# Extensions of Superalgebras of Krichever-Novikov type

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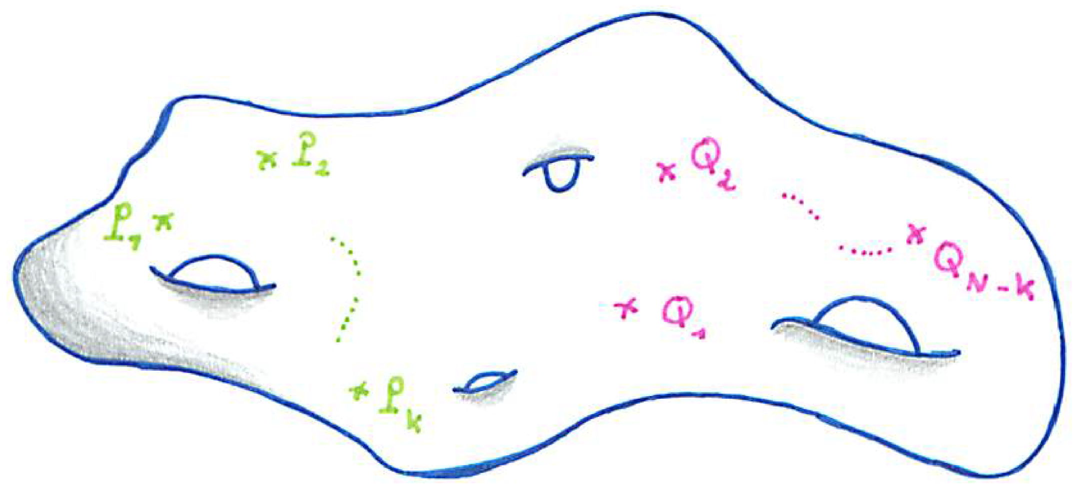
## State of the art and geometrical set-up

Krichever and Novikov introduced in 1987 a family of Lie algebras generalizing the Virasoro algebra. Further, Schlichenmaier studied these algebras in some more general cases around 1990 and continue to work currently on that in the superspaces case. In 2011, the notion of Lie antialgebra, which is a particular case of Jordan superalgebras and related to Lie superalgebras, was introduced by Ovsienko. Leidwanger and Morier-Genoud found in 2012 an important example of Lie antialgebra in the theory of algebras of Krichever-Novikov type. We give an explicit construction of central extensions of Lie superalgebras of K-N type and in the case of Lie antialgebra we calculate a 1-cocycle with coefficients in the dual space. In a particular case, a base can be found so that computations can be made.

The geometrical set-up is the following:

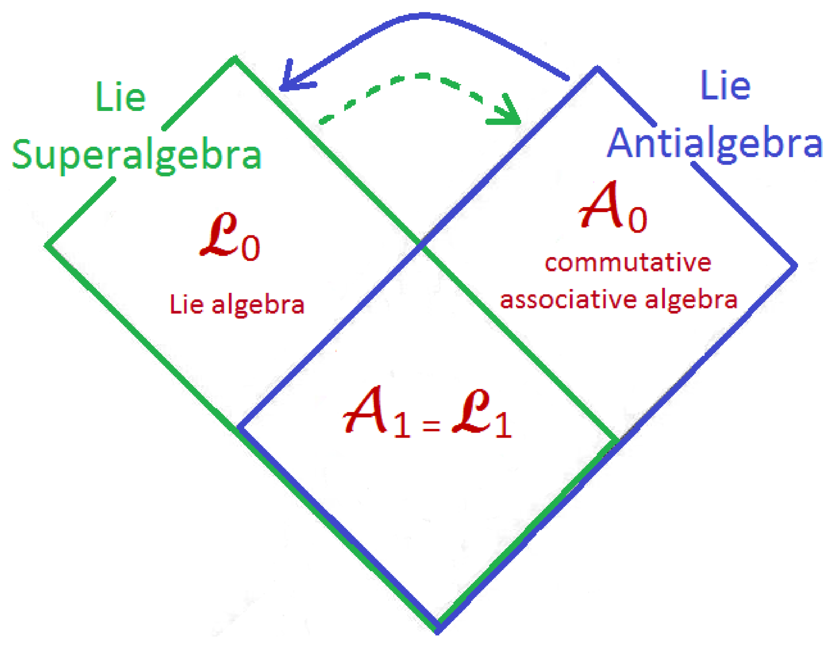
- Let $M$ be a compact Riemann surface of genus $g$;
- Fix a square root of $K$, where $K$ is the canonical line bundle of $M$;
- Let $A = \{P_1, \dots, P_K\} \cup \{Q_1, \dots, Q_{N-K}\}$ be a set of $N$ points;
- Denote $\mathcal{F}_\lambda$ the vector space of tensor densities of weight $\lambda$.




## Lie superalgebras and Lie antialgebras of K-N type

$$\mathcal{L}_{g,N} = \mathcal{F}_{-1} \oplus \mathcal{F}_{-1/2}$$

The space $\mathcal{F}_{-1} \cong \mathfrak{g}_{g,N}$ is the Lie algebra of meromorphic vector fields on $M$ which are holomorphic outside of $A$. The $\mathbb{Z}_2$-graded vector space $\mathcal{L}_{g,N}$ is equipped with a Lie superbracket.




$$\mathcal{J}_{g,N} = \mathcal{F}_0 \oplus \mathcal{F}_{-1/2}$$

The space $\mathcal{F}_0 \cong \mathfrak{a}_{g,N}$ is the associative algebra of meromorphic functions on $M$ which are holomorphic outside of $A$. The space $\mathcal{J}_{g,N}$ is equipped with a super commutative product satisfying the axioms of Lie antialgebras.

## Lie superalgebra $\mathcal{L}_{0,3}$

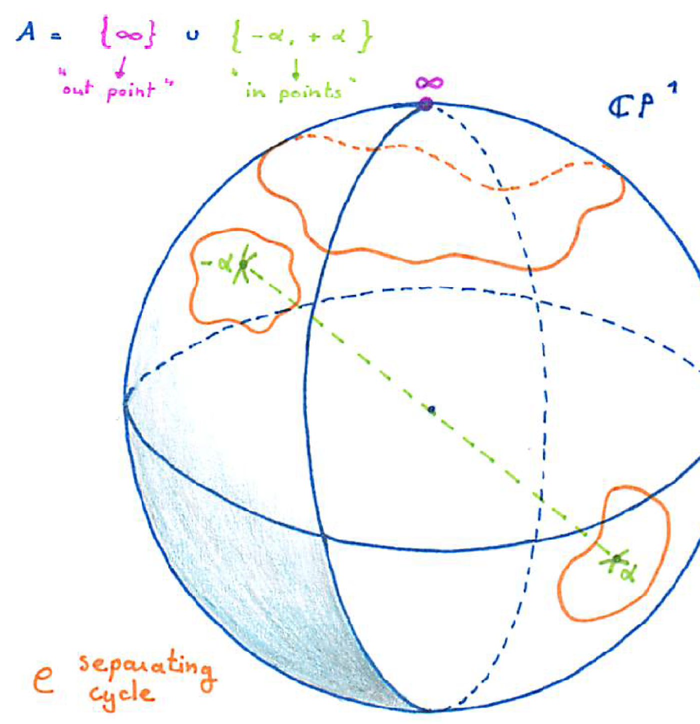
In local coordinates we have the
- even elements of the basis:

$$V_{2k}(z) = z(z-\alpha)^k(z+\alpha)^k \tfrac{d}{dz},$$
$$V_{2k+1}(z) = (z-\alpha)^{k+1}(z+\alpha)^{k+1} \tfrac{d}{dz},$$

- odd elements of the basis:

$$\varphi_{2k+\frac{1}{2}}(z) = \sqrt{2}z(z-\alpha)^k(z+\alpha)^k\, dz^{-1/2},$$
$$\varphi_{2k-\frac{1}{2}}(z) = \sqrt{2}(z-\alpha)^k(z+\alpha)^k\, dz^{-1/2}.$$

## Special case: $g = 0$, $N = 3$




## Lie antialgebra $\mathcal{J}_{0,3}$

In local coordinates we have the
- even elements of the basis:

$$G_{2k}(z) = (z-\alpha)^k(z+\alpha)^k,$$
$$G_{2k+1}(z) = z(z-\alpha)^k(z+\alpha)^k,$$

- odd elements of the basis:

$$\varphi_{2k+\frac{1}{2}}(z) = \sqrt{2}z(z-\alpha)^k(z+\alpha)^k\, dz^{-1/2},$$
$$\varphi_{2k-\frac{1}{2}}(z) = \sqrt{2}(z-\alpha)^k(z+\alpha)^k\, dz^{-1/2}.$$

## Results on $\mathcal{L}_{g,N}$ and $\mathcal{J}_{g,N}$

**Theorem 1**

The even bilinear map $c : \mathcal{L}_{g,N} \times \mathcal{L}_{g,N} \to \mathbb{C}$ given by

$$\begin{aligned} c\left(e(z)\tfrac{d}{dz}, f(z)\tfrac{d}{dz}\right) &= \frac{-1}{2i\pi}\int_{\mathcal{C}} \frac{1}{2}(e^{'''}f - ef^{'''}) - R(e'f - ef')dz, \\ c\left(\varphi(z)dz^{-1/2}, \psi(z)dz^{-1/2}\right) &= \frac{1}{2i\pi}\int_{\mathcal{C}} \frac{1}{2}\left(\varphi^{''}\psi + \varphi\psi^{''}\right) - \frac{1}{2}R\varphi\psi dz, \\ c\left(e(z)\tfrac{d}{dz}, \psi(z)dz^{-1/2}\right) &= 0 \end{aligned}$$

is a well defined almost-graded non trivial local 2-cocyle, where $\mathcal{C}$ is a separating cycle and $R$ is a projective connection.

**Corollary**

With respect to the splitting, an almost-graded local 1-cocycle on $\mathcal{L}_{g,N}$ with coefficient in the dual space $\mathcal{L}^*_{g,N}$ is given by

$$\begin{aligned} C\left(e(z)\tfrac{d}{dz}\right) &= -\left(e^{'''} - 2Re' - R'e\right)dz^2, \\ C\left(\varphi(z)dz^{-1/2}\right) &= \left(\varphi^{''} - \tfrac{1}{2}R\varphi\right)dz^{3/2}. \end{aligned}$$

It was proved by Ovsienko that a Lie antialgebra has no non-trivial central extensions, provided the even part contains a unit element. However, fixing a K-N pairing, there exists a nice construction of 1-cocycle that has similar properties that the one on the Lie superalgebras.

**Theorem 2**

With respect to the splitting, an almost-graded local 1-cocycle on $\mathcal{J}_{g,N}$ with coefficients in $\mathcal{J}^*_{g,N}$ is given by

$$\begin{aligned} \mathcal{C}\left(\varepsilon(z)\right) &= -\varepsilon'(z)dz, \\ \mathcal{C}\left(\psi(z)dz^{-1/2}\right) &= \left(\psi''(z) - \tfrac{1}{2}R\psi(z)\right)dz^{3/2}. \end{aligned}$$

**Remarks on the special case:** In the case where $g = 0$ with $N = 3$, we can choose the projective connection $R \equiv 0$ and up to isomorphism we can calculate explicitly the cocycles thanks to the elements of the basis, see [1] for more details.

**Reference:** [1] Kreusch M., "Extensions of Superalgebras of Krichever-Novikov type", preprint arXiv:1204.4338.

**Advisors:** Pierre Lecomte (Liege) & Valentin Ovsienko (Lyon).