Automatic Aircraft Cargo Load Planning with Pick-up and Delivery

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14ème conférence ROADEF
Société Française de Recherche Opérationnelle et Aide à la Décision
Université de Technologie de Troyes, 13-14-15 Février 2013

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Outline

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Context of the Research

⇒ **Problem Statement**:

"How to **optimally load** a **set of containers and pallets** (ULDs) into a **cargo aircraft** that has to serve **multiple destinations** under some safety, structural, economical, environmental and manoeuvrability **constraints?**"

- Transport of goods by air
- Sector has undergone changes since beginning of 2000s:
  - Important increase of competition (new Low Cost Cpnies)
  - Volatility and increasing trend in the oil prices

    → Change in mentality
  - Greater focus on environmental concerns
  - More attention to spendings
- Load planning has possibilities for costs cutting because it is still a manual task

Positioning

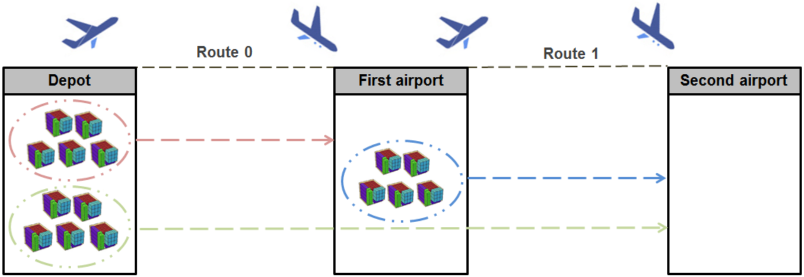
- In the case of transport of goods by air at multiple destinations, the questions we are asking are:

1. What are the associated costs ? $\rightarrow$ ECOnomic & ECOlogical model
2. What are the key factors we can act on ? $\rightarrow$ Mathematical model
3. How to optimize the decision? $\rightarrow$ Optimization method

Outline

Description of the Problem

- A cargo aircraft has to deliver goods at two consecutive airports[1]



- Find the optimal location for all ULDs into the cargo aircraft

  ⇒ To minimize the fuel consumption during the entire trip
  ⇒ To minimize the time required to unload and load ULDs at the intermediate destination

[1] Generalization could be easily done to more than two destinations

Summary of the model

Minimize (deviation most aft CG) and # ULDs to unload
(∀ route!)
subject to:

- Each ULD is loaded
- Each ULD fits in a position
- A position accepts only one ULD
- Some positions are overlapping: not simultaneously used
- Longitudinal stability: The CG is within certified limits
- Lateral balance
- Maximum weight per position
- Combined load limits
- Cumulative load limits
- Regulations for hazardous goods
- Two parts of larger ULDs in adjacent positions

⇒ "Assignment Problem / Combinatorial Problem"
⇒ Integer Linear Problem

Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific (dedicated to one specific aircraft,...)
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)

Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific (dedicated to one specific aircraft,...)
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)

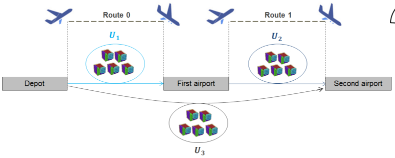
Main references for the basic problem (CG)

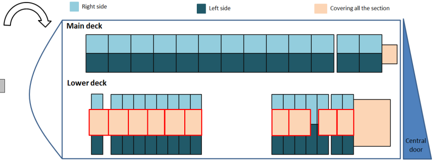
1. Limbourg, S., Schyns, M., and Laporte, G. (2011). Automatic Aircraft Cargo Load Planning. *Journal of the Operational Research Society*
2. Souffriau, W., Demeester, P. and Vanden Berghe, G. and De Causmaecker,P. (2008). The Aircraft Weight and Balance Problem. *Proceedings of ORBEL 22*, Brussels, pp. 44–45.
3. Mongeau, M. and Bès, C. (2003). Optimization of Aircraft Container Loading, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 39, pp. 140–150.

Outline

Main Parameters and Variables

- $\mathbb{K}$ is the **set of routes** $\Rightarrow$ parts of trip separating two successive airports
- $\mathbb{U}$ is the **set of ULDs** $\Rightarrow$ pallets and containers to be transported
- According to their origin and destination: three subsets of ULDs: $\mathbb{U}_1$, $\mathbb{U}_2$, $\mathbb{U}_3$
- $\mathbb{P}$ is the **set of all the positions** $\Rightarrow$ predefined spaces in the aircraft that may contain the ULDs
- There is **only one central door** situated at the extremity of the aircraft





Binary Variables

$$x_{ijk} = \begin{cases} 1 & \textit{if ULD i is in position j during the route k} \\ 0 & \textit{otherwise} \end{cases}$$

Outline

Objective function: Most Aft CG

- In terms of fuel consumption, the optimal location for the CG is the most aft
- We want to achieve the most aft CG under stability constraints
- We minimize, on the global trip, the absolute deviation between the most aft CG and the obtained CG

In mathematical terms, it gives:

$$\textbf{Min} \sum_{\forall k \in \mathbb{K}} \epsilon_k$$

Subject to:

$$\left.\begin{aligned} c_k - o_k - \epsilon_k &\leq 0 \\ c_k - o_k + \epsilon_k &\geq 0 \end{aligned}\right\} \forall k \in \mathbb{K}$$

where :

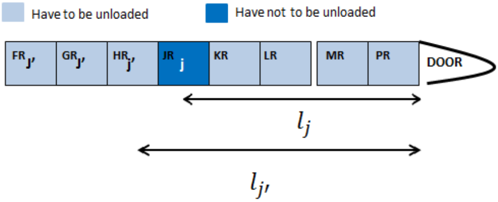
-c_k is the CG obtained after assignment of ULDs in the aircraft during the route k

-o_k is the optimal CG, i.e. most aft CG on the route k

Objective function: minimize # ULDs to Unload

- Loading time is function of the # of ULDs to be unloaded
- At the first airport, ULDs $\in \mathbb{U}_3$ have not to be unloaded
- If those ULDs can remain in the aircraft: time savings!
- So, what we want is:

  1. Locate the ULDs $\in \mathbb{U}_3$ that must be unloaded unnecessarily because they prevent the unloading of ULDs $\in \mathbb{U}_1$
  2. Minimize the # of ULDs in such location



Objective Function: Minimize # ULDs to Unload

In mathematical terms, we use the following expression to count the # of embarrassing positions:

$$\textbf{Min} \sum_{\forall j \in \mathbb{P}} n_j$$

Subject to:

$$\sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in \mathbb{P}_{ds} | l_{j'} > l_j} x_{i'j'1} - n_j N_j - (1 - x_{ij1}) N_j \leq 0 \quad \forall j \in \mathbb{P}_{ds}, \forall d \in \mathbb{D}, \forall s \in \mathbb{S}, \forall i \in \mathbb{U}_3$$

- N_j are constant numbers that give the number of positions behind each position j
- $0 \leq \sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in \mathbb{P}_{ds} | l_{j'} > l_j} x_{i'j'1} \geq N_j$
- n_j **are binary variables equal to 1 if the ULD in position j must be unloaded unnecessarily**

Objective Function: Minimize # ULDs to Unload

- Not sufficient to **min** (# ULDs from $\mathbb{U}_3$ with an ULD from $\mathbb{U}_1$ behind it)
- We have to be sure that:
  1. Each ULD $\in \mathbb{U}_3$ not unloaded keeps the same position for the second route
  2. Each ULD $\in \mathbb{U}_2$ (loaded at first airport) doesn't conduct to the unloading of ULD $\in \mathbb{U}_3$

It leads to the two following sets of constraints:

$$\begin{cases} x_{ij0} - n_j + y & \leq 1 \quad \forall j \in \mathbb{P}, \forall i \in \mathbb{U}_3 \\ x_{ij0} - x_{ij1} - y & \leq 0 \quad \forall j \in \mathbb{P}, \forall i \in \mathbb{U}_3 \end{cases}$$

And:

$$x_{ij0} - n_j + x_{i'j'1} \leq 1 \quad \forall j \in \mathbb{P}, \forall i \in \mathbb{U}_3, \forall j' \in \mathbb{P} \mid l_{j'} > l_j, \forall i' \in \mathbb{U}_2$$

Double Objective Function

Minimizing fuel consumption and # ULDs unloaded

$$\textbf{Min} \quad \underbrace{\alpha(\epsilon_1 + \epsilon_2)}_{\textit{Fuel consumption}} + \underbrace{\beta \sum_{j \in \mathbb{P}} n_j}_{\textit{Loading time}}$$

where :
-α is the additional cost (fuel + emissions) for a deviation of one inch from the most aft center of gravity
-β is the cost associated with the time required to unload one additional ULD (in terms of wages, fees to the airport for the usage of the runway...)

Outline

Summary of the model

$$\min \alpha \sum_{k \in \mathbb{K}} \epsilon_k + \beta \sum_{j \in \mathbb{P}} n_j$$

Subject to:

OF's

$$c_k - o_k - \epsilon_k \leq 0 \qquad \forall k \in \mathbb{K}$$
$$c_k - o_k + \epsilon_k \geq 0 \qquad \forall k \in \mathbb{K}$$
$$\sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in \mathbb{P}_{ds} | l_{j'} > l_j} x_{i'j'1} - n_j N_j - (1 - x_{ij1}) N_j \leq 0 \qquad \forall j \in \mathbb{P}_{ds}, \forall d \in \mathbb{D}, \forall s \in \mathbb{S}, \forall i \in \mathbb{U}_3$$

Lateral & longitudinal stability

$$min_k \leq c_k \leq max_k \qquad \forall k \in \mathbb{K}$$
$$-\bar{D} \leq \sum_{i(\mathbb{U}_1 \cup \mathbb{U}_3)} w_i \left(\sum_{j \in \mathbb{P}_R} x_{ij0} - \sum_{j \in \mathbb{P}_L} x_{ij0}\right) \leq \bar{D}$$
$$-\bar{D} \leq \sum_{i(\mathbb{U}_2 \cup \mathbb{U}_3)} w_i \left(\sum_{j \in \mathbb{P}_R} x_{ij1} - \sum_{j \in \mathbb{P}_L} x_{ij1}\right) \leq \bar{D}$$

Respect of routes

$$x_{ij0} = 0 \qquad \forall i \notin (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$x_{ij1} = 0 \qquad \forall i \notin (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$

Full load

$$\sum_{j \in \mathbb{P}} x_{ij0} = 1 \qquad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3)$$
$$\sum_{j \in \mathbb{P}} x_{ij1} = 1 \qquad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3)$$

Allowable positions

$$x_{ijk} = 0 \qquad \forall i \in \mathbb{U}, \forall j \in \mathbb{P},\ \forall k \in \mathbb{R}\ | U_i \ \textit{does not fit in}\ P_j$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} x_{ij0} \leq 1 \qquad \forall j \in \mathbb{P}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} x_{ij1} \leq 1 \qquad \forall j \in \mathbb{P}$$
$$x_{ij0} + x_{i'j'1} \leq 1 \qquad \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{O}_j$$
$$x_{ij1} + x_{i'j'2} \leq 1 \qquad \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{O}_j$$

Weight restrictions

$$w_i \times x_{ij0} \leq \bar{W}_j \qquad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$w_i \times x_{ij1} \leq \bar{W}_j \qquad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap O_a^d \neq \emptyset} x_{ij0} o_{ija}^d \leq \bar{O}_a^d \qquad \forall d \in \mathbb{D}^*, \forall a \in \mathbb{O}^d$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap O_a^d \neq \emptyset} x_{ij1} o_{ija}^d \leq \bar{O}_a^d \qquad \forall d \in \mathbb{D}^*, \forall a \in \mathbb{O}^d$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^a F_c \neq \emptyset} \sum_{l=1}^a x_{ij0} f_{ijl} \leq \bar{F}_a \qquad \forall a \in \mathbb{F}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^a F_c \neq \emptyset} \sum_{l=1}^a x_{ij1} f_{ijl} \leq \bar{F}_a \qquad \forall a \in \mathbb{F}$$
$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^a T_c \neq \emptyset} \sum_{l=1}^a x_{ij0} t_{ijl} \leq \bar{T}_a \qquad \forall a \in \mathbb{T}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^a T_c \neq \emptyset} \sum_{l=1}^a x_{ij1} t_{ijl} \leq \bar{T}_a \qquad \forall a \in \mathbb{T}$$

Larger ULDS

$$x_{ij1} - \sum_{j' \in \mathbb{P}_j^F} x_{f_i j'1} = 0 \qquad \forall i \in (\mathbb{U}^L \cap (\mathbb{U}_1 \cup \mathbb{U}_3)),\ \forall j \in \mathbb{P}$$
$$x_{ij2} - \sum_{j' \in \mathbb{P}_j^F} x_{f_i j'2} = 0 \qquad \forall i \in (\mathbb{U}^L \cap (\mathbb{U}_2 \cup \mathbb{U}_3)),\ \forall j \in \mathbb{P}$$

Hazardous goods

$$x_{ij1} + x_{i'j'1} \leq 1 \qquad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \textit{and}\ \forall j, j' \in \mathbb{P}$$
$$x_{ij2} + x_{i'j'2} \leq 1 \qquad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \textit{and}\ \forall j, j' \in \mathbb{P}$$

Constraints

Constraints linked to OF

$$
\begin{array}{ll}
c_k - o_k - \epsilon_k \leq 0 & \forall k \in \mathbb{K} \\
c_k - o_k + \epsilon_k \geq 0 & \forall k \in \mathbb{K} \\
\sum_{\forall i' \in \mathbb{U}_1} \sum_{\forall j' \in \mathbb{P}_{ds} | l_{j'} > l_j} x_{i'j'1} - n_j N_j - (1 - x_{ij1}) N_j \leq 0 & \forall j \in \mathbb{P}_{ds}, \forall d \in \mathbb{D}, \forall s \in \mathbb{S}, \\
& \forall i \in \mathbb{U}_3
\end{array}
$$

Constraints for stability

$$
\begin{array}{ll}
min_k \leq c_k \leq max_k & \forall k \in \mathbb{K} \\
-\bar{D} \leq \sum_{i(\mathbb{U}_1 \cup \mathbb{U}_3)} w_i (\sum_{j \in \mathbb{P}_R} x_{ij0} - \sum_{j \in \mathbb{P}_L} x_{ij0}) \leq \bar{D} & \\
-\bar{D} \leq \sum_{i(\mathbb{U}_2 \cup \mathbb{U}_3)} w_i (\sum_{j \in \mathbb{P}_R} x_{ij1} - \sum_{j \in \mathbb{P}_L} x_{ij1}) \leq \bar{D} &
\end{array}
$$

Constraints for routes

$$
\begin{array}{ll}
x_{ij0} = 0 & \forall i \notin (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P} \\
x_{ij1} = 0 & \forall i \notin (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}
\end{array}
$$

Constraints

Constraints for full load

$$\sum_{j\in\mathbb{P}} x_{ij0} = 1 \quad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3)$$
$$\sum_{j\in\mathbb{P}} x_{ij1} = 1 \quad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3)$$

Constraints for allowable positions

$$x_{ijk} = 0 \qquad \forall i \in \mathbb{U}, \forall j \in \mathbb{P},\ \forall k \in \mathbb{R}\ |U_i \ \ \textit{does not fit in} \ \ P_j$$

$$\sum_{i\in(\mathbb{U}_1\cup\mathbb{U}_3)} x_{ij0} \leq 1 \quad \forall j \in \mathbb{P}$$
$$\sum_{i\in(\mathbb{U}_2\cup\mathbb{U}_3)} x_{ij1} \leq 1 \quad \forall j \in \mathbb{P}$$

$$x_{ij0} + x_{i'j'1} \leq 1 \qquad \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{O}_j$$
$$x_{ij1} + x_{i'j'2} \leq 1 \qquad \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P},\ \forall j' \in \mathbb{O}_j$$

Constraints

Constraints for load limits

$$w_i \times x_{ij0} \leq \bar{W}_j \qquad \forall i \in (\mathbb{U}_1 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$
$$w_i \times x_{ij1} \leq \bar{W}_j \qquad \forall i \in (\mathbb{U}_2 \cup \mathbb{U}_3),\ \forall j \in \mathbb{P}$$

$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap O_a^d \neq \emptyset} x_{ij0} o_{ija}^d \leq \bar{O}_a^d \qquad \forall d \in \mathbb{D}^*, \forall a \in \mathbb{O}^d$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap O_a^d \neq \emptyset} x_{ij1} o_{ija}^d \leq \bar{O}_a^d \qquad \forall d \in \mathbb{D}^*, \forall a \in \mathbb{O}^d$$

$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} F_c \neq \emptyset} \sum_{l=1}^{a} x_{ij0} f_{ijl} \leq \bar{F}_a \qquad \forall a \in \mathbb{F}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} F_c \neq \emptyset} \sum_{l=1}^{a} x_{ij1} f_{ijl} \leq \bar{F}_a \qquad \forall a \in \mathbb{F}$$

$$\sum_{i \in (\mathbb{U}_1 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} T_c \neq \emptyset} \sum_{l=1}^{a} x_{ij0} t_{ijl} \leq \bar{T}_a \qquad \forall a \in \mathbb{T}$$
$$\sum_{i \in (\mathbb{U}_2 \cup \mathbb{U}_3)} \sum_{j \in \mathbb{P} | P_j \cap \bigcup_{c=1}^{a} T_c \neq \emptyset} \sum_{l=1}^{a} x_{ij1} t_{ijl} \leq \bar{T}_a \qquad \forall a \in \mathbb{T}$$

Constraints

Constraints for dangerous goods and larger ULDs

$$x_{ij1} + x_{i'j'1} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (\mathbb{U}_1 \cup \mathbb{U}_3), \text{ and } \forall j, j' \in \mathbb{P}$$
$$x_{ij2} + x_{i'j'2} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (\mathbb{U}_2 \cup \mathbb{U}_3), \text{ and } \forall j, j' \in \mathbb{P}$$

$$x_{ij1} - \sum_{j' \in \mathbb{P}_j^F} x_{f_i j' 1} = 0 \quad \forall i \in (\mathbb{U}^L \cap (\mathbb{U}_1 \cup \mathbb{U}_3)), \ \forall j \in \mathbb{P}$$
$$x_{ij2} - \sum_{j' \in \mathbb{P}_j^F} x_{f_i j' 2} = 0 \quad \forall i \in (\mathbb{U}^L \cap (\mathbb{U}_2 \cup \mathbb{U}_3)), \ \forall j \in \mathbb{P}$$

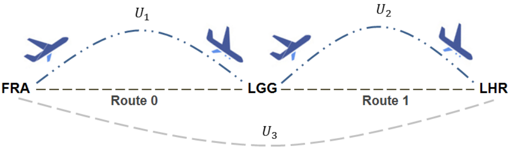
Outline

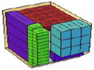
Model tested on a set of real data

- Mathematical model tested on a realistic case.
- Set of real-world data provided by an industrial partner.
- Objective: find a feasible and optimal position for each ULD within a minimal amount of time.
- Optimal solution = (CG to the aft) & (No ULDs unnecessarily unloaded).
- Model implemented in Java using IBM ILOG CPLEX: classical branch-and-cut CPLEX Solver library.

Situation

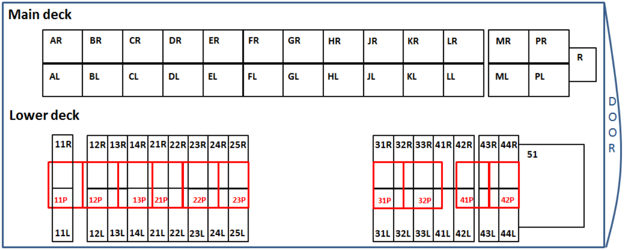
2 successive destinations, and so 2 routes





Set of U ULDs

Boeing 777
60 "normal" positions
+ 10 overlying ones

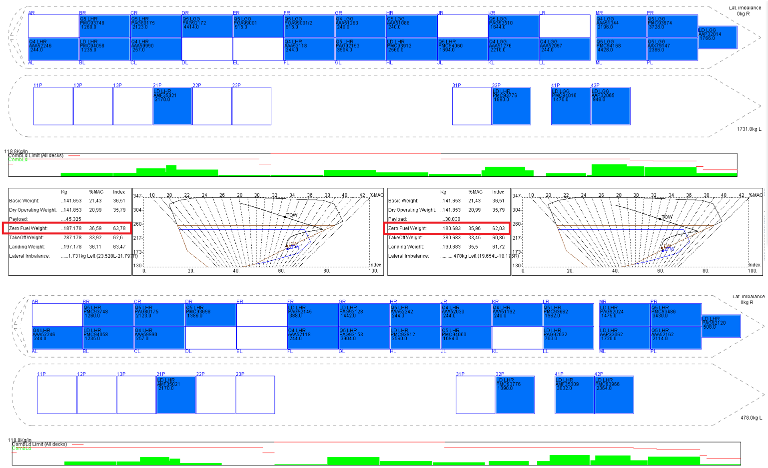


Results (II)

	Test 1	**Test 2**	**Test 3**
# ULDs $\mathbb{U}_1$	5	8	15
# ULDs $\mathbb{U}_2$	5	6	15
# ULDs $\mathbb{U}_3$	2	7	11
Total # ULDs	12	21	41

Results	Test 1		Test 2		Test 3	
	route 0	route 1	route 0	route 1	route 0	route 1
# ULDs	7	7	15	13	26	26
ZFW	152 441	150 521	170 962	162 146	187 178	180 683
Most aft CG	54.43	53.91	59.41	57.04	63.78	62.03
Obtained CG	54.43	53.91	59.41	57.04	63.78	62.03
Epsilon	0.0002	0.0012	0	0	0	0
$\sum n_j$	0	0	0	0	0	0
Time	**9 sec**		**25 min**		**1h 57 min**	

Graphical Representation of the Results (Test 3)



Outline

Conclusion and outlooks

To do list

- √ Mathematical formulation of the model
- ☐ Additional tests
- ☐ Pursue ongoing work on economic and ecological impacts (α and β)
- ☐ The load doesn't seem compressed naturally : include an inertia component in the model ?
- ☐ Introduction of multiple doors
- ☐ Development of Heuristics ?
- ☐ Extension of the model to other modes of transport: ships, trains, ...

Contact me

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Thank you for your attention !