Lagrange multiplier method
  \[
  \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \Phi^T_q(\mathbf{q}, t)\lambda = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\
  \Phi(\mathbf{q}, t) = 0,
  \]
  with the initial conditions
  \[
  \mathbf{q}(0) = \mathbf{q}_0 \text{ and } \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0.
  \]
**Time Integration**

- The set of nonlinear DAE is solved using the generalized-α method (Chung and Hulbert, 1993)
- Definition of a pseudo acceleration vector \( a \):
  \[
  (1 - \alpha_m) a_{n+1} + \alpha_m a_n = (1 - \alpha_f) \ddot{q}_{n+1} + \alpha_f \ddot{q}_n,
  \]
- Newmark integration formulae
  \[
  q_{n+1} = q_n + h \dot{q}_n + h^2 (1/2 - \beta) a_n + h^2 \beta a_{n+1},
  \]
  \[
  \dot{q}_{n+1} = \dot{q}_n + h (1 - \gamma) a_n + h \gamma a_{n+1},
  \]
- Solve iteratively the linearized dynamic equation system (Newton-Raphson scheme)
  \[
  M \Delta \ddot{q} + C_t \Delta \dot{q} + K_t \Delta q + \Phi_q^T \Delta \lambda = \Delta r
  \]
  \[
  \Phi_q \Delta q = \Delta \Phi
  \]
  where \( r = M \ddot{q} + \Phi_q^T \lambda - g \)
The Equivalent Static Load method
The EQSL Method

- Difficulties of dealing with dynamic constraints and loadings

- Definition of the Equivalent Static Load:
  
  *When a dynamic load is applied to a structure, the equivalent static load is defined as the static load that produces the same displacement field as the one created by the dynamic load at an arbitrary time. (Kang, Park & Arora, 2005)*

- Introduction of the concept on a linear structure

  **Equilibrium equation:**
  
  \[ \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t) + \mathbf{K}(\mathbf{x})\mathbf{y}(t) = \mathbf{s}(t) \]

  \[ \Leftrightarrow \mathbf{K}(\mathbf{x})\mathbf{y}(t) = \mathbf{s}(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t) \]

  The EQSL:

  \[ f_{eq}(t) = \mathbf{s}(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t) \]

  - In a discrete time domain, it exists one EQSL for each integration time step.
  - The dynamic response optimization problem is transformed in a static response optimization problem with multiple load cases.
The EQSL Method for MBS optimization

- Equations of motion for body $i$


\[
\begin{bmatrix}
    m_{RR}^i & m_{R\theta}^i & m_{Rf}^i \\
    m_{\theta R}^i & m_{\theta f}^i & \text{sym.} \\
    m_{f R}^i & m_{f \theta}^i & m_{ff}^i
\end{bmatrix}
\begin{bmatrix}
    \dddot{R}^i \\
    \dddot{\theta}^i \\
    \dddot{q}_f^i
\end{bmatrix}
\begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & K_{ff}^i
\end{bmatrix}
\begin{bmatrix}
    R^i \\
    \theta^i \\
    q_f^i
\end{bmatrix}
= -
\begin{bmatrix}
    C_{Ri}^T \\
    C_{\theta i}^T \\
    C_{q_f}^T
\end{bmatrix}
\lambda +
\begin{bmatrix}
    g_R^i \\
    g_\theta^i \\
    g_f^i
\end{bmatrix}
\]

EQSL for body $i$ at time $t$
(Kang, Park & Arora, 2005)

- The EQSL method is tailored to a floating frame formalism.
- Each body is optimized independently.

- Linearized equations of the equations of motion

\[
M(t_i) \Delta \ddot{q}(t_i) + C(t_i) \Delta \dot{q}(t_i) + K(t_i) \Delta q(t_i) + \Phi_q^T(t_i) \Delta \lambda(t_i) = 0
\]

\[
K(t_i) \Delta q(t_i) = -M(t_i) \Delta \ddot{q}(t_i) - C(t_i) \Delta \dot{q}(t_i) - \Phi_q^T(t_i) \Delta \lambda(t_i)
\]

While the structure of the equations seems similar to the equilibrium equation of a static linear structure, the optimization process cannot be directly based on this equation.
Differences between the two MBS approaches

**Floating Frame**
- Decoupling between the component flexibility
  - One stiffness matrix $K^i$ is defined per component.
- The matrix $K^i$ is constant with respect to the system configuration in the body attached frame.
- Decoupling between rigid body motions and deformations

**Inertial Frame**
- No decoupling between the component flexibility
  - $K_t$ is related to the whole system.
- The matrix $K_t$ evolves with respect to system configuration.
- No decoupling between rigid body motions and deformations in the displacement vector $q$.

**In general**
- Originally developed for rigid MBS
  - Flexibility introduced later
  - Unable to represent geometric stiffening
- Developed to obtain an integrated approach of the flexibility in MBS
  - For instance, stress analysis is straightforward
A post-processing step to define the EQSL with an inertial frame approach

1. For each component, it is possible to extract its tangent stiffness matrix by selecting suitable generalized coordinates.

2. To avoid storing $K_t$ at each time step, a reference state is considered ($t_{\text{ref}}$) → Need of suitable transformations.

3. Key point: introduction of a corotational frame in a post processing step for each component:
   - Enables to define the deformation in the attached-body frame.
   - Enables to define the appropriate transformations to go back to the reference state.

Using the cororotational frame:

\[
K^b_{t}(t_{\text{ref}})u^b(t) = g_{eq}^b(t)
\]

\[
K^b_{t}(t_{\text{ref}}) = -M(t_i)\Delta \ddot{q}(t_i) - C^b_{t}(t_i)\Delta \dot{q}(t_i) - \Phi_{q}^{T}(t_i)\Delta \lambda(t_i)
\]
Flowchart of the optimization process using the EQSL method

1. Initialize the design variables and set \( \text{it} = 0 \)
2. MBS simulation
3. EQSL Computation
4. \[ \| \text{EQSL}(\text{it}) - \text{EQSL}(\text{it} - 1) \| < \epsilon \]
   - True: Stop
   - False: Update \( \text{d.v} \)
5. Static response optimization with multiple load cases

\( \text{it} = \text{it} + 1 \)
Update \( \text{d.v} \)
The “fully integrated” method
Evolution of virtual prototyping

- Structural optimization
- Flexible multibody systems

**Static or quasi-static loading**

**Dynamic loading**

- Integrated optimization of flexible components in multibody systems
General form of the optimization problem

- Design problem casted in a mathematical programming problem

\[
\begin{align*}
\text{minimize} \quad & \varphi(x) \\
\text{subject to} \quad & \text{Equilibrium equation} \\
& c_j(x) \leq \bar{c}_j, \quad j = 1, \ldots, n_c, \\
& \underline{x}_v \leq x_v \leq \bar{x}_v, \quad v = 1, \ldots, n_v,
\end{align*}
\]

- Provides a general and robust framework to the solution procedure
- Various efficient solvers can be used.

- **Integrated method** - Formulation:

\[
\begin{align*}
\text{minimize} \quad & \varphi(x) \\
\text{subject to} \quad & M(q)\ddot{q} + \Phi^T_q(q,t)\lambda = g(\dot{q}, q, t), \\
& \Phi(q, t) = 0, \\
& c_j(x, t) \leq \bar{c}_j, \quad j = 1, \ldots, n_c, \\
& \underline{x}_v \leq x_v \leq \bar{x}_v, \quad v = 1, \ldots, n_v.
\end{align*}
\]
Sensitivity analysis

- Finite difference scheme can be CPU-time consuming.

- A semi-analytical method has been developed by O. Brüls and P. Eberhard (2008) which can be integrated in the generalized-α scheme.

\[
\begin{align*}
M \frac{d\dot{q}}{dp_u} + C_t \frac{d\dot{q}}{dp_u} + K_t \frac{dq}{dp_u} + \Phi_q^T \frac{d\lambda}{dp_u} &= - \frac{\partial r}{\partial p_u} \\
\Phi_q \frac{dq}{dp_u} &= - \frac{\partial}{\partial p_u} \Phi
\end{align*}
\]

- Sensitivity equations are linear with respect to \( \frac{dq}{dp_u} \) and \( \frac{d\lambda}{dp_u} \).

- Same structure as the linearized equations of motion:
  - Same integration procedure except for the residuals
  - Tangent iteration matrix is the same as the one of the original problem
  - No need to apply a Newton-Raphson procedure
Numerical Applications
A 2-dof robot subject to tracking trajectory constraints

- Minimize the mass
- 4 beam elements
- Design variables: diameters
- Imposed rotations at hinges
- Time step: 0.0005 [s]

\[ \Delta x_{tip}(t) = \Delta y_{tip}(t) = \frac{0.5}{T} \left( t - \frac{T}{2\pi} \sin \left( \frac{2\pi t}{T} \right) \right) \]

(Kang, Park & Arora, 2005)
First numerical application

\[
\begin{align*}
\text{minimize} & \quad m(x) \\
\text{subject to} & \quad \sqrt{\delta y_a^2(t_n) + \delta y_{tip}^2(t_n)} \leq 0.001 \text{ [m]}, \quad n = 1, \ldots, 67, \\
& \quad 0.02 \text{ [m]} \leq x_v \leq 0.06 \text{ [m]}, \quad v = 1, \ldots, 4.
\end{align*}
\]

where $\delta y_a(t_n)$ and $\delta y_{tip}(t_n)$ are respectively the vertical deflections in the inertial frame of the first link at the hinge A and of the second link at the tip.
First numerical application - Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Mass [kg]</th>
<th>Iterations</th>
<th>Inner iterations</th>
<th>d_1 [mm]</th>
<th>d_2 [mm]</th>
<th>d_3 [mm]</th>
<th>d_4 [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQSL Method</td>
<td>1.213</td>
<td>6</td>
<td>61</td>
<td>45.40</td>
<td>32.76</td>
<td>37.99</td>
<td>26.83</td>
</tr>
<tr>
<td>Integrated Method</td>
<td>1.214</td>
<td>13</td>
<td>/</td>
<td>45.44</td>
<td>32.69</td>
<td>38.08</td>
<td>26.78</td>
</tr>
</tbody>
</table>

**EQSL Approach**

**Integrated Approach**

- Mass Evolution
- Deviation Constraint

- EQSL Approach
- Integrated Approach

Upper bound

Second numerical application

$$\text{minimize } \quad m(x)$$

subject to

$$\sqrt{\delta x_{tip}^2(t_n) + \delta y_{tip}^2(t_n)} \leq 0.001 \text{ [m]}, \quad n = 1, \ldots, 51,$$

$$0.02 \text{ [m]} \leq x_v \leq 0.06 \text{ [m]}, \quad i = 1, \ldots, 4.$$ 

where $\delta x_{tip}(t_n)$ and $\delta y_{tip}(t_n)$ are respectively the horizontal and vertical deflections of the robot tip in the inertial frame.

- Only the extremity of the second robot link is concerned by the optimization constraint.
- It is a constraint on the global system behavior.
- With the EQSL method, the components are optimized independently. The first link does not appear in the constraint formulation while it is obvious that its flexibility has a contribution to the tip displacement.
- The problem can be overcome by using a sum over the deflection of all the components.
- This problem does not appear with the fully integrated method as the system is also treated as a whole during the optimization process.
Second numerical application - Results

<table>
<thead>
<tr>
<th></th>
<th>Mass [kg]</th>
<th>Iterations</th>
<th>Inner iterations</th>
<th>d₁ [mm]</th>
<th>d₂ [mm]</th>
<th>d₃ [mm]</th>
<th>d₄ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQSL Method</td>
<td>1.411</td>
<td>4</td>
<td>38</td>
<td>47.88</td>
<td>34.51</td>
<td>42.11</td>
<td>30.08</td>
</tr>
<tr>
<td>Integrated Method</td>
<td>1.408</td>
<td>15</td>
<td>/</td>
<td>48.59</td>
<td>34.82</td>
<td>41.60</td>
<td>29.02</td>
</tr>
</tbody>
</table>

EQSL Approach

Integrated Approach

EQSL Approach

Integrated Approach

Upper bound
Conclusions

- We proposed a method to derive the EQSL adapted to the nonlinear finite element based MBS formalism.

- Both methods can converge towards the same optimum for the considered example.

- Fundamental difference:
  - Fully integrated method: 1 dynamic analysis per iteration
  - EQSL method: 1 dynamic analysis + a set of static analysis per cycle

- For slowly varying body loads, the EQSL method normally requires less dynamic simulations and one dynamic analysis is more time consuming than one static analysis.

- The formulation of global behavior constraint can become rather complex with the EQSL method as the components are decoupled (e.g. multiple loop system).
Perspectives

- Ongoing work investigates systems with design dependent loading and more advanced cases as different behaviors are expected for the methods.

- A Lie group formulation enables to have a constant tangent stiffness matrix in the material frame and enables to have a measure of the deformation in the material frame.
Thank You Very Much For Your Attention

I acknowledge the Lightcar project sponsored by the Walloon Region of Belgium for its support.
Contact

Emmanuel TROMME

Automotive Engineering
- Aerospace and Mechanical Engineering Department
University of Liège

Chemin des Chevreuils, 1 building B52
4000 Liège Belgium

Email: emmanuel.tromme@ulg.ac.be
Tel: +32 4 366 91 73
Fax: +32 4 366 91 59