Multiperiod vehicle loading with stochastic release dates

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1 Context

Production scheduling and transportation planning are well-known processes in operations management. Although these tasks are consecutive in the supply chain, few optimization models simultaneously tackle the associated issues (see, e.g., Chen [2]). A most common situation, in practice, is actually that transportation management is disconnected from production planning: when production items or batches have been completely processed by the manufacturing plant, they become available for shipping, and they are consequently handled by the transportation managers. From a global managerial perspective, and with a view towards coordination of the product flows and customer satisfaction, this is not an ideal process. It is by far preferable, indeed, to set up an integrated production-transportation plan taking into account, among other constraints, the capacity of the plants and the customer due-dates.

Even when such a plan exists, however, many elements can concur to create significant differences between the provisions of the tactical plan and the actual situation faced by transportation managers on a day-to-day basis. Production delays are frequent in most industrial environments, and customer orders may not coincide with the forecasts used to establish the plans.

As a consequence, operational shipping decisions often rely solely on available (deterministic) data about items in physical inventory. The main objective of this paper is to examine whether and how transportation decisions can be improved when information about future releases of items is taken into account. We consider both expected cost and robustness to be important criteria when evaluating the quality of the transportation decisions. The problem formulation is based on a real-world application arising in the steel industry (see Cornillier et al. [3,4] for a related model).

2 Formulation

We consider the following multiperiod stochastic optimization formulation of a vehicle loading problem. A set of items must be delivered by trucks to M customers over a discrete (rolling) horizon consisting of T decision periods (typically, days). The objective is to minimize the total expected logistical costs.

Data relative to the first (current) period $t = 1$ is deterministic. The subsequent periods contain forecasts about the availability of items to be released from production. We represent this information by probabilistic distributions of release dates: $p_{it} \in [0, 1]$ $(i = 1, \ldots, N; t = 1, \ldots, T)$ denotes the probability that item i is released at period t and hence, is available for shipment in periods $t, t + 1, \ldots$ We assume that $\sum_{t=1}^{T} p_{it} \leq 1$ and $p_{i1} \in \{0, 1\}$ for all i (information relative to the first period is fully revealed).

Beside its possible release dates, each item i has several deterministic attributes:
- its weight w_i and a time window $[E_i, L_i] \subseteq \{1, \ldots, T\}$ for delivery;
- the warehouse location d_i where the item must be picked up;
- the customer location c_i where the item must be delivered;
- the travel time (expressed as an integer number of periods) from d_i to c_i.

There is an unlimited number of trucks. The maximum total weight that can be loaded on any truck is equal to C.

These, and a number of auxiliary parameters, allow us to compute the cost of a truck picking up a given subset of items at their respective warehouses and transporting them to their respective destinations. In our applications, all warehouses are located around the same plant and customers are close to each other, and we are primarily concerned with long-haul transportation; therefore, the routing aspects are very easy to handle (as each truck visits a handful of warehouses and customers). The total cost generated by a truck only depends on:
- the composition of the load;
- the total distance between the warehouses and the customers visited by the truck;
- the transportation cost per ton and per kilometer;
- an inventory cost, or opportunity cost, depending on the number of periods that each item spends in the warehouse after it has been released from production;
- penalties linked to early or late deliveries of items to customers.

This broad definition allows us to integrate various specific features of the cost function.

The decisions to be made represent the truckloads to be composed and shipped in period $t = 1$. As a general rule, grouping items on a same truck is beneficial, and a good shipping decision is based on the following insights: It may be appropriate to ship an item early (with respect to its due-date), or conversely, to wait before shipping it (even though it has been released or its due-date will be missed) if this results in an reduction of the expected number of trucks required and, more generally, in smaller expected total logistical costs.

Since the horizon is rolling, we actually want to solve an (infinite) sequence of optimization problems P_ℓ, one for each horizon $\{\ell+1,\ldots,\ell+T\}$, where $\ell = 1, 2, \ldots$ The objective is to minimize the expected cost of the decision policy by time period, or by unit amount shipped.

3 Algorithms

Consider a set of items to be shipped, say I, and their release dates, say $r(I) = \{r_i \,|\, i \in I\}$. Each pair $(I, r(I))$ is viewed as a possible *scenario* (Birge and Louveaux [1]).

Given a scenario $(I, r(I))$, optimizing the transportation cost for the items in I can be expressed as a large set covering (or bin packing) problem, say $SC(I, r(I))$, where each column corresponds to a feasible truckload. This reduction holds even when the items in I are released in several distinct periods. Based on these simple observations, several strategies have been developed for the stochastic version of the problem.

First period optimization: We solve to optimality the loading problem $SC(I, r(I))$ associated with the items that are available in period 1: $I = \{i \,|\, p_{i1} = 1\}$. Then, these items are removed from further consideration and the process is repeated for period 2, period 3, and so on (after observing the random variables corresponding to each period).

Expected release dates: We let $I = N$. For each item i, the release date is assumed to be fixed and equal to $r_i = \sum_{t \in T} t p_{it}$ (rounded to the nearest period). This defines a deterministic scenario. We solve the loading problem $SC(I, r(I))$, implement all shipping decisions which only involve items available in period 1 and iterate the process.

Most likely release dates: Similar to the previous strategy, but here r_i is the modal value of the distribution p_{it}: $r_i = \text{argmax}\{p_{it} \,|\, 1 \leq t \leq T\}$.

Earliest release dates: Similar to the previous strategies, but here r_i is the earliest possible release date of item i: $r_i = \min\{t \in T \,|\, p_{it} > 0\}$.

The next approach is *consensus-based* (Van Hentenryck and Bent [5]):

Consensus strategy: A sample of scenarios $(I, r(I))$ is generated, and the corresponding set covering problems $SC(I, r(I))$ are solved. Then, items that have been "frequently" selected to be shipped in period 1 are retained to constitute a new scenario, and an optimal loading plan is computed for these items.

Finally, a more complex, look-ahead strategy has also been implemented:

Restricted evaluation strategy: A sample of scenarios is considered as before.

After solving each set covering problem $SC(I, r(I))$, the quality of the optimal decision $x(I)$ obtained for period 1 is cross-evaluated on the remaining scenarios. The decision $x(I)$ that leads to the smallest overall cost is implemented in period 1.

4 Computational results and conclusions

The above policies have been extensively tested on randomly-generated instances which share the main characteristics of the industrial application. As we are dealing with stochastic optimization problems, particular attention has been paid to the estimation of the objective function (expected cost over a rolling horizon), to the statistical significance of the comparisons, and to the robustness of the results.

Our main conclusions are as follows:

1. Certain policies are clearly dominated by others. In particular, the **First period optimization** approach, which is frequently used in practice, shows poor performance.
2. Policies based on **Earliest release dates** perform surprisingly well and are robust under a variety of assumptions regarding the probability distributions of release dates.
3. The expected cost incurred by the best policies is closer to the cost of the optimal plan (computed under conditions of perfect information) than to the cost of the worst policies.

These conclusions establish the benefit to be drawn from the stochastic optimization approach.

References

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