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Chiara Canta* and Pierre Pestieau

# Long-Term Care, Insurance, and Family Norms

**Abstract:** Long-term care (LTC) is mainly provided by the family and subsidiarily by the market and the government. To understand the role of these three institutions, it is important to understand the motives and the working of family solidarity. In this paper, we focus on the case when LTC is provided by children to their dependent parents out of some norm that has been inculcated to them during their childhood by some exemplary behavior of their parents towards their own parents. In the first part, we look at the interaction between the family and the market in providing for LTC. The key parameters are the probability of dependence, the probability of having a norm-abiding child and the loading factor. In the second part, we introduce the government which has a double mission: correct for a prevailing externality and redistribute resources across heterogeneous households.

**Keywords:** norm transmission, long-term care, health insurance, optimal taxation

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## 1 Introduction

Our societies face, at present, a serious problem with long-term care (LTC). Defined as a mix of medical and support services for those with disabilities and chronic-care needs, LTC can be delivered at home, in an adult day care center, or through another type of community program, in an assisted living facility, or in a nursing home.[1] The source of this problem is twofold, demographic, and societal. On the one hand, populations are aging, and the number of people aged 80+ is rising. Their relative importance in the European Union

1 The present paper will not consider LTC for younger individuals, rather it will focus on LTC of the elderly.

will go from 4.41% in 2008 to 12.13% in 2060. The highest figures concern Italy: 5.50 and 14.91.[2] The issue of dependency arises precisely in that age bracket. On the other hand, with the drastic change in family values, the increasing number of childless households, and the mobility of children, the number of dependent elderly who cannot count on the assistance of a family member is increasing. Those two parallel evolutions explain why there is a mounting demand on the government and the market to provide alternatives to the family.

LTC is the nexus of intense and complex interactions among three institutions: the state, the market, and – naturally – the family.[3] Important empirical work, particularly in the US, has been devoted to the crowding-out effect that social assistance can have on either the market or the family. The answers to these questions depend closely on the nature of family solidarity. Is it based on pure altruism from children to dependent parents?[4] Does it rely on some sort of market or strategic exchange between the parents and the children, as presented by Kotlikoff and Spivak (1981) or Bernheim, Shleifer, and Summers (1985)?[5] Or does it depend on some sort of social norm that comes from the prevailing culture or from parental education? Those three motives for family solidarity are likely to coexist. They have to be well understood to grasp the role of the market in LTC and to design optimal policies.

We consider a framework where children can assist parents, but this entails a cost. This cost is incurred ex ante, before the degree of disability of the parents has been revealed. One can think of irreversible occupational or residential choices. For instance, children might buy a house close to their parents, or pursue certain studies or career paths allowing them to assist their parents if needed. All these investment decisions are taken before the degree of disability can be known. If parents turn out to be autonomous in their old age, the investment in family help made by the children is unproductive; it is treated like a sunk cost. In this respect, the paper is consistent with the findings of Konrad et al. (2002), who show that the some children locate close to their parents in order to provide assistance if dependence occurs.

In the present paper, we focus on the idea that children's assistance to dependent parents is motivated by a family norm that is inculcated to them by parents providing an explicit example of giving behavior during their offspring's

2 Eurostat, EUROPOP2008 convergence scenario of the 27 member states.

Q1 3 See Brown and Finkelstein (2004a, 2004b), Brown, Coe, and Finkelstein (2006), Finkelstein and McGarry (2003, 2004).

4 See Pestieau and Sato (2006, 2008) and Jousten et al. (2005).

5 There exist a number of papers studying exchanges within the family. See, e.g., Stern and Engers (2002). For surveys, see Norton (2000) and Grabowsky, Norton Houtven and Van (2012).

childhood. This behavior is modeled by Stark (1995) and Cox and Stark (1996, 2005) under the concept of demonstration effect. Accordingly, parents make transfers to their own parents when children are present to observe such transfers. This in return conditions the children's own behavior when their parents age. The conjecture that the parents' behavior is aimed at inculcating desirable behavior in their children generates testable hypotheses about transfers that have been investigated via household survey microdata. For instance, Cox and Stark (1996) find that elderly parents receive more visits and phone calls from their adult children if the latter have children themselves.[6] We use this idea for the issue of LTC, but with an important difference mentioned above: since the need for LTC is uncertain, it occurs only in case of loss of autonomy. It is a sunk cost if the individual does not need LTC in his old days.

As we will discuss presently, an individual has three ways of providing for LTC needs. First, he can hope to get help from his child. The amount of this aid will not only depend on the demonstration effect on the one hand, but also on the chance of having a traditional child. In other words, investing in demonstration is risky in two ways: it is only operative if the individual becomes dependent and if his child happens to be traditional. The second way is through the private insurance market. The plus of insurance is that it is targeted to the state of dependence; its minuses are the prevalence of loading costs and the fact that it does not provide the same quality of LTC service as the family. The third way is traditional saving, that is, saving for retirement. If the loading costs are prohibitive, the individual can choose to self-insure instead of buying LTC insurance.

In reality, informal LTC services provided by the family are often of a different nature than formal services provided by either the state or the market. In general, familial services are most effective in the early stages of dependency and state- or market-based ones in the late and more severe stages of dependency. Here, we make the assumption that there is only one degree of dependency, and leave the analysis of many dependency levels to further research.

The paper is both positive and normative. First, we want to understand the interplay between the market and the family, and we characterize the steady-state family norm. The level of family solidarity depends positively on the probability of dependency, on the probability of having a traditional child (that is, a child who adheres to the family norm), and on the level of

**6** Pezzin, Pollak, Schone (2009) study couples where one of the spouses is disabled. The nondisabled spouses is more likely to provide care to the disabled one if the couple has children. This evidence is in line with a demonstration effect at work.

the insurance loading cost. Thus, the model predicts that a more traditional society with imperfect insurance markets will have higher degrees of family solidarity.

We then characterize the optimal LTC policy when the family norm is endogenous. We show that the market outcome is not optimal even in the case of identical productivities, because in their choice of investment, individuals only partially internalize the benefit of this investment for their elderly parents. This creates an externality that can be corrected by a Pigouvian tax on labor. In addition to this, if the private insurance is not actuarially fair, individuals might overinvest in family help. In such a case, public LTC insurance can be a useful instrument for the social planner. We show that optimal public insurance has to make a trade-off between the insurance motive and the correction for the family norm externality. For instance, if family help is discouraged by the introduction of public LTC insurance, the social planner might provide less than full insurance in order to enhance the family norm. Introducing heterogeneous individuals with uneven incomes brings another role for the government: to not only correct for the above externality but also to transfer resources from high- to low-income households. We characterize the optimal tax schedule where the social planner can use a linear income tax, and a flat-benefit public LTC insurance.

The rest of the paper is organized as follows. In Section 2, we describe the model of family norm and the equilibrium allocation. In Section 3, we analyze the optimal allocation when all individuals in the population have the same productivity. We also discuss the role of a linear income tax and of public health insurance. In Section 4, we consider the case when individuals differ in productivity. In Section 5, we conclude.

## 2 A model of family norm

We consider an overlapping generations model in which people live through two periods. The first period corresponds to youth: each individual has one child, allocates time between family help and work, and devotes his earnings to consumption, savings, and long-term care insurance. The second period corresponds to old age: the individual consumes his savings. Furthermore, with probability $\pi$, the individual is dependent. In this case, besides the proceeds of savings, he receives family help and LTC insurance compensation.

To analyze the transmission of the family norm, we assume that parents can shape the preferences of their children through demonstration. This modeling strategy was first proposed by Stark (1995), who also found empirical evidence of the existence of such a demonstration effect (Cox and Stark 1996, 2005).

An individual active in time $t$ belongs to generation $t$. At the beginning of period $t$, before the dependency status of the parent has realized, the individual sets the family norm $\gamma_t \in [0, 1]$. This variable can be interpreted as an irreversible investment in the family that will be operative only if the parent turns out to be disabled. For instance, children might choose jobs and sectors that do not require too much traveling, or to move far away. They might also choose an education leading to careers which are compatible with family help.[7] All these decisions limit the career prospects of the children. Under this interpretation, $\gamma_t$ is a parameter reducing individual productivity and wage, $w$. With probability $\pi$, the parent is dependent and the investment in $\gamma_t$ is productive. With probability $(1 - \pi)$, this investment does not increase the utility of the parent. However, it still works as a demonstration device for children.

With probability $\pi$, a parent is dependent and receives from his child a transfer $\mu(\gamma_{t+1})$, where $\mu(.)$ is a strictly increasing and strictly concave function representing how valuable is children's help for dependent parents. This translates the idea that the nature of the transfer is not just monetary.

Let us define $\rho$ as the exogenous probability that a child conforms to the behavior of his parent by adopting the same rate of intrafamily transfer, $\gamma_t$. We will call this child traditional. With probability $(1 - \rho)$ the child is modern, and is not influenced by tradition. He chooses the investment that maximizes his own expected utility. For this type, the quantity $\gamma_{t+1}$ is set optimally and does not depend on $\gamma_t$.[8] Importantly, the type of the parent does not affect the probability that his child will be traditional. The utility function of an individual depends on whether the child turns out to be traditional or not. To simplify, we will assume that the productivity of each individual is equal to $w$. We limit the analysis to a small, open economy where the productivity of labor and the interest rate $r$ are assumed to be constant.

Summarizing, the timing is as follows:

1. At the beginning of period $t$, each individual has one child, and $\gamma_t$ is set depending upon the type of the individual.

7 We adopt an "asexual" setting in which each young adult makes individual choices. We thus abstract from the decision process within the household. In reality, it is clear that the choice of $\gamma$ is made at the level of the household.

8 An interesting extension would be to endogenize $\rho$, for example, by making it dependent on the behavior of the parents.

2. The individual inelastically supplies one unit of labor for a wage $(1-\gamma_t)w$.
3. The disability of the parent is revealed. In case of disability, the parent receives $\mu(\gamma_t)$. The individual's income is allocated between current consumption $c_t$, savings $s_t$, and a premium $P(I_t)$ for long-term care (LTC) insurance $I_t$.
4. In period $t+1$, the individual is old. He consumes the gross return of his savings, $(1+r)s_t$, where $r \geq 0$ is the interest rate. If disabled, he receives from his child $\mu(\gamma_{t+1})$ and an insurance benefit equal to $I_t$.

The insurance premium is

$$P(I_t) = \frac{\lambda \pi I_t}{(1+r)},$$

where $\lambda \geq 1$ is the insurance company's loading factor.

Individuals in each generation can be of two types, traditionals (denoted by T) and moderns (denoted by M).

Some comments are in order concerning the meaning of $\gamma$. In our model, $\gamma$ is an investment that reduces the productivity of the young individual and in turn will enable him to help his parent in case of dependency. The young adult is not interested in the benefit $\mu(\gamma)$ that his dependent parent will enjoy, but by the example that $\gamma$ may set for his own child. This investment in time made ex ante could be viewed as the opportunity cost of living close to one's parent or choosing an occupation that makes one more available in case the parent becomes dependent.[9] Quite clearly, such an investment is lost if the parent stays healthy and autonomous. As said above, we also assume that the function $\mu(.)$ is strictly increasing and concave, and that $\mu'(0) = \infty$. Compared to other types of aid (public or private), aid from children is viewed as highly valuable, yet with decreasing returns (hence the concavity of $\mu(\gamma)$). However, it is important to underline that the LTC provided by the family has here the same nature of the LTC provided by the market, in the sense that it is substituable to formal care.

Another interpretation of our setup could be that $w\gamma$ would correspond to some insurance premium that would provide an income to the aiding child in case his parent becomes disabled. In that case the premium is $w\gamma$ and the compensation $w\gamma/\pi$ allows the child to provide aid of size $w\gamma/\pi$.

**9** An alternative specification could have been that the individual provides aid of length $\gamma$ just in case of dependency of his parent, with the expectation that in case of his own dependency, he would get $\gamma$. This specification happened to be more complex analytically. Furthermore, such a modeling strategy would not be compatible with the demonstration effect: only children whose grandparents were dependent would be exposed to a family norm.

## 2.1 Traditional individuals' behavior

A traditional young adult adopts the family norm chosen by his own parent, namely $\gamma_t^T = \gamma_{t-1}$. His expected utility function takes the form

$$u(c_t^T) + \beta\left[(1-\pi)u(d_t^T) + \pi\left[\rho H(m_t^T) + (1-\rho)H(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right]\right], \qquad [1]$$

where $c_t^T = (1-\gamma_t^T)w - P(I^T) - s_t^T$, $d_t^T = s_t^T(1+r)$ and $m_t^T = s_t^T(1+r) + I_t^T + \mu(\gamma_t^T)$. The individual maximizes the sum of present utility and future expected utility discounted by the factor $\beta \in (0, 1)$. With probability $(1-\rho)$, the child is modern, and his parent receives a transfer $\gamma_{t+1}^M$ in case of dependency. We assume that $H(x) \leq u(x) \ \ \forall x$, that is to say that, given any consumption level, individuals are always worse off if dependent.

The traditional individual will choose $s_t^T$ and $I_t^T$ in order to maximize [1]. The first-order conditions are

$$\begin{aligned} u'(c_t^T) &= (1+r)\beta(1-\pi)u'(d_t^T) \\ &\quad + \pi(1+r)\beta\left[\rho H'(m_t^T) + (1-\rho)H'(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right], \end{aligned}$$

and

$$u'(c_t^T) = \frac{\beta}{\lambda}(1+r)\left[\rho H'(m_t^T) + (1-\rho)H'(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right].$$

Combining these two equations, we get the following condition:

$$\left(\frac{1}{\lambda} - \pi\right)\left[\rho H'(m_t^T) + (1-\rho)H'(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right] = (1-\pi)u'(d_t^T).$$

The individual fully insures himself if and only if $\lambda = 1$. If $\lambda > 1$, insurance is not full and the marginal utility of the individual when disabled is greater than the marginal utility in the case of autonomy. In particular, if $\lambda \geq 1/\pi$, the individual buys no insurance and relies on self-insurance. In the following, we assume interior solutions by considering a loading factor $\lambda < 1/\pi$.

## 2.2 Modern individuals' behavior

A modern young adult (denoted by *M*) born in *t* chooses $\gamma_t^M$, $I_t^M$, and $s_t^M$ in order to maximize

$$\begin{aligned} &u(c_t^M) + \beta(1-\pi)u(d_t^M) \\ &\quad + \pi\beta\pi\left[\rho H(m_t^M) + (1-\rho)H(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M))\right] \end{aligned}$$

where $c_t^M = w(1-\gamma_t^M) - P(I_t^M) - s_t^M$, $d_t^M = s_t^M(1+r)$ and $m_t^M = s_t^M(1+r) + I_t^M + \mu(\gamma_t^M)$.

Note that $\gamma_t$ depends on the belief that each individual holds about the behavior of future generations, and in particular about $\gamma_{t+1}^M$. In fact, each individual plays an intergenerational game with his offspring. A strategy for any individual (in any generation) is $\gamma_t \in [0,1]$. In this game, the environment is stationary since $w$, $r$, and the function $\mu(.)$ are the same across generations.

The first-order condition with respect to $\gamma_t^M$ is

$$wu'(c_t^M) = \beta\rho\pi\mu'(\gamma_t^M)H'(m_t^M). \qquad [2]$$

The left-hand side of this condition represents the opportunity cost of the family norm. The right-hand side is the marginal benefit deriving from the presence of traditional children who will reproduce the family norm. If the child is modern (with probability $1-\rho$), the family norm chosen at $t$ will have no influence on $\gamma_{t+1}$.

The first-order condition with respect to savings and LTC insurance are respectively[10]

$$\begin{aligned} u'(c_t^M) = {} & (1+r)\beta\left[(1-\pi)u'(d_t^M)\right] \\ & +\pi(1+r)\beta\left[\rho H'(m_t^M) + (1-\rho)H'(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M))\right] \end{aligned} \qquad [3]$$

and

$$u'(c_t^M) = \frac{\beta}{\lambda}(1+r)\left[\rho H'(m_t^M) + (1-\rho)H'(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M))\right], \qquad [4]$$

where the interpretation is the same as for the traditional type.

Using [4], the first order condition with respect to $\gamma_t$, [2], becomes

$$\begin{aligned} & w\frac{\beta}{\lambda}(1+r)\left[\rho H'(m_t^M) + (1-\rho)H'(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M))\right] \\ & = \beta\rho\pi\mu'(\gamma_t^M)H'\left(m_t^M\right). \end{aligned} \qquad [5]$$

This expression implicitly defines the best response function for each individual born in $t$, $\gamma(\gamma_{t+1}^M)$, with $\gamma : [0,1] \to [0,1]$. Note that, due to the stationarity of the problem, the function $\gamma(.)$ is the same for all generations.

In Appendix A, we prove the following proposition:

**Proposition 1** *The intergenerational game admits a unique equilibrium. This equilibrium is stationary: each modern individual in each generation chooses the same family norm $\gamma^M$, where $\gamma^M$ is implicitly defined by*

10 All along, we assume interior solutions, i.e., $\lambda\pi<1$.

$$\mu'(\gamma^M) = w\frac{1+r}{\lambda\pi\rho}. \qquad [6]$$

*Furthermore*, $0 < \gamma^M < 1$.

The equilibrium norm is constant over time, so that modern individuals in different generations will all choose the same level of family norm, $\gamma^M$. Since $\mu(.)$ is strictly concave, $\gamma^M$ decreases if the interest rate or the productivity increase. On the one hand, if the interest rate increases, this makes LTC insurance and savings a better way to transfer consumption across periods compared to the family norm. On the other hand, an increase in productivity corresponds to an increase in the opportunity cost of time devoted to the family.[11]

The family norm increases with the probability of having a traditional child and the probability of being disabled. In fact, when choosing $\gamma^M$, modern individuals consider that this help will be productive with probability $\rho\pi$. However, we know that the help set by the children has an impact on the utility of the parents with probability $\pi$. This discrepancy creates what we call an externality here below. Note also that, if there are no traditional individuals ($\rho = 0$), then the family norm is equal to zero.

The family norm $\gamma^M$ is also increasing with the loading factor. If the loading factor is very high, it becomes more interesting for the individual to substitute the family norm, which acts as an informal insurance, for LTC insurance. However, note that even if $\lambda = 1$, that is if LTC insurance is actuarially fair, the assumption $\mu'(0) = \infty$ implies that the family norm is always positive. Intuitively, under our assumptions, the first unit of children's help has very high returns, exceeding that of insurance. To put it differently, for small values of $\gamma$, assistance from children is always more valuable than assistance from strangers.

## 2.3 Steady state Q3

In the steady state, $\gamma_t = \gamma_{t+1} = \gamma^{SS}$ $\forall t$ and for each dynasty.

Assume that the initial norm is $\gamma_0$. In each dynasty, as soon as one individual is modern, $\gamma = \gamma^M$ for all subsequent generations. Traditional individuals will just reproduce $\gamma^M$, while modern ones face exactly the same incentives as

11 This result is in line with Cox and Stark (1996), who find that the number of contacts with elderly parents decreases in the children's income.

their first modern ancestor. Even if the occurrence of a modern individual does not affect the incidence of traditionalism in subsequent generations ($\rho$ being exogenous), all individuals behave like their modern ancestor and set the same level of family norm. After $t$ number of periods, the probability that at least one individual is modern in a given dynasty is $1-\rho^t$. Thus, in period $t$, $(1-\rho^t)$ dynasties set $\gamma_t=\gamma^M$. As $t$ tends to infinity, $(1-\rho^t)$ tends to one, and the economy reaches the steady state, with $\gamma^{SS}=\gamma^M$.

In the following, we suppose that $t$ is large enough so that the proportion of dynasties that do not set the family norm equal to $\gamma^{SS}$ is negligible. We will denote the steady-state family norm in the absence of government intervention $\gamma^{LF}$, with $\gamma^{LF}=\gamma^{SS}=\gamma^M$. Interestingly, $\gamma^{SS}$ could very well be higher than $\gamma_0$. Modern children abandon tradition and might find it optimal to generate a higher family norm.

It is worth discussing two extreme cases. First, all individuals in the society might be modern ($\rho=0$). Then, as discussed above, the family norm at the steady state would be equal to zero. Second, all individuals might be traditional ($\rho=1$). In this case, there would be no dynamics, and $\gamma_0$ would be the steady state value of the family norm.

# 3 Normative analysis: identical productivities

In this section, we concentrate on the design of the optimal LTC policy when the family norm is endogenous. We consider a population where all dynasties have the same productivities, and we characterize the first-best allocation. We then study how the first best could be decentralized through a linear income tax and public LTC insurance. We also analyze the second-best allocation when only a limited set instruments is available. We show that, in such a second-best scenario, the social planner faces a trade-off between insurance provision and family norm enhancement.

## 3.1 First best

In the steady state, whatever the type of children, the consumption in case of dependency is equal to $m$. The first-best allocation is characterized by the solution of the problem of a utilitarian social planner maximizing the utility of the representative generation in the steady state under the economy resource constraint. In doing so, the social planner takes into account the fact that $\gamma$ is only operative with probability $\pi$.[12] The first-best problem is:

$$\max_{c,d,m,\gamma} \; u(c) + \beta[(1-\pi)u(d) + \pi H(m)]$$
$$s.t. \;\; c(1+r) + (1-\pi)d + \pi m \leq w(1-\gamma)(1+r) + \pi\mu(\gamma).$$

The first-order conditions with respect to the consumption levels yield the following equations

$$u'(c^{FB}) = \beta(1+r)u'(d^{FB}) = \beta(1+r)H'(m^{FB}) = (1+r)\psi,$$

where $\psi$ is the multiplier associated with the resource constraint. In the first-best case, there is perfect consumption smoothing across time and states.

The first-order condition with respect to $\gamma$ is

$$w(1+r) = \pi\mu'(\gamma^{FB})$$
$$\Longleftrightarrow \mu'(\gamma^{FB}) = w\frac{(1+r)}{\pi}.$$

Contrasting this expression with [6], we can compare $\gamma^{FB}$ with $\gamma^{LF}$. First note that, in the first-best case, $\gamma$ does not depend on $\rho$. The social planner internalizes the fact that the help of modern children positively affects their parents, so that the social benefit of $\gamma$ equals $\pi\mu(\gamma)$. With laissez-faire, individuals only take into account the benefit of the family investment due to the imitation behavior of traditional children. They thus internalize only a share of the social benefit, $\rho\pi\mu(\gamma)$. It is important to note that this result holds only if $\rho$ is strictly greater than zero and smaller than one, that is to say, if the family norm exhibits some dynamics.

Second, $\gamma^{FB}$ does not depend on $\lambda$, since we assume here that the government can transfer consumption freely across periods and states of the world. Overall, the relationship between $\gamma^{FB}$ and $\gamma^{LF}$ is ambiguous. Because of the positive externality on parents, $\gamma^{LF}$ tends to be too small if insurance is actuarially fair. However, if $\lambda > 1$, LTC insurance becomes less attractive and is substituted for by the family norm. More precisely $\gamma^{FB} \geq \gamma^{LF}$, if and only if $\lambda\rho \leq 1$.

Remember that $\rho$ represents the proportion of individuals engaging in traditional behavior. It can be considered as a proxy for the traditionalism of a society. Consequently, our comparison between first best and equilibrium family

12 Alternatively, one could analyze the case in which the social planner is able to impose a mutualization of family help. In such a case, $\gamma$ is never wasted, since individuals with healthy parents are forced to help the dependent elderly not belonging to their family. This specification would be more relevant for traditional societies with extended family. Our model applies to nuclear families.

norms has an easy interpretation. Societies where the loading factor and/or traditionalism are low will display family norms that are lower than the first best level. On the contrary, traditional societies that do not have access to an efficient insurance market will display an excessive use of family help.

### 3.1.1 Decentralization of the first best

The first best can be decentralized by a linear income tax and a demogrant, if the loading factor is equal to one ($\lambda = 1$). In this case, the only distortion in the laissez-faire allocation comes from underprovision of $\gamma$. A linear tax $\theta$ on individual income and a lump-sum transfer $L$ decentralize the first best. The optimal $\theta$ decentralizing the first-best family norm is a Pigouvian tax. This tax induces the individuals to internalize the impact of the full social benefit of the family norm:

$$\mu'(\gamma^*) = (1-\theta^*)w\frac{(1+r)}{\rho\pi} = w\frac{(1+r)}{\pi} = \mu'(\gamma^{FB})$$
$$\iff \theta^* = 1-\rho.$$

The optimal lump-sum transfer is $L^* = \theta^* w(1-\gamma^{FB})$, so that the disposable income is not affected by the government intervention. As noted above, this result only holds if $0 < \rho < 1$. If all individuals were modern, then agents would have no private benefit from investing in family help. There would be no use for a payroll tax. Conversely, if all individuals were traditional, then the family norm would be equal to $\gamma_0$, and it would be impossible to obtain the optimal level of family norm. Modern individuals are necessary to generate some dynamics of the family norm, and are necessary to make policy effective. Intuitively, any policy is powerless if individuals are unable to abandon a tradition.

If the loading factor is higher than one, such a system does not decentralize the first best. However, the first best can be achieved if flat-benefit social LTC insurance is introduced. In this case, individuals pay a share $\theta$ of their income, receive a lump-sum transfer when young, and receive a transfer $B$ in case of dependency when old. Public LTC insurance is assumed to be actuarially fair, so that the resource constraint of the social planner is

$$L \leq \theta w(1-\gamma) - \frac{\pi B}{(1+r).}$$

Given the tax schedule, the representative individual chooses $\gamma^*(\theta, L, B)$, $s^*(\theta, L, B)$, and $I^*(\theta, L, B)$, such that the individual first order conditions are satisfied:

$$(1-\theta)wu'(c^*) = \beta\rho\pi\mu'(\gamma^*)H'(m^*)$$
$$u'(c^*) = (1+r)\beta[(1-\pi)u'(d^*) + \pi H'(m^*)]$$
$$\lambda u'(c^*) \geq \beta(1+r)H'(m^*),$$

where $c^* = (1-\gamma^*)(1-\theta)w + L - P(I^*) - s$, $d^* = s^*(1+r)$, and $m^* = s^*(1+r) + I^* + \mu(\gamma^*) + B$. Note that the condition on $I^*$ allows for corner solutions, which may take place if public LTC insurance crowds out private. The problem of the social planner is

$$\max_{\theta,L,B} u(c^*(\theta,L,B)) + \beta[(1-\pi)u(d^*(\theta,L,B)) + \pi H(m^*(\theta,L,B))]$$
$$s.t.\ \ L \leq \theta w(1-\gamma) - \frac{\pi B}{(1+r)}.$$

Since the resource constraint is always saturated at the optimum, we can rewrite the problem as

$$\max_{\theta,B} u(c^*(\theta,B)) + \beta[(1-\pi)u(d^*(\theta,B)) + \pi H(m^*(\theta,B))],$$

where $c^*(\theta,B) = (1-\gamma^*)w - \pi B/(1+r) - \lambda\pi I^*/(1+r) - s^*$, while $d^*$ and $m^*$ are defined as above.

In Appendix B, we prove the following result:

**Proposition 2** *Assume that a social LTC benefit is financed by a payroll tax and a lump-sum tax on the young. Then:*

*(i) the optimal social LTC benefit is such that individuals are fully insured:*

$$\beta(1+r)H'(m^*) = u'(c^*).$$

*(ii) the optimal payroll tax is* $\theta^* = 1-\rho$.

Again, the payroll tax corrects for the family norm externality.[13] Through public LTC insurance, it is possible to smooth consumption across states. Full insurance is possible, since the tax schedule includes a lump-sum transfer and public insurance is non-distortionary (see Cremer and Pestieau 2011). Furthermore, public insurance is actuarially fair. Note that, if $\lambda = 1$, public and private LTC insurance are perfect substitutes: as we show above, in this case, public insurance is not necessary to reach the first best.

**13** If instead children made a monetary investment in family help, this would not affect their productivity. In this case, a lump-sum transfer, instead than a payroll tax, would be necessary to implement the optimal level of family norm.

## 3.2 Second best: public LTC insurance

We now look at the optimal public LTC insurance scheme when the number of instruments of the social planner is limited. We assume that the social LTC benefit is funded either through a lump-sum tax on the young or a proportional income tax. Furthermore, we limit the analysis to the case in which the private LTC insurance is not actuarially fair, i.e., $\lambda > 1$.

First, suppose that the only instrument available to the social planner is a transfer $B$ to dependent individuals financed by a lump-sum tax $L$ on the young. Given this tax, the individual chooses $\gamma^*(L,B)$, $s^*(L,B)$ and $I^*(L,B)$.

The problem of the social planner is now:

$$\max_{L,B} u(c^*(L,B)) + \beta[(1-\pi)u(d^*(L,B)) + \pi H(m^*(L,B))]$$
$$s.t.\ \ L \geq \frac{\pi B}{(1+r)}.$$

Since the resource constraint is always binding, the problem can be rewritten as

$$\max_{B} u(c^*(B)) + \beta[(1-\pi)u(d^*(B)) + \pi H(m^*(B))],$$

where $c^* = (1-\gamma^*)w - \pi B/(1+r) - \lambda\pi I^*/(1+r) - s^*$, and $m^* = s^*(1+r) + I^* + \mu(\gamma^*) + B$, and $d^*$ is defined as above. The first-order condition with respect to $B$, after using the envelope theorem, reduces to

$$\begin{aligned}&[-u'(c^*) + \beta(1+r)H'(m^*)]\\&\quad+\beta(1-\rho)\pi H'(m^*)\mu'(\gamma^*)\frac{d\gamma^*}{dB} = 0,\end{aligned} \qquad [7]$$

Condition (7) is easy to interpret. The first terms in brackets represents the insurance concern of the social planner. If this term is equal to zero, insurance is full. The second term represents the family norm externality. Two cases might arise. If public LTC insurance does not crowd out private, then $\gamma$ is given by [6], and $\partial\gamma/\partial B$ is equal to zero. If public LTC insurance crowds out private, it is reasonable to assume that the family norm is decreasing in $B$, since public LTC insurance transfers resources from the young to the dependent.[14] Consequently, the second term of [7] is always smaller or equal to zero. For condition [7] to hold, one needs $[-u'(c^*) + \beta(1+r)H'(m^*)] > 0$, which implies less than full insurance. In fact, suppose that the social planner chooses a social LTC

**14** This is not necessarily true, since public LTC insurance also discourages savings, and savings and family norm are substitutes.

benefit equalizing the marginal utilities in all states of the world. Then, from [2], we get

$$\mu'(\gamma^*) = w\frac{1+r}{\pi\rho} > \mu'(\gamma^{FB}),$$

implying a family norm smaller than in the first best. In this case, public LTC insurance crowds out private, so that $\partial\gamma/\partial B$ is negative. Consequently, to enhance the family norm, the social planner optimally chooses a smaller benefit, such that $[-u'(c^*) + \beta(1+r)H'(m^*)] > 0$, and individuals are less than fully insured. The family norm is given by [2], and is such that $\mu'(\gamma^*) < w(1+r)/\rho\pi$. We have thus established the following result:

**Proposition 3** *Assume that $\lambda > 1$, and a social LTC benefit is financed by a lump-sum tax on the young. Then, the second-best allocation is characterized by underinsurance.*

This result is a consequence of the trade-off between insuring disability and correcting for the family norm externality.[15]

Suppose now that the transfer $B$ to dependent individuals is financed by a payroll tax $\theta$. Given this tax, the individual chooses $\gamma^*(\theta, B)$, $s^*(\theta, B)$ and $I^*(\theta, B)$ in such a way that the individual first-order conditions are satisfied.

The problem of the social planner is now:

$$\max_{\theta, B} u(c^*(\theta, B)) + \beta[(1-\pi)u(d^*(\theta, B)) + \pi H(m^*(\theta, B))]$$

$$s.t.\ \ w\theta(1-\gamma) \geq \frac{\pi B}{(1+r)}.$$

Since the resource constraint is always binding, the problem can be rewritten as

$$\max_{\theta} u(c^*(\theta)) + \beta[(1-\pi)u(d^*(\theta)) + \pi H(m^*(\theta))],$$

where $c^* = (1-\gamma^*)(1-\theta)w - \lambda\pi I^*/(1+r) - s^*$, and $m^* = s^*(1+r) + I^* + \mu(\gamma^*) + w\theta(1-\gamma)(1+r)/\pi$, and $d^*$ is defined as above.

15 If $\lambda = 1$, individuals purchase full LTC insurance on the private market. In this case, $\mu'(\gamma^*) = w(1+r)/\pi\rho$, irrespective of the level of $B$. Thus, public LTC insurance cannot affect the level of the family norm, and the optimal $B$ is equal to zero.

The first-order condition with respect to $\theta$, after using the envelope theorem, reduces to

$$\begin{aligned} & w(1-\gamma^*)[-u'(c^*)+\beta(1+r)H'(m^*)] \\ & +\beta\pi H'(m^*)\left[(1-\rho)\mu'(\gamma^*)-\theta w\frac{(1+r)}{\pi}\right]\frac{d\gamma^*}{d\theta}=0. \end{aligned} \quad [8]$$

The first term represents the insurance concern of the social planner. This term is positive whenever there is less than full insurance. The second term represents the family norm externality. Its sign is ambiguous, since $d\gamma/d\theta$ might be either positive or negative. On the one hand, the tax reduces the opportunity cost of investing in family help. On the other hand, the tax reduces the disposable income of young individuals and reduces the marginal benefit of family help received in old age. Overall, it is not clear which effect dominates. Two cases might arise. If public LTC insurance does not crowd out private, then eq. [4] holds, and the family norm is implicitly given by condition [6]. In this case, $d\gamma/d\theta > 0$. Conversely, if public LTC insurance crowds out private, $\gamma$ is given by [2], and $d\gamma/d\theta \gtreqless 0$.

To get some intuition, suppose that the social planner chooses a tax level $\tilde{\theta}$ such that $[-u'(c^*)+\beta(1+r)H'(m^*)]=0$, so that individuals are fully insured. Given this tax level, the steady state family norm is such that

$$\mu'(\gamma^*)=w(1-\tilde{\theta})\frac{(1+r)}{\rho\pi}.$$

Thus, the left-hand side of first-order condition [8], is equal to

$$\beta\pi H'(m^*)\left[(1-\rho)\mu'(\gamma^*)-\tilde{\theta}w(1+r)/\pi\right](d\gamma^*/d\theta),$$

which is equal to zero if and only if $\tilde{\theta}=(1-\rho)$. This is not in general true.

First consider the case where $\tilde{\theta}>(1-\rho)$. In this case, the family norm is too high with respect to the first best, and $\left[(1-\rho)\mu'(\gamma^*)-\tilde{\theta}\, w(1+r)/\pi\right]$ is negative. If $d\gamma/d\theta>0$, then the sign of the left-hand side of [8] evaluated at $\tilde{\theta}$ is negative, so that the optimal $\theta$ is smaller than $\tilde{\theta}$. Such a tax level induces less than full insurance (i.e., $[-u'(c^*)+\beta(1+r)H'(m^*)]>0$). Consequently, the social planner gives up some insurance in order to reduce the family norm. Conversely, if $d\gamma/d\theta<0$, the left-hand side of [8] evaluated at $\tilde{\theta}$ is positive, so that the optimal $\theta$ is greater than $\tilde{\theta}$. In this case, the social planner insures individuals more than fully, in order to keep down the family norm.

If $\tilde{\theta} < (1-\rho)$, we can use similar reasoning. The equilibrium family norm is too small. If $d\gamma/d\theta > 0$ (resp. $d\gamma/d\theta < 0$), the social planner overinsures (resp. underinsures) individuals in order to enhance the family norm.[16]

Finally, note that $\theta^{SB} > 0$. Suppose this were not true. Public LTC insurance would consist of a transfer from the dependent to the young, and there would be no crowding out of private insurance. Therefore, insurance would be less than full, and the first term of [8] would be positive. Since eq. [6] would hold, $d\gamma/d\theta > 0$ Then the left-hand side of the above expression would be positive, which contradicts $\theta^{SB}<0$ being optimal.

We have thus established the following result.

**Proposition 4** *Assume that $\lambda > 1$ and a social LTC benefit is financed by a payroll tax $\theta$ on the young. Then, two cases are possible*

*(i) if the family norm is decreasing in $\theta$, the second-best allocation is characterized either by overinsurance and a family norm greater than $\gamma^{FB}$, or by underinsurance and a family norm smaller than $\gamma^{FB}$.*

*(ii) if the family norm is increasing in $\theta$, the second-best allocation is characterized either by overinsurance and a family norm smaller than $\gamma^{FB}$, or by underinsurance and a family norm greater than $\gamma^{FB}$.*

All in all, our results underline that the social planner faces a trade-off: it is not possible to provide the optimal amount of insurance because this would lead to suboptimal levels of family help. Since help from family members is highly efficient (as captured by the function $\mu(\gamma)$), public insurance and the family norm are not perfect substitutes, as long as individuals attribute a high value to help from their children. In a second-best setting, the social planner optimally deviates from the full insurance provision in order to provide the right incentive to modern children.

# 4 Normative analysis: heterogeneous productivities

We now turn to the case of dynasties characterized by different productivities. We assume that there is a finite number $n$ of productivity types in the

16 If $\lambda = 1$, individuals purchase full insurance LTC on the private market. Thus, the first term in [8] is equal to zero. A tax $\theta = (1-\rho)$ decentralizes the first-best family norm. Then, a public LTC benefit financed by a payroll tax decentralizes the first best whenever private LTC insurance is actuarially fair.

population. A particular productivity type is denoted by $w_i$, with $i = 1, 2, ..., n$, and $w_1 < w_2 < ... < w_n$. Each productivity level $w_i$ occurs with probability $p_i$ with $\sum_{i=1}^{n} p_i = 1$. Individuals in the same dynasty have the same productivities.

In the steady state, an individual of type $i$ has a family norm $\gamma_i = \gamma(w_i, .)$. Furthermore, eq. [6] implies that $\partial\gamma/\partial w \le 0$. More productive individuals have a lower family norm, since their opportunity cost of devoting time to the family is higher. In Appendix C, we show that savings increase with $w$, while it is not possible to sign the derivative of private insurance purchases with respect to $w$.

## 4.1 First best

The problem of a utilitarian social planner is to maximize the sum of individual utilities in the steady state:

$$\max_{c_i, d_i, m_i, \gamma_i} \; E_w\{u(c) + \beta[(1-\pi)u(d) + \pi H(m)]\}$$
$$s.t. \quad E_w\{c(1+r) + (1-\pi)d + \pi m + \gamma w(1+r) - \pi\mu(\gamma)\} \le (1+r)E_w[w],$$

where $E_w[u(w)] = \sum_{i=1}^{n} p_i u(w_i)$. The first-order conditions with respect to the consumption levels are the following:

$$u'(c_i^{FB}) = \beta(1+r)u'(d_i^{FB}) = \beta(1+r)H'(m_i^{FB}) = (1+r)\psi \quad \forall i, \qquad [9]$$

where $\psi$ is the multiplier associated with the resource constraint. These conditions imply that $c_i^{FB} = c^{FB}, d_i^{FB} = d^{FB}$, and $m_i^{FB} = m^{FB}$. In the first best, the allocation is characterized by perfect consumption smoothing across productivities, periods, and health states.

The first-order condition with respect to the family norm can be written as

$$w_i u'(c^{FB}) = \beta\pi\mu'(\gamma_i^{FB})H'(m^{FB}).$$

Combining this condition with [9], one gets

$$u'(\gamma_i^{FB}) = w_i \frac{(1+r)}{\pi},$$

implying that more productive individuals should set a smaller family norm, since it is more efficient for them to devote time to formal labor activities. In the first best thus, the optimal family norm decreases with the individual productivity, while consumption levels are uniform across types.

## 4.2 Second best: linear income tax

Consider now a situation where the social planner can only use a linear tax. She collects a fraction $\theta$ of individuals' income and redistributes the tax revenue through a lump sum transfer $L$. Given this tax schedule, each individual $i$ optimally chooses $\gamma_i^*(\theta, L)$, $s_i^*(\theta, L)$, and $I_i^*(\theta, L)$.

The problem of the social planner is now:

$$\max_{\theta, L} E_w\{u(c^*(\theta, L)) + \beta[(1-\pi)u(d^*(\theta, L)) + \pi H(m^*(\theta, L))]\}$$
$$s.t.\;\; L \leq E_w[w(1-\gamma^*)\theta].$$

Since the budget constraint is binding at the optimal allocation, one can replace $L$ with $E_w[w(1-\gamma)\theta]$ in the problem and maximize with respect to $\theta$ only. The first-order condition with respect to $\theta$, is equal to

$$\begin{aligned} &E_w\{-w(1-\gamma^*)u'(c^*) + E_w[w(1-\gamma^*)]u'(c^*)\} \\ &\quad + E_w\left\{\beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{d\gamma^*}{d\theta} - E_w\left[w\theta\frac{d\gamma^*}{d\theta}\right]u'(c^*)\right\} = 0. \end{aligned}$$

After manipulations of the first-order condition, we obtain the following result.

**Proposition 5** *Assume that the social planner can only use a payroll tax and a lump sum transfer to the young. The optimal payroll tax is given by the following expression*

$$\theta^* = \frac{-Cov_w[u',\,(1-\gamma^*)w] + \beta\pi(1-\rho)E_w\left[\mu' H'\frac{d\gamma^*}{d\theta}\right]}{E_w[wu']E_w\left[\frac{d\gamma^*}{d\theta}\right]} > 0. \qquad [10]$$

In setting the tax, the social planner takes into account not only the usual trade-off between redistribution (the first term of the numerator) and efficiency (the denominator), but also the family norm externality (the second term of the numerator). Since the level of $\gamma$ is suboptimal in the laissez-faire, the social planner will set a greater tax than the one he/she would choose in the absence of family norms. The tax reduces labor wages and consequently enhances the time devoted to family help. Note also that the tax cannot correct for the lack of full insurance due to the loading factor. If public insurance is not available, then the social planner cannot improve on the individual insurance choices $I^*$.

## 4.3 Second best: payroll tax and public LTC insurance

We now characterize the optimal public LTC insurance when individuals are heterogeneous. We consider a setting where individuals pay a share $\theta$ of their income, and receive a transfer $B$ in case of dependency in old age. Public LTC insurance is assumed to be actuarially fair. The resource constraint of the social planner is thus

$$E_w[\theta w(1-\gamma)] \geq \frac{\pi B}{(1+r)}.$$

Given the tax schedule, each individual $i$ optimally chooses $\gamma_i^*(\theta, B)$, $s_i^*(\theta, B)$, and $I_i^*(\theta, B)$. Note that the first-order condition with respect to $I$, [4] allows for corner solutions, which may take place if public LTC insurance entirely crowds out private insurance. In particular, starting at $B=0$, $I_i^*$ decreases in $B$, and $I_i^*=0$ if

$$\lambda u'(c_i^*) > \beta(1+r)H'(m_i^*).$$

If $I^* > 0$, then $\gamma$ is implicitly defined by [6] and does not depend on the transfer $B$. If $I^*=0$, however, the level of the family norm might be affected by such a transfer. The problem of the social planner is

$$\max_{\theta, B} E_w\{u(c^*) + \beta[(1-\pi)u(d^*) + \pi H(m^*)]\}$$
$$s.t.\ \ E_w[\theta w(1-\gamma^*)] \geq \frac{\pi B}{(1+r)}.$$

We solve this problem in Appendix D, establishing the following result.

**Proposition 6** *Assume that the social planner can only use a payroll tax to finance a flat-rate-benefit LTC insurance. The optimal payroll tax is given by the following expression:*

$$\theta^* = \frac{\left[\begin{array}{c} -Cov_w[u',\ (1-\gamma^*)w] + E_w[\beta(1+r)H' - u']E_w[(1-\gamma^*)w] \\ +\beta\pi(1-\rho)E_w\left[\mu' H' \frac{\partial \gamma^C}{\partial \theta}\right] \end{array}\right]}{\psi E_w\left[w \frac{\partial \gamma^C}{\partial \theta}\right]}. \qquad [11]$$

When setting the tax, the social planner takes into account not only the the usual trade-off between redistribution (the first term of the numerator) and efficiency (the denominator), but also the insurance motive and the family norm externality (the second and third terms of the numerator). With respect to the formula [10], the optimal tax here also depends on an insurance term: the social planner can affect the level of insurance through $B$.

It can be easily shown that $\theta^* > 0$. Assume, though, that this was not true. In this case, the government would make a transfer from disabled individuals to the young (and more intensively to the high-income young). In this case, no crowding out of the private LTC insurance takes place and [6] holds, so that $\partial\gamma^C/\partial\theta > 0$ for each individual. Furthermore, insurance would not be full and $[\beta(1+r)H' - u'] > 0$. Thus, the right-hand side of [11] is positive, which is a contradiction of $\theta^* < 0$.

Note that, in the absence of a demogrant, public insurance plays a redistributive role. If a demogrant were available, $B$ would have no redistributive role. In addition, if $\lambda = 1$, a demogrant would ensure redistribution and public LTC insurance would be a redundant instrument.

# 5 Conclusion

The purpose of this paper was to analyze a particular example of caring initiative taken by children for the benefit of their aged and disabled parents. The motivation of children is not altruism but the hope that their caring behavior will influence their own children in doing the same if later they also need help. The caring initiative we have in mind is an investment or a decision that is made before the occurrence of disability. This can be a particular residential location, an occupational choice, or some type of training that can be highly useful in case parents become disabled. If parents remain healthy, those investments are of little value and can be treated as sunk costs. Besides the uncertainty over disability, there is a second uncertainty that concerns the tradition-abiding behavior of children. Indeed, we can realistically expect that a fraction of children will not follow the example of their parents in making such a caring decision. In addition to this ex ante investment, individuals can provide for their golden years by saving and by buying private LTC insurance.

Given this setting, we first look at the case where all individuals are alike ex ante. We show that the laissez-faire solution is not optimal because in making their decisions, individuals neglect the future actions of the non-traditional children. This calls for a Pigouvian instrument. If private LTC insurance is not actuarially fair, we show that a linear tax and public LTC insurance decentralize the first best. If the social planner can only rely on public LTC insurance (funded through a lump-sum transfer or a proportional linear tax), then it is not possible to decentralize the first best. There is a trade-off between providing adequate insurance coverage and giving incentives for family help.

Then, we turn to the case where individuals differ in earnings. The role of the government in this case, is to not only correct for the above externality but also to redistribute resources. We consider a couple of instruments: a linear income tax and a flat LTC social benefit. We obtain the second-best values of these instruments. Not surprisingly, the optimality of social insurance dependup on the loading costs. The payroll tax plays a double role: it not only finances public LTC expenditures, but it is also a subsidy on the caring investment.

This paper focuses on the role of family norms in long-termcare decisions, leaving aside altruistic motivation and strategic behaviors. These three sources of family help are likely to coexistin practice, and the transmission of a family norm through a demonstrationeffect is expected to reinforce altruism and strategic exchanges. Our qualitative results would still hold if individuals had othermotivation for family help. In general, the presence of a demonstration effect would make altruistic (or strategic) individuals provide more care than they otherwise would.

# Appendix A: Proof of Proposition 1

We will first prove the existence and uniqueness of a stationary equilibrium, in which $\gamma_t^M = \gamma^M$ for all $t$. Then we will show that this is indeed the unique equilibrium of the intergenerational game.

## Stationary equilibrium: existence

The stationary equilibrium is given by the fixed point of the best response function $\gamma: \ [0,1] \to [0,1]$ implicitly defined in [5]. Since $u(.)$ and $H(.)$ are continuous functions, $\gamma(.)$ is also continuous. Furthermore, $[0,1]$ is convex and compact. Then $\gamma(.)$, has a fixed point by Brouwer's theorem, and there exists a stationary equilibrium of the intergenerational game.

Setting $\gamma_t^M = \gamma_{t+1}^M$ in [5] yields

$$\mu'(\gamma^M) = w\frac{1+r}{\lambda\pi\rho}.$$

Since $\mu(.)$ is strictly concave, this expression implicitly defines the unique fixed point of $\gamma(\gamma_{t+1}^M)$. Thus, the intergenerational game admits a unique stationary equilibrium, with $\gamma^M$ defined by [6].

Under our assumption that $\mu'(0) = \infty, \gamma^M$ is strictly greater than zero. Furthermore, $\gamma^M$ is strictly smaller than one. To see this, it is sufficient to verify that the first order condition [2] at $\gamma^M = 1$ is strictly negative. This is always the case if $u(.)$ satisfies the standard Inada condition $u'(0) = \infty$. Consequently, $\gamma^M \in [\gamma(0), \gamma(1)] \subset [0,\ 1]$.

## Uniqueness

The first order condition with respect to $\gamma_t^M$ can be rewritten as

$$\rho H'(m_t^M)\left[\pi\mu'(\gamma_t^M) - \frac{w(1+r)}{\lambda}\right] - \frac{w(1+r)}{\lambda}(1-\rho)H'(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M)) = 0$$

Using the implicit function theorem, we can write

$$\frac{\partial \gamma_t}{\partial \gamma_{t+1}} = \frac{\frac{w}{\lambda}(1+r)(1-\rho)H''(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M))}{\rho H''(m_t^M)\mu'(\gamma_t^M)\left[\pi\mu'(\gamma_t^M) - \frac{w}{\lambda}(1+r)\right] + \rho H'(m_t^M)\pi\mu''(\gamma_t^M)} \geq 0$$

Thus, the best response function $\gamma(\gamma_{t+1}^M)$ is monotonically increasing. Furthermore, it is easy to show that $\gamma(0) > 0$ and $\gamma(1) < 1$, so that setting $\gamma_t^M$ equal to zero or one is never a best response for any generation.

Since $\gamma(\gamma_{t+1}^M)$ has a unique fixed point, it has to cross the identity line $l(\gamma_{t+1}^M) = \gamma_{t+1}^M$ from above. Thus, $\gamma(\gamma_{t+1}^M) \in (\gamma_{t+1}^M,\ \gamma^M)$ if and only if $\gamma_{t+1}^M < \gamma^M$, where $\gamma^M$ is the fixed point of $\gamma(.)$. Conversely, $\gamma(\gamma_{t+1}^M) \in (\gamma^M,\ \gamma_{t+1}^M)$ if and only if $\gamma_{t+1}^M > \gamma^M$.

Given these features of the best response function, we can show that there does not exist any equilibrium such that at least one generation chooses a family norm different from $\gamma^M$. We will consider two cases.

1. Suppose that there exists an equilibrium such that $\gamma_t^M > \gamma^M$. Since $\gamma_t^M$ is an equilibrium strategy, it is a best response to $\gamma_{t+1}^M$. Due to the monotonicity of the best response function, $\gamma_t^M > \gamma^M$ implies $\gamma_{t+1}^M > \gamma_t^M > \gamma^M$. Repeating this argument, it is possible to prove that the best responses of generations $t, ..., t+n$ satisfy $\gamma^M < \gamma_t^M < \gamma_{t+1}^M ... < \gamma_{t+n}^M$. For an $n$ high enough, $\gamma_{t+n}^M = \gamma(1)$. Thus, in equilibrium $\gamma_{t+n+1}^M = 1$. However, setting the family norm equal to one is never a best response, so that this cannot be an equilibrium strategy of the intergenerational game.
2. A similar reasoning can be applied for the case $\gamma_t^M < \gamma^M$. If this is an equilibrium strategy, then for an $n$ high enough, $\gamma_{t+n+1}^M = 0$. However, setting the family norm equal to zero is never a best response, so that this cannot be an equilibrium of the intergenerational game.

# Appendix B: Proof of Proposition 2

The first-order conditions with respect to $\theta$ and $B$ are

$$\begin{aligned}
&[\beta(1-\rho)\pi H'(m^*)\mu'(y^*) - w\theta u'(c^*)]\frac{\partial y^*}{\partial \theta}\\
&\quad +[\beta\rho\pi H'(m^*)\mu'(y^*) - w(1-\theta)u'(c^*)]\frac{\partial y^*}{\partial \theta}\\
&\quad +[(1+r)\beta((1-\pi)u'(d^*)+\pi H'(m^*)) - u'(c)]\frac{\partial s^*}{\partial \theta}\\
&\quad +\left[\beta\pi H'(m^*) - \frac{\lambda\pi}{(1+r)}u'(c^*)\right]\frac{\partial I^*}{\partial \theta} = 0,
\end{aligned}$$

and

$$\begin{aligned}
&\beta\pi H'(m^*) - \frac{\pi}{(1+r)}u'(c^*)\\
&\quad +[\beta(1-\rho)\pi H'(m^*)\mu'(y^*) - w\theta u'(c^*)]\frac{\partial y^*}{\partial B}\\
&\quad +[\beta\rho\pi H'(m^*)\mu'(y^*) - w(1-\theta)u'(c^*)]\frac{\partial y^*}{\partial B}\\
&\quad +[(1+r)\beta((1-\pi)u'(d^*)+\pi H'(m^*)) - u'(c)]\frac{\partial s^*}{\partial B}\\
&\quad +\left[\beta\pi H'(m^*) - \frac{\lambda\pi}{(1+r)}u'(c^*)\right]\frac{\partial I^*}{\partial B} = 0.
\end{aligned}$$

Using the envelope theorem and observing that either $\beta\pi H'(m) = \lambda\pi u'(c)/(1+r)$, or $I^* = 0$ (implying $\partial I^*/\partial\theta = \partial I^*/\partial B = 0$), we can rewrite the conditions above as

$$[\beta(1-\rho)\pi H'(m^*)\mu'(y^*) - w\theta u'(c^*)]\frac{\partial y^*}{\partial \theta} = 0, \qquad [12]$$

and

$$\beta\pi H'(m^*) - \frac{\pi}{(1+r)}u'(c^*) + [\beta(1-\rho)\pi H'(m^*)\mu'(y^*) - w\theta u'(c^*)]\frac{\partial y^*}{\partial B} = 0. \qquad [13]$$

Substituting [12] in [13], we get

$$\beta H'(m^*) - \frac{1}{(1+r)}u'(c^*) = 0,$$

so that consumption is smoothed across states. Furthermore,

$$\mu'(y^*)\beta(1-\rho)\pi H'(m^*) - w\theta u'(c^*) = 0$$

Since the individual first-order condition with respect to $\gamma^*$ is $(1-\theta)wu'(c^*) = \beta\rho\pi\mu'(\gamma^*)H'(m^*)$, we can rewrite this condition as

$$\frac{(1-\rho)}{\rho} = \frac{\theta^*}{(1-\theta^*)} \Longleftrightarrow \theta^* = 1-\rho.$$

# Appendix C: comparative statics with respect to *w*

Equation [6] permits us to recover $\partial\gamma/\partial w = ((1+r)/\lambda\rho\pi)/\mu''(\gamma) < 0$. Total derivation of [3] and [4] yields

$$\frac{\partial I}{\partial w}U_{sI} + \frac{\partial s}{\partial w}U_{ss} = 0$$
$$\frac{\partial I}{\partial w}U_{II} + \frac{\partial s}{\partial w}U_{Is} = 0$$

In order to solve this system, define

$$A \equiv \begin{bmatrix} U_{II} U_{Is} \\ U_{sI} U_{ss} \end{bmatrix} = \begin{bmatrix} \beta\pi H''(m) + (\lambda\pi)^2 u''(c)/(1+r)^2 & \beta\pi(1+r)H''(m) + \lambda\pi u''(c)/(1+r) \\ \beta\pi(1+r)H''(m) + \lambda\pi u''(c)/(1+r) & (1+r)^2\beta[(1-\pi)u''(d) + \pi H''(m)] + u''(c) \end{bmatrix},$$

and

$$B \equiv \begin{bmatrix} -\frac{\partial U_I}{\partial w} - \frac{\partial U_s}{\partial w} \end{bmatrix} = \begin{bmatrix} (1-\gamma)\frac{\pi\lambda}{(1+r)}u''(c) - w\frac{\partial\gamma}{\partial w}\frac{\pi\lambda}{(1+r)}u''(c) - \beta\pi H''(m)\mu'(\gamma)\frac{\partial\gamma}{\partial w} \\ (1-\gamma)u''(c) - w\frac{\partial\gamma}{\partial w}u''(c) - \beta\pi(1+r)H''(m)\mu'(\gamma)\frac{\partial\gamma}{\partial w} \end{bmatrix}.$$

Using this notation, we can write

$$\frac{\partial I}{\partial w} = \frac{det\begin{bmatrix} -\frac{\partial U_I}{\partial w} U_{Is} \\ -\frac{\partial U_s}{\partial w} U_{ss} \end{bmatrix}}{det[A]}$$

and

$$\frac{\partial s}{\partial w} = \frac{det\begin{bmatrix} U_{II} - \frac{\partial U_I}{\partial w} \\ U_{sI} - \frac{\partial U_s}{\partial w} \end{bmatrix}}{det[A]}.$$

Straightforward calculations yield (under the assumption that $\lambda\pi < 1$)

$$\begin{aligned}\det[A] &= \beta^2\pi(1-\pi)(1+r)^2 H''(m)u''(d) + \beta\pi H''(m)u''(c)\\ &\quad + \beta\lambda^2\pi^2(1-\pi)u''(c)u''(d) + \beta\lambda\pi^2(2+\lambda\pi)u''(c)H''(m) > 0,\end{aligned}$$

$$\begin{aligned}\det\begin{bmatrix} -\frac{\partial U_I}{\partial w} & U_{Is}\\ -\frac{\partial U_s}{\partial w} & U_{ss}\end{bmatrix} &= \beta\lambda\pi(1+r)(1-\pi)(1-\gamma)u''(c)u''(d) - \beta\lambda\pi(1-\pi)(1+r)w\frac{\partial\gamma}{\partial w}u''(c)u''(d)\\ &\quad -\beta^2\pi(1-\pi)(1+r)^2\mu'(\gamma)\frac{\partial\gamma}{\partial w}H''(m)u''(d)\\ &\quad +\beta\pi u''(c)H''(m)(\lambda\pi-1)\left((1+r)(1-\gamma)-(1+r)w\frac{\partial\gamma}{\partial w}+\mu'(\gamma)\frac{\partial\gamma}{\partial w}\right) \lesseqgtr 0.\end{aligned}$$

and

$$\det\begin{bmatrix} U_{II} & -\frac{\partial U_I}{\partial w}\\ U_{sI} & -\frac{\partial U_s}{\partial w}\end{bmatrix} > 0.$$

Therefore, savings increase in the productivity parameter, while the sign of $\partial I/\partial w$ is ambiguous.

# Appendix D: Proof of Proposition 6

The Lagrange expression for the planning problem is

$$\begin{aligned}\mathscr{L} &= E_w\{u(c^*) + \beta[(1-\pi)u(d^*) + \pi H(m^*)]\}\\ &\quad -\psi\left\{-E_w[\theta w(1-\gamma^*)] + \frac{\pi B}{(1+r)}\right\},\end{aligned}$$

where $\psi \geq 0$ is the Lagrangian multiplier associated with the revenue constraint. The first-order conditions with respect to $\theta$ and $B$ yield

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial\theta} &= E_w\left[-w(1-\gamma^*)u'(c^*) + \beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{\partial\gamma^*}{\partial\theta}\right]\\ &\quad -\psi E_w\left[w\theta\frac{\partial\gamma^*}{\partial\theta} - w(1-\gamma^*)\right] = 0,\end{aligned} \tag{14}$$

and

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial B} &= E_w\left[\beta\pi H'(m^*) + \beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{\partial\gamma^*}{\partial B}\right]\\ &\quad -\psi E_w\left[\frac{\pi}{(1+r)} + w\theta\frac{\partial\gamma^*}{\partial B}\right] = 0.\end{aligned} \tag{15}$$

Using the above first-order conditions, we can write

$$\frac{\partial \tilde{\mathscr{L}}}{\partial \theta} \equiv \frac{\partial \mathscr{L}}{\partial \theta} + \frac{\partial \mathscr{L}}{\partial B}\frac{\partial B}{\partial \theta} = 0, \qquad [16]$$

where $\partial B/\partial \theta = (1+r)E_w[w(1-\gamma^*)]/\pi$ is obtained from the resource constraint of the government. We define

$$\frac{\partial \gamma^C}{\partial \theta} \equiv \frac{\partial \gamma^*}{\partial \theta} + \frac{\partial \gamma^*}{\partial B}\frac{\partial B}{\partial \theta}.$$

The sign of $\partial \gamma^c/\partial \theta$ is ambiguous whenever public insurance crowds out private. Combining [14] and [15], we can rewrite [16] as

$$\frac{\partial \tilde{\mathscr{L}}}{\partial \theta} = E_w\left[-w(1-\gamma^*)u'(c^*) + \beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{\partial \gamma^C}{\partial \theta}\right]$$
$$+E_w[\beta(1+r)H'(m^*)]E_w[w(1-\gamma^*)] - \psi E_w\left[w\theta\frac{\partial \gamma^C}{\partial \theta}\right] = 0.$$

After simplifications, this expression yields [11].

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