Automatic Aircraft Cargo Load Planning with Pick-up and Delivery

V. Lurkin and M. Schyns

University of Liège
QuantOM

14ème conférence ROADEF
Société Française de Recherche Opérationnelle et Aide à la Décision
Université de Technologie de Troyes, 13-14-15 Février 2013
Outline

1. Motivation
2. Problem Description
3. Model
4. Results
5. Conclusion and outlooks
Outline

1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
⇒ Problem Statement:

“How to optimally load a set of containers and pallets (ULDs) into a cargo aircraft that has to serve multiple destinations under some safety, structural, economical, environmental and manoeuvrability constraints?”

- Transport of goods by air
- Sector has undergone changes since beginning of 2000s:
  - Important increase of competition (new Low Cost Companies)
  - Volatility and increasing trend in the oil prices
    - Change in mentality
  - Greater focus on environmental concerns
  - More attention to spendings
- Load planning has possibilities for costs cutting because it is still a manual task
In the case of transport of goods by air at multiple destinations, the questions we are asking are:

1. What are the associated costs? → ECONomic & ECOlogical model
2. What are the key factors we can act on? → Mathematical model
3. How to optimize the decision? → Optimization method
Outline

1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Description of the Problem

- A cargo aircraft has to deliver goods at two consecutive airports\(^1\)

- Find the optimal location for all ULDs into the cargo aircraft
  - To minimize the fuel consumption during the entire trip
  - To minimize the time required to unload and load ULDs at the intermediate destination

\(^1\)Generalization could be easily done to more than two destinations
Summary of the model

Minimize

\( \text{minimize} \), (deviation most aft CG) and \( \# \) ULDs to unload

(\( \forall \) route!)

subject to:

Each ULD is loaded
Each ULD fits in a position
A position accepts only one ULD
Some positions are overlapping: not simultaneously used
Longitudinal stability: The CG is within certified limits
Lateral balance
Maximum weight per position
Combined load limits
Cumulative load limits
Regulations for hazardous goods
Two parts of larger ULDs in adjacent positions

⇒ “Assignment Problem / Combinatorial Problem”
⇒ Integer Linear Problem
Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific (dedicated to one specific aircraft,...)
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)
Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific (dedicated to one specific aircraft,...)
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)

Main references for the basic problem (CG)


1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Outline

1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Main Parameters and Variables

- $\mathcal{K}$ is the set of routes $\Rightarrow$ parts of trip separating two successive airports.
- $\mathcal{U}$ is the set of ULDs $\Rightarrow$ pallets and containers to be transported.
- According to their origin and destination: three subsets of ULDs: $\mathcal{U}_1$, $\mathcal{U}_2$, $\mathcal{U}_3$.
- $\mathcal{P}$ is the set of all the positions $\Rightarrow$ predefined spaces in the aircraft that may contain the ULDs.
- There is only one central door situated at the extremity of the aircraft.

Binary Variables

$$x_{ijk} = \begin{cases} 1 & \text{if ULD } i \text{ is in position } j \text{ during the route } k \\ 0 & \text{otherwise} \end{cases}$$
Outline

1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Objective function: Most Aft CG

- In terms of fuel consumption, the optimal location for the CG is the most aft
- We want to achieve the most aft CG under stability constraints
- We minimize, on the global trip, the absolute deviation between the most aft CG and the obtained CG

In mathematical terms, it gives:

\[
\min \sum_{\forall k \in K} \epsilon_k
\]

**Subject to:**

\[
\begin{align*}
  c_k - o_k - \epsilon_k & \leq 0 \\
  c_k - o_k + \epsilon_k & \geq 0
\end{align*}
\] \( \forall k \in K \)

where:
- \(c_k\) is the CG obtained after assignment of ULDs in the aircraft during the route \(k\)
- \(o_k\) is the optimal CG, i.e. most aft CG on the route \(k\)
Objective function: minimize the # ULDs to Unload

- Loading time is function of the # of ULDs to be unloaded
- At the first airport, $\text{ULDs} \in U_3$ have not to be unloaded
- If those ULDs can remain in the aircraft: time savings!
- So, what we want is:
  1. Locate the $\text{ULDs} \in U_3$ that must be unloaded unnecessarily because they prevent the unloading of $\text{ULDs} \in U_1$
  2. Minimize the # of ULDs in such location
Objective Function: Minimize # ULDs to Unload

In mathematical terms, we use the following expression to count the # of embarrassing positions:

\[
\text{Min} \sum_{\forall j \in P} n_j
\]

Subject to:

\[
\sum_{\forall i' \in U_1} \sum_{\forall j' \in P_{ds} | l_{j'} > l_j} x_{i'j'1} - n_j N_j - (1 - x_{ij1}) N_j \leq 0 \quad \forall j \in P_{ds}, \forall d \in D, \forall s \in S, \forall i \in U_3
\]

- \( N_j \) are constant numbers that give the number of positions behind each position \( j \)
- \( 0 \leq \sum_{\forall i' \in U_1} \sum_{\forall j' \in P_{ds} | l_{j'} > l_j} x_{i'j'1} \geq N_j \)
- \( n_j \) are binary variables equal to 1 if the ULD in position \( j \) must be unloaded unnecessarily
Objective Function: Minimize \# ULDs to Unload

- Not sufficient to \textbf{min} (\# ULDs from $U_3$ with an ULD from $U_1$ behind it)
- We have to be sure that:
  
  1. Each ULD $\in U_3$ not unloaded keeps the same position for the second route
  2. Each ULD $\in U_2$ (loaded at first airport) doesn’t conduct to the unloading of ULD $\in U_3$

It leads to the two following sets of constraints:

\[
\begin{aligned}
  x_{ij_0} - n_j + y &\leq 1 & \forall j \in P, \forall i \in U_3 \\
  x_{ij_0} - x_{ij_1} - y &\leq 0 & \forall j \in P, \forall i \in U_3
\end{aligned}
\]

And:

\[
x_{ij_0} - n_j + x_{ij'}_{j'1} \leq 1 & \forall j \in P, \forall i \in U_3, \forall j' \in P \mid l_{j'} > l_j, \forall i' \in U_2
\]
Minimizing fuel consumption and \# ULDs unloaded

\[
\text{Min } \alpha (\epsilon_1 + \epsilon_2) + \beta \sum_{j \in P} n_j
\]

where:
- $\alpha$ is the additional cost (fuel + emissions) for a deviation of one inch from the most aft center of gravity
- $\beta$ is the cost associated with the time required to unload one additional ULD (in terms of wages, fees to the airport for the usage of the runway...)
Outline

1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Summary of the model

\[
\begin{align*}
\min & \quad \alpha \sum_{k \in K} c_k + \beta \sum_{j \in \mathcal{P}} n_j \\
\text{Subject to:} & \\
& c_k - o_k - c_k \leq 0 \quad \forall k \in K \\
& c_k - o_k + c_k \geq 0 \quad \forall k \in K \\
& \sum_{j \in \mathcal{P}, v \in \mathcal{P}, t \in \mathcal{P}} \sum_{j \in \mathcal{P}, v \in \mathcal{P}, t \in \mathcal{P}} x_{i+j} - n_j \cdot N_j - (1 - x_{i+j}) \cdot N_j \leq 0 \\
& \forall j \in \mathcal{P}, \forall d \in \mathcal{D}, \forall a \in \mathcal{S}, \forall i \in \mathcal{U}_3
\end{align*}
\]

- \( \min_k \leq c_k \leq \max_k \) \quad \forall k \in K

- \( -D \leq \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} w_i \cdot \left( \sum_{j \in \mathcal{P}} x_{i+j} - \sum_{j \in \mathcal{P}} x_{i+j} \right) \leq D \)

- \( -D \leq \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} w_i \cdot \left( \sum_{j \in \mathcal{P}} x_{i+j} - \sum_{j \in \mathcal{P}} x_{i+j} \right) \leq D \)

- \( x_{i+j} = 0 \) \quad \forall i \in \mathcal{U}_1, \forall j \in \mathcal{P}

- \( \sum_{j \in \mathcal{P}} x_{i+j} = 1 \) \quad \forall i \in \mathcal{U}_1, \forall j \in \mathcal{P}

- \( x_{i+j} = 0 \) \quad \forall i \in \mathcal{U}_2, \forall j \in \mathcal{P}

- \( \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} x_{i+j} \leq 1 \) \quad \forall i \in \mathcal{U}_1, \forall j \in \mathcal{P}

- \( x_{i+j} \leq 1 \) \quad \forall i \in \mathcal{U}_1, \forall j \in \mathcal{P}

- \( x_{i+j} + x_{i+j'} \leq 1 \) \quad \forall i, j \in \mathcal{U}_1, \forall j' \in \mathcal{P}

- \( x_{i+j} + x_{i+j'} \leq 1 \) \quad \forall i, j \in \mathcal{U}_1, \forall j' \in \mathcal{P}

- \( u_i \times x_{i+j} \leq W_j \) \quad \forall i \in \mathcal{U}_1, j \in \mathcal{P}

- \( u_i \times x_{i+j} \leq W_j \) \quad \forall i \in \mathcal{U}_2, j \in \mathcal{P}

- \( \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} \sum_{j \in \mathcal{P}} x_{i+j} \leq \bar{G}_d \) \quad \forall d \in \mathcal{D}, \forall a \in \mathcal{A}_d

- \( \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} \sum_{j \in \mathcal{P}} x_{i+j} \leq \bar{G}_d \) \quad \forall d \in \mathcal{D}, \forall a \in \mathcal{A}_d

- \( \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} \sum_{j \in \mathcal{P}} x_{i+j} \leq \bar{G}_d \) \quad \forall d \in \mathcal{D}, \forall a \in \mathcal{A}_d

- \( \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} \sum_{j \in \mathcal{P}} x_{i+j} \leq \bar{G}_d \) \quad \forall d \in \mathcal{D}, \forall a \in \mathcal{A}_d

- \( \sum_{i \in \mathcal{U}_1, j \in \mathcal{P}} \sum_{j \in \mathcal{P}} x_{i+j} \leq \bar{G}_d \) \quad \forall d \in \mathcal{D}, \forall a \in \mathcal{A}_d

- \( x_{i+j} + x_{i+j'} = 0 \) \quad \forall i, j \in \mathcal{U}_1, \forall j' \in \mathcal{P}

- \( x_{i+j} + x_{i+j'} = 0 \) \quad \forall i, j \in \mathcal{U}_1, \forall j' \in \mathcal{P}

- \( x_{i+j} \leq 1 \) \quad \forall i, j \in \mathcal{U}_1, \forall j' \in \mathcal{P}

- \( x_{i+j} \leq 1 \) \quad \forall i, j \in \mathcal{U}_1, \forall j' \in \mathcal{P}
Constraints linked to OF

\[ c_k - o_k - \epsilon_k \leq 0 \quad \forall k \in K \]
\[ c_k - o_k + \epsilon_k \geq 0 \quad \forall k \in K \]
\[ \sum_{i' \in U_1} \sum_{j' \in P_{ds}, l_{i'} > l_j} x_{i'j'1} - n_j N_j - (1 - x_{ij1}) N_j \leq 0 \quad \forall j \in P_{ds}, \forall d \in D, \forall s \in S, \forall i \in U_3 \]

Constraints for stability

\[ \min_k \leq c_k \leq \max_k \quad \forall k \in K \]
\[ -\bar{D} \leq \sum_i (U_1 \cup U_3) w_i \left( \sum_{j \in P_R} x_{ij0} - \sum_{j \in P_L} x_{ij0} \right) \leq \bar{D} \]
\[ -\bar{D} \leq \sum_i (U_2 \cup U_3) w_i \left( \sum_{j \in P_R} x_{ij1} - \sum_{j \in P_L} x_{ij1} \right) \leq \bar{D} \]

Constraints for routes

\[ x_{ij0} = 0 \quad \forall i \notin (U_1 \cup U_3), \forall j \in P \]
\[ x_{ij1} = 0 \quad \forall i \notin (U_2 \cup U_3), \forall j \in P \]
Constraints

Constraints for full load

\[ \sum_{j \in \mathbb{P}} x_{ij0} = 1 \quad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3) \]
\[ \sum_{j \in \mathbb{P}} x_{ij1} = 1 \quad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3) \]

Constraints for allowable positions

\[ x_{ijk} = 0 \quad \forall i \in \mathbb{U}, \forall j \in \mathbb{P}, \forall k \in \mathbb{R} \mid \text{U}_i \text{ does not fit in } P_j \]
\[ \sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} x_{ij0} \leq 1 \quad \forall j \in \mathbb{P} \]
\[ \sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} x_{ij1} \leq 1 \quad \forall j \in \mathbb{P} \]
\[ x_{ij0} + x_{i'j'1} \leq 1 \quad \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3), \forall j \in \mathbb{P}, \forall j' \in \mathbb{O}_j \]
\[ x_{ij1} + x_{i'j'2} \leq 1 \quad \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3), \forall j \in \mathbb{P}, \forall j' \in \mathbb{O}_j \]
Constraints for load limits

\[ w_i \times x_{ij0} \leq \bar{W}_j \quad \forall i \in (U_1 \cup U_3), \quad \forall j \in \mathcal{P} \]
\[ w_i \times x_{ij1} \leq \bar{W}_j \quad \forall i \in (U_2 \cup U_3), \quad \forall j \in \mathcal{P} \]

\[ \sum_{i \in (U_1 \cup U_3)} \sum_{j \in \mathcal{P} | P_j \cap O_a^d \neq \emptyset} x_{ij0} o_{ija}^d \leq \bar{O}_a^d \quad \forall d \in D^*, \forall a \in O^d \]
\[ \sum_{i \in (U_2 \cup U_3)} \sum_{j \in \mathcal{P} | P_j \cap O_a^d \neq \emptyset} x_{ij1} o_{ija}^d \leq \bar{O}_a^d \quad \forall d \in D^*, \forall a \in O^d \]

\[ \sum_{i \in (U_1 \cup U_3)} \sum_{j \in \mathcal{P} | P_j \cap U_c^a \neq \emptyset} F_c \neq \emptyset \sum_{l=1}^{a} x_{ij0} f_{ijl} \leq \bar{F}_a \quad \forall a \in F \]
\[ \sum_{i \in (U_2 \cup U_3)} \sum_{j \in \mathcal{P} | P_j \cap U_c^a \neq \emptyset} F_c \neq \emptyset \sum_{l=1}^{a} x_{ij1} f_{ijl} \leq \bar{F}_a \quad \forall a \in F \]

\[ \sum_{i \in (U_1 \cup U_3)} \sum_{j \in \mathcal{P} | P_j \cap U_c^a \neq \emptyset} T_c \neq \emptyset \sum_{l=1}^{a} x_{ij0} t_{ijl} \leq \bar{T}_a \quad \forall a \in T \]
\[ \sum_{i \in (U_2 \cup U_3)} \sum_{j \in \mathcal{P} | P_j \cap U_c^a \neq \emptyset} T_c \neq \emptyset \sum_{l=1}^{a} x_{ij1} t_{ijl} \leq \bar{T}_a \quad \forall a \in T \]
### Constraints for dangerous goods and larger ULDs

\[
\begin{align*}
    x_{ij1} + x_{i'j'1} & \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (U_1 \cup U_3), \text{ and } \forall j, j' \in P \\
    x_{ij2} + x_{i'j'2} & \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (U_2 \cup U_3), \text{ and } \forall j, j' \in P \\
    x_{ij1} - \sum_{j' \in P_j} x_{f_i j'1} & = 0 \quad \forall i \in (U^L \cap (U_1 \cup U_3)), \forall j \in P \\
    x_{ij2} - \sum_{j' \in P_j} x_{f_i j'2} & = 0 \quad \forall i \in (U^L \cap (U_2 \cup U_3)), \forall j \in P
\end{align*}
\]
Outline

1 Motivation

2 Problem Description

3 Model
   - Main Parameters
   - Objective Function
   - Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Model tested on a set of real data

- Mathematical model tested on a realistic case.
- Set of real-world data provided by an industrial partner.
- Objective: find a feasible and optimal position for each ULD within a minimal amount of time.
- Optimal solution = (CG to the aft) & (No ULDs unnecessarily unloaded).
2 successive destinations, and so 2 routes

Set of $U$ ULDs

Boeing 777
60 “normal” positions
+ 10 overlying ones
### Results (II)

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td># ULDs $U_1$</td>
<td>5</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td># ULDs $U_2$</td>
<td>5</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td># ULDs $U_3$</td>
<td>2</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td><strong>Total # ULDs</strong></td>
<td><strong>12</strong></td>
<td><strong>21</strong></td>
<td><strong>41</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>route 0</td>
<td>route 1</td>
<td>route 0</td>
</tr>
<tr>
<td># ULDs</td>
<td>7</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>ZFW</td>
<td>152 441</td>
<td>150 521</td>
<td>170 962</td>
</tr>
<tr>
<td>Most aft CG</td>
<td>54.43</td>
<td>53.91</td>
<td>59.41</td>
</tr>
<tr>
<td>Obtained CG</td>
<td>54.43</td>
<td>53.91</td>
<td>59.41</td>
</tr>
<tr>
<td>Epsilon</td>
<td>0.0002</td>
<td>0.0012</td>
<td>0</td>
</tr>
<tr>
<td>$\sum n_j$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Time</td>
<td>9 sec</td>
<td>25 min</td>
<td>1h 57 min</td>
</tr>
</tbody>
</table>

V. Lurkin and M. Schyns (ULg)  
Automatic Cargo Load Planning  
ROADEF 2013 28 / 32
Graphical Representation of the Results (Test 3)

V. Lurkin and M. Schyns (ULg)

Automatic Cargo Load Planning

ROADEF 2013 29 / 32
Outline

1 Motivation

2 Problem Description

3 Model
   • Main Parameters
   • Objective Function
   • Constraints in Complete Model

4 Results

5 Conclusion and outlooks
Conclusion and outlooks

To do list

√ Mathematical formulation of the model

□ Additional tests

□ Pursue ongoing work on economic and ecological impacts ($\alpha$ and $\beta$)

□ The load doesn’t seem compressed naturally: include an inertia component in the model?

□ Introduction of multiple doors

□ Development of Heuristics?

□ Extension of the model to other modes of transport: ships, trains, ...
Contact me

My email address: vlurkin@ulg.ac.be

QuantOM website: http://www.quantom.hec.ulg.ac.be

Thank you for your attention!